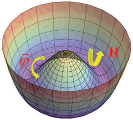
A.P., hep-ph/9806303, arXiv:1804.05664



Effective Field Theory

Antonio Pich

IFIC, Univ. Valencia – CSIC

- Basic concepts in EFT
- SMEFT
- EWET (HEFT)

PREFIT – School, DESY, Hamburg, 2–13 March 2020

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies $(E_\gamma \ll m_e)$

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ / m_e)

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$

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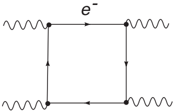

$$a = -\frac{1}{36}\,\alpha^2 \qquad , \qquad b = \frac{7}{90}\,\alpha^2$$

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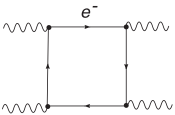


$$\Longrightarrow \qquad a = -\frac{1}{36}\,\alpha^2 \quad , \qquad b = \frac{7}{90}\,\alpha^2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \;\propto\; \frac{\alpha^4 E^6}{m_e^8}$$

Dimensions

$$S = \int d^4x \, \mathcal{L}(x) \quad \longrightarrow \quad [\, \mathcal{L} \,] = E^4$$

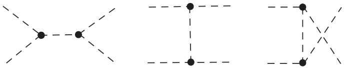
$$\mathcal{L}_{\mathrm{KG}} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \, \phi^\dagger \phi \quad \longrightarrow \quad [\, \phi \,] = [\, V^\mu \,] = [\, A^\mu \,] = E$$

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi} \, (i \gamma^\mu \partial_\mu - m) \, \psi \quad \longrightarrow \quad [\, \psi \,] = E^{3/2}$$

$$[\, \sigma \,] = E^{-2} \qquad , \qquad [\, \Gamma \,] = E$$

Scalar Field Theory

- $\mathcal{L}_{\rm I} = -\frac{\boldsymbol{\lambda}}{3!}\,\phi^3 \quad \Longrightarrow \quad [\lambda] = E$



$$\sigma(1+2\to 3+4) \;\sim\; \frac{\lambda^4}{s^3}\,\left\{1+\mathcal{O}\left(\frac{\lambda^2}{s}\right)+\cdots\right\} \qquad\qquad (s \gg m^2)$$

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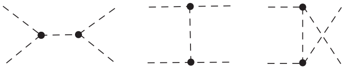

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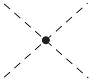

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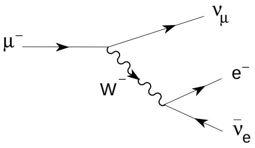
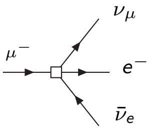
Fermi Theory

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, M_W^2}$$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \quad \xrightarrow{q^2 \ll M_W^2} \quad \frac{g_{\mu\nu}}{M_W^2}$$



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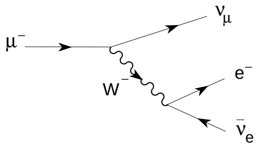

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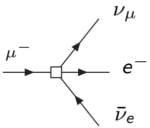

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$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$

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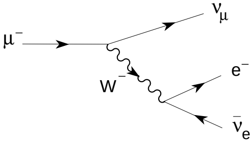

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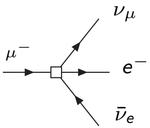

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Exp: $(17.82 \pm 0.04)\%$

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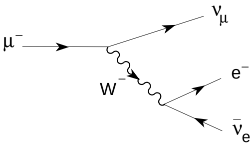

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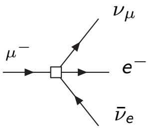

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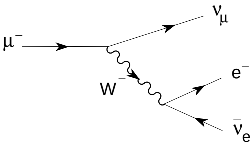

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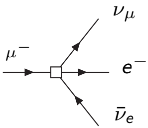

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- $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \sim G_F^2\, s$ $\qquad$ **Violates unitarity at high energies**

Relevant, Irrelevant & Marginal

$$\mathcal{L} = \sum_i c_i \, O_i \qquad , \qquad [O_i] = d_i \quad \longrightarrow \quad c_i \sim \frac{1}{\Lambda^{d_i - 4}}$$

Low-energy behaviour:

- **Relevant** ($\mathrm{d_i} < 4$): $\quad I \, , \, \phi^2 \, , \, \phi^3 \, , \, \bar\psi\psi$

  Enhanced by $(\Lambda/E)^{4-d_i}$

- **Marginal** ($\mathrm{d_i} = 4$): $\quad \phi^4 \, , \, \phi \, \bar\psi\psi \, , \, V_\mu \, \bar\psi\gamma^\mu\psi$

- **Irrelevant** ($\mathrm{d_i} > 4$): $\quad \bar\psi\psi \, \bar\psi\psi \, , \, \partial_\mu\phi \, \bar\psi\gamma^\mu\psi \, , \, \phi^2 \, \bar\psi\psi \, , \, \cdots$

  Suppressed by $(E/\Lambda)^{d_i-4}$

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log\left(Q^2/Q_0^2\right)}$$

QED: $\beta_1 = \frac{2}{3} \sum_f Q_f^2 N_f > 0$ ➡ $\lim_{Q^2 \to 0} \alpha(Q^2) = 0$

Quantum corrections make QED irrelevant at low energies

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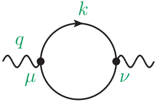
Quantum corrections make QED irrelevant at low energies

QCD: $\beta_1 = \frac{2\,N_F \;-\; 11\,N_C}{6} \;<\; 0$ ➡ $\lim_{Q^2\to 0}\alpha_s(Q^2) \;=\; \infty$

Quantum corrections make QCD relevant at low energies

Vacuum Polarization $(m_f = 0)$



$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3}\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$

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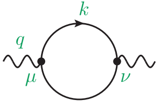

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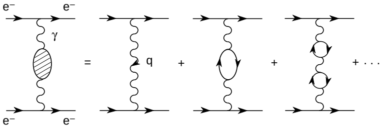

$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

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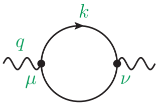

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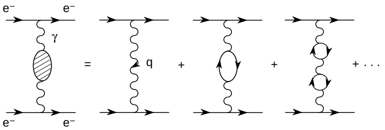

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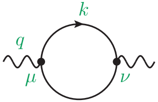
Vacuum Polarization $(m_f \neq 0)$



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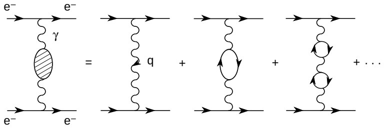

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Mass-Dependent Scheme: $\Delta\Pi_\epsilon(\mu^2) \;\equiv\; \Pi(-\mu^2)$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\,x(1-x)\,\log\left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2+\mu^2 x(1-x)}\right]$$

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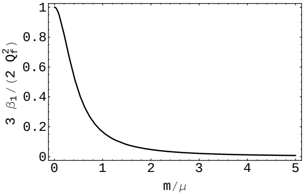

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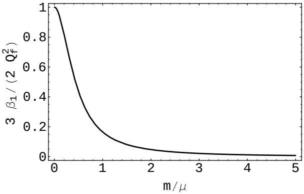

- $m_f^2 \ll \mu^2\,,\ q^2$: $\beta_1 = \frac{2}{3}\,Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2\,\frac{\alpha}{3\pi}\,\log\left(-q^2/\mu^2\right)$

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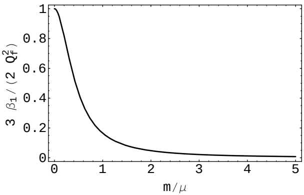

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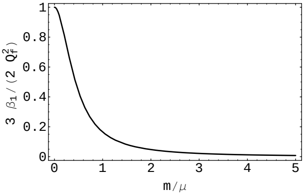
- $m_f^2 \gg \mu^2, q^2$: $\quad \beta_1 \sim \frac{2}{15}\,Q_f^2\,\frac{\mu^2}{m_f^2} \quad , \quad \Pi_R(q^2/\mu^2) \sim Q_f^2\,\frac{\alpha}{15\pi}\,\frac{q^2+\mu^2}{m_f^2}$

Mass-Dependent Scheme: $\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2)$

$$\Pi_R(q^2/\mu^2) = -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\,x(1-x)\,\log\left[\frac{m_f^2 - q^2x(1-x)}{m_f^2+\mu^2x(1-x)}\right]$$

$$\beta_1 = 4\,Q_f^2\,\int_0^1 dx\,\frac{\mu^2x^2(1-x)^2}{m_f^2+\mu^2x(1-x)}$$



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→ **DECOUPLING** **(Appelquist-Carazzone Theorem)**

$\overline{\rm MS}$ **Scheme:** $\Delta\Pi_\epsilon(\mu^2) \;\equiv\; -Q_f^2\,\frac{\alpha_0}{3\pi}\,\Delta_\infty(\mu)$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\,\int_0^1 dx\; x(1-x)\;\log\left[\frac{m_f^2-q^2x(1-x)}{\mu^2}\right]$$

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Heavy fermions do not decouple

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Perturbation theory breaks down

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Perturbation theory breaks down

SOLUTION: Integrate Out Heavy Particles

Matching

μ

EFT with heavy fermion $\qquad \beta_1 = \frac{2}{3}\, Q_f^2 + \beta_1^{\rm light}$

m_f - - - - - - - - - - - - - - - - -

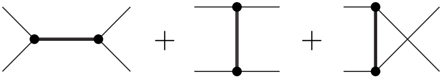
EFT without heavy fermion $\qquad \beta_1 = \beta_1^{\rm light}$

- **Two different EFTs (with and without the heavy fermion f)**
- **Same S-matrix elements for light-particle scattering at $\mu \sim m_f$**
- **Different β function ➔ Different coupling**

  Couplings are auxiliary parameters (not observables)
- **Same IR behaviour. Different UV behaviour**

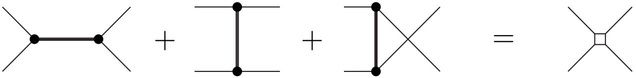
$$\mathcal{L}(\phi,\Phi) \;=\; \frac{1}{2}\,(\partial\phi)^2 + \frac{1}{2}\,(\partial\Phi)^2 - \frac{1}{2}\,m^2\phi^2 - \frac{1}{2}\,M^2\Phi^2 - \frac{\lambda}{2}\,\phi^2\Phi$$

$$\sigma(\phi\phi\to\phi\phi) \;\sim\; \frac{1}{E^2}\;\times\;\begin{cases} (\lambda/E)^4\;, & (m \ll M \ll E) \\ (\lambda/M)^4\;, & (m, E \ll M) \end{cases}$$



$$\mathcal{L}(\phi,\Phi) \;=\; \frac{1}{2}\,(\partial\phi)^2 + \frac{1}{2}\,(\partial\Phi)^2 - \frac{1}{2}\,m^2\phi^2 - \frac{1}{2}\,M^2\Phi^2 - \frac{\lambda}{2}\,\phi^2\Phi$$

$$\sigma(\phi\phi\to\phi\phi) \;\sim\; \frac{1}{E^2}\;\times\;\begin{cases} (\lambda/E)^4 \;, & (m \ll M \ll E) \\ (\lambda/M)^4 \;, & (m, E \ll M) \end{cases}$$

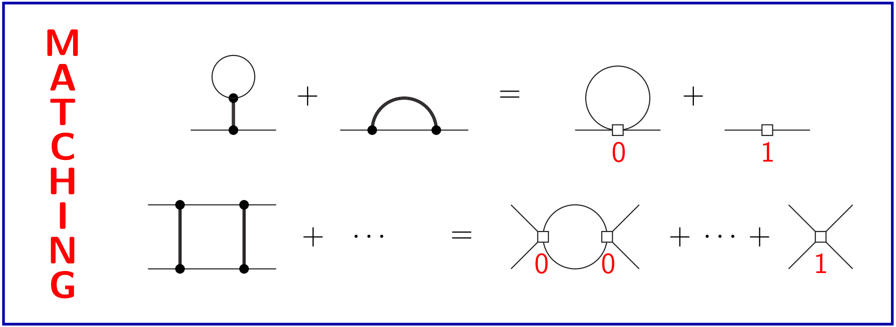


E, m $\ll$ M

$$\frac{\lambda^2}{s-M^2} = -\frac{\lambda^2}{M^2}\sum_{n=0}\frac{s}{M^2} \quad\longrightarrow\quad \mathcal{L}_{\mathrm{eff}}(\phi) \;=\; \sum_i c_i\, O_i(\phi)$$

$$[O_i] = d_i \qquad ; \qquad c_i \sim \frac{\lambda^2}{M^2}\,\frac{1}{M^{d_i-4}}$$

One Loop: $\mathcal{L}_{\text{eff}} = \frac{1}{2}\, a\, (\partial\phi)^2 - \frac{1}{2}\, b\, \phi^2 + c\, \frac{\lambda^2}{8M^2}\, \phi^4 + \cdots$



$$a = 1 + a_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad b = m^2 + b_1\, \frac{\lambda^2}{16\pi^2} + \cdots$$

$$c = 1 + c_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad \cdots$$

Principles of Effective Field Theory

- **Low-energy dynamics** independent of details at high energies
- Appropriate physics description at the analyzed scale **(degrees of freedom)**
- **Energy gaps:** $0 \leftarrow m \ll E \ll M \rightarrow \infty$
- Non-local heavy-particle exchanges replaced by a **tower of local interactions** among the light particles
- **Accuracy:** $(E/M)^{d_i - 4} \gtrsim \epsilon \quad \Longleftrightarrow \quad d_i \lesssim 4 + \frac{\log(1/\epsilon)}{\log(M/E)}$
- **Same infrared** (but different ultraviolet) **behaviour** than the underlying fundamental theory
- The only remnants of the high-energy dynamics are in the **low-energy couplings** and in the **symmetries** of the EFT

Evolution from High to Low Scales

Large μ

$\mathcal{L}(\phi_i) + \mathcal{L}(\phi_i, \Phi)$ ϕ_i, Φ

Renormalization Group

$\mu = M$ **Matching**

$\mathcal{L}(\phi_i) + \delta\mathcal{L}(\phi_i)$ ϕ_i

Renormalization Group

Small μ

SMEFT

I. Brivio, M. Trott, arXiv:1706.08945

Energy Scale	Fields	Effective Theory
$\Lambda_{\rm NP} \sim \rm TeV$	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Underlying Dynamics**

Energy Gap

Energy Scale	Fields	Effective Theory
$\rm M_W$	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Standard Model**

SM Effective Field Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}} \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**

$$\mathrm{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

- **Light** ($m \ll \Lambda \equiv \Lambda_{NP}$) **SM fields only**

- **The SM Lagrangian corresponds to D = 4**

- $c_i^{(D)}$ **contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_X \, (\bar q_L \gamma^\mu q_L) \, X_\mu \quad \longrightarrow \quad \frac{g_X^2}{M_X^2} \, (\bar q_L \gamma^\mu q_L) \, (\bar q_L \gamma_\mu q_L)$$

- **Assumes that H(125) belongs to an** $\mathrm{SU(2)_L}$ **doublet**

Linear Realization of the $SU(2)_L \otimes U(1)_Y$ symmetry

- **H and the electroweak Goldstones combine into an $SU(2)_L$ doublet:**

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad , \qquad U(\vec{\varphi}) = \exp\left\{ i\,\vec{\sigma}\cdot\frac{\vec{\varphi}}{v} \right\}$$

- **The SM Lagrangian is the low-energy effective theory with D = 4**

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **1 operator with D = 5:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ **(violates L by 2 units)**

  Weinberg

- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)**

  Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek

- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)**

  Weinberg, Wilczek–Zee, Abbott–Wise

- **3 generations: 1350 CP-even and 1149 CP-odd operators with D = 6**

  Alonso–Jenkins–Manohar-Trott

$D=6$ Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

X^3		Φ^6 and $\Phi^4 D^2$		$\psi^2\Phi^3$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_\Phi$	$(\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger\Phi)(\bar{l}_p e_r \Phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger\Phi)\,\Box\,(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger\Phi)(\bar{q}_p u_r \widetilde{\Phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{\Phi D}$	$(\Phi^\dagger D^\mu\Phi)^\star\,(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger\Phi)(\bar{q}_p d_r \Phi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\Phi^2$		$\psi^2 X\Phi$		$\psi^2\Phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger\Phi\; G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\,\tau^I\Phi\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\Phi\widetilde{G}}$	$\Phi^\dagger\Phi\; \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu^I\Phi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger\Phi\; W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A u_r)\,\widetilde{\Phi}\, G_{\mu\nu}^A$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\Phi\widetilde{W}}$	$\Phi^\dagger\Phi\; \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\,\tau^I\widetilde{\Phi}\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger\Phi\; B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\,\widetilde{\Phi}\, B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu^I\Phi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\Phi\widetilde{B}}$	$\Phi^\dagger\Phi\; \widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A d_r)\,\Phi\, G_{\mu\nu}^A$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger\tau^I\Phi\; W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\,\tau^I\Phi\, W_{\mu\nu}^I$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\Phi\widetilde{W}B}$	$\Phi^\dagger\tau^I\Phi\; \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\,\Phi\, B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\widetilde{\Phi}^\dagger D_\mu\Phi)(\bar{u}_p\gamma^\mu d_r)$

$q = q_L\,,\; l = l_L\,,\; u = u_R\,,\; d = d_R\,,\; e = e_R$, $\overleftrightarrow{D}_\mu^I \equiv \tau^I \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \tau^I$, $p, r =$ generation indices

D=6 Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\left[(d_p^\alpha)^T C\, u_r^\beta\right]\left[(q_s^{\gamma j})^T C\, l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(u_s^\gamma)^T C\, e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\varepsilon_{mn}\,\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C\, l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}\,(\tau^I\varepsilon)_{jk}\,(\tau^I\varepsilon)_{mn}\,\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C\, l_t^n\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\,\left[(d_p^\alpha)^T C\, u_r^\beta\right]\left[(u_s^\gamma)^T C\, e_t\right]$

$q = q_L\ ,\ l = l_L\ ,\ u = u_R\ ,\ d = d_R\ ,\ e = e_R\quad ,\quad p, r, s, t =$ generation indices

Shifts of SM parameters:

$$\mathcal{L}_{\mathrm{SMEFT}}^{(D=6)} = \sum_i \frac{C_i}{\Lambda^2}\, \mathcal{O}_i$$

- $V(H) = \lambda \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 - \frac{C_\Phi}{\Lambda^2} \left(\Phi^\dagger\Phi\right)^3 \quad \longrightarrow \quad v_T = \left(1 + \frac{3C_\Phi}{8\lambda}\,\frac{v^2}{\Lambda^2}\right) v$

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- **Normalization:** $H \to \left(1 + c_H^{\rm kin}\right) H$, $c_H^{\rm kin} = \left(C_{\Phi\Box} - \frac{1}{4}\, C_{\Phi D}\right)\frac{v^2}{\Lambda^2}$

➡ $M_H^2 = 2\lambda\, v_T^2 \left(1 - \frac{3C_\Phi}{2\lambda}\,\frac{v^2}{\Lambda^2} + 2\, c_H^{\rm kin}\right)$

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$$\longrightarrow \qquad M_H^2 = 2\lambda\, v_T^2 \left(1 - \frac{3C_\Phi}{2\lambda}\,\frac{v^2}{\Lambda^2} + 2\, c_H^{\rm kin}\right)$$

- **Flavour:** $\mathcal{Y}_f \not\propto M_f$

$$\mathcal{L} = -\bar{Q}_L^r\,[Y_d]_{rs}\,\Phi\, d_R^s + \cdots \quad \longrightarrow$$

$$[M_f]_{rs} = \frac{v_T}{\sqrt{2}}\left([Y_\psi]_{rs} - \frac{v^2}{2\Lambda^2}\,[C_{f\Phi}]_{rs}\right)$$

$$[\mathcal{Y}_f]_{rs} = \frac{1}{v_T}\,[M_f]_{rs}\,\left(1 + c_H^{\rm kin}\right) - \frac{v^2}{\Lambda^2}\,[C_{f\Phi}]_{rs}$$

Shifts of SM parameters:

$$\mathcal{L}_{\rm SMEFT}^{(D=6)} = \sum_i \frac{C_i}{\Lambda^2}\, \mathcal{O}_i$$

- $V(H) = \lambda \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 - \frac{C_\Phi}{\Lambda^2}\left(\Phi^\dagger\Phi\right)^3$ $\quad\longrightarrow\quad$ $v_T = \left(1 + \frac{3C_\Phi}{8\lambda}\,\frac{v^2}{\Lambda^2}\right) v$

- **Normalization:** $H \to \left(1 + c_H^{\rm kin}\right) H \quad , \quad c_H^{\rm kin} = \left(C_{\Phi\Box} - \frac{1}{4}\, C_{\Phi D}\right) \frac{v^2}{\Lambda^2}$

$$\longrightarrow \qquad M_H^2 = 2\lambda\, v_T^2 \left(1 - \frac{3C_\Phi}{2\lambda}\,\frac{v^2}{\Lambda^2} + 2\, c_H^{\rm kin}\right)$$

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$$[M_f]_{rs} = \frac{v_T}{\sqrt{2}} \left([Y_\psi]_{rs} - \frac{v^2}{2\Lambda^2}\, [C_{f\Phi}]_{rs}\right)$$

$$[\mathcal{Y}_f]_{rs} = \frac{1}{v_T}\, [M_f]_{rs} \left(1 + c_H^{\rm kin}\right) - \frac{v^2}{\Lambda^2}\, [C_{f\Phi}]_{rs}$$

- **Fermi:** $\frac{4\, G_F}{\sqrt{2}} = \frac{2}{v_T^2} + \frac{1}{\Lambda^2}\left(2\left[C_{\Phi l}^{(3)}\right]_{ee} + 2\left[C_{\Phi l}^{(3)}\right]_{\mu\mu} - \left[C_{ll}\right]_{\mu e, e\mu} - \left[C_{ll}\right]_{e\mu, \mu e}\right)$

Gauge Fields:

$$\mathcal{L}_{\rm eff}^{(D=6)} = \sum_i \frac{C_i}{\Lambda^2}\, \mathcal{O}_i$$

Alonso et al

$$\bar{g}' = g' \left(1 + C_{\Phi B}\, \frac{v_T^2}{\Lambda^2}\right) \quad , \quad \bar{g} = g \left(1 + C_{\Phi W}\, \frac{v_T^2}{\Lambda^2}\right) \quad , \quad \bar{g}_s = g_s \left(1 + C_{\Phi G}\, \frac{v_T^2}{\Lambda^2}\right)$$

$$\tan\bar{\theta}_W = \frac{\bar{g}'}{\bar{g}} + \frac{v_T^2}{2\Lambda^2} \left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right) C_{\Phi WB}$$

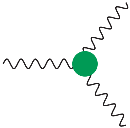
$$M_W^2 = \frac{v_T^2}{4}\, \bar{g}^2 \quad , \quad M_Z^2 = \frac{v_T^2}{4} \left(\bar{g}^2 + \bar{g}'^2\right) + \frac{\bar{v}_T^4}{8\Lambda^2} \left\{\left(\bar{g}^2 + \bar{g}'^2\right) C_{\Phi D} + 4\, \bar{g}\, \bar{g}'\; C_{\Phi WB}\right\}$$

$$\bar{e} = \bar{g}\, \sin\bar{\theta}_W \left\{1 - \cot\bar{\theta}_W\, \frac{\bar{v}_T^2}{2\Lambda^2}\, C_{\Phi WB}\right\} \quad , \quad \bar{g}_Z = \frac{\bar{e}}{\sin\bar{\theta}_W \cos\bar{\theta}_W} \left\{1 + \frac{\bar{g}^2 + \bar{g}'^2}{2\bar{g}\bar{g}'}\, \frac{\bar{v}_T^2}{\Lambda^2}\, C_{\Phi WB}\right\}$$

$$\rho \equiv \frac{\bar{g}^2 M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{\bar{v}_T^2}{2\Lambda^2}\, C_{\Phi D}$$

Anomalous Triple Gauge Couplings

Lorentz, $\mathcal{P}$, $\mathcal{C}$ and e.m. gauge invariance: $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$



$$\mathcal{L}_{WWV} = -i\, g_{WWV} \Big\{ g_1^V \left(W^+_{\mu\nu} W^{-\mu} V^\nu - W^+_\mu W^{-\mu\nu} V_\nu \right) + \kappa_V\, W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2}\, W^+_{\mu\nu} W^{-\nu\rho} V_\rho^{\ \mu} \Big\}$$

$g_{WW\gamma} = e = g\ \sin\theta_W$, $g_{WWZ} = g\ \cos\theta_W$, $g_1^\gamma = 1$, $g_1^Z = 1 + \delta g_1^Z$, $\kappa_V = 1 + \delta\kappa_V$

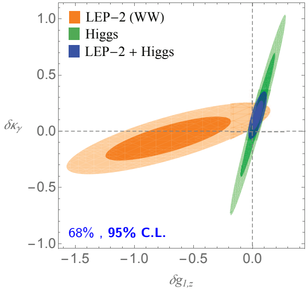
- **D = 6 relations:** $\lambda_\gamma = \lambda_Z$, $\delta\kappa_Z = \delta g_1^Z - \tan^2\theta_W\, \delta\kappa_\gamma$

  ➡ 3 free parameters: δg_1^Z , $\delta\kappa_\gamma$, λ_Z (SM: $\delta g_1^Z = \delta\kappa_\gamma = \lambda_Z = 0$)

- **SMEFT:**

$$\delta g_1^Z = \frac{1}{\sqrt{2} G_F \sin 2\theta_W} \frac{C_{\Phi WB}}{\Lambda^2} \quad , \quad \delta\kappa_\gamma = \frac{\cot\theta_W}{\sqrt{2} G_F} \frac{C_{\Phi WB}}{\Lambda^2} \quad , \quad \lambda_Z = \frac{6\cos\theta_W M_W^2}{g_{WWZ}} \frac{C_W}{\Lambda^2}$$

Constraints on Anomalous Triple Gauge Couplings



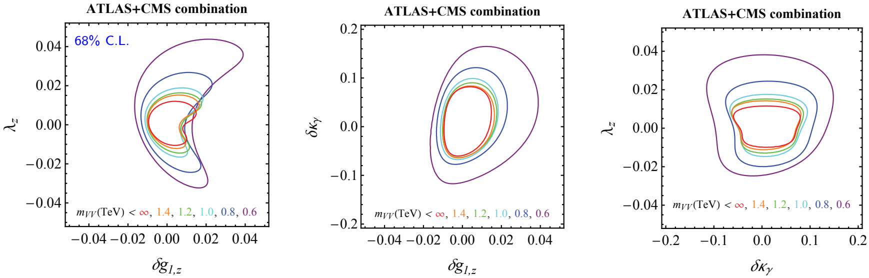
Falkowski, González-Alonso, Greljo, Marzocca, 1508.00581

$$\begin{pmatrix} \delta g_{1,z} \\ \delta\kappa_\gamma \\ \lambda_z \end{pmatrix} = \begin{pmatrix} 0.043 \pm 0.031 \\ 0.142 \pm 0.085 \\ -0.162 \pm 0.073 \end{pmatrix}$$

$$\rho = \begin{pmatrix} 1 & 0.74 & -0.85 \\ 0.74 & 1 & -0.88 \\ -0.85 & -0.88 & 1 \end{pmatrix}$$, MFV assumed

$pp \to WZ\,(WW) \to \ell\nu\ell^+\ell^-\;(\ell\nu\ell\nu)$

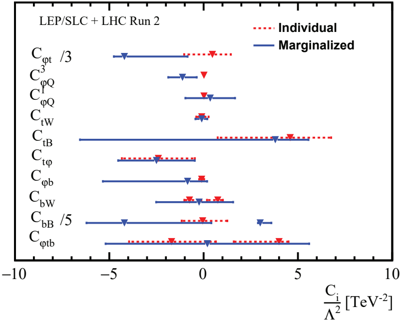
Falkowski, González-Alonso, Greljo, Marzocca, Son, 1609.06312



Global Fit to Top Data

Durieux et al., 1907.10619

68% C.L. limits



	Λ^{-2} and Λ^{-4} terms	Λ^{-2} term only
$C_{\varphi t}/\Lambda^2$	$(-16, -2.4)$	$(-2.1, +4.5)$
$C^3_{\varphi Q}/\Lambda^2$	$(-1.9, -0.4)$	$(-0.7, +0.5)$
$C^1_{\varphi Q}/\Lambda^2$	$(-1, +1.7)$	$(-0.6, +0.7)$
C_{tW}/Λ^2	$(-0.4, +0.2)$	$(-0.42, +0.24)$
C_{tB}/Λ^2	$(-6.8, +5.6)$	$(-9.6, +38.4)$
$C_{t\varphi}/\Lambda^2$	$(-4.6, -0.4)$	$(-4.42, 0)$
$C_{\varphi b}/\Lambda^2$	$(-5.4, +0.2)$	$(-0.6, +0.2)$
C_{bW}/Λ^2	$(-2.6, +2.1)$	—
C_{bB}/Λ^2	$(-31.2, +2.4)$, $(+14.4, +18)$	—
$C_{\varphi tb}/\Lambda^2$	$(-5.2, 5.6)$	—

$$O^1_{\varphi Q} \equiv \frac{y_t^2}{2}\; \bar{q}\gamma^\mu q \;\; \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi,$$
$$O^3_{\varphi Q} \equiv \frac{y_t^2}{2}\; \bar{q}\tau^I\gamma^\mu q \;\; \varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi,$$
$$O_{\varphi u} \equiv \frac{y_t^2}{2}\; \bar{u}\gamma^\mu u \;\; \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi,$$
$$O_{\varphi d} \equiv \frac{y_t^2}{2}\; \bar{d}\gamma^\mu d \;\; \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi,$$
$$O_{\varphi ud} \equiv \frac{y_t^2}{2}\; \bar{u}\gamma^\mu d \;\; \varphi^T\epsilon i D_\mu\varphi,$$

$$O_{uW} \equiv y_t g_W\; \bar{q}\tau^I\sigma^{\mu\nu}u \;\; \epsilon\varphi^* W^I_{\mu\nu},$$
$$O_{dW} \equiv y_t g_W\; \bar{q}\tau^I\sigma^{\mu\nu}d \;\; \epsilon\varphi^* W^I_{\mu\nu},$$
$$O_{uB} \equiv y_t g_Y\; \bar{q}\sigma^{\mu\nu}u \;\; \epsilon\varphi^* B_{\mu\nu},$$
$$O_{dB} \equiv y_t g_Y\; \bar{q}\sigma^{\mu\nu}d \;\; \epsilon\varphi^* B_{\mu\nu},$$

$$O_{u\varphi} \equiv \bar{q}u \;\; \epsilon\varphi^* \;\; \varphi^\dagger\varphi$$
$$O_{d\varphi} \equiv \bar{q}d \;\; \epsilon\varphi^* \;\; \varphi^\dagger\varphi$$

See also Hartland et al., 1901.05965

Backup Slides

Why the sky looks blue?

Why the sky looks blue?

Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \;\ll\; \Delta E \sim \alpha^2 m_e \;\ll\; a_0^{-1} \sim \alpha m_e \;\ll\; M_A$$

- Neutral atom + gauge invariance ➡ $F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description: $\mathcal{L} = \psi^\dagger \left(i\,\partial_t + \frac{1}{2M}\,\vec{\nabla}^2 \right) \psi + \mathcal{L}_{\rm int}$

$$\mathcal{L}_{\rm int} \;=\; a_0^3\; \psi^\dagger\psi\, \left(c_1 \vec{E}^2 + c_2 \vec{B}^2 \right) + \dots \qquad , \qquad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \;\sim\; c_i\, a_0^3\, E_\gamma^2 \qquad \longrightarrow \qquad \boldsymbol{\sigma} \;\propto\; a_0^6\, E_\gamma^4$$

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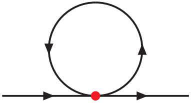
Blue light is scattered more strongly than red one

Quantum Loops

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi - \frac{a}{\Lambda^2}\,(\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4}\,(\bar{\psi}\,\Box\psi)(\bar{\psi}\psi) + \cdots$$



$$\delta m \;\sim\; 2i\,\frac{a}{\Lambda^2}\,m \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m^2}$$

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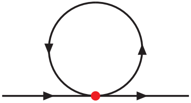

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- **Cut-off regularization:** $\delta m \sim m\,\frac{a}{\Lambda^2}\,\Lambda^2 \sim m\,a$ **Not suppressed!**

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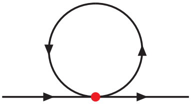

$$\delta m \;\sim\; 2i\,\frac{a}{\Lambda^2}\,m \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m^2}$$

- **Cut-off regularization:** $\delta m \sim m\,\frac{a}{\Lambda^2}\,\Lambda^2 \sim m\,a$ **Not suppressed!**
- **Dimensional regularization:** **Mass independent**

$$\delta m \;\sim\; 2a\,m\,\frac{m^2}{16\pi^2\Lambda^2}\left\{\Delta_\infty(\mu) + \log\left(\frac{m^2}{\mu^2}\right) - 1 + \mathcal{O}(D-4)\right\}$$

Well-defined expansion

$$\Delta_\infty(\mu) = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

Decoupling:

Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

SM: $M_W = M_Z \cos\theta_W = \frac{1}{2}\, g\, v \quad , \quad M_H = \sqrt{2\lambda}\, v \quad , \quad M_f = \frac{1}{\sqrt{2}}\, y_f\, v$

- **Decoupling occurs when** $v \to \infty$**, keeping the couplings fixed**

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{2v^2} \;\longrightarrow\; 0$$

- **There is no decoupling if some** $M_i \to \infty$**, keeping** $v = 246$ **GeV** $(g,\, \lambda,\, y_f \to \infty)$

QCD Matching

$$(\mu > M) \qquad \mathcal{L}_{\rm QCD}^{(N_F)} \quad \Longleftrightarrow \quad \mathcal{L}_{\rm QCD}^{(N_F-1)} + \sum_{d_i>4} \frac{c_i}{M^{d_i-4}}\, O_i \qquad (\mu < M)$$

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$$\alpha_s^{(N_F)}(\mu^2) \; = \; \alpha_s^{(N_F-1)}(\mu^2) \, \left\{ 1 + \sum_{k=1}^{\infty} C_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

$$L \equiv \ln \left(\mu^2/M^2\right)$$

$$m_q^{(N_F)}(\mu^2) \; = \; m_q^{(N_F-1)}(\mu^2) \, \left\{ 1 + \sum_{k=1}^{\infty} H_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi} \right]^k \right\}$$

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- **Matching conditions known to 4 loops:** $C_{1,2,3,4}\;,\;H_{1,2,3,4}$

  (Schroder-Steinhauser, Chetyrkin et al, Larin et al, Liu-Steinhauser)

- $\alpha_s(\mu^2)$ **is not continuous at threshold**

Wilson Coefficients: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}}\, O_i$

$$\langle O_i \rangle_B = Z_i(\epsilon,\mu)\, \langle O_i(\mu)\rangle_R \qquad ; \qquad \mu\, \frac{d}{d\mu}\, \langle O_i \rangle_B = 0$$

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$\longrightarrow$

$$\begin{aligned} c_i(\mu) &= c_i(\mu_0)\;\exp\left\{\int_{\alpha_0}^{\alpha}\frac{d\alpha}{\alpha}\,\frac{\gamma_{O_i}(\alpha)}{\beta(\alpha)}\right\} \\ &= c_i(\mu_0)\,\left[\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right]^{\gamma_{O_i}^{(1)}/\beta_1}\;\left\{1+\cdots\right\} \end{aligned}$$

Operator Mixing: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} O_i$

$$\langle O_i \rangle_B = \sum_j Z_{ij}(\epsilon, \mu) \langle O_j(\mu) \rangle_R \qquad ; \qquad \gamma_O \equiv Z^{-1} \mu \frac{d}{d\mu} Z$$

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Diagonalization: $\left(U^{-1} \gamma_O^T \, U\right)_{ij} = \tilde{\gamma}_{O_i} \, \delta_{ij} \qquad ; \qquad \tilde{c}_i = U^{-1}_{ij} \, c_j$

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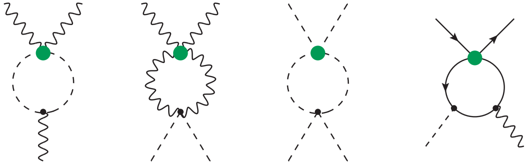
$$\rightarrow \quad c_i(\mu) = \sum_{j,k} U_{ij} \exp\left\{ \int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \frac{\tilde{\gamma}_{O_j}(\alpha)}{\beta(\alpha)} \right\} U_{jk}^{-1} c_k(\mu_0)$$

Renormalization Group Equations

$$\mathcal{L}_{\mathrm{eff}}^{(D=6)} = \sum_i \frac{C_i(\mu)}{\Lambda^2}\, \mathcal{O}_i(\mu)$$

Loop Corrections



$$\mu\,\frac{\mathrm{d}C_i}{\mathrm{d}\mu} = \sum_j \frac{\gamma_{ij}}{16\pi^2}\, C_j \quad \longrightarrow \quad C_i(\mu) = C_i(\Lambda) - \sum_j \frac{\gamma_{ij}}{16\pi^2}\, C_j(\Lambda)\, \log\left(\frac{\Lambda}{\mu}\right)$$

Anomalous Dimensions: γ_{ij}

Anomalous Dimensions γ_{ij}

$$\gamma \sim O\left(1, g^2, \lambda, y^2, g^4, g^2\lambda, g^2y^2, \lambda^2, \lambda y^2, y^4, g^6, g^4\lambda, g^6\lambda\right)$$

Alonso–Jenkins–Manohar–Trott

	g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
g^3X^3	g^2	0	0	1	0	0	0	0
H^6	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

Complete 59×59 matrix computed in the BW-GIMR basis