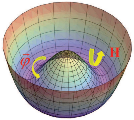
A.P., hep-ph/9806303, arXiv:1804.05664



Effective Field Theory

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- **Basic concepts in EFT**
- **SMEFT**
- **EWET (HEFT)**

PREFIT – School, DESY, Hamburg, 2–13 March 2020

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim \mathrm{TeV}$	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Underlying Dynamics**
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Standard Model**

Energy Gap

SMEFT

I. Brivio, M. Trott, arXiv:1706.08945

SM Effective Field Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}} \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**

$$\mathrm{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

- **Light** $(\mathrm{m} \ll \Lambda \equiv \Lambda_{\mathrm{NP}})$ **SM fields only**

- **The SM Lagrangian corresponds to D = 4**

- $\mathrm{c_i^{(D)}}$ **contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_X \, (\bar{q}_L \gamma^\mu q_L) \, X_\mu \quad \longrightarrow \quad \frac{g_X^2}{M_X^2} \, (\bar{q}_L \gamma^\mu q_L) \, (\bar{q}_L \gamma_\mu q_L)$$

- **Assumes that H(125) belongs to an** $\mathrm{SU(2)_L}$ **doublet**

Linear Realization of the $SU(2)_L \otimes U(1)_Y$ symmetry

- **H and the electroweak Goldstones combine into an $SU(2)_L$ doublet:**

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} (v + H)\, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad , \qquad U(\vec{\varphi}) = \exp\left\{ i\, \vec{\sigma} \cdot \frac{\vec{\varphi}}{v} \right\}$$

- **The SM Lagrangian is the low-energy effective theory with D = 4**

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **1 operator with D = 5:** $\mathcal{O}^{(5)} = \bar{L}_L \tilde{\Phi} \tilde{\Phi}^T L_L^c$ **(violates L by 2 units)**
  Weinberg

- **59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)**
  Buchmuller–Wyler, Grzadkowski–Iskrzynski–Misiak–Rosiek

- **5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)**
  Weinberg, Wilczek–Zee, Abbott–Wise

- **3 generations: 1350 CP-even and 1149 CP-odd operators with D = 6**
  Alonso–Jenkins–Manohar-Trott

$D=6$ Operators (other than 4-fermion ones)

Grzadkowski–Iskrzynski–Misiak–Rosiek

X^3		Φ^6 and $\Phi^4 D^2$		$\psi^2\Phi^3$	
$\mathcal{O}_G$	$f^{ABC}\, G_\mu^{A\nu}\, G_\nu^{B\rho}\, G_\rho^{C\mu}$	$\mathcal{O}_\Phi$	$(\Phi^\dagger\Phi)^3$	$\mathcal{O}_{e\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{l}_p e_r \Phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\, \widetilde{G}_\mu^{A\nu}\, G_\nu^{B\rho}\, G_\rho^{C\mu}$	$\mathcal{O}_{\Phi\Box}$	$(\Phi^\dagger\Phi)\,\Box\,(\Phi^\dagger\Phi)$	$\mathcal{O}_{u\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{q}_p u_r \widetilde{\Phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK}\, W_\mu^{I\nu}\, W_\nu^{J\rho}\, W_\rho^{K\mu}$	$\mathcal{O}_{\Phi D}$	$\left(\Phi^\dagger D^\mu\Phi\right)^\star\,\left(\Phi^\dagger D_\mu\Phi\right)$	$\mathcal{O}_{d\Phi}$	$(\Phi^\dagger\Phi)\,(\bar{q}_p d_r \Phi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\, \widetilde{W}_\mu^{I\nu}\, W_\nu^{J\rho}\, W_\rho^{K\mu}$				

$X^2\Phi^2$		$\psi^2 X\Phi$		$\psi^2\Phi^2 D$	
$\mathcal{O}_{\Phi G}$	$\Phi^\dagger\Phi\;G_{\mu\nu}^A\, G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\;\tau^I\Phi\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi l}^{(1)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)\,(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\Phi\widetilde{G}}$	$\Phi^\dagger\Phi\;\widetilde{G}_{\mu\nu}^A\, G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu} e_r)\;\Phi\; B_{\mu\nu}$	$\mathcal{O}_{\Phi l}^{(3)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu^I\Phi)\,(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\Phi W}$	$\Phi^\dagger\Phi\;W_{\mu\nu}^I\, W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A u_r)\;\widetilde{\Phi}\; G_{\mu\nu}^A$	$\mathcal{O}_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)\,(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\Phi\widetilde{W}}$	$\Phi^\dagger\Phi\;\widetilde{W}_{\mu\nu}^I\, W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\;\tau^I\widetilde{\Phi}\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi q}^{(1)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)\,(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\Phi B}$	$\Phi^\dagger\Phi\;B_{\mu\nu}\, B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu} u_r)\;\widetilde{\Phi}\; B_{\mu\nu}$	$\mathcal{O}_{\Phi q}^{(3)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu^I\Phi)\,(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\Phi\widetilde{B}}$	$\Phi^\dagger\Phi\;\widetilde{B}_{\mu\nu}\, B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu} T^A d_r)\;\Phi\; G_{\mu\nu}^A$	$\mathcal{O}_{\Phi u}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)\,(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\Phi WB}$	$\Phi^\dagger\tau^I\Phi\;W_{\mu\nu}^I\, B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\;\tau^I\Phi\; W_{\mu\nu}^I$	$\mathcal{O}_{\Phi d}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)\,(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\Phi\widetilde{W}B}$	$\Phi^\dagger\tau^I\Phi\;\widetilde{W}_{\mu\nu}^I\, B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu} d_r)\;\Phi\; B_{\mu\nu}$	$\mathcal{O}_{\Phi ud}$	$i(\widetilde{\Phi}^\dagger D_\mu\Phi)\,(\bar{u}_p\gamma^\mu d_r)$

$q = q_L\,,\; l = l_L\,,\; u = u_R\,,\; d = d_R\,,\; e = e_R$, $\overleftrightarrow{D}_\mu^I \equiv \tau^I\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\tau^I$, $p,\, r =$ generation indices

$D=6$ Four-Fermion Operators

Grzadkowski–Iskrzynski–Misiak–Rosiek

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

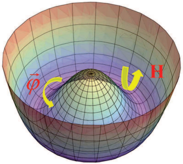
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(d_p^\alpha)^T C\, u_r^\beta\right]\left[(q_s^{\gamma j})^T C\, l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(u_s^\gamma)^T C\, e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\,\varepsilon_{jk}\,(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\,\varepsilon_{jk}\,\varepsilon_{mn}\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C\, l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k u_t)$	$\mathcal{O}_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}\,(\tau^I\varepsilon)_{jk}\,(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C\, q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C\, l_t^n\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\,\varepsilon_{jk}\,(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C\, u_r^\beta\right]\left[(u_s^\gamma)^T C\, e_t\right]$

$q = q_L$, $l = l_L$, $u = u_R$, $d = d_R$, $e = e_R$, $p, r, s, t =$ generation indices

EWET

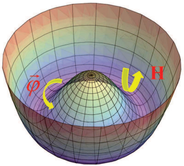
(HEFT)

A.P., arXiv:1804.05664



$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix}$$

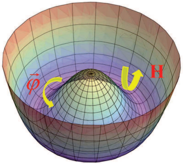


$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

$$= \frac{1}{2}\,\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu\Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2$$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$



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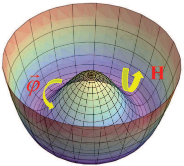
$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\,\Sigma\,g_R^\dagger$

Custodial Symmetry

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} (v + H)\; U(\vec{\varphi}\,)$$

$$U(\vec{\varphi}\,) \equiv \exp\left\{ i\, \vec{\sigma} \cdot \frac{\vec{\varphi}}{v} \right\}$$



$$\begin{aligned}
\mathcal{L}_\Phi &= (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \\
&= \frac{1}{2} \mathrm{Tr}\left[(D^\mu \Sigma)^\dagger D_\mu \Sigma \right] - \frac{\lambda}{4} \left(\mathrm{Tr}\left[\Sigma^\dagger \Sigma \right] - v^2 \right)^2 \\
&= \frac{v^2}{4} \mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U \right] + O(H/v)
\end{aligned}$$

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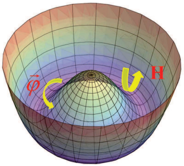

$$\begin{aligned} \mathcal{L}_\Phi &= (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2} \mathrm{Tr}\left[(D^\mu \Sigma)^\dagger D_\mu \Sigma \right] - \frac{\lambda}{4} \left(\mathrm{Tr}\left[\Sigma^\dagger \Sigma \right] - v^2 \right)^2 \\ &= \frac{v^2}{4} \mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U \right] + O(H/v) \end{aligned}$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\qquad \Sigma \to g_L\, \Sigma\, g_R^\dagger$

Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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- **Goldstone Bosons**

$\langle 0 | \bar{q}^j_R q^i_L | 0 \rangle$ **(QCD)**, Φ **(SM)** ➡ $U_{ij}(\phi) = \{ \exp(i\vec{\sigma}\cdot\vec{\varphi}/v) \}_{ij}$

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- **Expansion in powers of momenta** ⬌ **derivatives**

Parity ➡ even dimension ; $U\,U^\dagger = 1$ ➡ $2n \geq 2$

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U ➡ $g_L\, U\, g_R^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

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$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$

Derivative Coupling

EFFECTIVE LAGRANGIAN: $\mathcal{L}(U) = \sum_n \mathcal{L}_{2n}$

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Derivative Coupling

Goldstones become free at zero momenta

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) = \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \equiv \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\varphi^0 \end{pmatrix}$$

$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger \quad , \quad \langle A\rangle \equiv \mathrm{Tr}(A)$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

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$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) = \exp\left\{\frac{i\sqrt{2}}{v}\Phi\right\} \quad , \quad \Phi \equiv \frac{1}{\sqrt{2}}\vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\varphi^0 \end{pmatrix}$$

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$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\quad U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

Goldstone Electroweak Effective Theory

$$\mathcal{L}_{\mathrm{EW}}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) = \exp\left\{\frac{i\sqrt{2}}{v}\Phi\right\} \quad , \quad \Phi \equiv \frac{1}{\sqrt{2}}\vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\varphi^0 \end{pmatrix}$$

$$D^\mu U = \partial^\mu U - i\hat{W}^\mu U + i U\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i U^\dagger \hat{W}^\mu - i\hat{B}^\mu U^\dagger \quad , \quad \langle A\rangle \equiv \mathrm{Tr}(A)$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i[\hat{W}^\mu,\hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i[\hat{B}^\mu,\hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \to g_L \hat{W}^\mu g_L^\dagger + i\, g_L \partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \to g_R \hat{B}^\mu g_R^\dagger + i\, g_R \partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\hat{W}^\mu = -\frac{g}{2}\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2}\sigma_3 B^\mu$

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

$$\mathrm{M_W} = \mathrm{M_Z}\cos\theta_W = \tfrac{1}{2}\,\mathrm{g\,v}$$

$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\,\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger$$

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(explicit symmetry breaking)

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- **EW Goldstones are responsible for** $\mathrm{M_{W,Z}}$ **(not the Higgs!)**
- **QCD pions also generate small** $\mathrm{W, Z}$ **masses:** $\delta_\pi \mathrm{M_W} = \frac{1}{2}\,\mathrm{g\,f_\pi}$

Goldstone interactions are determined by the underlying symmetry

$$U(\varphi) \;=\; \exp\left\{\frac{i\sqrt{2}}{v}\,\Phi\right\} \qquad , \qquad \Phi \;\equiv\; \frac{1}{\sqrt{2}}\,\vec{\sigma}\cdot\vec{\varphi} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\,\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\,\varphi^0 \end{pmatrix}$$

$$\begin{aligned}
\frac{v^2}{4}\,\langle\partial_\mu U^\dagger \partial^\mu U\rangle \;&=\; \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0 \\
&+\; \frac{1}{6v^2}\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\} \\
&+\; O\left(\varphi^6/v^4\right)
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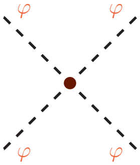

$$T\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) \;=\; \frac{s+t}{v^2}$$

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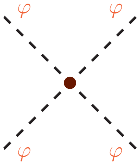
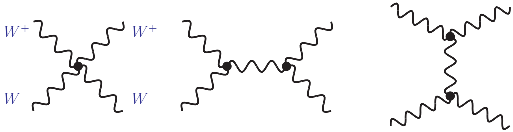


$$T\left(\varphi^+\varphi^- \to \varphi^+\varphi^-\right) \;=\; \frac{s+t}{v^2}$$

Non-Linear Lagrangian: $2\varphi \to 2\varphi,\; 4\varphi\;\cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$
$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ derivative interactions

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \quad \Longrightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W}\,\left(|\vec{k}|, 0, 0, k^0\right) \;=\; \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

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One naively expects $\quad T(W_L^+ W_L^- \to W_L^+ W_L^-) \sim g^2 \, \frac{|\vec{k}|^4}{M_W^4}$

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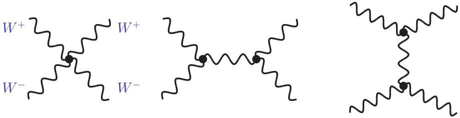
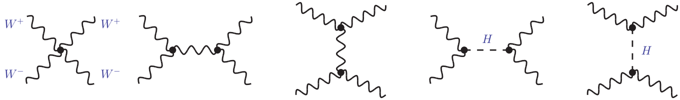


Gauge Cancelation

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) \;=\; \frac{s+t}{v^2} \;+\; O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$=\; T(\varphi^+\varphi^- \to \varphi^+\varphi^-) \;+\; O\left(\frac{M_W}{\sqrt{s}}\right)$$

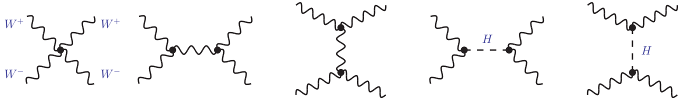
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



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Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\rm SM} \approx -\dfrac{2M_H^2}{v^2} \quad , \quad a_0 \equiv \dfrac{1}{32\pi}\displaystyle\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\dfrac{M_H^2}{8\pi v^2}$

Perturbative unitarity:

Lee–Quigg–Thacker

$$|a_0| \le 1 \quad \Rightarrow \quad \mathrm{M_H} < \sqrt{8\pi}\,\mathrm{v}\;\underbrace{\sqrt{2/3}}_{\mathrm{W^+W^-,\,ZZ,\,HH}} \approx 1 \;\mathrm{TeV}$$

What happens in QCD?

- **QCD satisfies unitarity** **(it is a renormalizable theory)**
- **Pion scattering unitarized by exchanges of resonances (composite objects):**
  - **P-wave ($J = 1$) unitarized by ρ exchange**
  - **S-wave ($J = 0$) unitarized by σ exchange**
- **The σ meson is the QCD equivalent of the SM Higgs**
- **BUT, the σ is an 'effective' object generated through $\pi\pi$ rescattering (summation of pion loops)**

Does not seem to work this way in the EW case, but . . .

Energy Scale	**Fields**	**Effective Theory**
$\Lambda_{\rm NP} \sim \mathrm{TeV}$	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Underlying Dynamics**

Energy Gap

M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	**Standard Model**

EWET with a Light (singlet) Higgs

Assumptions:

- **Spontaneous Symmetry Breaking:** $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$
- **h(125)** **is an** $\mathrm{SU(2)_{L+R}}$ **scalar singlet**

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \longrightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h/v)\, \mathcal{O}_X$$

$$\mathcal{F}_X(h/v) = \sum_{n=0} c_n^{(X)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h/v) = v^4 \sum_{n=2} c_n^{(V)} \left(\frac{h}{v}\right)^n$$

Low-Energy Effective Theory ➡ Power Counting

- **Momentum expansion:** $\Lambda \sim 4\pi v\,,\ M_X$

$$\mathcal{A} = \sum_n \mathcal{A}_n \left(\frac{p}{\Lambda}\right)^n$$

- $U(\varphi)\,,\ \varphi\,,\ h \sim O(p^0)$

  $D_\mu U\,,\ \hat{W}_\mu\,,\ \hat{B}_\mu \sim O(p^1) \quad , \quad \hat{W}_{\mu\nu}\,,\ \hat{B}_{\mu\nu} \sim O(p^2)$

- **A general connected diagram with N_d vertices of $O(p^d)$ and L Goldstone loops has a power dimension:** Weinberg

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

➡ **Finite number of divergences / counterterms**

Electroweak Effective Theory

$$\mathcal{L}_{\rm EWET} = \underbrace{\mathcal{L}_{YM} + i \sum_f \bar{f}\gamma^\mu D_\mu f + \Delta\mathcal{L}_2}_{\mathcal{L}_{\rm EW}^{(2)}} + \mathcal{L}_{\rm EW}^{(4)} + \cdots$$

$$\Delta\mathcal{L}_2^{\rm Bosonic} = \frac{1}{2}\,\partial_\mu h\,\partial^\mu h - \frac{1}{2}\,m_h^2\,h^2 - V(h/v) + \frac{v^2}{4}\,\mathcal{F}_u(h/v)\,\langle (D^\mu U)^\dagger D_\mu U\rangle$$

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$$V(h/v) \;=\; v^4 \sum_{n=3} c_n^{(V)} \left(\frac{h}{v}\right)^n \qquad , \qquad \mathcal{F}_u(h/v) \;=\; 1 \;+\; \sum_{n=1} c_n^{(u)} \left(\frac{h}{v}\right)^n$$

Electroweak Effective Theory

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SM: $c_3^{(V)} = \frac{m_h^2}{2v^2}$, $c_4^{(V)} = \frac{m_h^2}{8v^2}$, $c_{n>4}^{(V)} = 0$; $c_1^{(u)} = 2$, $c_2^{(u)} = 1$, $c_{n>2}^{(u)} = 0$

Yukawa Couplings

$$\Delta\mathcal{L}_2^{\mathrm{Ferm.}} = -v \left\{ \bar{Q}_L \, U(\varphi) \left[\hat{\mathrm{Y}}_{\mathrm{u}} \, \mathcal{P}_+ + \hat{\mathrm{Y}}_{\mathrm{d}} \, \mathcal{P}_- \right] Q_R \; + \; \bar{L}_L \, U(\varphi) \, \hat{\mathrm{Y}}_{\ell} \, \mathcal{P}_+ \, L_R \; + \; \mathrm{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad , \qquad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L \, U(\varphi) \, g_R^\dagger \quad , \quad Q_L \to g_L \, Q_L \quad , \quad Q_R \to g_R \, Q_R \quad , \quad \mathcal{P}_\pm \to g_R \, \mathcal{P}_\pm \, g_R^\dagger$$

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- **Flavour Structure:** $\hat{\mathrm{Y}}_{\mathrm{u,d},\ell}$ 3×3 **matrices in flavour space**

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- **Flavour Structure:** $\hat{\mathrm{Y}}_{\mathrm{u,d},\ell}$ 3×3 **matrices in flavour space**
- **Higgs field:** $\hat{\mathrm{Y}}_{\mathrm{u,d},\ell}(h/v) = \sum_{n=0} \hat{\mathrm{Y}}^{(\mathrm{n})}_{\mathrm{u,d},\ell} \left(\frac{h}{v}\right)^n$

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\mathrm{EW}}^{(4)}\Big|_{\mathrm{Bosonic}} = \sum_i \mathcal{F}_i(h/v)\, \mathcal{O}_i \qquad\qquad \mathcal{F}_i(h/v) = \sum_{n=0} \mathcal{F}_{i,n} \left(\frac{h}{v}\right)^n$$

Appelquist-Bernard, Longhitano, Buchalla et al, Alonso et al, Pich et al...

$\mathcal{O}(\mathrm{p}^4)$ $\mathcal{P}$-even bosonic operators

A.P., Rosell, Santos, Sanz-Cillero

$$\mathcal{O}_1 = \langle\, U^\dagger \hat{W}_{\mu\nu} U \hat{B}^{\mu\nu} \,\rangle \qquad \mathcal{O}_2 = \langle\, \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} + \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \,\rangle$$

$$\mathcal{O}_3 = i\,\langle\, \hat{W}_{\mu\nu} D^\mu U\, D^\nu U^\dagger + \hat{B}_{\mu\nu} D^\mu U^\dagger D^\nu U \,\rangle \qquad \mathcal{O}_4 = \langle\, D_\mu U^\dagger D_\nu U \,\rangle\, \langle\, D^\mu U^\dagger D^\nu U \,\rangle$$

$$\mathcal{O}_5 = \langle\, D_\mu U^\dagger D^\mu U \,\rangle^2 \qquad \mathcal{O}_6 = \frac{1}{v^2}\, (\partial_\mu h)(\partial^\mu h)\, \langle\, D_\nu U^\dagger D^\nu U \,\rangle$$

$$\mathcal{O}_7 = \frac{1}{v^2}\, (\partial_\mu h)(\partial_\nu h)\, \langle\, D^\mu U^\dagger D^\nu U \,\rangle \qquad \mathcal{O}_8 = \frac{1}{v^4}\, (\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$$

$$\mathcal{O}_9 = -\frac{i}{v}\, (\partial^\mu h)\, \langle\, \hat{W}_{\mu\nu} D^\nu U\, U^\dagger + \hat{B}_{\mu\nu} D^\nu U^\dagger U \,\rangle$$

Custodial symmetry assumed

Unitary Gauge: $U = 1$

All invariants reduce to polynomials of h and gauge fields

- **Bilinear gauge terms:** $\mathcal{O}_1, \mathcal{O}_2$

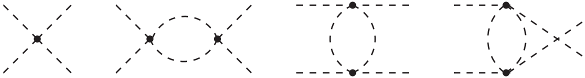
  ➡ **Oblique corrections** $(\Delta r, \Delta\rho, \Delta k \longleftrightarrow S, T, U)$

- **Trilinear gauge couplings:** $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$

- **Quartic gauge couplings:** $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$

- **Higgs interactions:** $\mathcal{O}_{1\text{–}9}$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$:

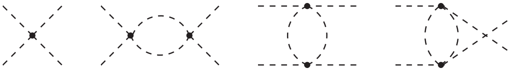


$$\mathcal{A}(\varphi^a\varphi^b \to \varphi^c\varphi^d) \;=\; A(s,t,u)\,\delta_{ab}\,\delta_{cd} + A(t,s,u)\,\delta_{ac}\,\delta_{bd} + A(u,t,s)\,\delta_{ad}\,\delta_{bc}$$

$$\begin{aligned} A(s,t,u) \;=\; & \frac{s}{v^2} + \frac{4}{v^2}\left[\mathcal{F}^r_{4,0}(\mu)\,(t^2+u^2) + 2\,\mathcal{F}^r_{5,0}(\mu)\,s^2\right] \\ & + \frac{1}{16\pi^2 v^2}\left\{\frac{5}{9}\,s^2 + \frac{13}{18}\,(t^2+u^2) + \frac{1}{12}\,(s^2-3t^2-u^2)\,\log\left(\frac{-t}{\mu^2}\right)\right. \\ & \left. + \frac{1}{12}\,(s^2-t^2-3u^2)\,\log\left(\frac{-u}{\mu^2}\right) - \frac{1}{2}\,s^2\,\log\left(\frac{-s}{\mu^2}\right)\right\} \end{aligned}$$

$$\mathcal{F}_{i,0} \;=\; \mathcal{F}^r_{i,0}(\mu) + \frac{\gamma^{\mathcal{O}}_{i,0}}{32\pi^2}\left[\frac{2\,\mu^{D-4}}{D-4} - \log(4\pi) + \gamma_E\right] \quad , \quad \gamma^{\mathcal{O}}_{4,0} = \frac{1}{6} \quad , \quad \gamma^{\mathcal{O}}_{5,0} = \frac{1}{12}$$

$$\varphi^a \varphi^b \to \varphi^c \varphi^d$$



+ Higgs (tree + 1-loop) contributions

$$\mathcal{L} = \frac{v^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle \left[1 + 2\, a\, \frac{h}{v} + b\, \frac{h^2}{v^2} \right]$$

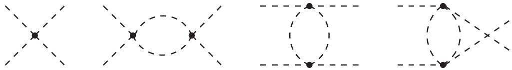
$$a = \tfrac{1}{2}\, c_1^{(u)} \quad , \quad b = c_2^{(u)}$$

Espriu–Mescia–Yencho, Delgado–Dobado–Llanes-Estrada

$$\begin{aligned} A(s,t,u) = {} & \frac{s}{v^2}\,(1-a^2) + \frac{4}{v^2}\left[\mathcal{F}^r_{4,0}(\mu)\,(t^2+u^2) + 2\,\mathcal{F}^r_{5,0}(\mu)\, s^2\right] \\ & + \frac{1}{16\pi^2 v^2}\left\{ \frac{1}{9}\,(14\,a^4 - 10\,a^2 - 18\,a^2 b + 9\,b^2 + 5)\, s^2 + \frac{13}{18}\,(1-a^2)^2\,(t^2+u^2) \right. \\ & \qquad - \frac{1}{2}\,(2\,a^4 - 2\,a^2 - 2\,a^2 b + b^2 + 1)\, s^2 \log\left(\frac{-s}{\mu^2}\right) \\ & \qquad \left. + \frac{1}{12}\,(1-a^2)^2 \left[(s^2 - 3t^2 - u^2)\log\left(\frac{-t}{\mu^2}\right) + (s^2 - t^2 - 3u^2)\log\left(\frac{-u}{\mu^2}\right)\right]\right\} \end{aligned}$$

$$\gamma_4 = \tfrac{1}{6}\,(1-a^2)^2 \quad , \quad \gamma_5 = \tfrac{1}{24}\,(2 + 5\,a^4 - 4\,a^2 - 6\,a^2 b + 3\,b^2)$$

$\varphi^a \varphi^b \to \varphi^c \varphi^d$



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$\gamma_4 = \frac{1}{6} (1 - a^2)^2 \quad , \quad \gamma_5 = \frac{1}{24} (2 + 5\, a^4 - 4\, a^2 - 6\, a^2 b + 3\, b^2)$

SM: $a = b = 1 \quad , \quad \mathcal{F}_{4,0} = \mathcal{F}_{5,0} = 0$ ➡ $A(s,t,u) \sim \mathcal{O}(M_H^2/v^2)$

EW Resonance Effective Theory

- **Towers of heavy states are usually present in strongly-coupled models of EWSB: Technicolour, Walking TC...**

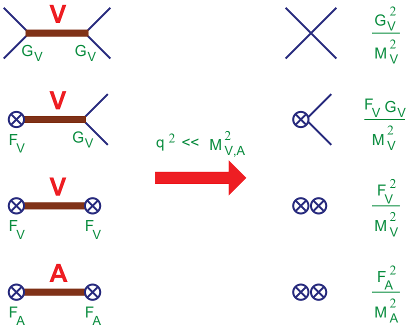
- **The low-energy constants (LECs) of the Goldstone Lagrangian contain information on the heavier states. The lightest states not included in the Lagrangian dominate**

1. Build $\mathcal{L}_{\text{eff}}(\varphi_i, R_k)$ with the lightest R_k coupled to the φ_i
2. Require a good UV behaviour ➡ Low # of derivatives
3. Match the two effective Lagrangians ➡ LECs

This program works in QCD: RχT **(Ecker–Gasser–Leutwyler–Pich–de Rafael)**

Good dynamical understanding at large N_C

Resonance Exchange



Pich, Rosell, Santos, Sanz-Cillero

$$\mathcal{F}_1 = \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2} \quad , \quad \mathcal{F}_2 = -\frac{F_A^2}{8M_A^2} - \frac{F_V^2}{8M_V^2} \quad , \quad \mathcal{F}_3 = -\frac{F_V\, G_V}{2M_V^2}$$

$$\mathcal{F}_4 = \frac{G_V^2}{4M_V^2} \quad , \quad \mathcal{F}_5 = \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2} \quad , \quad \mathcal{F}_6 = -\frac{(\lambda_1^{hA})^2 v^2}{M_A^2}$$

$$\mathcal{F}_7 = \frac{d_P^2}{2M_P^2} + \frac{(\lambda_1^{hA})^2 v^2}{M_A^2} \quad , \quad \mathcal{F}_8 = 0 \quad , \quad \mathcal{F}_9 = -\frac{F_A \lambda_1^{hA} v}{M_A^2}$$

Short-distance constraints bring sharper predictions

Pich, Rosell, Santos, Sanz-Cillero

$$\mathcal{F}_1 = \frac{F_A^2}{4M_A^2} - \frac{F_V^2}{4M_V^2} = -\frac{v^2}{4}\left(\frac{1}{M_V^2}+\frac{1}{M_A^2}\right)$$

$$\mathcal{F}_2 = -\frac{F_A^2}{8M_A^2} - \frac{F_V^2}{8M_V^2} = -\frac{v^2(M_V^4+M_A^4)}{8M_V^2M_A^2(M_A^2-M_V^2)}$$

$$\mathcal{F}_3 = -\frac{F_V\,G_V}{2M_V^2} = -\frac{v^2}{2M_V^2}$$

$$\mathcal{F}_4 = \frac{G_V^2}{4M_V^2} = \frac{(M_A^2-M_V^2)v^2}{4M_V^2M_A^2}$$

$$\mathcal{F}_5 = \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2} = \frac{c_d^2}{4M_{S_1}^2} - \frac{(M_A^2-M_V^2)v^2}{4M_V^2M_A^2}$$

$$\mathcal{F}_6 = -\frac{(\lambda_1^{hA})^2v^2}{M_A^2} = -\frac{M_V^2(M_A^2-M_V^2)v^2}{M_A^6}$$

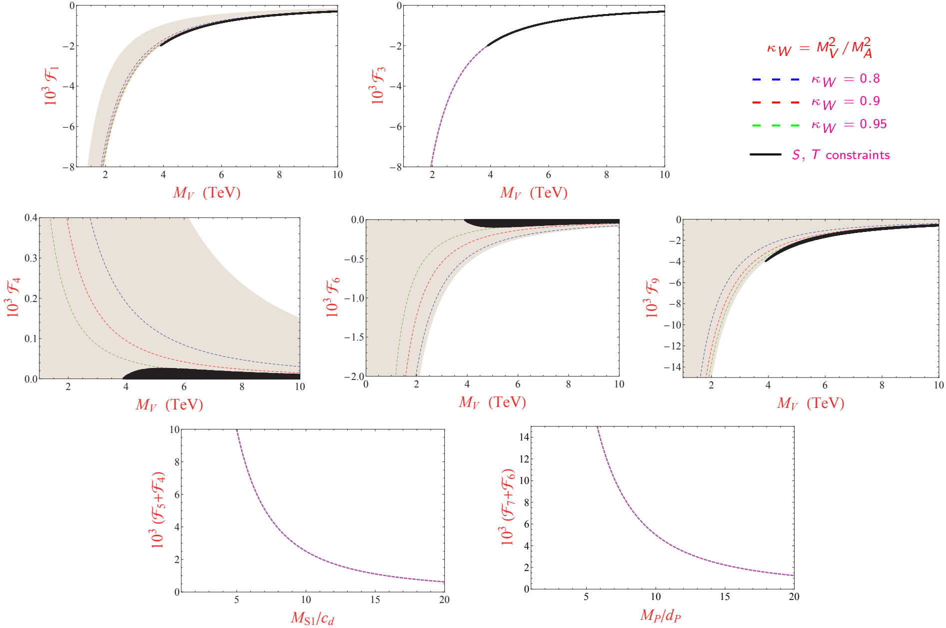
$$\mathcal{F}_7 = \frac{d_P^2}{2M_P^2} + \frac{(\lambda_1^{hA})^2v^2}{M_A^2} = \frac{d_P^2}{2M_P^2} + \frac{M_V^2(M_A^2-M_V^2)v^2}{M_A^6}$$

$$\mathcal{F}_8 = 0$$

$$\mathcal{F}_9 = -\frac{F_A\lambda_1^{hA}v}{M_A^2} = -\frac{M_V^2v^2}{M_A^4}$$

Asymptotically-Free Theories

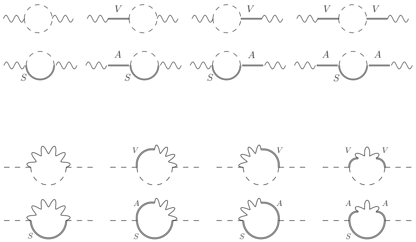
A.P., Rosell, Santos, Sanz-Cillero, arXiv:1510.03114



Gauge Boson Self-Energies @ NLO

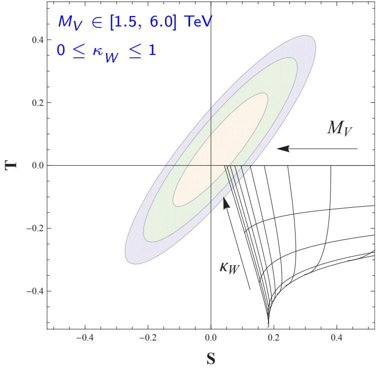
Sensitive to the light scalar h(125)

AP, Rosell, Sanz-Cillero, 1212.6769



$$\kappa_W \equiv \frac{g_{hWW}}{g_{hWW}^{SM}} = \frac{M_V^2}{M_A^2} \in [0.94,\, 1]$$

$M_A \approx M_V > 4$ TeV (95% CL)



$1^{st} + 2^{nd}$ **WSRs**

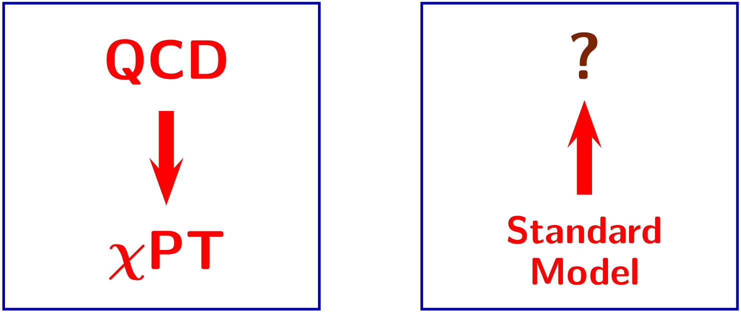
OUTLOOK

- **Effective Field Theory: powerful low-energy tool**
- **Mass Gap:** $E, m_{\text{light}} \ll \Lambda_{\text{NP}}$
- **Assumption: relevant symmetries (breakings) & light fields**
- **Most general** $\mathcal{L}_{\text{eff}}(\phi_{\text{light}})$ **allowed by symmetry**
- **Short-distance dynamics encoded in LECs**
- **LECs constrained phenomenologically**
- **Goal: get hints on the underlying fundamental dynamics**

$\Longrightarrow$ **New Physics**

Learning from QCD experience. EW problem more difficult

Fundamental Underlying Theory unknown

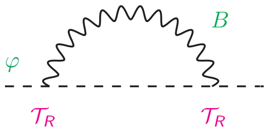


Additional dynamical input (fresh ideas!) needed

PREFIT-School
PRecision Effective FIeld Theory School
2-13 March 2020 at DESY, Hamburg

Backup Slides

Custodial Symmetry Breaking: $\hat{B}_\mu \equiv -g' \frac{\sigma_3}{2} B_\mu$



$$\langle U^\dagger D^\mu U \mathcal{T}_R \, U^\dagger D_\mu U \mathcal{T}_R \rangle$$

$$U^\dagger D_\mu U = i \frac{\sqrt{2}}{v} D_\mu \Phi + \cdots \quad , \quad \mathcal{T}_R \to g_R \, \mathcal{T}_R \, g_R^\dagger \quad , \quad \mathcal{T}_R = -g' \frac{\sigma_3}{2}$$

$$\langle U^\dagger D^\mu U \mathcal{T}_R \, U^\dagger D_\mu U \mathcal{T}_R \rangle = \langle U^\dagger D^\mu U \mathcal{T}_R \rangle \langle U^\dagger D_\mu U \mathcal{T}_R \rangle + \frac{1}{2} \langle (D_\mu U)^\dagger D_\mu U \rangle \langle \mathcal{T}_R \mathcal{T}_R \rangle$$

Power-Counting Rules:

A.P., Rosell, Santos, Sanz-Cillero, 1609.06659

$$v\,,\ \frac{\varphi}{v}\,,\ u(\varphi)\,,\ U(\varphi)\,,\ \frac{h}{v}\,,\ \frac{\vec{W}_\mu}{v}\,,\ \frac{B_\mu}{v} \quad\sim\quad \mathcal{O}\left(p^0\right)$$

$$\frac{\psi}{v}\,,\ \frac{\bar{\psi}}{v} \quad\sim\quad \mathcal{O}\left(p^{1/2}\right)$$

$$D_\mu U\,,\ u_\mu\,,\ \partial_\mu\,,\ \hat{W}_\mu\,,\ \hat{B}_\mu\,,\ m_h\,,\ m_W\,,\ m_Z\,,\ m_\psi\,,\ g\,,\ g'\,,\ \mathcal{Y}\,,\ \mathcal{T}_R \quad\sim\quad \mathcal{O}\left(p\right)$$

$$\hat{W}_{\mu\nu}\,,\ \hat{B}_{\mu\nu}\,,\ f_{\pm\,\mu\nu}\,,\ c_n^{(V)} \quad\sim\quad \mathcal{O}\left(p^2\right)$$

$$\partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_n}\,\mathcal{F}(h/v) \quad\sim\quad \mathcal{O}\left(p^n\right)$$

$$\left(\frac{\bar{\psi}'\,\Gamma\,\psi}{v^2}\right)^n \quad\sim\quad \mathcal{O}\left(p^{2n}\right)$$

$$\Gamma \;\sim\; p^{\hat{d}_\Gamma} \qquad , \qquad \hat{d}_\Gamma \;=\; 2 + 2L + \sum_{\hat{d}} (\hat{d}-2)\, N_{\hat{d}}$$

CP-Invariant Bosonic Operators

A.P., Rosell, Santos,
Sanz-Cillero
1609.06659

i	$\mathcal{O}_i$	$\widetilde{\mathcal{O}}_i$
1	$\frac{1}{4}\langle\, f_+^{\mu\nu} f_{+\,\mu\nu} - f_-^{\mu\nu} f_{-\,\mu\nu}\,\rangle$	$\frac{i}{2}\langle\, f_-^{\mu\nu}[u_\mu, u_\nu]\,\rangle$
2	$\frac{1}{2}\langle\, f_+^{\mu\nu} f_{+\,\mu\nu} + f_-^{\mu\nu} f_{-\,\mu\nu}\,\rangle$	$\langle\, f_+^{\mu\nu} f_{-\,\mu\nu}\,\rangle$
3	$\frac{i}{2}\langle\, f_+^{\mu\nu}[u_\mu, u_\nu]\,\rangle$	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, f_+^{\mu\nu} u_\nu\,\rangle$
4	$\langle\, u_\mu u_\nu\,\rangle\,\langle\, u^\mu u^\nu\,\rangle$	—
5	$\langle\, u_\mu u^\mu\,\rangle^2$	—
6	$\frac{1}{v^2}\,(\partial_\mu h)(\partial^\mu h)\,\langle\, u_\nu u^\nu\,\rangle$	—
7	$\frac{1}{v^2}\,(\partial_\mu h)(\partial_\nu h)\,\langle\, u^\mu u^\nu\,\rangle$	—
8	$\frac{1}{v^4}\,(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	—
9	$\frac{1}{v}\,(\partial_\mu h)\,\langle\, f_-^{\mu\nu} u_\nu\,\rangle$	—
10	$\langle\mathcal{T} u_\mu\rangle^2$	—
11	$\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}$	—

$$\mathcal{L}_4^{\mathrm{Bosonic}} = \sum_{i=1}^{11} \mathcal{F}_i(h/v)\,\mathcal{O}_i \;+\; \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i(h/v)\,\widetilde{\mathcal{O}}_i$$

A.P., Rosell, Santos, Sanz-Cillero, 1609.06659

$$J_\Gamma = \begin{cases} \bar\psi_L \Gamma \psi_L + \bar\psi_R \Gamma \psi_R & (\Gamma = \gamma^\mu, \gamma^\mu\gamma_5) \\ \bar\psi_L \Gamma\, U\, \psi_R + \bar\psi_R \Gamma\, U^\dagger \psi_L & (\Gamma = I, i\gamma_5, \sigma^{\mu\nu}) \end{cases}$$

CP-Invariant Fermionic Operators

i	$\mathcal{O}_i^{\psi^2}$	$\widetilde{\mathcal{O}}_i^{\psi^2}$	$\mathcal{O}_i^{\psi^4}$	$\widetilde{\mathcal{O}}_i^{\psi^4}$
1	$\langle J_S \rangle \langle u_\mu u^\mu \rangle$	$\langle J_T^{\mu\nu} f_{-\,\mu\nu} \rangle$	$\langle J_S J_S \rangle$	$\langle J_V^\mu J_{A,\mu} \rangle$
2	$i \langle J_T^{\mu\nu} [u_\mu, u_\nu] \rangle$	$\frac{1}{v} (\partial_\mu h) \langle u_\nu J_T^{\mu\nu} \rangle$	$\langle J_P J_P \rangle$	$\langle J_V^\mu \rangle \langle J_{A,\mu} \rangle$
3	$\langle J_T^{\mu\nu} f_{+\,\mu\nu} \rangle$	$\langle J_V^\mu \rangle \langle u_\mu \mathcal{T} \rangle$	$\langle J_S \rangle \langle J_S \rangle$	—
4	$\hat{X}_{\mu\nu} \langle J_T^{\mu\nu} \rangle$	—	$\langle J_P \rangle \langle J_P \rangle$	—
5	$\frac{1}{v} (\partial_\mu h) \langle u^\mu J_P \rangle$	—	$\langle J_V^\mu J_{V,\mu} \rangle$	—
6	$\langle J_A^\mu \rangle \langle u_\mu \mathcal{T} \rangle$	—	$\langle J_A^\mu J_{A,\mu} \rangle$	—
7	$\frac{1}{v^2} (\partial_\mu h)(\partial^\mu h) \langle J_S \rangle$	—	$\langle J_V^\mu \rangle \langle J_{V,\mu} \rangle$	—
8	—	—	$\langle J_A^\mu \rangle \langle J_{A,\mu} \rangle$	—
9	—	—	$\langle J_T^{\mu\nu} J_{T\,\mu\nu} \rangle$	—
10	—	—	$\langle J_T^{\mu\nu} \rangle \langle J_{T\,\mu\nu} \rangle$	—

$$\mathcal{L}_4^{\text{Ferm.}} = \sum_{i=1}^{7} \mathcal{F}_i^{\psi^2}(h/v)\, \mathcal{O}_i^{\psi^2} + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i^{\psi^2}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^2} + \sum_{i=1}^{10} \mathcal{F}_i^{\psi^4}(h/v)\, \mathcal{O}_i^{\psi^4} + \sum_{i=1}^{2} \widetilde{\mathcal{F}}_i^{\psi^4}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^4}$$

LO Resonance EW Lagrangian:

A.P., Rosell, Santos, Sanz-Cillero

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{EWET}} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \cdots$$

Heavy Triplets: V(1^{--}) , A(1^{++}) , P(1^{++}) ; **Heavy Singlet:** $S_1(0^{++})$

$$\sum_R \mathcal{L}_R = \frac{v}{2}\,\kappa_W\, h\,\langle u^\mu u_\mu\rangle + \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\, f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\,[u^\mu u^\nu]\rangle$$
$$+ \frac{F_A}{2\sqrt{2}}\,\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + \sqrt{2}\,\lambda_1^{hA}\,\partial_\mu h\,\langle A^{\mu\nu} u_\nu\rangle$$
$$+ \frac{d_P}{v}\,\partial_\mu h\,\langle P\, u^\mu\rangle + \frac{c_d}{\sqrt{2}}\, S_1\,\langle u^\mu u_\mu\rangle + \lambda_{hS_1}\, v\, h^2\, S_1$$

$$U = u^2 = \exp\left\{\tfrac{i}{v}\,\vec{\sigma}\,\vec{\varphi}\right\} \quad , \quad u_\mu \equiv i\,u\,(D_\mu U)^\dagger u = u_\mu^\dagger \quad , \quad f_\pm^{\mu\nu} = u^\dagger \hat{W}^{\mu\nu} u \pm u\,\hat{B}^{\mu\nu} u^\dagger$$

Antisymmetric $V_{\mu\nu}$ and $A_{\mu\nu}$ fields (better UV properties):

$$\mathcal{L}_{\mathrm{Kin}} = -\frac{1}{2}\sum_{R=V,A}\langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{1}{2}\, M_R^2\, R_{\mu\nu} R^{\mu\nu}\rangle$$