

SMEFT Hands-on

Ilaria Brivio

What you need to have installed

follow instructions on the **TWiki**

- ▶ Mathematica
- ▶ FeynRules
- ▶ VirtualBox with `Delphes2020.vdi`
- ▶ material in the TWiki:
 - ▶ download on your laptop: **SMEFT_HandsOn_pt1.tar**
 - ▶ for the second part, on the VM: `SMEFT_HandsOn_pt2.tar`

Part I – theory

The SMEFT

- theory valid if:
- ▶ new physics nearly decoupled: $\Lambda \gg (\nu, E)$
 - ▶ at the accessible scale: **SM** fields + symmetries

☞ a Taylor expansion in canonical dimensions ($\delta = \nu/\Lambda$ or E/Λ):

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \frac{1}{\Lambda^3} \mathcal{L}_7 + \frac{1}{\Lambda^4} \mathcal{L}_8 + \dots$$

$$\mathcal{L}_n = \sum_i C_i \mathcal{O}_i^{d=n}$$

C_i free parameters (Wilson coefficients)

\mathcal{O}_i invariant operators that form
a complete, non redundant basis

The SMEFT

- theory valid if:
- ▶ new physics nearly decoupled: $\Lambda \gg (v, E)$
 - ▶ at the accessible scale: **SM** fields + symmetries

☞ a Taylor expansion in canonical dimensions ($\delta = v/\Lambda$ or E/Λ):

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}_6$$

$$\mathcal{L}_n = \sum_i C_i \mathcal{O}_i^{d=n}$$

C_i free parameters (Wilson coefficients)

\mathcal{O}_i invariant operators that form
a complete, non redundant basis

The Warsaw basis

Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

1	X^3	2	φ^6 and $\varphi^4 D^2$	3	$\psi^2 \varphi^3$	5
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$	
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$	
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^*$ $(\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$	
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					
4	$X^2 \varphi^2$	6	$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	7
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$	
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$	
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$	
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$	
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$	
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$	
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$	
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$	

The Warsaw basis

Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

8a	$(\bar{L}L)(\bar{L}L)$	8b	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$		8c
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$	
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$	
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$	
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$	
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$	
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$	
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$	
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$	

8d $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$

B-violating

Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^\gamma)^T C l_t^k]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

The Warsaw basis

Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

8a	$(\bar{L}L)(\bar{L}L)$	8b	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$		8c
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$	
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$	
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$	
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$	
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$	
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$	
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$	
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$	

8d $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$

B-violating

Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C \gamma^\beta \gamma^\gamma]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(\gamma^\gamma \gamma^\gamma \gamma^\gamma \gamma^\gamma) C e_t]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(\gamma^\gamma \gamma^\gamma \gamma^\gamma \gamma^\gamma) [(q_s^\gamma)^T C l_t^n]]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(\gamma^\gamma \gamma^\gamma \gamma^\gamma \gamma^\gamma) C u_r^\beta] [(u_s^\gamma)^T C e_t]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$		omitted

A code for LO SMEFT: SMEFTsim

today we will be using **SMEFTsim**

Brivio, Jiang, Trott 1709.06492

- ▶ website: <http://feynrules.irmp.ucl.ac.be/wiki/SMEFT>
- ▶ a FeynRules model with the complete Warsaw basis
- ▶ it comes also in pre-exported UFO models
- ▶ implements **3** flavor assumptions \times **2** input parameter schemes

general

MFV

U35

$\{\alpha_{\text{em}}, m_Z, G_F\}$

$\{m_W, m_Z, G_F\}$

- ▶ works in Mathematica and MC generators (MadGraph)

- ▶ allows MC generation at **LO**

→ for NLO use **SMEFT@NLO**

- ▶ SM $hgg, h\gamma\gamma, hZ\gamma$ vertices implemented in the $m_t \rightarrow \infty$ limit

A closer look at some operators

In the `SMEFT_HandsOn.nb` notebook in Mathematica:

- ▶ load `FeynRules`
- ▶ define the variables

```
Flavor = U35
Scheme = MwScheme
```

- ▶ load the model `SMEFTsim_A_main.fr`
- ▶ output the Feynman rules for some operators, e.g.:

```
OH13
OdH
OHd
OHB
...
```

Flavor assumptions

The SMEFT Lagrangian is defined with free flavor indices

→ this means **2499 real parameters**

e.g. $\mathcal{L} \supset (C_{He})_{pr} (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_{R,p} \gamma^\mu e_{R,r})$

→ hermitian: $(C_{He})_{pr} = (C_{He})_{rp}^*$ → 3 ℝ + 3 ℂ ≡ **9** real. par.

e.g. $\mathcal{L} \supset (C_{le})_{prst} (\bar{l}_{L,p} \gamma_\mu l_{L,r})(\bar{e}_{R,s} \gamma^\mu e_{R,t})$

→ hermitian: $(C_{le})_{prst} = (C_{le})_{rpst}^* = (C_{le})_{prts}^* = (C_{le})_{rpts}$

→ 9 ℝ + 36 ℂ ≡ **45** real par.

e.g. $\mathcal{L} \supset (C_{ledq})_{prst} (\bar{l}_{L,p}^j \gamma_\mu e_{R,r})(\bar{d}_{R,s} \gamma^\mu q_{R,t}^j)$

→ **not** hermitian: → 81 ℂ ≡ **162** real par.

Flavor assumptions

one can assume a flavor symmetry → **only invariant contractions allowed**

$U(3)^5$

maximal: for each SM field $\psi = \{q_L, u_R, d_R, l_L, e_R\}$:

$\psi_p \mapsto \Omega_{\psi,pr} \psi_r$ with Ω_ψ a 3×3 unitary matrix

Flavor assumptions

one can assume a flavor symmetry → only invariant contractions allowed

$U(3)^5$

maximal: for each SM field $\psi = \{q_L, u_R, d_R, l_L, e_R\}$:

$\psi_p \mapsto \Omega_{\psi,pr} \psi_r$ with Ω_ψ a 3×3 unitary matrix

invariants:

e.g. $\bar{u}_R \gamma^\mu u_R \rightarrow \bar{u}_{Rp} \gamma^\mu (\Omega_u^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr} = \bar{u}_{Rp} \gamma^\mu \delta_{pr} u_{Rr}$ diagonal

$\bar{q}_L u_R ? \rightarrow \bar{q}_{Lp} (\Omega_q^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr}$ not invariant!

Flavor assumptions

one can assume a flavor symmetry → only invariant contractions allowed

$U(3)^5$

maximal: for each SM field $\psi = \{q_L, u_R, d_R, l_L, e_R\}$:

$\psi_p \mapsto \Omega_{\psi,pr} \psi_r$ with Ω_ψ a 3×3 unitary matrix

invariants:

e.g. $\bar{u}_R \gamma^\mu u_R \rightarrow \bar{u}_{Rp} \gamma^\mu (\Omega_u^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr} = \bar{u}_{Rp} \gamma^\mu \delta_{pr} u_{Rr}$ diagonal

$\bar{q}_L u_R ? \rightarrow \bar{q}_{Lp} (\Omega_q^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr}$ not invariant!

Yukawas inserted as spurions : constants transforming under the sym.:

$Y_u \mapsto \Omega_u Y_u \Omega_q^\dagger, \quad Y_d \mapsto \Omega_d Y_d \Omega_q^\dagger, \quad Y_e \mapsto \Omega_e Y_e \Omega_l^\dagger$

in this way $(\bar{u}_R Y_u q_L), \quad (\bar{d}_R Y_d q_L), \quad (\bar{e}_R Y_e l_L)$ are allowed

→ all **chirality-changing** currents require inserting a Yukawa!

Flavor assumptions

one can assume a flavor symmetry → only invariant contractions allowed

$U(3)^5$

maximal: for each SM field $\psi = \{q_L, u_R, d_R, l_L, e_R\}$:

$\psi_p \mapsto \Omega_{\psi,pr} \psi_r$ with Ω_ψ a 3×3 unitary matrix

invariants:

e.g. $\bar{u}_R \gamma^\mu u_R \rightarrow \bar{u}_{Rp} \gamma^\mu (\Omega_u^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr} = \bar{u}_{Rp} \gamma^\mu \delta_{pr} u_{Rr}$ diagonal

$\bar{q}_L u_R ? \rightarrow \bar{q}_{Lp} (\Omega_q^\dagger)_{ps} (\Omega_u)_{sr} u_{Rr}$ not invariant!

Yukawas inserted as spurions : constants transforming under the sym.:

$Y_u \mapsto \Omega_u Y_u \Omega_q^\dagger, \quad Y_d \mapsto \Omega_d Y_d \Omega_q^\dagger, \quad Y_e \mapsto \Omega_e Y_e \Omega_l^\dagger$

in this way $(\bar{u}_R Y_u q_L), \quad (\bar{d}_R Y_d q_L), \quad (\bar{e}_R Y_e l_L)$ are allowed

→ all **chirality-changing** currents require inserting a Yukawa!

→ **81 real parameters**

Predictions in the SMEFT

From $\mathcal{L}_{\text{SMEFT}}$ to observables

*so, we have chosen a flavor assumption and written down the Lagrangian.
Are we ready to calculate?*

Nope. Not yet.

From $\mathcal{L}_{\text{SMEFT}}$ to observables

*so, we have chosen a flavor assumption and written down the Lagrangian.
Are we ready to calculate?*

Nope. Not yet.

Some Lagrangian manipulations are needed first:

- ▶ normalize and diagonalize all **kinetic terms**
- ▶ define a **input** parameter scheme (“tree-level renormalization scheme”)

From $\mathcal{L}_{\text{SMEFT}}$ to observables

so, we have chosen a flavor assumption and written down the Lagrangian.
Are we ready to calculate?

Nope. Not yet.

Some Lagrangian manipulations are needed first:

- ▶ normalize and diagonalize all **kinetic terms**
- ▶ define a **input** parameter scheme (“tree-level renormalization scheme”)

corrections stemming from these operations are induced by \mathcal{L}_6 : $\propto C_i$

→ when applied to \mathcal{L}_6 itself: $\mathcal{O}(C_i^2)$ effects → neglected

→ only need to be evaluated on \mathcal{L}_{SM}

Kinetic term normalization and field redefinitions

Some $d = 6$ operators give corrections to **kinetic terms**

e.g. $C_{HB} (H^\dagger H) B_{\mu\nu} B^{\mu\nu} \xrightarrow{\text{unitary g.}} C_{HB} \frac{v^2}{2} B_{\mu\nu} B^{\mu\nu} + C_{HB} \frac{2vh + h^2}{2} B_{\mu\nu} B^{\mu\nu}$

$$C_{HD} (H^\dagger D_\mu H)(D^\mu H^\dagger H) \xrightarrow{\text{unitary g.}} C_{HD} \frac{v^2}{4} \partial_\mu h \partial^\mu h + \dots$$

$$\begin{aligned} C_{H\square} (H^\dagger H) D_\mu D^\mu (H^\dagger H) &\xrightarrow{\text{unitary g.}} C_{H\square} \left[\frac{v^2}{2} \partial_\mu h \partial^\mu h + \frac{3}{2} v^2 h \partial_\mu \partial^\mu h \right] + \dots \\ &= -C_{H\square} \partial_\mu h \partial^\mu h + \dots \end{aligned}$$

Calculating with non-canonically normalized kinetic terms is complicated
→ requires modifying LSZ formula

Kinetic term normalization and field redefinition: B_μ

an easier solution: redefine the fields

eg. B_μ $\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\left[1 - 2v^2C_{HB}\right]$

Kinetic term normalization and field redefinition: B_μ

an easier solution: redefine the fields

eg. B_μ $\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\left[1 - 2v^2C_{HB}\right]$

replace everywhere $\begin{cases} B_\mu \rightarrow B_\mu [1 + v^2 C_{HB}] \\ g' \rightarrow g' [1 - v^2 C_{HB}] \end{cases}$ and expand linearly in C_{HB}

Kinetic term normalization and field redefinition: B_μ

an easier solution: redefine the fields

eg. B_μ $\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\left[1 - 2v^2C_{HB}\right]$

replace everywhere $\begin{cases} B_\mu \rightarrow B_\mu [1 + v^2 C_{HB}] \\ g' \rightarrow g' [1 - v^2 C_{HB}] \end{cases}$ and expand linearly in C_{HB}

$$\rightarrow -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\left[1 - 2v^2C_{HB}\right]\left[1 + 2v^2C_{HB}\right] = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \mathcal{O}(C_{HB}^2)$$

$\rightarrow D_\mu \sim g'B_\mu$ unchanged up to $\mathcal{O}(C_{HB}^2)$

$\rightarrow \mathcal{L}_6$ unchanged up to $\mathcal{O}(C_{HB}^2)$

$\rightarrow C_{HB}$ only remains in $C_{HB}\frac{2vh + h^2}{2}B_{\mu\nu}B^{\mu\nu}$

Kinetic term normalization and field redefinitions: h

an easier solution: redefine the fields

eg. h $\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{2} \partial_\mu h \partial^\mu h \left[1 + \frac{v^2}{2} C_{HD} - 2v^2 C_{H\square} \right] = \frac{1}{2} \partial_\mu h \partial^\mu h [1 + \Delta_h]$

Kinetic term normalization and field redefinitions: h

an easier solution: redefine the fields

eg. h $\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{2} \partial_\mu h \partial^\mu h \left[1 + \frac{v^2}{2} C_{HD} - 2v^2 C_{H\square} \right] = \frac{1}{2} \partial_\mu h \partial^\mu h [1 + \Delta_h]$

replace everywhere $h \rightarrow h \left[1 - \frac{v^2}{4} C_{HD} + v^2 C_{H\square} \right]$, expand linearly in C_{HD} , $C_{H\square}$

Kinetic term normalization and field redefinitions: h

an easier solution: redefine the fields

eg. h $\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{2} \partial_\mu h \partial^\mu h \left[1 + \frac{v^2}{2} C_{HD} - 2v^2 C_{H\square} \right] = \frac{1}{2} \partial_\mu h \partial^\mu h [1 + \Delta_h]$

replace everywhere $h \rightarrow h \left[1 - \frac{v^2}{4} C_{HD} + v^2 C_{H\square} \right]$, expand linearly in C_{HD} , $C_{H\square}$

$$\rightarrow \frac{1}{2} \partial_\mu h \partial^\mu h [1 + \Delta_h] [1 - \Delta_h] = \frac{1}{2} \partial_\mu h \partial^\mu h + \mathcal{O}(\Delta_h^2)$$

$\rightarrow \mathcal{L}_6$ unchanged up to $\mathcal{O}(\Delta_h^2)$

\rightarrow **SM Higgs couplings:** $h^3, h^4, hVV, hhVV, h\bar{\psi}\psi$

with n h -legs are rescaled by [1 - $n \Delta_h$]

A special kinetic term correction: \mathcal{O}_{HWB}

$$C_{HWB} (H^\dagger W_{\mu\nu} H) B^{\mu\nu} \xrightarrow{\text{unitary g.}} -C_{HWB} \frac{v^2}{2} W_{\mu\nu}^3 B^{\mu\nu} + \dots$$

introduces a **kinetic mixing** between W^3, B — needs to be diagonalized!

A special kinetic term correction: \mathcal{O}_{HWB}

$$C_{HWB} (H^\dagger W_{\mu\nu} H) B^{\mu\nu} \xrightarrow{\text{unitary g.}} -C_{HWB} \frac{v^2}{2} W_{\mu\nu}^3 B^{\mu\nu} + \dots$$

introduces a **kinetic mixing** between W^3, B — needs to be diagonalized!

3 subsequent operations:

- (1) normalize kin. term for B C_{HB} and W^i C_{HW}
- (2) rotate to diagonalize kin. term in (W^3, B) C_{HWB}
- (3) rotate to diagonalize mass term $\rightarrow (Z, A)$

doing (2), (3) it at once:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} 1 & -v^2 C_{HWB}/2 \\ -v^2 C_{HWB}/2 & 1 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A special kinetic term correction: \mathcal{O}_{HWB}

$$C_{HWB} (H^\dagger W_{\mu\nu} H) B^{\mu\nu} \xrightarrow{\text{unitary g.}} -C_{HWB} \frac{v^2}{2} W_{\mu\nu}^3 B^{\mu\nu} + \dots$$

introduces a **kinetic mixing** between W^3, B — needs to be diagonalized!

3 subsequent operations:

- (1) normalize kin. term for B  C_{HB} and W^i 
- (2) rotate to diagonalize kin. term in (W^3, B)  C_{HWB}
- (3) rotate to diagonalize mass term $\rightarrow (Z, A)$

doing (2), (3) it at once:

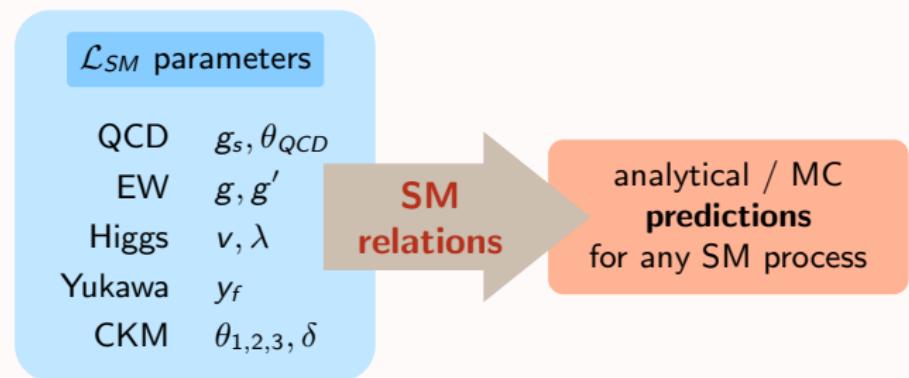
$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} 1 & -v^2 C_{HWB}/2 \\ -v^2 C_{HWB}/2 & 1 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

→ correction to the **Weinberg angle** → enters **SM γ, Z couplings**

Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters

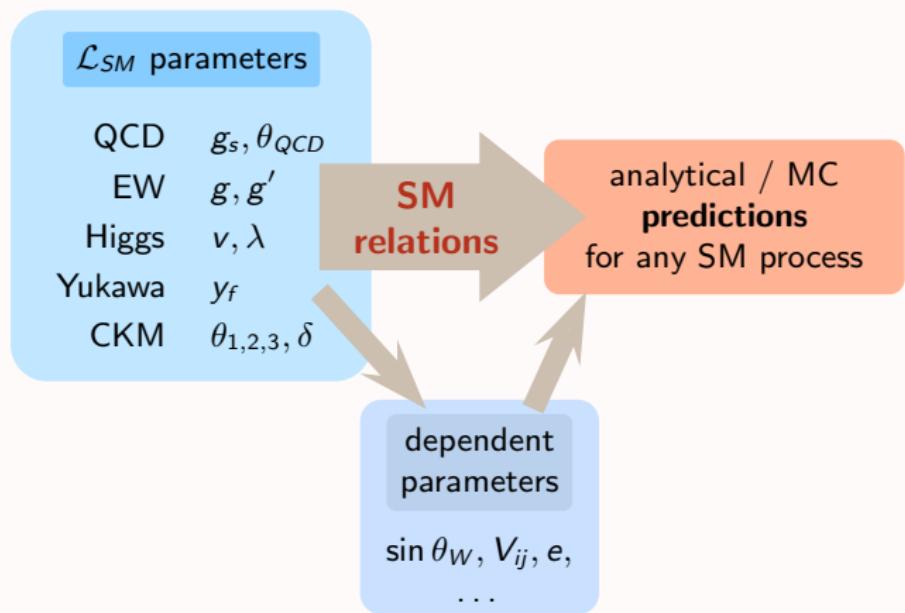
SM



Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters

SM



Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters

SM

input
measurements

α_s , d_N
 m_Z , α_{em}
 $\mu \rightarrow e\nu\nu$, m_h
 m_f
meson decay/osc

\mathcal{L}_{SM} parameters

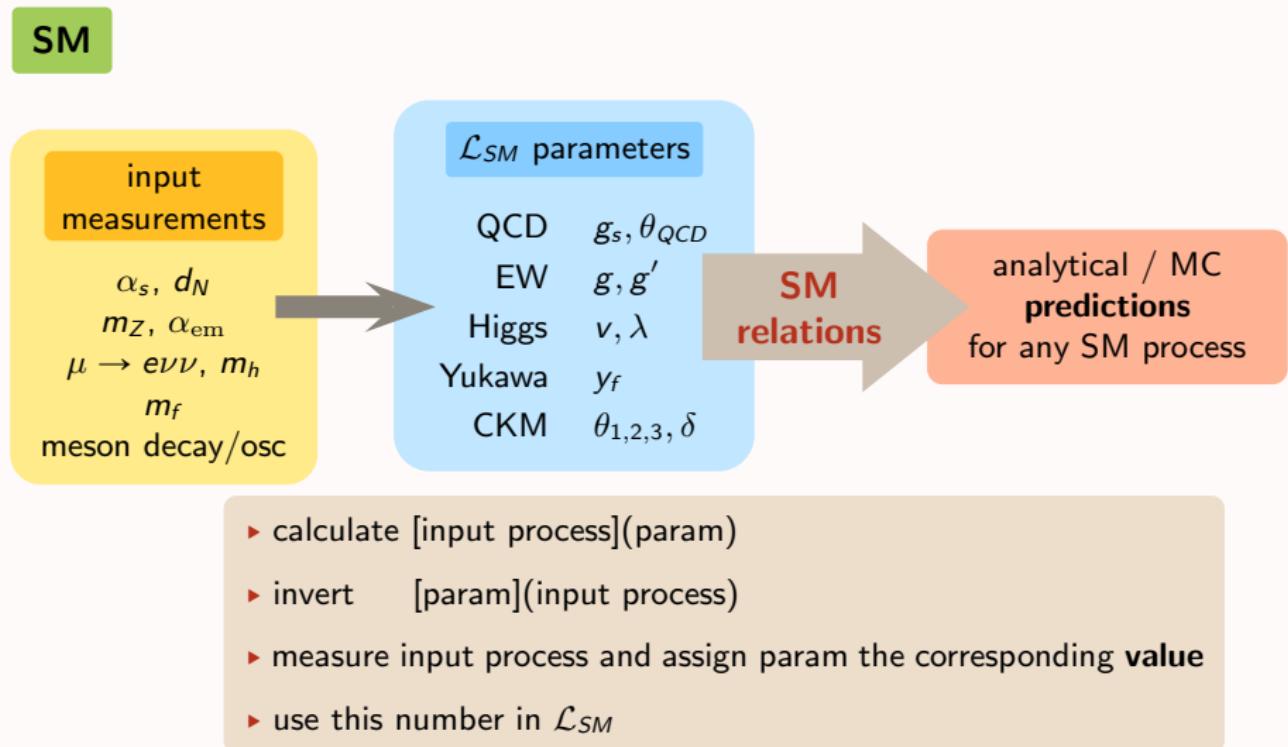
QCD	g_s, θ_{QCD}
EW	g, g'
Higgs	v, λ
Yukawa	y_f
CKM	$\theta_{1,2,3}, \delta$

SM
relations

analytical / MC
predictions
for any SM process

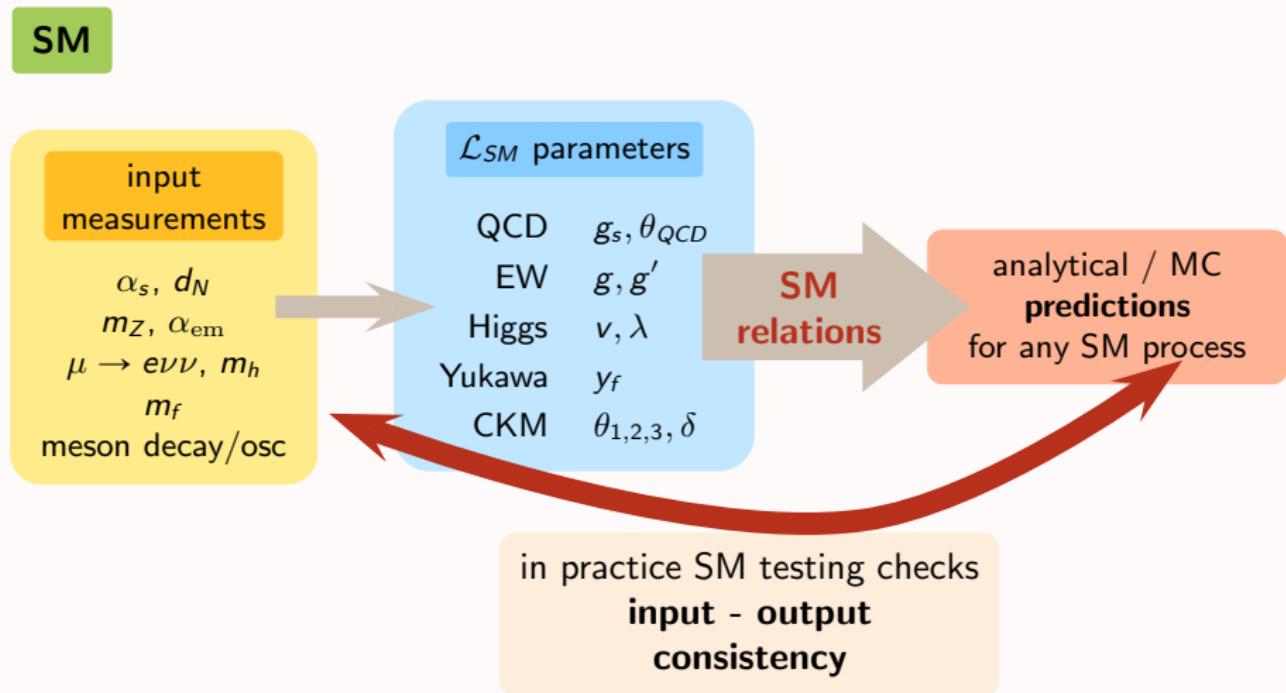
Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters



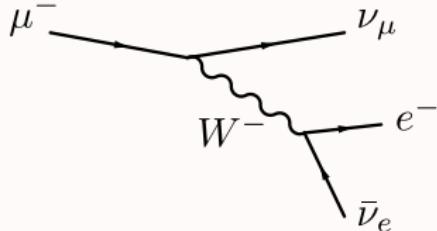
Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters



Input parameters example: G_F

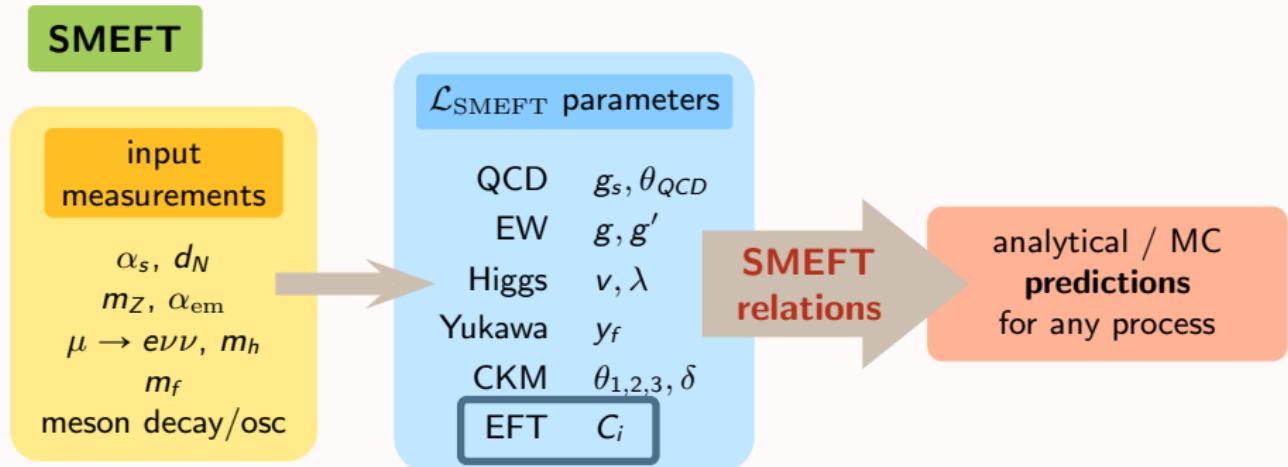
The Fermi constant is precisely measured from muon decay



$$\Gamma_{SM}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

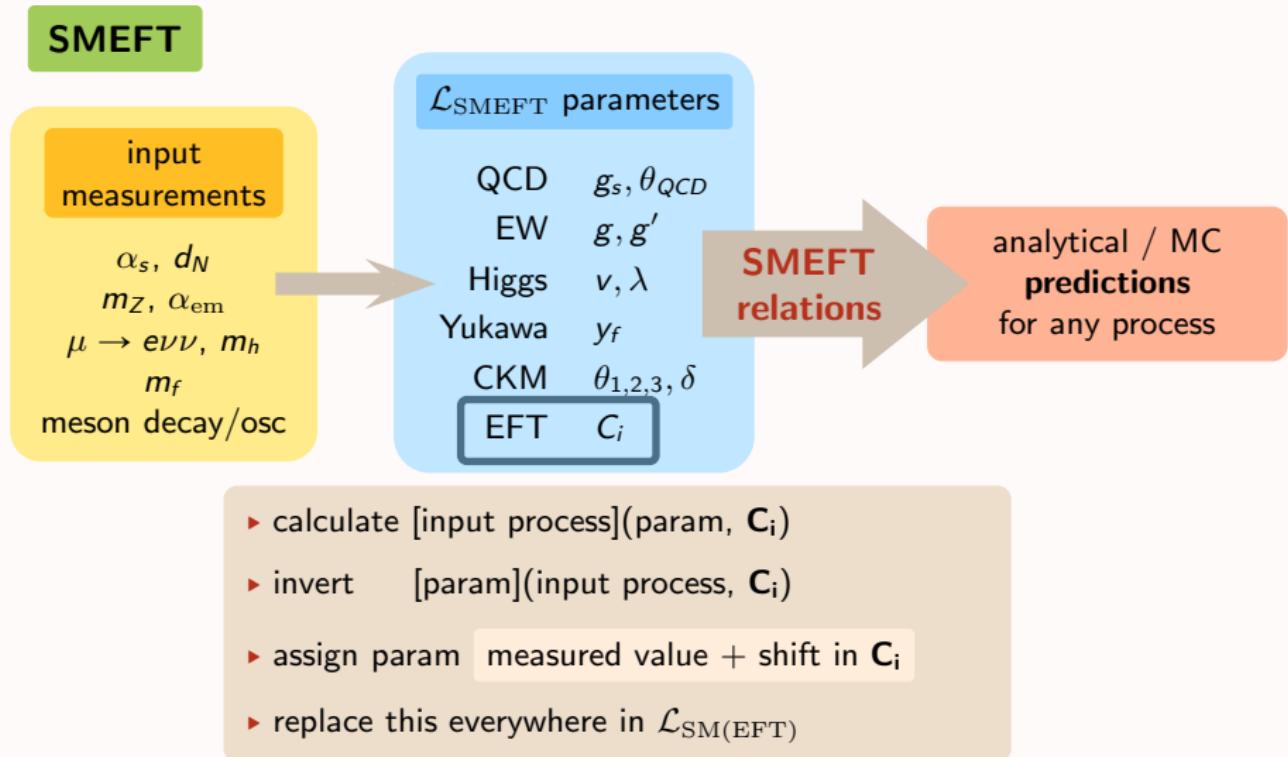
Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters



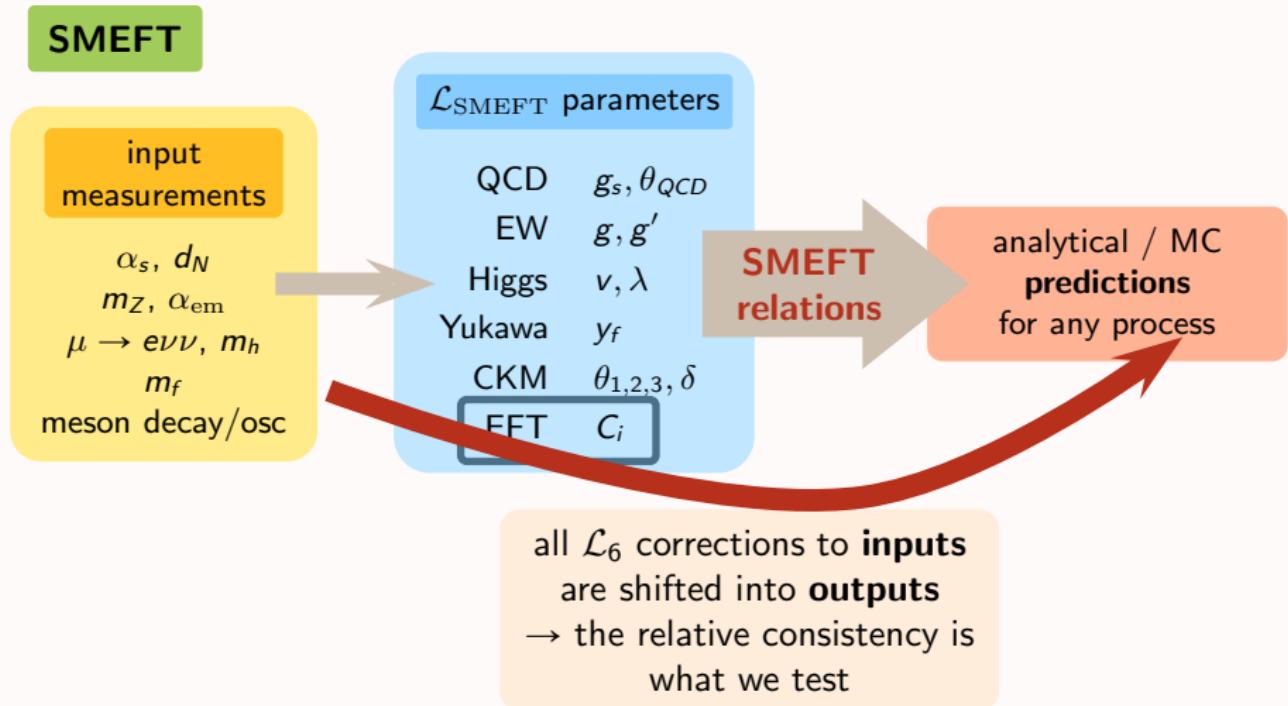
Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters



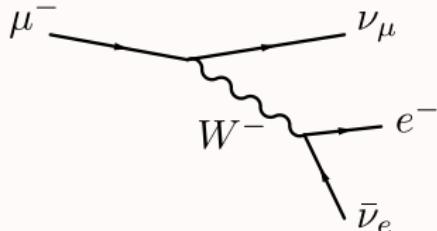
Input parameters

In order to make numerical predictions we need to assign numerical values to the Lagrangian parameters

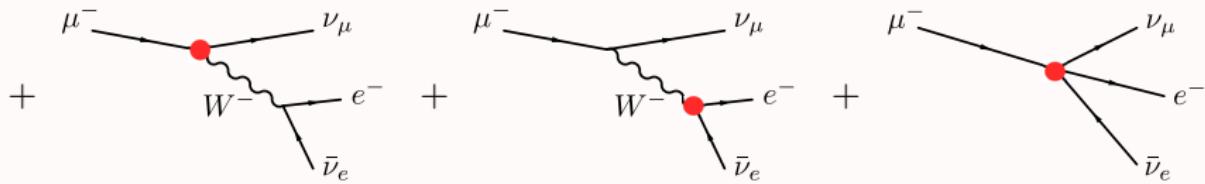


Input parameters example: G_F

The Fermi constant is precisely measured from muon decay



$$\Gamma_{SM}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$



$$\begin{aligned}\Gamma_{\text{SMEFT}}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) &= \Gamma_{SM} \left[1 + 2\nu^2 \left((C_{HI}^{(3)})_{22} + (C_{HI}^{(3)})_{11} - (C_{II})_{1221} \right) \right] \\ &\stackrel{U(3)^5}{=} \Gamma_{SM} \left[1 + 4\nu^2 \left(C_{HI}^{(3)} - \frac{1}{2} C'_{II} \right) \right]\end{aligned}$$

Input parameters example: G_F

$$\begin{aligned}\Gamma_{\text{SMEFT}}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) &= \Gamma_{SM} \left[1 + 2v^2 \left((C_{HI}^{(3)})_{22} + (C_{HI}^{(3)})_{11} - (C_{II})_{1221} \right) \right] \\ &\stackrel{U(3)^5}{=} \Gamma_{SM} \left[1 + 4v^2 \left(C_{HI}^{(3)} - \frac{1}{2} C_{II}' \right) \right]\end{aligned}$$

$$\begin{aligned}\rightarrow 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} &= G_F \left[1 + 2v^2 \left(C_{HI}^{(3)} - \frac{1}{2} C_{II}' \right) \right] \\ &= G_F \left[1 + \sqrt{2} \boxed{\Delta G_F} \right]\end{aligned}$$

$$\rightarrow \bar{v} = \frac{1}{2^{1/4} \sqrt{G_F}} = \hat{v} \left[1 + \frac{\Delta G_F}{\sqrt{2}} \right]$$

$$\hat{v} \equiv 246.22 \text{ GeV}$$

\bar{v} \equiv parameter in $\mathcal{L} \rightarrow \boxed{\Delta G_F}$ enters all **vertices with v** in \mathcal{L}_{SM}

Input parameters for the EW sector

a more correct analysis: the **EW sector** has 3 independent parameters

$$\{v, g, g'\}$$

that are fixed by **3** input measurements, usually chosen among

$$\{m_Z, m_W, G_F, \alpha_{\text{em}}\}$$

Input parameters for the EW sector

a more correct analysis: the **EW sector** has 3 independent parameters

$$\{\nu, g, g'\}$$

that are fixed by 3 input measurements, usually chosen among

$$\{m_Z, m_W, G_F, \alpha_{\text{em}}\}$$

i.e. one chooses 3 equations among

$$\hat{m}_Z^2 = [91.1876 \text{ GeV}]^2 = \frac{\bar{v}^2}{4} (\bar{g}^2 + \bar{g}'^2) \left[1 + \frac{\nu^2 C_{HD}}{2} + \frac{2\nu^2 \bar{g} \bar{g}'}{\bar{g}^2 + \bar{g}'^2} C_{HWB} \right]$$

$$\hat{m}_W^2 = [80.387 \text{ GeV}]^2 = \frac{\bar{v}^2 \bar{g}^2}{4}$$

$$\hat{G}_F^2 = 1.1663787 \times 10^{-5} \text{ GeV}^{-2} = \frac{1}{\sqrt{2}\bar{v}^2} \left[1 + 2\nu^2 C_{HII}^{(3)} - \nu^2 C_{II}' \right]$$

$$\hat{\alpha}_{\text{em}}(m_Z) = 1/127.95 = \frac{1}{4\pi} \frac{\bar{g}^2 \bar{g}'^2}{\bar{g}^2 + \bar{g}'^2} \left[1 - \frac{\nu^2 \bar{g} \bar{g}'}{\bar{g}^2 + \bar{g}'^2} C_{HWB} \right]$$

solves in $\{\bar{v}, \bar{g}, \bar{g}'\}$ and **replaces** the solution $\bar{x} \rightarrow \hat{x}(1 + \delta x/x)$ in \mathcal{L}_{SM}

Input parameters for the EW sector

example: $\{m_Z, G_F, \alpha_{\text{em}}\}$ scheme

$$\bar{v}^2 = \hat{v}^2 \left[1 + \sqrt{2} \Delta G_F \right],$$

$$\bar{g}^2 = \hat{g}^2 \left[1 - \frac{c_\theta^2}{c_{2\theta}} \left(\frac{\Delta m_Z^2}{m_Z^2} + \frac{\Delta G_F}{\sqrt{2}} \right) - \frac{s_\theta^2}{c_{2\theta}} \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} \right],$$

$$\bar{g}'^2 = \hat{g}'^2 \left[1 + \frac{s_\theta^2}{c_{2\theta}} \left(\frac{\Delta m_Z^2}{m_Z^2} + \frac{\Delta G_F}{\sqrt{2}} \right) + \frac{c_\theta^2}{c_{2\theta}} \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} \right],$$

$$\hat{v}^2 = \frac{1}{\sqrt{2} \hat{G}_F}$$

$$\hat{g}^2 = \frac{4\pi \hat{\alpha}_{\text{em}}}{s_\theta^2}$$

$$\hat{g}'^2 = \frac{4\pi \hat{\alpha}_{\text{em}}}{c_\theta^2}$$

$$\frac{\Delta m_Z^2}{m_Z^2} = v^2 \left(\frac{C_{HD}}{2} + s_{2\theta} C_{HWB} \right)$$

with

$$\frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}^2} = -\frac{v^2 s_{2\theta}}{2} C_{HWB}$$

$$\sqrt{2} \Delta G_F = v^2 \left(2 C_{HI}^{(3)} - C_{II}' \right)$$

$$\text{and } s_\theta^2 = \frac{1}{2} \left[1 - \sqrt{1 - \frac{2\sqrt{2}\pi \hat{\alpha}_{\text{em}}}{\hat{G}_F \hat{m}_Z^2}} \right]$$

Input parameters for the EW sector

example: $\{m_Z, G_F, \alpha_{\text{em}}\}$ scheme

$$\bar{v}^2 = \hat{v}^2 \left[1 + \sqrt{2} \Delta G_F \right],$$

$$\hat{v}^2 = \frac{1}{\sqrt{2} \hat{G}_F}$$

$$\bar{g}^2 = \hat{g}^2 \left[1 - \frac{c_\theta^2}{c_{2\theta}} \left(\frac{\Delta m_Z^2}{m_Z^2} + \frac{\Delta G_F}{\sqrt{2}} \right) - \frac{s_\theta^2}{c_{2\theta}} \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} \right],$$

$$\hat{g}^2 = \frac{4\pi \hat{\alpha}_{\text{em}}}{s_\theta^2}$$

$$\bar{g}'^2 = \hat{g}'^2 \left[1 + \frac{s_\theta^2}{c_{2\theta}} \left(\frac{\Delta m_Z^2}{m_Z^2} + \frac{\Delta G_F}{\sqrt{2}} \right) + \frac{c_\theta^2}{c_{2\theta}} \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} \right],$$

$$\hat{g}'^2 = \frac{4\pi \hat{\alpha}_{\text{em}}}{c_\theta^2}$$

→ corrections to m_W^2

$Z\bar{f}f$

$W\bar{f}f$

TGC

QGC

...

Input parameters for the EW sector

example: $\{m_Z, G_F, m_W\}$ scheme

$$\bar{v}^2 = \hat{v}^2 \left[1 + \sqrt{2} \Delta G_F \right],$$

$$\hat{v}^2 = \frac{1}{\sqrt{2} \hat{G}_F}$$

$$\bar{g}^2 = \hat{g}^2 \left[1 - \frac{\Delta G_F}{\sqrt{2}} \right],$$

$$\hat{g}^2 = 4\sqrt{2} G_F m_Z^2 c_\theta^2$$

$$\bar{g}'^2 = \hat{g}'^2 \left[1 - \frac{\Delta G_F}{\sqrt{2}} - \frac{1}{s_\theta^2} \frac{\Delta m_Z^2}{m_Z^2} \right],$$

$$\hat{g}'^2 = 4\sqrt{2} G_F m_Z^2 s_\theta^2$$

with

$$\frac{\Delta m_Z^2}{m_Z^2} = v^2 \left(\frac{C_{HD}}{2} + s_{2\theta} C_{HWB} \right)$$

$$\text{and } s_\theta^2 = 1 - \frac{\hat{m}_W^2}{\hat{m}_Z^2}$$

$$\sqrt{2} \Delta G_F = v^2 \left(2C_{HI}^{(3)} - C_{II}' \right)$$

Input parameters for the EW sector

example: $\{m_Z, G_F, m_W\}$ scheme

$$\bar{v}^2 = \hat{v}^2 \left[1 + \sqrt{2} \Delta G_F \right],$$

$$\hat{v}^2 = \frac{1}{\sqrt{2} \hat{G}_F}$$

$$\bar{g}^2 = \hat{g}^2 \left[1 - \frac{\Delta G_F}{\sqrt{2}} \right],$$

$$\hat{g}^2 = 4\sqrt{2} G_F m_Z^2 c_\theta^2$$

$$\bar{g}'^2 = \hat{g}'^2 \left[1 - \frac{\Delta G_F}{\sqrt{2}} - \frac{1}{s_\theta^2} \frac{\Delta m_Z^2}{m_Z^2} \right],$$

$$\hat{g}'^2 = 4\sqrt{2} G_F m_Z^2 s_\theta^2$$

→ corrections to $\gamma \bar{f} f$

$Z \bar{f} f$

$W \bar{f} f$

TGC

QGC

...

Input parameter example: m_b

assuming $U(3)^5 \rightarrow \mathcal{O}_{dH} = (H^\dagger H)(\bar{q}_L Y_D^\dagger H^\dagger d_R)$ → take the b terms:

$$\begin{aligned}\mathcal{L}_{\text{SMEFT}} &\supset -\frac{\bar{y}_b \bar{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} \right] \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.} \\ &= -\frac{\bar{y}_b \hat{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} + \frac{\Delta G_F}{\sqrt{2}} \right] \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.}\end{aligned}$$

Input parameter example: m_b

assuming $U(3)^5 \rightarrow \mathcal{O}_{dH} = (H^\dagger H)(\bar{q}_L Y_D^\dagger H^\dagger d_R)$ → take the b terms:

$$\begin{aligned}\mathcal{L}_{\text{SMEFT}} &\supset -\frac{\bar{y}_b \bar{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} \right] \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.} \\ &= -\underbrace{\frac{\bar{y}_b \hat{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} + \frac{\Delta G_F}{\sqrt{2}} \right]}_{\text{measured as } \hat{m}_b} \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.}\end{aligned}$$

measured as \hat{m}_b

$$\rightarrow \bar{y}_b = \hat{m}_b \frac{\sqrt{2}}{\hat{v}} \left[1 + \frac{v^2}{2} C_{dH} - \frac{\Delta G_F}{\sqrt{2}} \right]$$

Input parameter example: m_b

assuming $U(3)^5 \rightarrow \mathcal{O}_{dH} = (H^\dagger H)(\bar{q}_L Y_D^\dagger H^\dagger d_R)$ → take the b terms:

$$\begin{aligned}\mathcal{L}_{\text{SMEFT}} &\supset -\frac{\bar{y}_b \bar{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} \right] \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.} \\ &= \underbrace{-\frac{\bar{y}_b \hat{v}}{\sqrt{2}} \left[1 - \frac{v^2}{2} C_{dH} + \frac{\Delta G_F}{\sqrt{2}} \right]}_{\text{measured as } \hat{m}_b} \bar{b}_L b_R - \frac{\bar{y}_b}{\sqrt{2}} \left[1 - \frac{3v^2}{2} C_{dH} \right] h \bar{b}_L b_R + \text{h.c.}\end{aligned}$$

measured as \hat{m}_b

$$\rightarrow \bar{y}_b = \hat{m}_b \frac{\sqrt{2}}{\hat{v}} \left[1 + \frac{v^2}{2} C_{dH} - \frac{\Delta G_F}{\sqrt{2}} \right]$$

replacing \bar{y}_b back in \mathcal{L} :

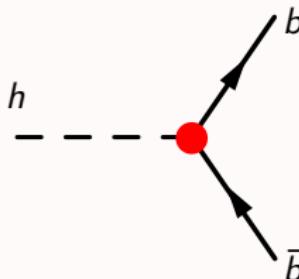
$$\mathcal{L}_{\text{SMEFT}} \supset -\hat{m}_b \bar{b}_L b_R - \frac{\hat{m}_b}{\hat{v}} \left[1 - v^2 C_{dH} - \frac{\Delta G_F}{\sqrt{2}} \right] h \bar{b}_L b_R + \text{h.c.}$$

correction to Yukawa coupling

$$h \rightarrow b\bar{b}$$

LO SMEFT corrections to $h \rightarrow b\bar{b}$

for NLO check Cullen,Gauld,Pecjak,Scott 1512.02508, 1607.06354, 1904.06358



LO SMEFT contributions are all SM-like:

$$\Gamma_{\text{SMEFT}}(h \rightarrow b\bar{b}) = \Gamma_{SM}(h \rightarrow b\bar{b}) \left[1 + 2 \operatorname{Re} \delta g_{hbb} \right]$$

direct \mathcal{O}_{dH} contribution $-\frac{3v^2}{2} C_{dH}$

input shifts $+\frac{v^2}{2} C_{dH} - \frac{\Delta G_F}{\sqrt{2}}$

Higgs kinetic term redefinition $-\frac{v^2}{4} C_{HD} + v^2 C_{H\square}$

$\frac{\delta g_{hbb}}{v^2} - \frac{C_{HD}}{4} + C_{H\square} - C_{dH} - C_{HI}^{(3)} + \frac{C'_{II}}{2}$