

# Data Interpretation

## Lecture 2

*PREFIT school, DESY (2020)*

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# Rationale

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# Rationale for New Physics

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Even if we had no evidence for BSM, there would be a rationale for new physics

**Rationale**

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graph TD; R(Rationale) --> H(Hierarchies  
gauge, mass, flavor); R --> E(End of the road  
unitarity, triviality, stability); H --- S(Symmetries and/or dynamics  
New states); E --- S;
```

**Hierarchies**  
gauge, mass, flavor

**End of the road**  
unitarity, triviality, stability

**Symmetries and/or dynamics**  
New states

# Rationale for New Physics

## Example: Naturalness

Predictive theory: quantum mechanical. In QFT, physical quantities *run* mass term in a Lagrangian, quantum corrections

$$\mathcal{L}_m = -m_\Psi \bar{\Psi}\Psi - m_\phi^2 \phi^2$$

## Fermions

Massless fermion, additional symmetry

$$\Psi \rightarrow e^{-i\gamma_5\theta} \Psi$$

if this chiral symmetry is preserved QM



$\delta m_\Psi \propto m_\Psi \log(\mu_1/\mu_2)$

chiral symmetry **protects** fermions masses from large UV corrections  
Light fermions are **technically natural**

## Energy

Quantum Gravity

some other new physics

some new physics

energies we can probe

# Rationale for New Physics

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### Energy

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### Scalars

Massless scalar, scale invariance

This classical symmetry is not preserved  
QM (is anomalous)

scalars are **not protected** by a symmetry,  
are UV sensitive, natural value for the  
mass is the highest scale it couples to

**Light scalars are unnatural**

# Rationale for New Physics

## Example: Naturalness

Energy

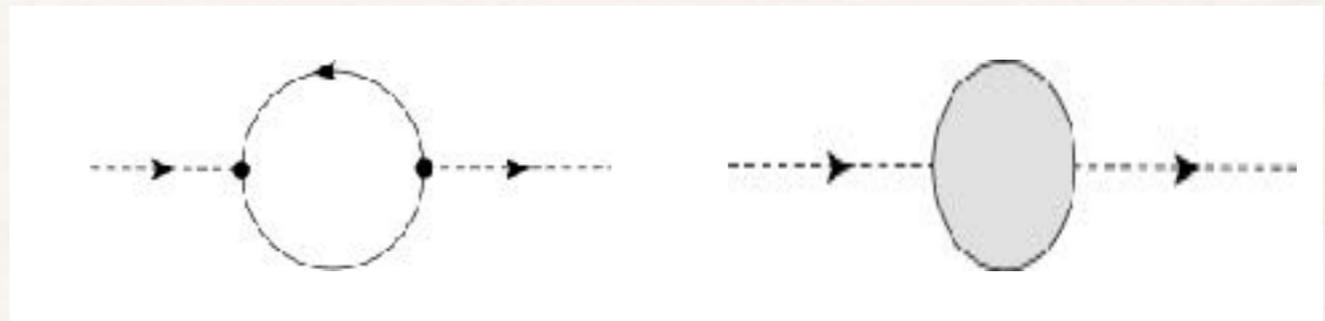
Quantum Gravity

some other new physics

some new physics

energies we can probe

Quantum corrections to scalars



threshold  
corrections

QGrav

$$\delta m_\phi^2 \propto c_1 \Lambda_{NP}^2 + c_2 M_{Pl}^2$$

$$(\text{Physical mass})^2 = (\text{bare mass})^2 + (\text{unsuppressed Qcorrections})^2$$

**light scalar = enormous fine-tuning**

The Higgs is a scalar, and there is no sight of new physics so far  
Should we just live with it?

# Is a tuning all there is?

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## Example: Naturalness

At the beginning of the EW theory, people were trying to figure out how to make sense of a gauge theory with massive W,Z

mass terms spoil renormalizability  
(predictivity) of the gauge theory

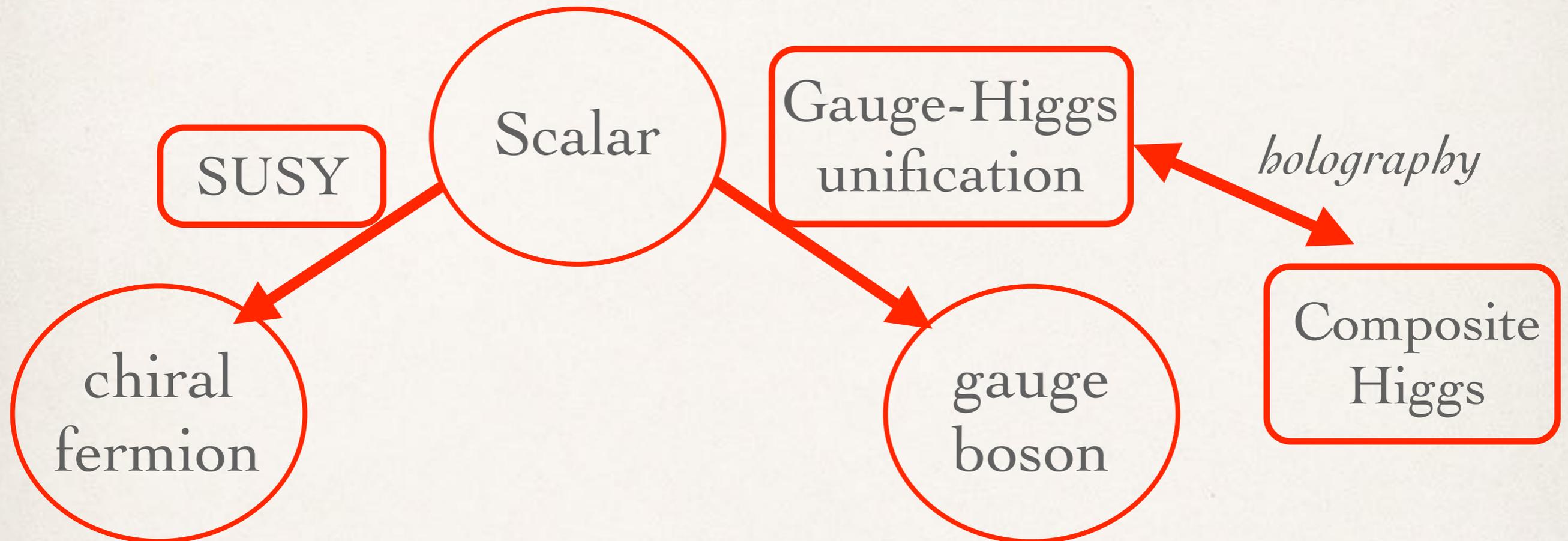
Feinberg (1958) proposed divergences were cancelled if a precise set of cancellations could happen (invoked fine-tuning)

At the end the story was more subtle  
The concept of Spontaneous Symmetry Breaking  
*secret renormalizability*

I view fine-tunings as calls for  
**new principles to be discovered**

# Light scalars

The light Higgs is a reality  
symmetry / duality arguments to explain its nature



Many, many possible realizations (phenomenology)  
Predict new states, to be discovered  
(SUSY partners, techni-baryons and mesons, spin-two...)  
AND induce **deviations in the Higgs behaviour**

# Back to the Higgs

The Higgs is a very special creature in the SM:  
a fundamental and light scalar

Quantum Gravity

some new physics

$M_{NP}$

energies we can probe

$h, W, Z, t \dots$

$$\delta m_h^2 \sim M_{NP}^2 \Rightarrow m_h^{phys} \sim M_{NP}$$

unless

1. There's *nothing* (DESERT)

2. Something *special* happens

2i.) fine-tuning (small=huge-huge)

$$m_{h,phys}^2 \simeq m_{h,bare}^2 + \delta m_h^2$$

2ii.) new symmetries

$\delta m_h^2 \propto$  parameter breaks the symm

2iii.) dynamics

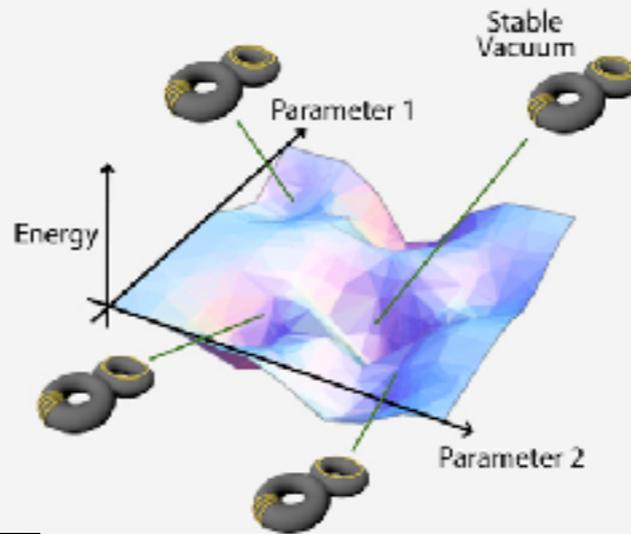
scalar=bound state of fermions or gauge fields

# Back to the Higgs

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What fundamental principle could be behind this behaviour?

Landscape of  
String Theory?



Something like  
Superconductivity?



New dimensions?  
Supersymmetry?

# Supersymmetry

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# Symmetries

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We build field theories imposing symmetries on the action

Example  $s=0, 1/2, 1, 2$

Klein-Gordon, Dirac, Yang-Mills, Fierz-Pauli

*great ref: Landau-Lifshitz ClassFT*

What is possible or not depends on whether a symmetry can be written for it

Coleman-Mandula **no-go theorem** [1962]:

Lie Algebra = Poincare  $\otimes$  Internal  
symmetries of (space-time, internal)  
S-matrix

=> internal and external (s-t) symmetries do not talk to each other

# Supersymmetry (SUSY)

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Supersymmetry is a way around that  
abandons the Lie group framework  
internal generators  $\Rightarrow$  fermionic  $Q$   
**super-Poincare algebra**

SUSY has important consequences

$$Q |B\rangle = |F\rangle$$

$$Q |F\rangle = |B\rangle$$

\*

Fermions and bosons are no longer  
two separate worlds

Normal field B or F  $\rightarrow$  SUSY field is both

e.g. Higgs  $\rightarrow$  SUSY Higgs  $(H, \tilde{H})$  Higgs (s=0)+Higgssino (s=1/2)

All fields in superfield are *degenerate*

$\Rightarrow$  Higgs should come with a 125 GeV fermion

*\*being sloppy with daggers*

# SUSY breaking

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=> Higgs should come with a 125 GeV fermion

=> electron should come with a 0.511 GeV charged scalar

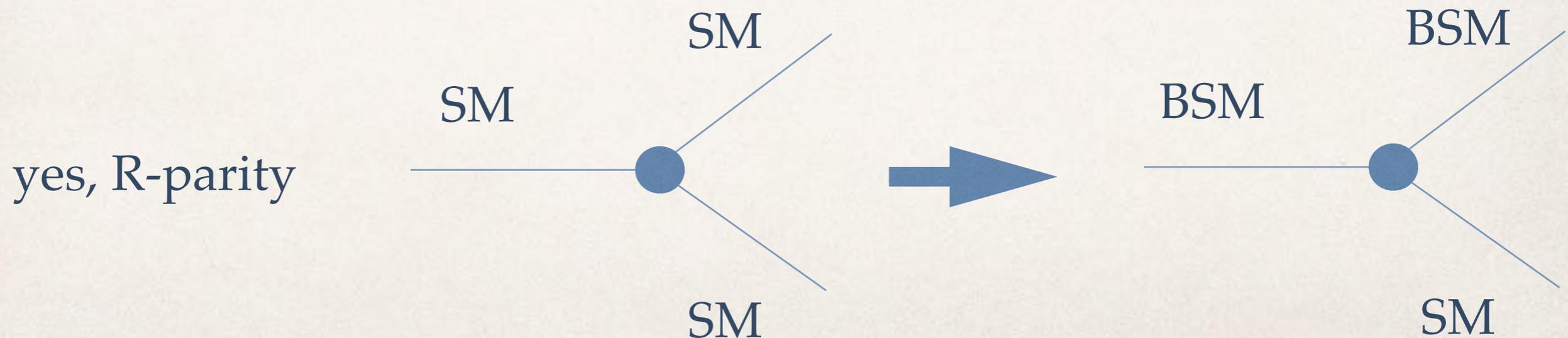
=> there should be a massless fermion (photino) force mediator

etc, etc

*All that is wrong!*

Then SUSY must be *broken* => splitting between partners  
in the superfield of order the SUSY breaking scale

*if SUSY is broken, does any symmetry survive?*



# SUSY breaking

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if SUSY is broken, does any symmetry survive?

yes, SUSY is still a good symmetry above SUSY breaking scale

**Higgsino** : chiral fermion  $\rightarrow$  protected by chiral symmetry

**Higgs**  $\rightarrow$  protected by chiral symmetry at high-energies

$$\delta m_h^2 \propto \text{parameter breaks the symm} \sim m_{soft}^2 \sim (TeV)^2$$

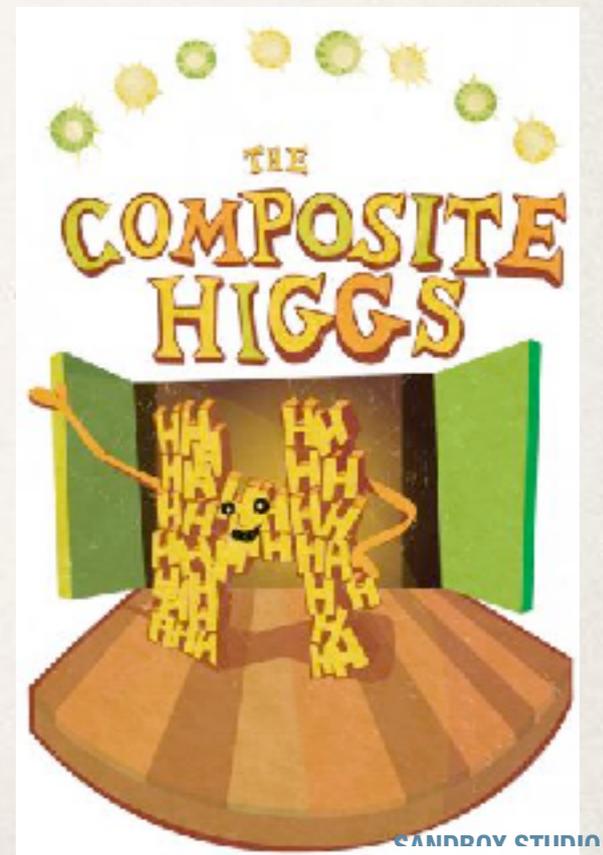
Higgs is *naturally light* in SUSY

as long as the SUSY particles are not too far from the EW scale

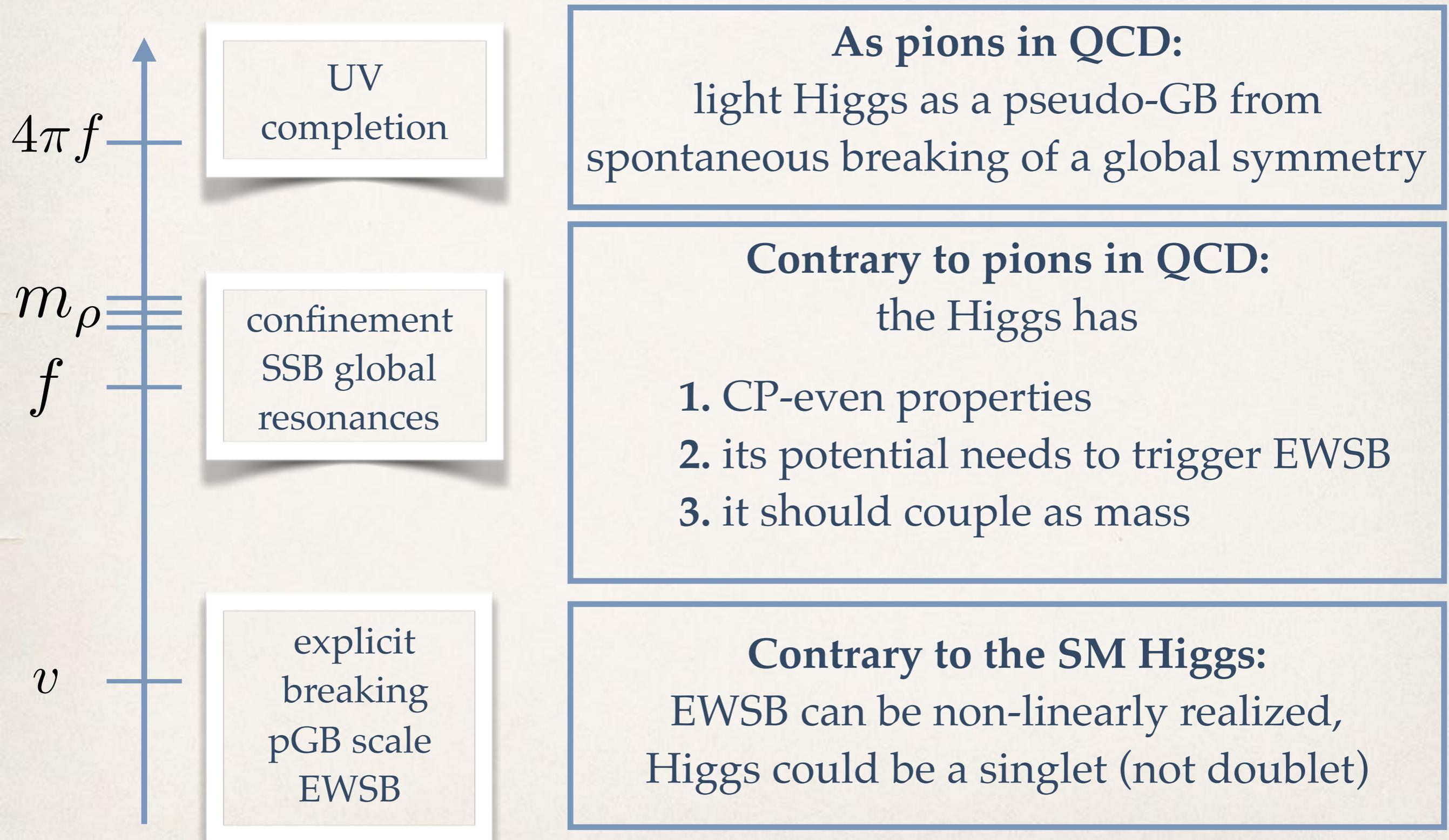
Naturalness in SUSY  $\Rightarrow$  light SUSY particles

# Compositeness

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# Composite Higgs in a nutshell



# Composite Higgs: Quantum numbers

$$\mathcal{G} \rightarrow \mathcal{H}$$

pGBs from SSB

$$\Sigma(x) = \exp(i\sqrt{2}h^a(x)X^a/f)\Sigma_0$$

The CP properties of the resulting pGBs depend on the CP properties of the strong sector

## A. Coupling to gauge

part of the global sym  $\mathcal{H}$  is weakly gauged  
depends on the embedding

$$\Pi_1(p^2)\Sigma^T A_\mu A_\nu \Sigma$$

## B. Coupling to fermions

many options for fermion rep

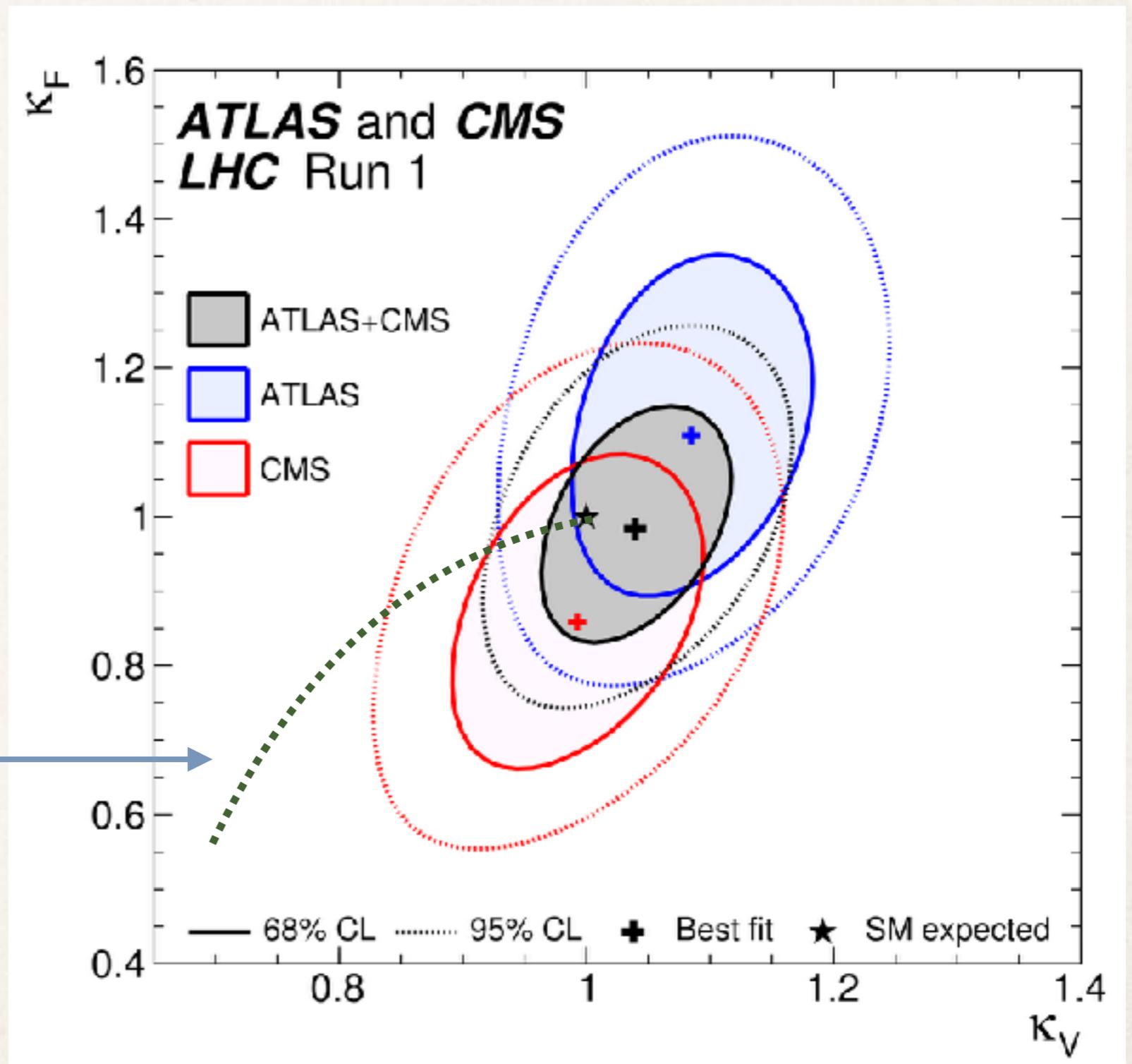
$$\bar{\Psi}\Gamma^i\Sigma_i\Psi$$

choice of global breaking and embedding: **CP-even scalar doublet**

**pheno:** Non-linear realization, Higgs couplings deviations

# Composite Higgs: Quantum numbers

coupling to fermions



different CHMs  
correspond to different lines  
the effect decreases as

$$\xi = v^2 / f^2$$

coupling to vectors

# Composite Higgs: Potential and EWSB

Usual paradigm:  
potential generated via **Coleman-Weinberg** contributions

e.g. GAUGE

$$V_{\text{eff}}(h) = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots$$

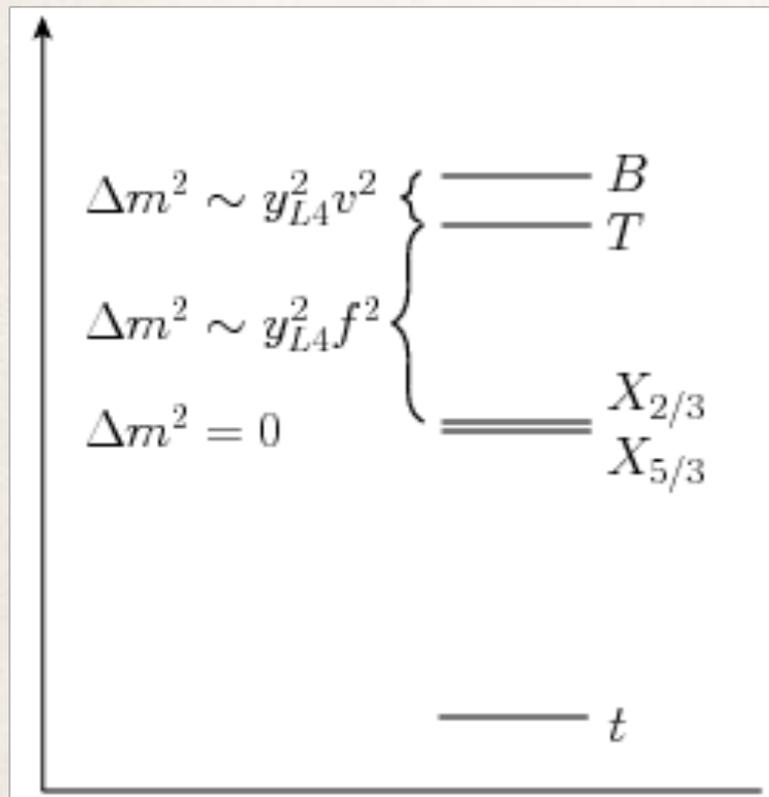
Georgi-Kaplan (80's)  
gauge-top *does not* trigger EWSB  
need new fermionic resonances  
**TOP-PARTNERS**

$$m_h^2 \sim \frac{N_c y_t^2}{16\pi^2} \frac{v^2}{f^2} m_T^2$$

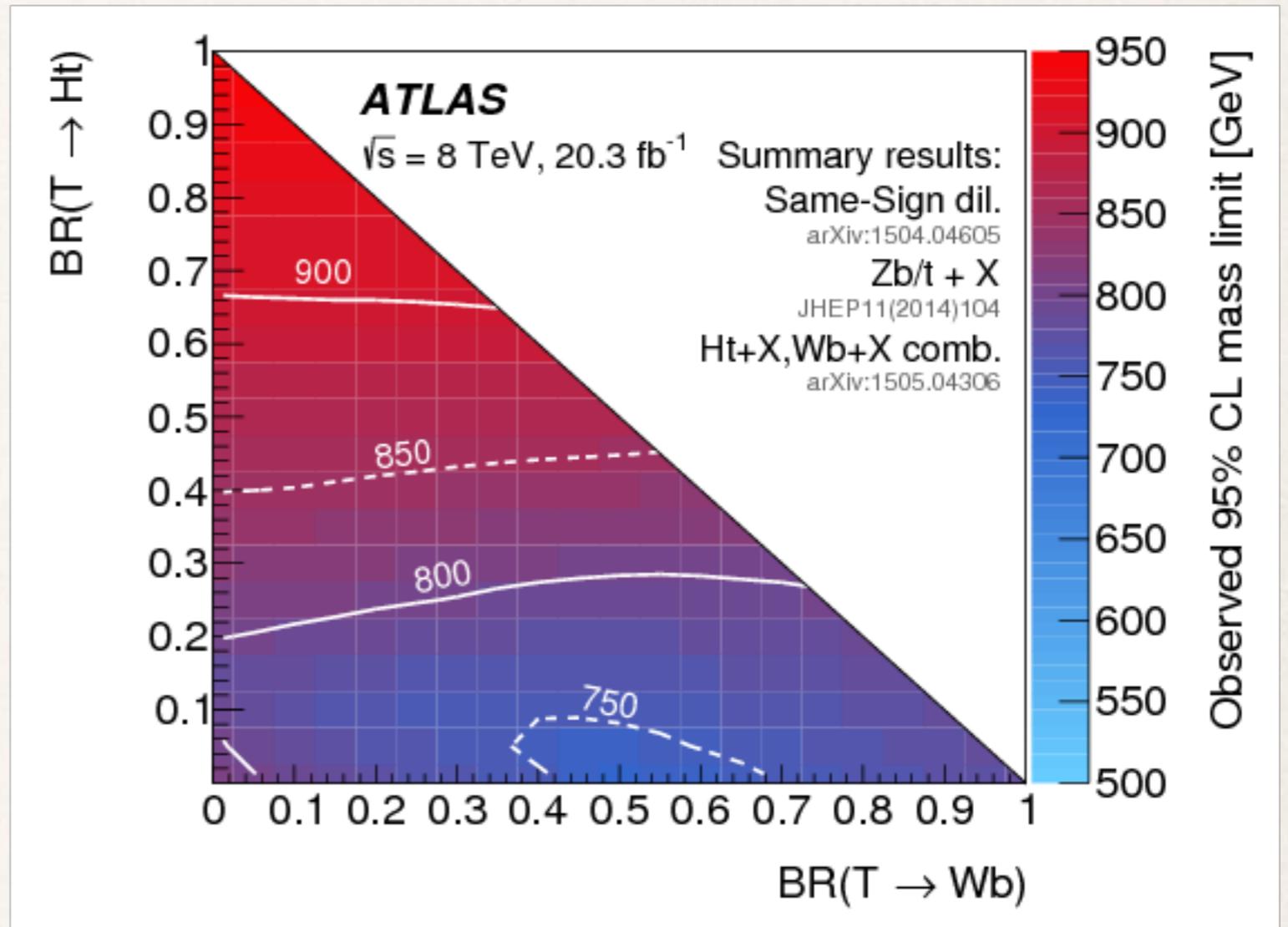
**pheno:** New, light (below TeV) techni-baryons  
should couple to the Higgs, W, Z

# Composite Higgs: Potential and EWSB

typical distribution  
of top-partners



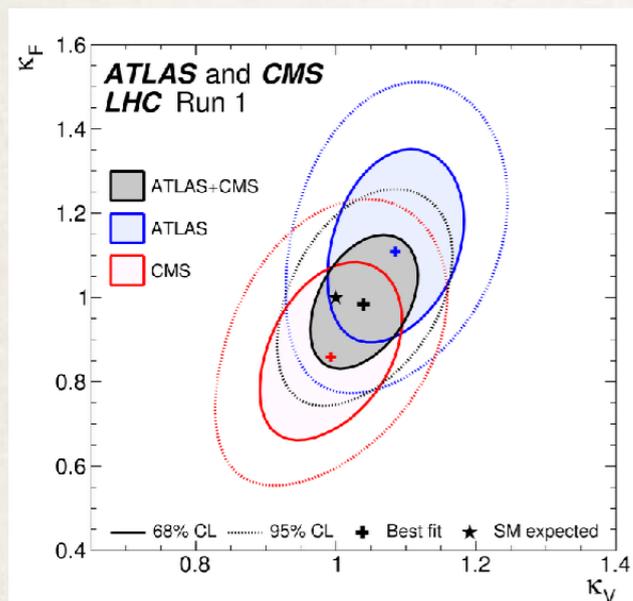
Panico et al. 2016



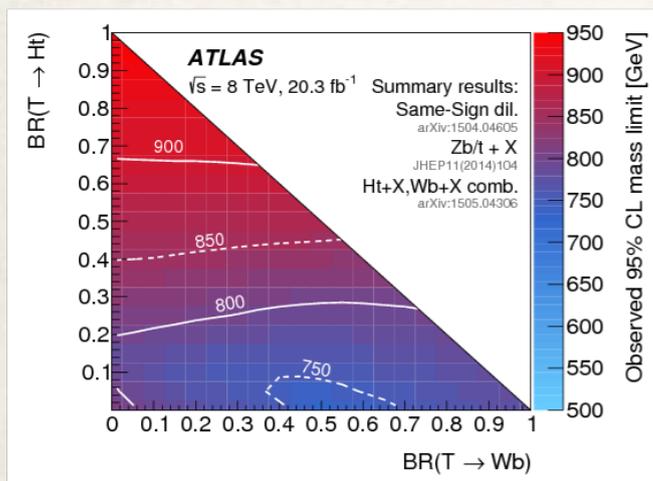
resonances below  $\sim 800 \text{ GeV}$  are excluded

$$m_h^2 \sim \frac{N_c y_t^2}{16\pi^2} \frac{v^2}{f^2} m_T^2 \quad \text{tuning in the Higgs potential severe}$$

# Status in model-building



Given the experimental constraints,  
lack of deviations in the Higgs behaviour and  
absence for new composite fermions  
**interest in more natural (non-minimal) models**



e.g. new ways to trigger EWSB and fermion  
mass generation, measure of tuning of the  
theory, un-coloured fermion resonances...

*examples:*

EWSB triggered by other scalars: see-saw CH

VS, SETFORD. 1508.06133

new symmetries in the global sector: Maximally symmetric CH

CSAKI, MA, SHU. 1702.00405

# *Additional material (Exercises)*

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# Gauge Coupling Unification and Split Supersymmetry

## 1 Unification

There are various arguments as to why a Supersymmetric extension of the Standard Model may be of interest for understanding TeV scale physics such as we will probe at the Large Hadron Collider. One motivation people often give is that SUSY 'predicts a unification of gauge couplings'. In this question, we'll see what this means...

We write the renormalisation group equation for the gauge couplings  $g_3, g_2, g_1$  of the Standard Model group  $SU(3) \times SU(2) \times U(1)$  as

$$\mu \frac{dg_i}{d\mu} = \frac{\beta_i}{16\pi^2} g_i^3 \quad (\text{no sum on } i) \quad (1)$$

where  $\mu$  here is the renormalisation scale, and  $\beta_i$  are the one-loop beta-function coefficients (real constants).

For  $SU(N)$  gauge groups, we calculated the  $\beta_i$  coefficients in the Standard Model course:

$$\beta_i = -\frac{11N}{3} + \frac{2}{3} \sum_f T_R(f) + \frac{1}{3} \sum_s T_R(s), \quad (2)$$

where  $f$  denotes a 2-component Weyl fermion and  $s$  a complex scalar.  $T_R$  is the Dynkin Index of the appropriate representation of  $SU(N)$  corresponding to the field  $f$  or  $s$ ; explicitly, this is  $1/2$  for the fundamental rep<sup>1</sup> and  $N$  for the adjoint rep.

For  $U(1)$  we have

$$\beta_1 = \frac{2}{3} \sum_f Y_f^2 + \frac{1}{3} \sum_s Y_s^2 \quad (3)$$

where  $Y_{f,s}$  is the hypercharge of a (2-component) fermion or complex scalar respectively.

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<sup>1</sup>This choice is just a convention — once fixed, all the other  $T_R$  values follow.

In tutorial 3 we saw that for the Standard Model, at one-loop order,

$$\beta_1 = \frac{41}{6} \quad \beta_2 = -\frac{19}{6} \quad \beta_3 = -7,$$

whereas for the MS2M

$$\beta_1 = 11 \quad \beta_2 = 1 \quad \beta_3 = -3. \quad (4)$$

- a) Defining  $\alpha_i(\mu) = \frac{g_i^2(\mu)}{4\pi}$ , solve the RG equation (1) to find a relationship between  $\alpha_i(M_Z)$  and  $\alpha_i(\mu_0)$  for a general scale  $\mu_0$ .

*Hint:* Equation (1) takes the simplest form when written in terms of  $\alpha^{-1}$ .

- b) Grand Unified Theories predict that at some scale  $\mu_0 = M_{GUT}$ ,

$$\frac{5}{3}\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}). \quad (5)$$

Assuming this, derive

$$\alpha_3(M_Z)^{-1} = \alpha_2(M_Z)^{-1} + \frac{\beta_3 - \beta_2}{3\beta_1/5 - \beta_2} \left[ \frac{3}{5}\alpha_1^{-1}(M_Z) - \alpha_2^{-1}(M_Z) \right]. \quad (6)$$

- c) Taking the (rough) experimental values  $g_1(M_Z) = 0.357$  and  $g_2(M_Z) = 0.652$ , and assuming all the Standard Model couplings unify at  $M_{GUT}$ , what value of  $g_3(M_Z)$  do we predict from equation (6)? Does the MS2M do any better, if we assume that SUSY is broken just above the electroweak scale?
- d) Show that if we introduce the fine-structure constant  $\alpha = \frac{e^2}{4\pi}$ , with  $e = g_2 \sin \theta_W$  and  $\tan \theta_W = \frac{g_1}{g_2}$ , then equation (6) can be recast as

$$\alpha_3(M_Z)^{-1} = \alpha^{-1} \left[ \sin^2 \theta_W + \frac{3 - 8 \sin^2 \theta_W}{5} \frac{b_3 - b_2}{b_1 - b_2} \right], \quad (7)$$

where  $b_3 = \beta_3$ ,  $b_2 = \beta_2$  and  $b_1 = \frac{3}{5}\beta_1$ . Furthermore, show that the unification scale is given by

$$\log \left( \frac{M_{GUT}}{M_Z} \right) = \frac{2\pi(3 - 8 \sin^2 \theta_W)}{5\alpha(b_1 - b_2)}, \quad (8)$$

and that at the unification scale, the value of the coupling is

$$\alpha_{GUT} = \frac{5\alpha(b_1 - b_2)}{5 \sin^2 \theta_W b_1 - 3 \cos^2 \theta_W b_2}. \quad (9)$$

- e) What is the Unification scale and value of the coupling at  $M_{GUT}$  predicted by:
- the Standard Model?
  - the MS2M?

## 2 Split Supersymmetry

The idea of Split Supersymmetry is to forget using SUSY as a solution to the hierarchy problem, but to still require that it leads the unification of gauge couplings and provides a dark matter candidate. We'll look at this idea, following reference [1]; their starting point was to note that the beta-function coefficients,  $b_i$ , can be written as

$$b_3 = \frac{1}{3} (4N_g - 33 + N_3) \quad (10a)$$

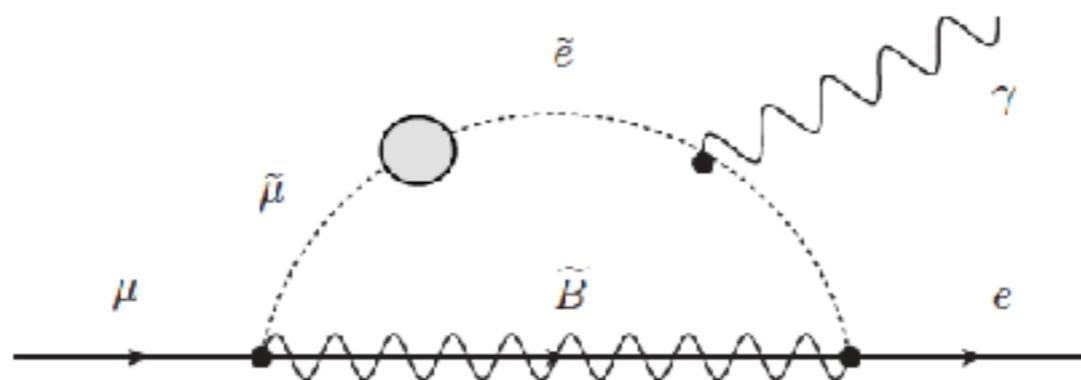
$$b_2 = \frac{1}{3} \left( 4N_g - 22 + \frac{n_H}{2} + N_2 \right) \quad (10b)$$

$$b_1 = \frac{1}{3} \left( 4N_g + \frac{3n_H}{10} + N_1 \right) \quad (10c)$$

where  $N_g$  counts the contribution to the  $\beta$ -functions from complete SU(5) irreps, and it is normalized such that the 3 families of SM quarks and leptons give  $N_g = 3$ .<sup>2</sup> For the MS2M one can easily show that  $N_g = \frac{9}{2}$ . The number of Higgs doublets is  $n_H$ , and  $N_i$  ( $i = 1, 2, 3$ ) give the contributions from matter in incomplete GUT multiplets (for example, in the MS2M, this includes contributions from the gauginos and higgsinos).

The important observation is that  $N_g$  actually cancels out in the equations (7) and (8), and so doesn't enter into the predictions for  $\alpha_s$  or  $M_{GUT}$ . Split SUSY makes use of this fact: All scalars in the MS2M can be very heavy, except one Higgs, and unification can still take place.<sup>3</sup> We still need the gauginos ( $\tilde{g}$ ,  $\tilde{W}$  and  $\tilde{B}$ ) and Higgsinos  $\tilde{h}_{u,d}$  to have masses of order the TeV scale in order to retain the nice features of unification, and to also have interesting dark matter candidates.

- a) If we send the scale of SUSY to the GUT scale, what are the natural values for the squark and slepton masses? What about the fermionic superpartners (gauginos and higgsinos)?
- b) Another interesting feature of split SUSY is that pushing the scalar masses to high scales alleviates the most pressing bounds from flavor-changing neutral currents (FCNCs), CP violation, proton decay and so on. The reason is that all those dangerous bounds are based on calculating a diagram that is suppressed by a factor of the scalar masses. For example, let's look at the  $M_{\text{scalar}}$  dependence of the  $\mu \rightarrow e \gamma$  bound: the SUSY particles typically contribute to this process through a diagram of the type:



where the mass insertion (grey blob) comes from a flavor-violating, soft SUSY-breaking term of the form  $-m_{\tilde{e}\tilde{\mu}}^2 \tilde{e}\tilde{\mu}$ . One can use naïve dimensional analysis (NDA) to estimate the size of this contribution to the branching ratio to be

$$\text{BR}(\mu \rightarrow e\gamma) \approx \frac{g^2 e^2}{16\pi^2} \left( \frac{m_{\tilde{e}\tilde{\mu}}^2}{m_{\tilde{t}}^2} \right)^2 \frac{v^2}{m_{\tilde{t}}^2} \frac{v^2}{m_{\tilde{B}}^2} \quad (11)$$

where  $m_{\tilde{t}}$  is the slepton mass, and we have used the fact that  $\mu$  decays are dominated by  $\mu \rightarrow e\nu_{\mu}\bar{\nu}_e$ , which goes as  $G_F^2$ . Is this formula dimensionally correct?

- c) Assume  $m_{\tilde{e}\tilde{\mu}}^2 \approx m_{\tilde{t}}^2$  (no flavor hierarchy) and  $m_{\tilde{B}} \approx v$ . Find the experimental constraint on the  $\text{BR}(\mu \rightarrow e\gamma)$  and use it to derive a lower bound on  $m_{\tilde{t}}$ .
- d) In split SUSY, gluinos (gluini?!) are lighter than squarks, so it is interesting to think about how gluinos decay. Use NDA to estimate the decay width  $\Gamma_{\tilde{g}}$ , and hence the decay length,  $c\tau$ , of the gluino as a function of  $m_{\tilde{g}}$  and  $m_{\tilde{q}}$  (assuming that  $m_{\tilde{g}} \gg m_{\text{LSP}}$ , so there are SUSY particles for  $\tilde{g}$  to decay into).

Long lived gluinos are a ‘smoking gun’ feature of split SUSY. The LHC is looking for them by keeping the detectors on when there are no collisions; as gluinos carry color charge, if they hang around long enough they end up getting bound up into  $R$ -hadrons (hadrons with non-trivial charge under  $R$ -parity) that can potentially be brought to rest by all the material in the detector. If the beams are colliding, the detector is too busy detecting other things to notice the intermittent decays of these  $R$ -hadrons, but when there are no collisions, one would only expect to register cosmic rays, and possibly the decay of interesting stuff trapped in the detector.

## References

- [1] G. F. Giudice and A. Romanino, “Split supersymmetry,” Nucl. Phys. B 699 (2004) 65 [Erratum-ibid. B 706 (2005) 65] [arXiv:hep-ph/0406088].

# 1 Goldstone Bosons

According to Goldstone's theorem,<sup>1</sup> whenever a global symmetry group  $G$  is spontaneously broken down to a smaller one  $H$ , it gives rise to  $\dim(G) - \dim(H)$  massless bosons known as *Goldstone bosons*.

Today we're going to look at what happens when we spontaneously break a global symmetry:

$$SU(N) \longrightarrow SU(N-1). \quad (1)$$

a) How many Goldstone bosons (GBs) are generated by this breaking?

There are many ways to parameterise the GB fields, but we will try to be smart and choose a representation which clearly shows how all the fields transform under  $SU(N)$  and  $SU(N-1)$ .

b) Explain how the  $N \times N$  matrix

$$U_{N-1} \equiv \begin{pmatrix} \hat{U}_{N-1} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{with } \hat{U}_{N-1} \text{ an } (N-1) \times (N-1) \text{ matrix} \quad (2)$$

provides a representation of the unbroken symmetry transformations.

Let's represent the GBs by introducing an  $N \times N$  matrix  $\Pi$  in the following way

$$\phi(x) = e^{i\Pi(x)/f} \phi_0(x) \quad (3)$$

where

$$\Pi(x) = \begin{pmatrix} 0_{(N-1) \times (N-1)} & \vec{\pi}(x) \\ \vec{\pi}^\dagger(x) & 0 \end{pmatrix} \quad \vec{\pi}(x) = \begin{pmatrix} \pi_1(x) \\ \vdots \\ \pi_{N-1}(x) \end{pmatrix} \in \mathbb{C}^{N-1} \quad (4)$$

$$\phi_0(x) = \frac{f}{\sqrt{2}} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \pi_0(x) \end{pmatrix} \quad \pi_0(x) \in \mathbb{R} \quad (5)$$

- c) How does  $\phi$  transform under the unbroken symmetries?
- d) Does  $\phi$  contain the right number of degrees of freedom?
- e) We would like to see how  $\phi$  transforms under the *broken* symmetries. We will first represent the broken symmetries by the transformation:

$$U_{\text{broken}} = \exp \left\{ i \begin{pmatrix} 0 & \vec{\alpha} \\ \vec{\alpha}^\dagger & 0 \end{pmatrix} \right\} \quad \vec{\alpha} \in \mathbb{C}^{N-1} \quad (6)$$

Show that  $\phi$  transforms as

$$\phi \rightarrow U_{\text{broken}} e^{i\Pi/f} \phi_0 = e^{i\Pi'/f} \phi_0 \quad (7)$$

to first order in  $\vec{\alpha}$ , where

- (i) The  $\vec{\pi}$  field shifts linearly:

$$\vec{\pi}' = \vec{\pi} + f \vec{\alpha}. \quad (8)$$

- (ii) The field  $\phi_0$  is invariant under  $SU(N-1)$  transformations.

- f) Although one says that the  $SU(N)$  symmetry has been spontaneously broken down to  $SU(N-1)$  what really happens is that the broken part of the symmetry is realized in a way that is different from the unbroken parts. To see this more clearly compare how the fields transform under a broken symmetry vs. how they transform under the unbroken ones. For the broken generators one says that the symmetries are “non-linearly” realized. Thus for infinitesimal transformations involving the broken generators one requires that the shifts in (8) are symmetries. Show that this statement is consistent with the statement of Goldstone’s theorem that the GBs are massless.
- g) This shift symmetry also implies that no potential is generated (no quartic coupling, no term made up of powers of the field) and only derivative interactions are allowed. To see this explicitly, expand the GB kinetic term

$$\partial_\mu \phi^\dagger \partial^\mu \phi \quad (9)$$

up to quartic order in the fields.