(1) The Drell-Yan Process and (2) Vector Boson Production in Hadron Collisions

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Outline

Lecture 1 : The Drell-Yan Process

(A) Introduction to the DY Process

Historical introduction

Related processes

(B) QCD and the DY process

(C) The DY process at the LHC

Lecture 2 : Vector Boson Production

The Drell-Yan process — $\mu^+ \mu^-$ production in hadron collisions — has been important for the development of QCD for ~ 50 years:

•• discoveries of new particles,

•• data for parton distribution functions,

•• tests of perturbative QCD.

This lecture will describe historical and modern examples of the DY pro cess and related processes.

(A) Introduction to the Drell-Yan process

∎ *Historical Introduction*

The classic Drell-Yan process is inclusive $\mu^+ \mu^-$ production in hadron collisions, with large invariant mass $m_{\mu\mu}$,

$$H_1 + H_2 \longrightarrow \mu^+ \; \mu^- + X$$

where X stands for "any other particles".

The earliest experiment was at Brookhaven, ca 1970

OBSERVATION OF MASSIVE MUON PAIRS IN HADRON COLLISIONS; Christenson, et al, Phys Rev Lett, 25, 1523 (1970).

- Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV/c^2, have been observed in collisions of high–energy protons with uranium nuclei...

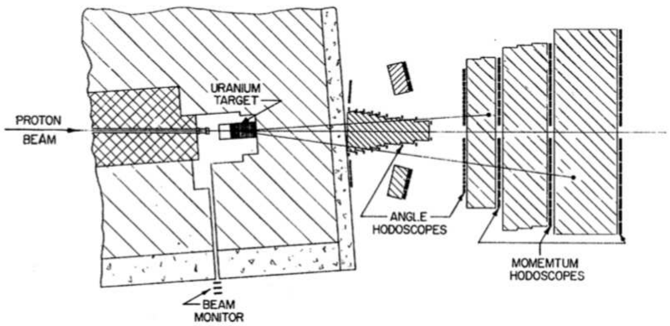
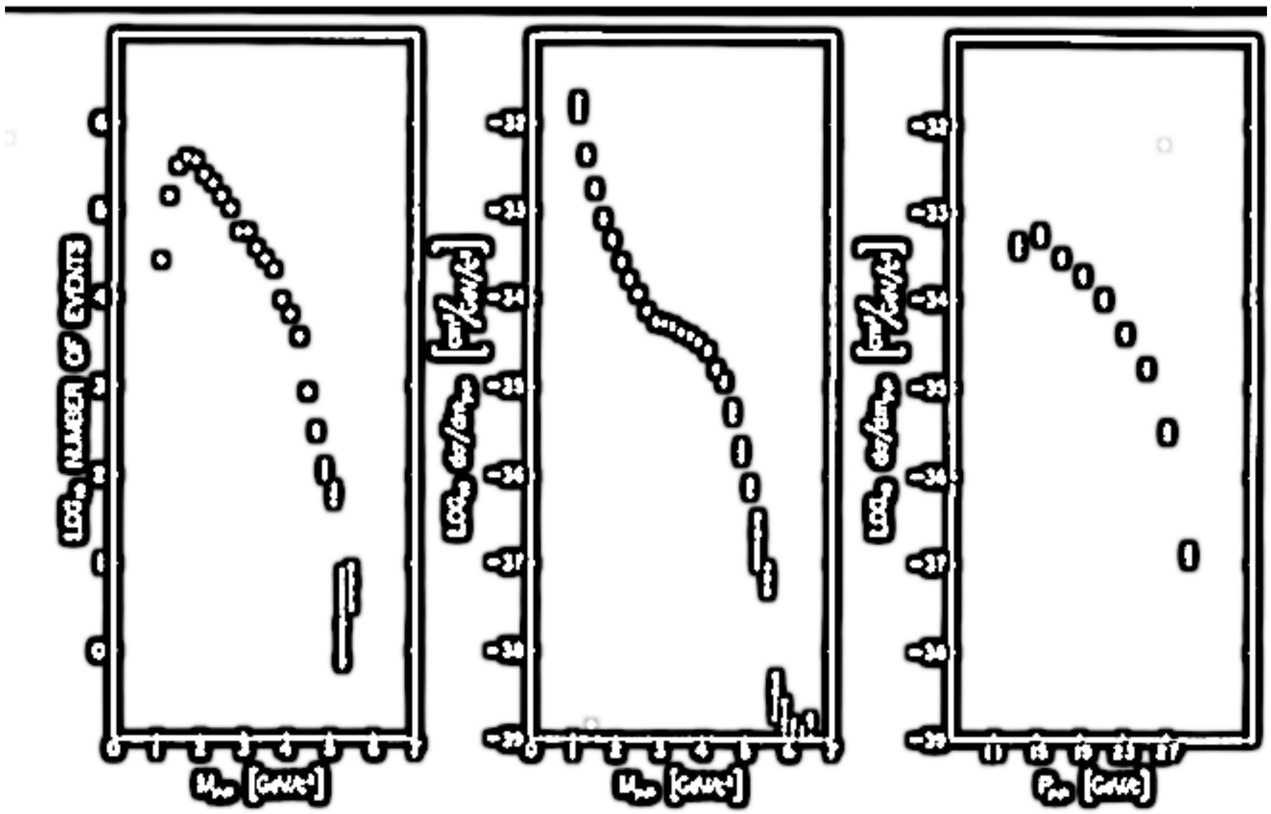


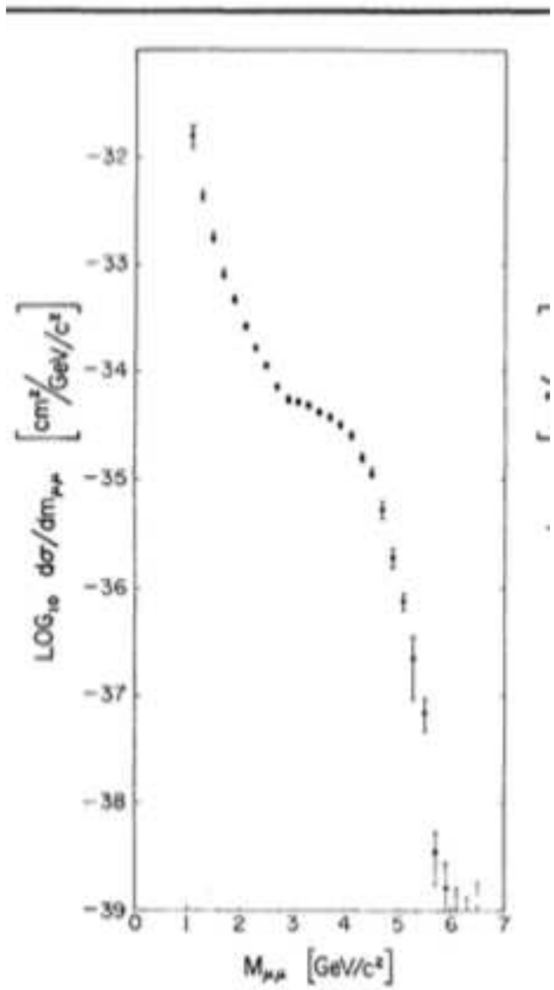
FIG. 1. Plan view of the apparatus.

Figure 2 from Christenson, et al; the cross section $d\sigma / dm_{\mu\mu}$



(a) Observed events as a function of the effective mass of the muon pair. (b) Cross sectio

What do you observe here?



tion of the effective mass of the

Two things:

(1)

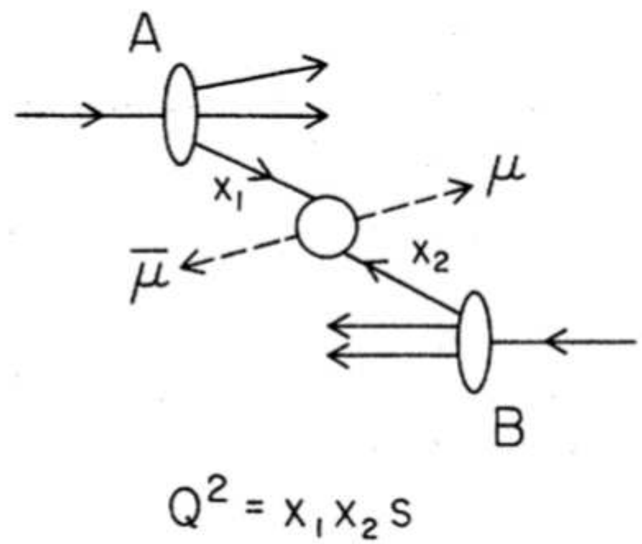
(2)

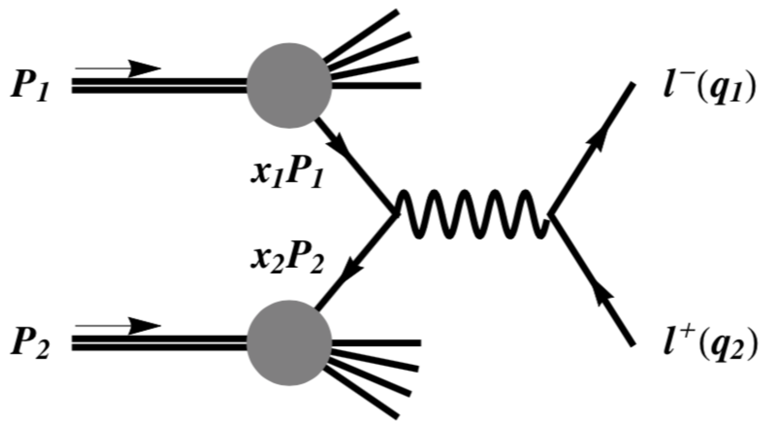
• *Theory — historical*

In 1970, Drell and Yan applied the parton model (*which had been developed earlier by Feynman for D.I.S.*) …

MASSIVE LEPTON–PAIR PRODUCTION IN HADRON–HADRON COLLISIONS AT HIGH ENERGIES;
S.D.Drell and T.–M.Yan, Phys Rev Lett, 25, 316 (1970).

● On the basis of a parton model studied earlier we consider the production process of large–mass lepton pairs from hadron–hadron collisions in the limiting region, $s \to \infty$, Q^2/s finite …





$s=(P_1+P_2)^2$
$Q^2=(q_1+q_2)^2$
$\hat{s}=(x_1P_1+x_2P_2)^2 \approx x_1x_2s$

The high-mass muon pair comes from QED annihilation of a quark antiquark pair, with momentum fractions x_1 and x_2 resp.

Parton subprocess	$\frac{d\hat{\sigma}}{dQ^2} = \frac{4\pi\alpha^2}{3\,Q^2}\, e_q^2\, \delta(Q^2 - \hat{s})$
Hadronic σ	$\frac{d\sigma}{dQ^2} = \frac{1}{3}\sum_q \int_0^1 dx_1 \int_0^1 dx_2\, f_q(x_1) f_{q'}(x_2) \frac{d\hat{\sigma}}{dQ^2}$
scaling function	$Q^4 \frac{d\sigma}{dQ^2} = F(\frac{Q^2}{s}) = F(\tau)$

The "impact approximation": $\sigma \propto \Sigma\, f_q \otimes f_{\overline{q}} \otimes \sigma$

Exercise : In quantum theory, M = $M_1 + M_2 + M_3 \ldots$;

then $\sigma \propto \; | \, M \, |^2 = \; | \, M_1 + M_2 + M_3 \ldots \, |^2 \neq \sigma_1 + \sigma_2 + \sigma_3 \; \ldots$; is the parton model consistent?

- ***Example: Calculate the "scaling function" as a function of τ***

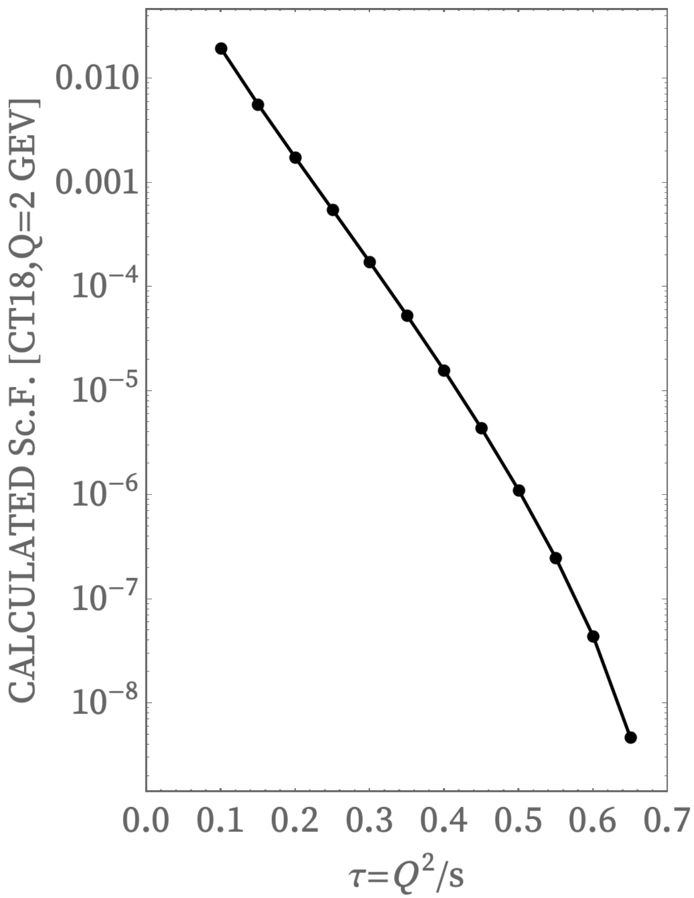
'CT18' PDFs are unpublished, preliminary PDFs, currently under development; used here for illustration purposes.

$$\left.\frac{d\sigma}{dQ^2}\right|_{had.} = \frac{1}{3}\cdot\frac{1}{3}\cdot 3 \int_0^1 dx_1 \int_0^1 dx_2 \; u(x_1)\,\bar{u}(x_2)\,\frac{d\hat{\sigma}}{dQ^2}$$

$$= \frac{4\pi\alpha^2}{9Q^4}\left(\frac{2}{3}\right)^2 \int_0^1 dx_1 \int_0^1 dx_2 \; u(x_1)\,\bar{u}(x_2)\; \delta\!\left(x_1 x_2 - \frac{Q^2}{s}\right)\cdot\frac{Q^2}{s}$$

$$\rightarrow 0 \text{ as } Q^2 \rightarrow s$$

For a single flavor $u\,\overline{u} \to \mu^{+}\,\mu^{-}$ the result is



Calculated from CT18 PDFs

The cross section decreases rapidly as Q increases, because f(x)→ 0 as x→1.

Related Processes

In general, consider

$$\mathbf{H_1 + H_2 \longrightarrow V + X}$$

$$\text{with } \mathbf{V \longrightarrow l_1 + \overline{l}_2}$$

H_1, H_2 : p $\overline{p}$ or p p { Tevatron or LHC }

V : γ^* or γ^*, Z^0 or $W^\pm$ or Z' etc

$l_1, \overline{l}_2$: $e^+ e^-$ or $\mu^+ \mu^-$ or $e^\pm \overset{(-)}{\nu}_e$ or $\mu^\pm \overset{(-)}{\nu}_\mu$ etc

In other words, the generalization of the classic Drell-Yan process is vector boson production with leptonic decay ($\Longleftarrow$ see Lecture 2).

A comment on the importance of lepton-pair production processes in hadron collisions…

∎ Search for resonances in the $l_1 \, \overline{l_2}$ final state. Think of these discoveries from $\mu^+ \, \mu^-$ production :

J/Ψ (+ other charmonium states ; p + Be)

Υ (+ other upsilon states ; p + { Cu, Pt })

Z^0 (an intermediate vector boson ; p + $\overline{p}$) .

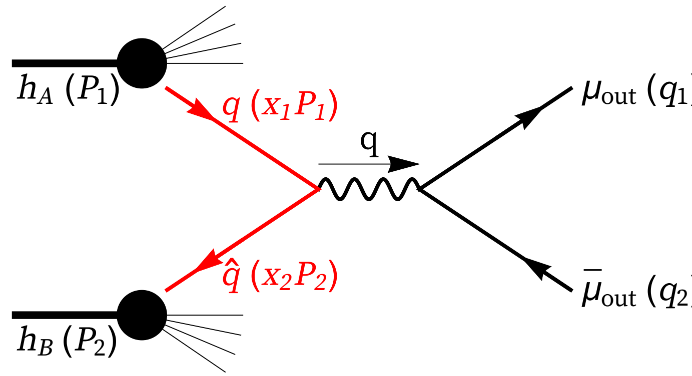
∎ The cross sections depend on

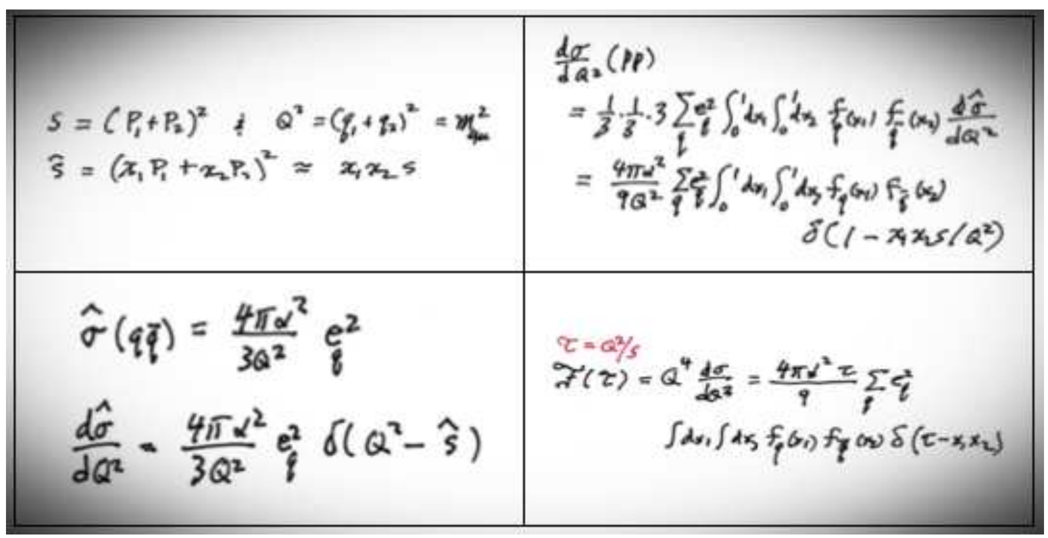
Parton Distribution Functions

∴ The *measurements* of Drell-Yan cross sections and related processes provide quantitative information about PDFs. For example: the FNAL/E866 NuSea experiment (discuss later)

(B) QCD perturbation theory and the DY process

So far the only interaction we have included in the theory is QED, calculated in the Born approximation. This is called ***the LO approximation*** ...





Scaling of the LO cross section

$$F(\tau) = Q^4 \left(\frac{d\sigma}{dQ^2} \right)$$

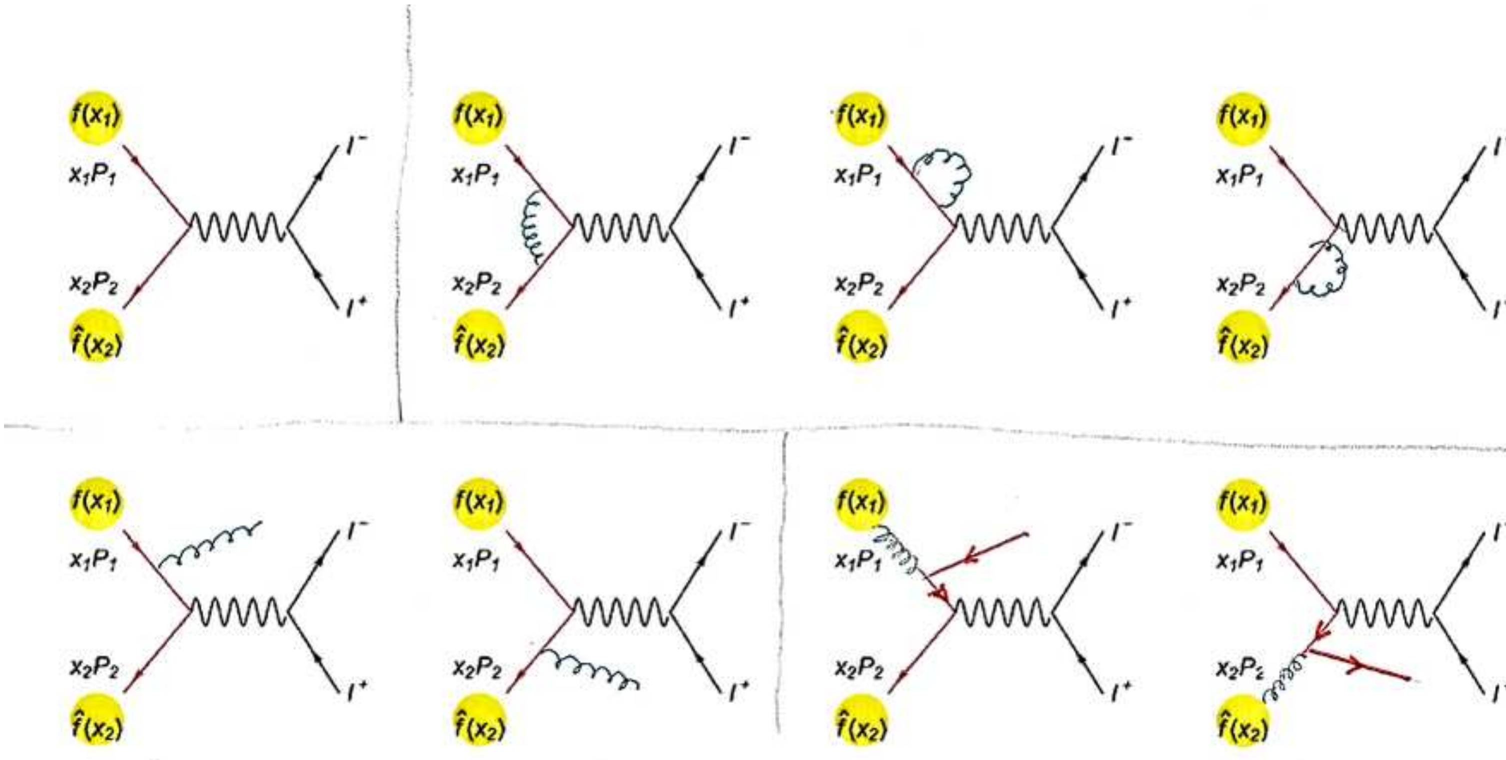
$$= \frac{4\,\pi\,\alpha^2}{9}\ \tau \sum_q e_q^2 \int_0^1 dx_1 \int_0^1 dx_2\ f_q\,(x_1)\ f_{\overline{q}}\,(x_2)\ \delta[\ \tau - x_1\ x_2\ ]$$

depends only on the ratio $\tau = Q^2/s$. “scaling”

Also, note that the τ dependence of σ is determined by $f_q(x)$ and $f_{\overline{q}}\,(x)$.

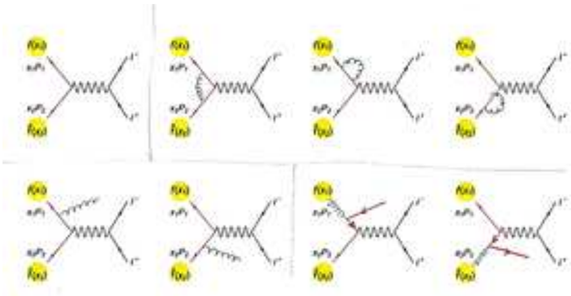
But the LO approximation is not accurate enough. So now we must calculate QCD corrections. We can use perturbation theory, justified by asymptotic freedom.

LO matrix element = M_0	Virtual corrections = $g^2\ M_2^{(v)}$
Real radiation ∊ $g\ M_1^{(r)}$	Compton style ∊ $g\ M_1^{(r)}$



$$d\sigma(2\to 2) \propto |\ M_0 + g^2\ M_2^{(v)}\ |^2 = M_0^2 + g^2(M_2^{(v)}\ M_0^* + cc) + O\ (g^4) \quad [\times\ d\Phi_2]$$

$$d\sigma(2\to 3) \propto |\ g\ M_1^{(r)}\ |^2 \quad [\times\ d\Phi_3]$$ //interferring where appropriate//



The 1-loop diagrams are familiar from QED, but with color factors. They have UV and IR divergences; ⇒ dimensional regularization, D = 4–2ϵ .

— The UV divergences { i.e., certain poles as $\epsilon \to 0$ } cancel with renormalization, which is familiar from QED.

— The IR divergences in $M_2^{(v)}$ { i.e., other poles as $\epsilon \to 0$ } cancel IR divergences in the real emission of soft gluons; familiar from the Bloch-Nordsieck cancellation in QED.

— But there remain some collinear divergences because the quark masses are “zero”. These are not familiar from QED, because the electron mass is *not* zero. But they are familiar from ***the KLN theorem***: for massless particles the cross section is finite (or, IR safe) for ***inclusive*** initial and final states; the initial states must include all degenerate states.

Kinoshita (1962); Lee and Nauenberg (1964)

These remaining collinear divergences will be absorbed into the Parton Distribution Functions.
⟹ *f(x)* { the LO PDF} will be replaced by $f(x, \mu_F)$ { the NLO PDF}.
TRICKY QUESTION : where did the variable μ_F come from? (tomorrow)

The final NLO result will look like this...

$$\frac{d^2\sigma}{dy\, dQ^2} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/A}(x_1, \mu_F)\;\; f_{b/B}(x_2, \mu_F)$$

$$H_{ab}(x_1, x_2, y, Q^2; \mu_F, \mu_R)$$

$$H = H_0 + \alpha_S H_1$$

The result is accurate to $O(\alpha_S)$.
Normally we take $\mu_F = \mu_R = Q = m_{\mu\mu}$. Making this choice (or any choice for μ_F and μ_R) leads to a theoretical uncertainty called "scale dependence", which is inherent in perturbation theory.

■ For the global analysis of QCD, the crucial points are ...

(i) factorization still holds;

(ii) the NLO PDFs for DY are the same as for DIS *(nontrivial and also holds to all orders in pQCD !)*

(iii) PDFs $f_{a/A}(x, \mu_F)$ obey the $O(\alpha_S)$ DGLAP evolution equations;

DGLAP evolution of parton distribution functions "Master formula" of factorization : the cross section for $h_1 + h_2 \to n + X$ is $\sigma_{had} = \Sigma \int_\cdot^\cdot \int_0^1 dx_1\, dx_2 \int f_a(x_1 , \mu_F)\, f_b(x_2, \mu_F)\, d\hat{\sigma}_{ab}$ where $d\hat{\sigma}_{ab}$ is the hard scattering function for a+b→n. ⇒PDF evolution at leading order	$\frac{\partial}{\partial \log Q^2} \begin{pmatrix} f_q(x, Q^2) \\ f_g(x, Q^2) \end{pmatrix} = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \left[M^{(1)}\left(\frac{z}{x}\right)\right] \begin{pmatrix} f_q(z, Q^2) \\ f_g(z, Q^2) \end{pmatrix}$ where $[M^{(1)}(\rho)] = \begin{bmatrix} P^{(1)}{}_{qq}(\rho) & P^{(1)}{}_{qg}(\rho) \\ P^{(1)}{}_{gq}(\rho) & P^{(1)}{}_{gg}(\rho) \end{bmatrix}$ "leading order splitting functions"; evolution $\propto \alpha_S$ ⇒ NLO PDFs ; generalize to higher orders ⇒ NNLO PDFs .

∴ Parametrize $f_{a/A}(x, Q_0)$ at some Q_0

$\Longrightarrow$ $f_{a/A}(x, Q)$ are known ∀ Q.

"... factorization, evolution, and universality"

NLO and NNLO calculations

I won' t try to do these calculations today. (That would take a week of lectures.) See Lecture 2. There are some details in "The Handbook of pQCD", but not complete calculations.

Refs.

Handbook of perturbative QCD;
George Sterman, John Smith, John C. Collins, James Whitmore, Raymond Brock, Joey Huston, Jon Pumplin, Wu-Ki Tung, Hendrik Weerts, Chien-Peng Yuan, Stephen Kuhlmann, Sanjib Mishra, Jorge G. Morfín, Fredrick Olness, Joseph Owens, Jianwei Qiu, and Davison E. Soper
Rev. Mod. Phys. 67, 157 – Published 1 January 1995

NLO calculations for the DY process

D. Politzer,
G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. B 157 (1979) 461.
J. Kubar-Andre and F. E. Paige, Phys. Rev. D 19 (1979) 221.
B. Humpert and W. L. van Neerven, Nucl. Phys. B 184 (1981) 225.

NNLO calculations for the DY process

R. Hamberg, W.L. van Neerven and T. Matsuura, Nucl. Phys. B 359 (1991) 343
[Erratum-ibid. B 644 (2002) 403].
Anastasiou, Dixon, Melnikov, Petrielli, Phys. Rev. D 69, 094008 (2004).
K. Melnikov and F. Petriello, Phys. Rev. D 74 (2006) 114017.

Global Analysis of QCD…
… how to determine the PDFs

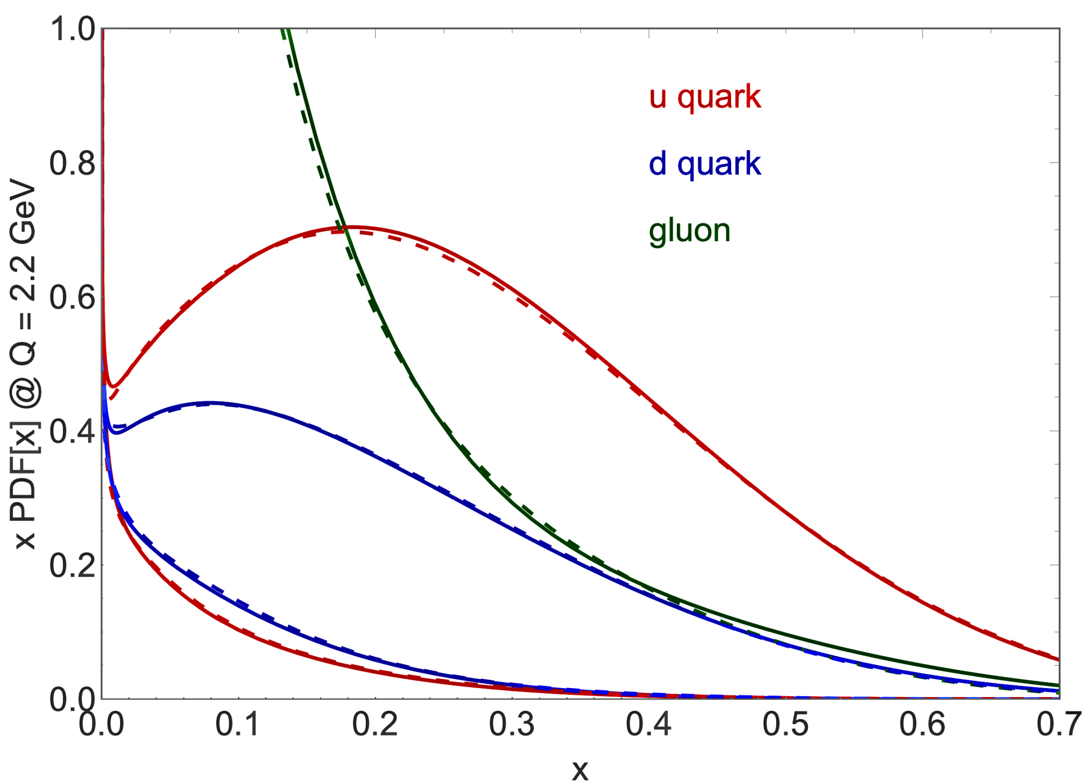
$$\sigma_{\rm had} = \Sigma \int_{\cdot}^{\cdot} \int_0^1 {\rm dx}_1\, {\rm dx}_2 \int f_a(x_1, \mu_F)\, f_b(x_2, \mu_F)\, d\hat{\sigma}_{\rm ab}$$

The hadronic cross section is measured.
The partonic hard-scattering function is calculated. (*)
The evolution of the PDFs, from momentum scale Q_0 to Q is known. (*)

(*) to NLO or NNLO

Use this information to "determine"
$f_{a/H}$ (x, Q_0 =1.3 GeV) , for the 7 partons.

- DIS : $d\sigma \propto \Sigma f_a \otimes d\hat{\sigma}_a$ –gives certain information about the $f(x, Q_0)$
- + Drell-Yan : $d\sigma \propto \Sigma f_a \otimes f_b \otimes d\hat{\sigma}_{ab}$ –gives different information
- + Other processes –e.g., sensitive to the gluon PDF



I'll show some preliminary results for 'CT18' PDFs (unpublished) for illustration purposes. These results are not qualitatively different from earlier CTEQ PDFs.

(C) Examples of experiments that have measured Drell-Yan processes

We will look at an LHC experiment, but first we'll consider some earlier experiments, which have been important in the development of PDFs.

- FNAL experiment E605 (ca 1990)
- The FNAL E866/NuSea Collaboration (ca 2000)
- The high mass Drell Yan process, ATLAS 7 TeV (2010-16)

Again, why are these important?

▪ ▪ for testing EW and QCD interactions

the experimental signal is clean

the theory is precise (NNLO QCD)

▪ ▪ for constraining parton distribution functions

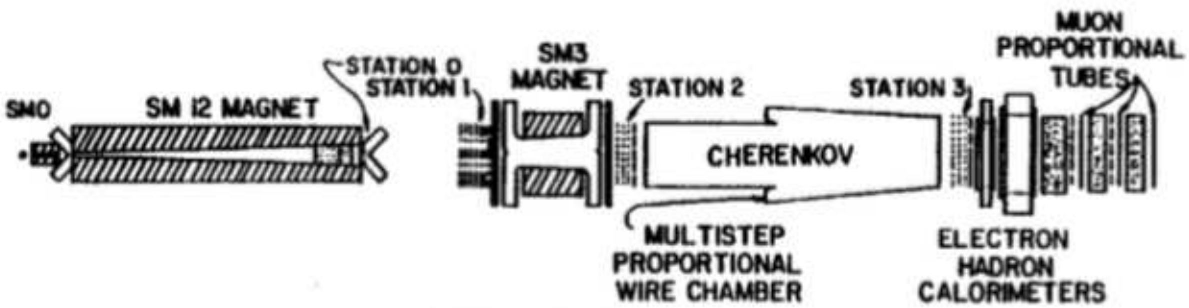
complementary to D.I.S.

§ FNAL experiment E605

Reference: Moreno et al, "Dimuon Production in Proton-Copper Collisions at $\sqrt{s}$ = 38.8 GeV", FERMILAB-PUB-90/223-E (1990).

Describe the experiment

- Observe Drell-Yan yields from 800 GeV protons on Cu targets
- muon pairs with $m_{\mu\mu} \in \{ 7 , 18 \}$ GeV
- the E605 Muon Spectrometer



PLAN VIEW E-605

Kinematics of the Drell-Yan process

LO Drell Yan

$$m^3 \frac{d^2\sigma}{dm\, dx_F} = \frac{8\pi\alpha^2}{9} \frac{x_1 x_2}{x_1 + x_2} \sum_i e_i^2 \left[q_i^A(x_1)\, \bar{q}_i^B(x_2) + \bar{q}_i^A(x_1)\, q_i^B(x_2) \right]$$

Kinematics

$$\tau \equiv \frac{m^2}{s} = x_1 x_2$$

$$x_F \equiv \frac{2 p_{long.}}{\sqrt{s}} = x_1 - x_2$$

← dimuon long. momentum in CM frame

Rapidity $y = \frac{1}{2} \ln \frac{E + p_{long.}}{E - p_{long.}}$

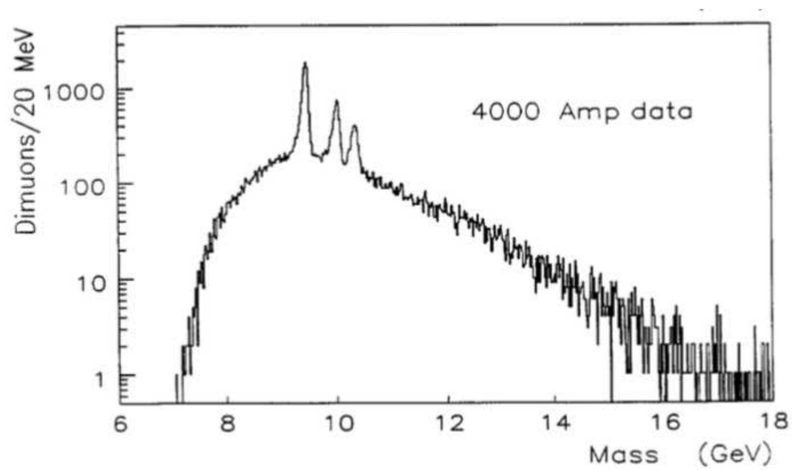
← dimuon energy "

<u>Exercise</u> $x_1 = \frac{m}{\sqrt{s}} e^{y}$ and $x_2 = \frac{m}{\sqrt{s}} e^{-y}$

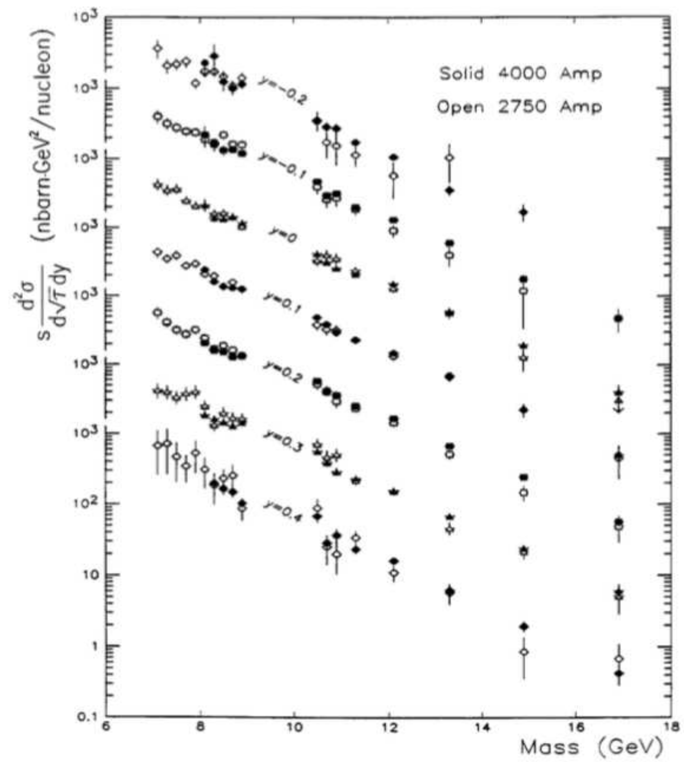
$$x_F = \frac{m}{\sqrt{s}} 2 \sinh y$$

Results published by E605

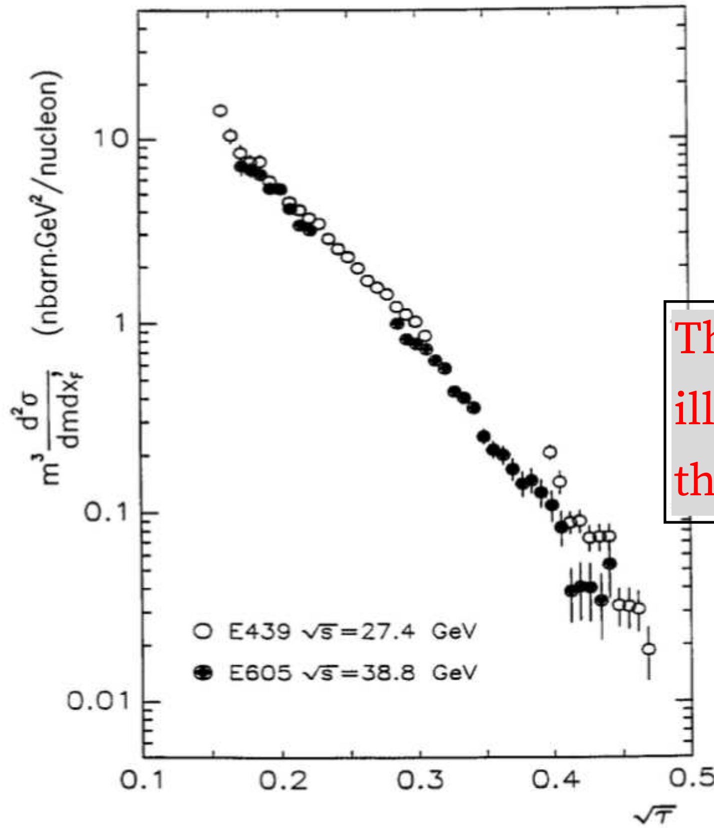
The raw mass spectrum



raw dimuon
m_{ll} spectrum



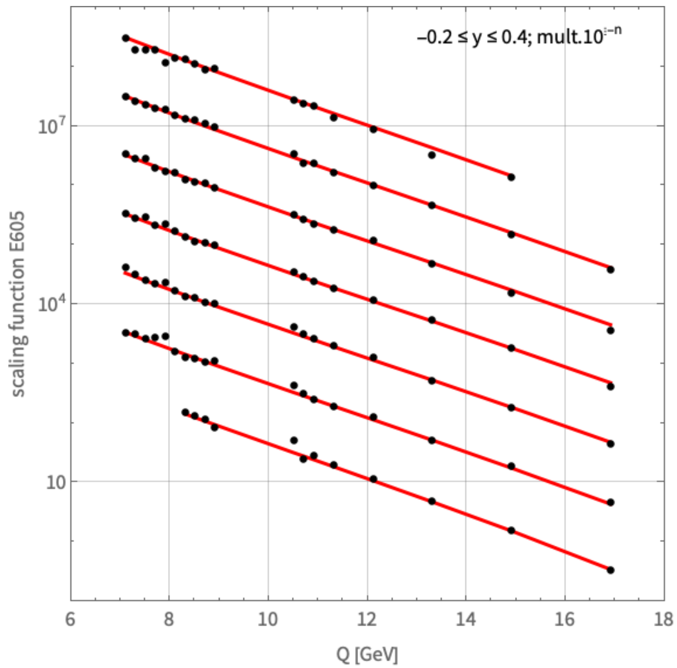
Plot the scaling function versus $m_{\mu\mu}$



This plot illustrates **scaling** of the DY cross section

The E605 data has contributed to all CTEQ PDFs.

Compare the E605 data to the 'CT18' Global Analysis



- $\chi^2/N = 0.869$
- N=113
- 7 y bins
- $Q = m_{\mu\mu}$

§ The FNAL E866/NuSea Collaboration

References:

- Hawker et al, hep-ex/0103030 (1998); *on the light antiquark flavor asymmetry*
- Webb et al, hep-ex/0302019 (2003); *the absolute cross sections*

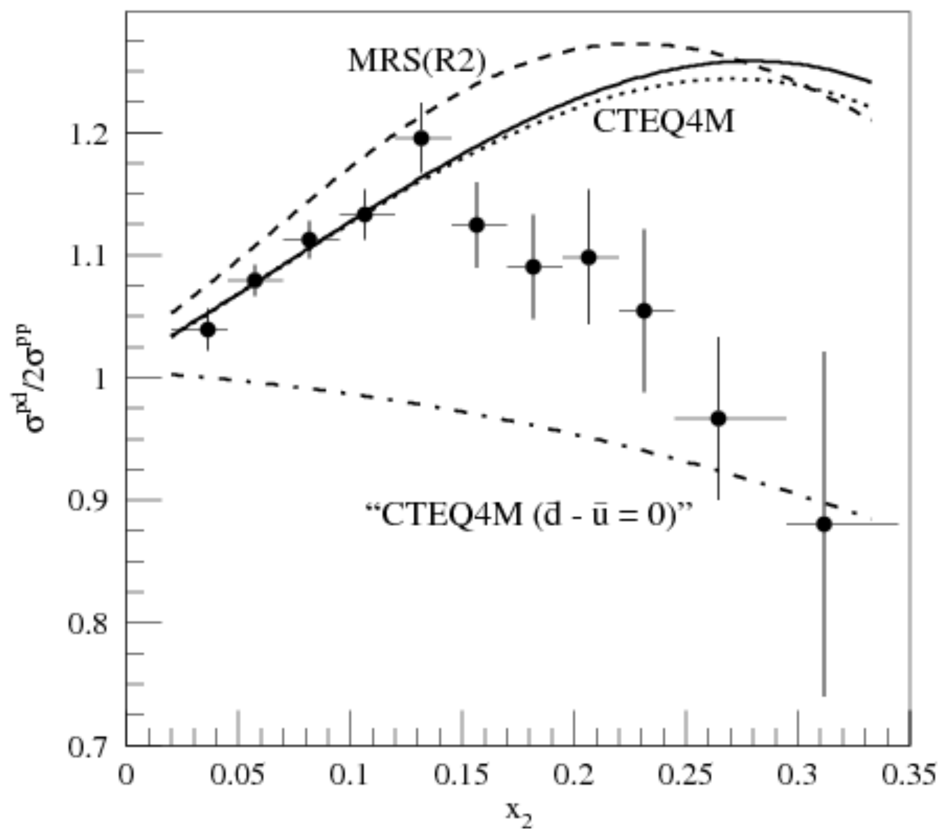
Description of the experiment:

- Drell-Yan yields from 800 GeV protons incident on hydrogen and deuterium targets, with $m_{\mu\mu} \geq 4.5\ \mathrm{GeV} / c^2$.

One result of the experiment is the ratio $\sigma^{\mathrm{pd}} / (2\,\sigma^{\mathrm{pp}})$

$$\sim \frac{\sigma(\mathrm{pp}) + \sigma(\mathrm{pn})}{2\,\sigma(\mathrm{pp})} = \frac{1}{2} + \frac{1}{2}\,\frac{\sigma(\mathrm{pn})}{\sigma(\mathrm{pp})}$$

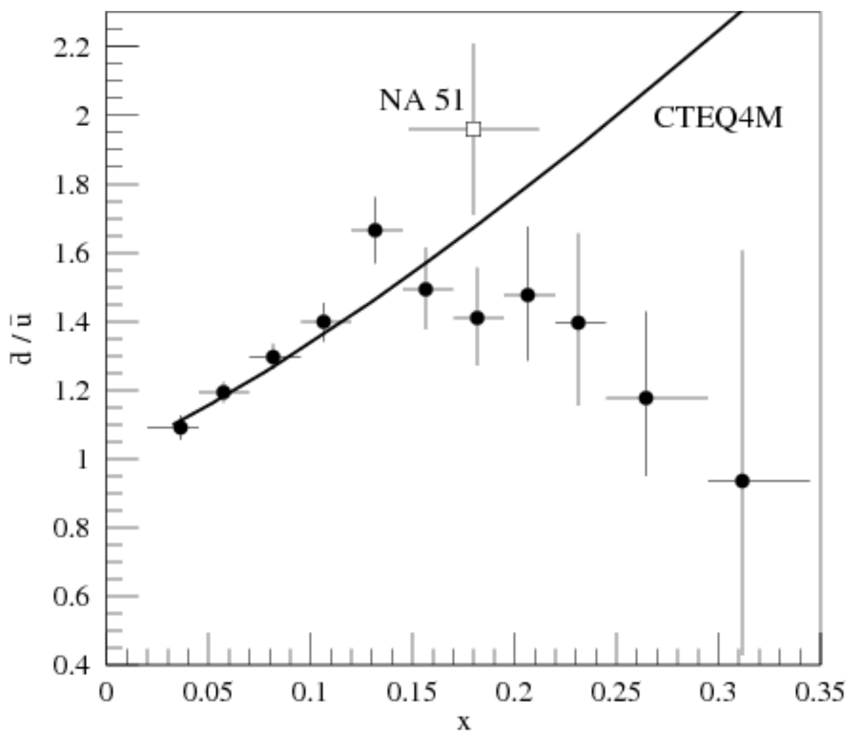
$$\frac{\sigma(\mathrm{pn})}{\sigma(\mathrm{pp})} \propto \frac{u_p\,\overline{u}_n + \overline{u}_p\,u_n}{u_p\,\overline{u}_p + \overline{u}_p\,u_p} + \{\mathrm{u} \to \mathrm{d}\}$$


MRS(R2)
CTEQ4M
"CTEQ4M (d̄ - ū = 0)"
$\sigma^{pd}/2\sigma^{pp}$
x_2
0.7
0.8
0.9
1
1.1
1.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

What!?

Analysis of the E866 result

Remember, this was 1998, before the use of NLO and NNLO QCD perturbation theory. So the experimenters analyzed the result in terms of the LO DY approximation...



based on a LO relation...

$(\sigma^{pd}/2\sigma^{pp})\,|_{x_1>>x_2}$

$\approx 1/2(1+\overline{d}_2/\overline{u}_2)$

The ratio $\sigma^{pd} / 2\, \sigma^{pp}$

$$m^3 \frac{d^2\sigma}{\mathrm{dm\, dx}_F} = \left(\frac{8\pi\alpha^2}{9}\right)\left(\frac{x_1 x_2}{x_1+x_2}\right) \Sigma\, e_i^2 \,[\, q_i^A(x_1)\, \overline{q}_i^B(x_2) + \overline{q}_i^A(x_1)\, q_i^B(x_2) \,]$$

parton luminosity for pp collision

$$\mathrm{L(pp)} = \tfrac{4}{9}\, u(x_1)\, \overline{u}(x_2) + \tfrac{1}{9}\, d(x_1)\, \overline{d}(x_2) + \tfrac{4}{9}\, \overline{u}(x_1)\, u(x_2) + \tfrac{1}{9}\, \overline{d}(x_1)\, d(x_2)$$

parton luminosity for pn collision (exercise; isospin symmetry!)

$$\mathrm{L(pn)} = \tfrac{4}{9}\, u(x_1)\, \overline{d}(x_2) + \tfrac{1}{9}\, d(x_1)\, \overline{u}(x_2) + \tfrac{4}{9}\, \overline{u}(x_1)\, d(x_2) + \tfrac{1}{9}\, \overline{d}(x_1)\, u(x_2)$$

parton luminosity for pd collision (d = p + n)

L(pd) = L(pp) + L(pn)

If x_1 is large, then I can neglect set $\overline{u}(x_1) = 0$ and $\overline{d}(x_1) = 0$. Then

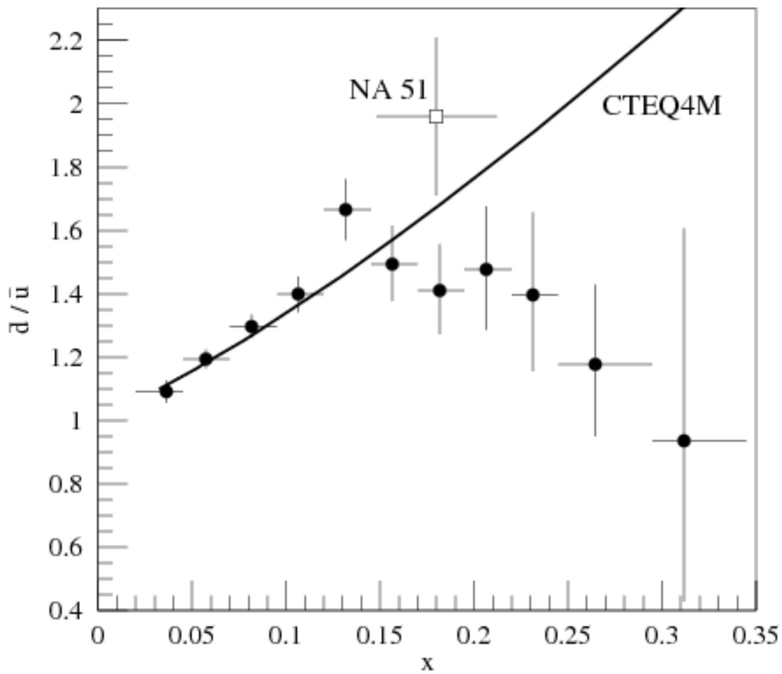
$$\mathrm{L(pp)} \approx \tfrac{4}{9}\, u(x_1)\, \overline{u}(x_2) + \tfrac{1}{9}\, d(x_1)\, \overline{d}(x_2)$$

$$\mathrm{L(pn)} \approx \tfrac{4}{9}\, u(x_1)\, \overline{d}(x_2) + \tfrac{1}{9}\, d(x_1)\, \overline{u}(x_2)$$

$$\frac{L(\mathrm{pd})}{L(\mathrm{pp})} = 1 + \frac{4\,\mathrm{u1\,db2} + \mathrm{d1\,ub2}}{4\,\mathrm{u1\,ub2} + \mathrm{d1\,db2}} = 1 + \frac{4\,\mathrm{db2/ub2} + \mathrm{d1/u1}}{4 + (\mathrm{d1/u1})(\mathrm{db2/ub2})} = 1 + \frac{4a+b}{4+a\,b} = 1 + \mathrm{a}\,\frac{4+b/a}{4+b\,a} \approx 1 + \mathrm{a}$$

$\left(a = \overline{d}(x_2) / \overline{u}(x_2)\right.$ is ≈ 1 and $\mathrm{b} = \mathrm{d}(x_1)/\mathrm{u}(x_1)$ is $\lesssim 1$)

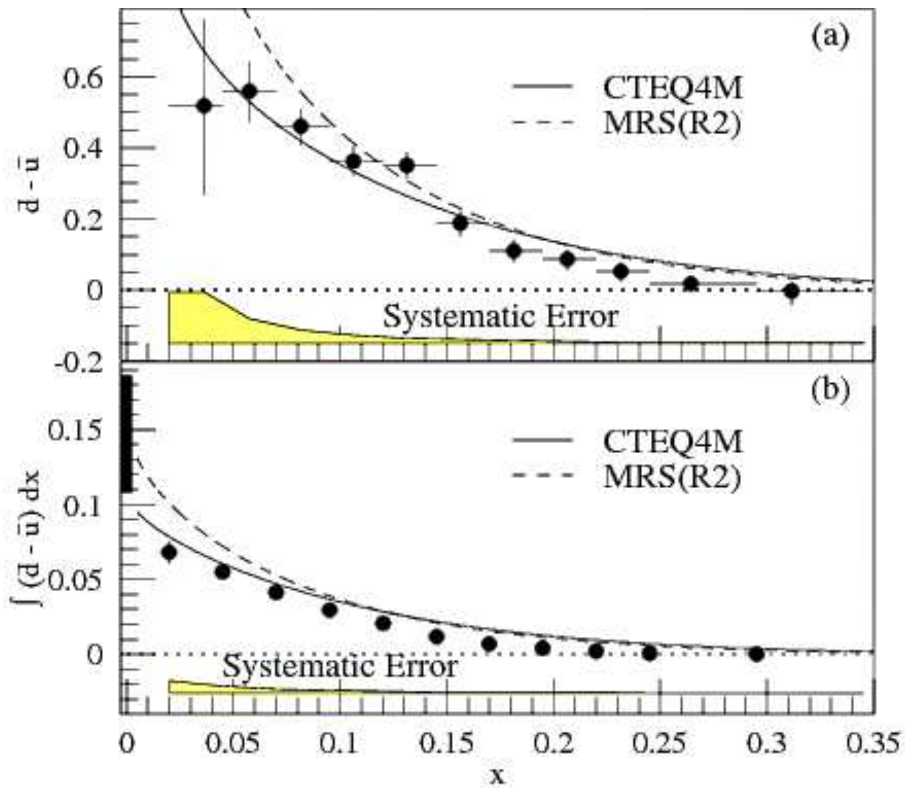
$$\therefore\ \frac{\sigma\,(\mathrm{pd})}{2\,\sigma(\mathrm{pp})} \approx \frac{1}{2}\left(1 + \frac{\overline{d}(x_2)}{\overline{u}(x_2)}\right)$$



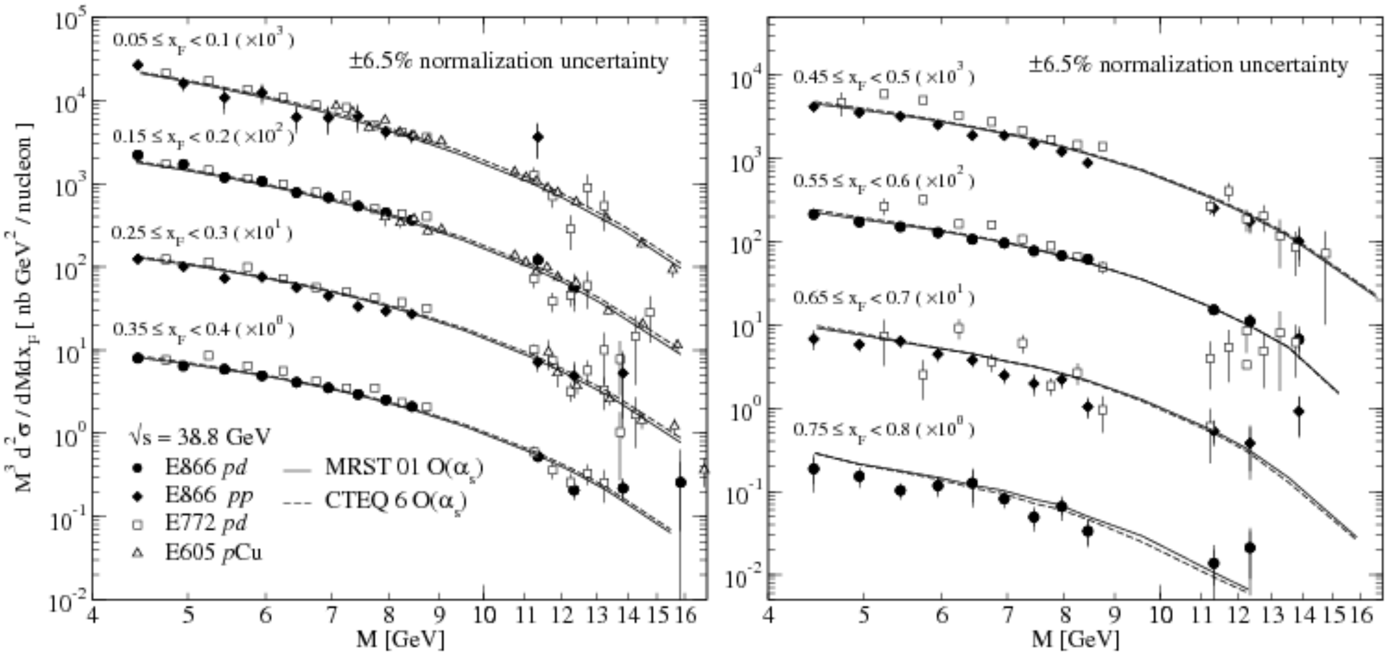
So, they claimed that $\overline{d}(x) / \overline{u}(x)$ is

- greater than 1 (ie, $\overline{d} > \overline{u}$)
- increases for $x \lesssim 0.15$
- decreases for $x \gtrsim 0.2$
- the PDFs of that day did not agree

One must be careful when considering PDFs at large x,



The E866 experiment also published absolute cross sections, σ^{pp} and σ^{pd} for $\sqrt{s}$ = 38.8 GeV.
Data: showing $M^3\ d^2\sigma/(\mathrm{dM\ dx}_F)$ versus M



Exercise: Calculate the maximum rapidity.

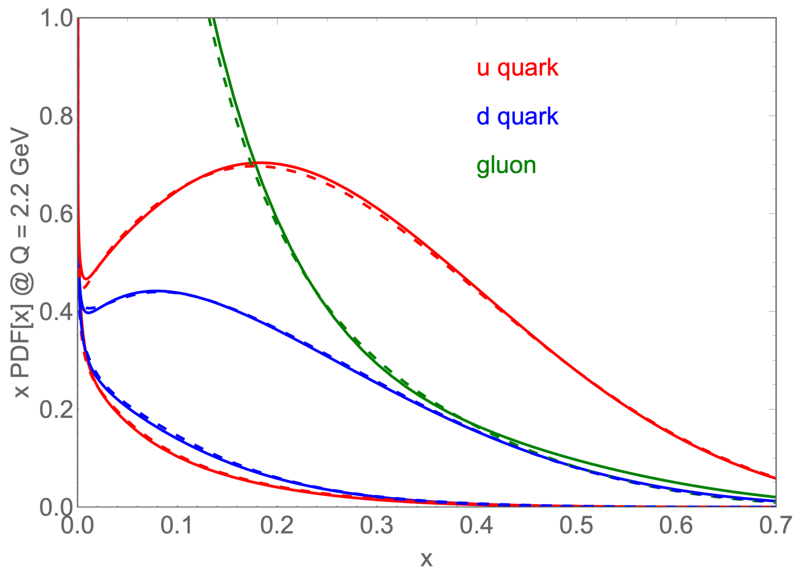
PDFs from the ‘CT18’ Global Analysis

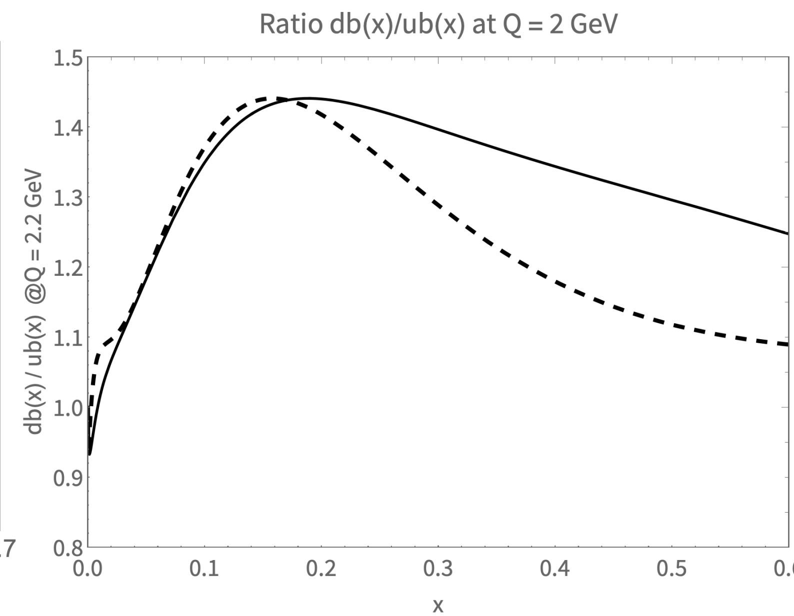
We'll look at two issues :

(1) Is $\overline{d}$(x,Q) / $\overline{u}$ (x,Q) > 1 ?

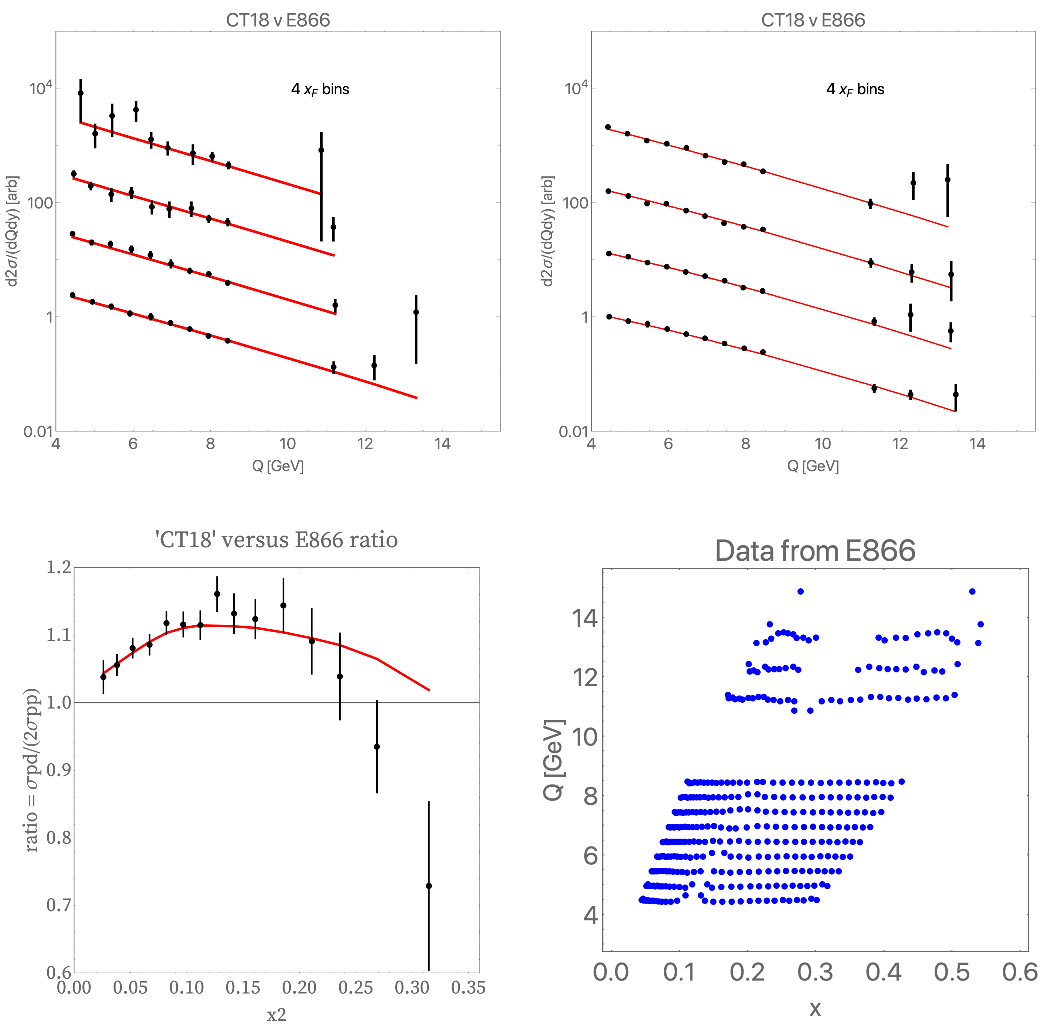
(2) Compare the DY data from E866, to a modern theoretical calculation (NNLO with ‘CT18’ PDFs)

(1)


u quark
d quark
gluon
x PDF[x] @ Q = 2.2 GeV
x
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.3
0.5
0.7


Ratio db(x)/ub(x) at Q = 2 GeV
db(x) / ub(x) @Q = 2.2 GeV
x
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6

(2)



CT18 v E866
4 x_F bins
d2σ/(dQdy) [arb]
Q [GeV]
CT18 v E866
4 x_F bins
d2σ/(dQdy) [arb]
Q [GeV]
'CT18' versus E866 ratio
ratio = σpd/(2σpp)
x2
Data from E866
Q [GeV]
x

Follow up experiment E906

§ The Drell-Yan process at the LHC

ATLAS high-mass lepton pair production at 8 TeV

Reference:

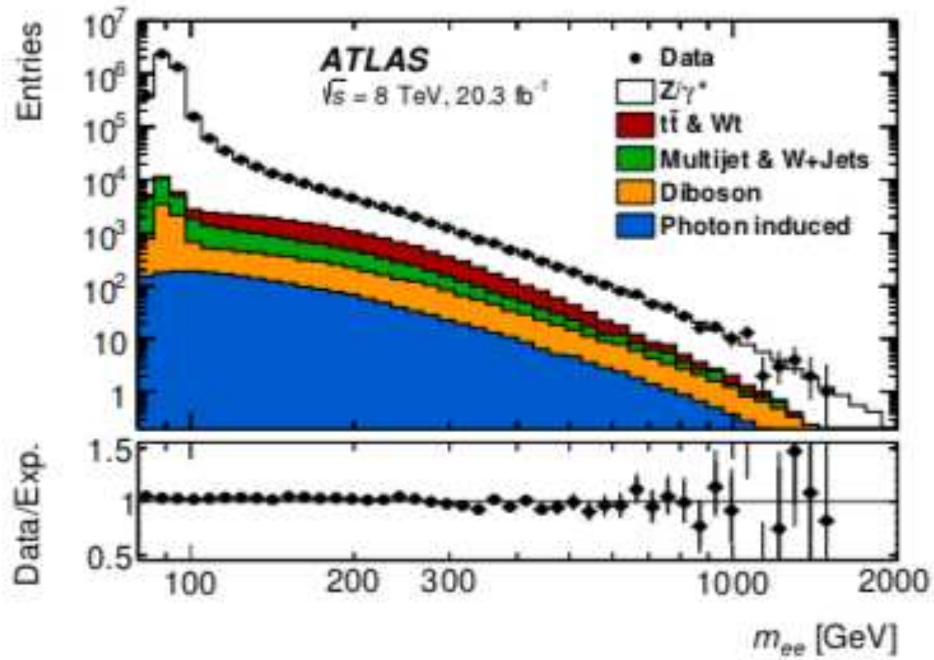
"Measurement of the double-differential high-mass Drell-Yan cross section in pp collisions at $\sqrt{s}$ = 8 TeV with the ATLAS detector"; JHEP 08 (2016) 009.

Description of the experiment

The number of data points = 48

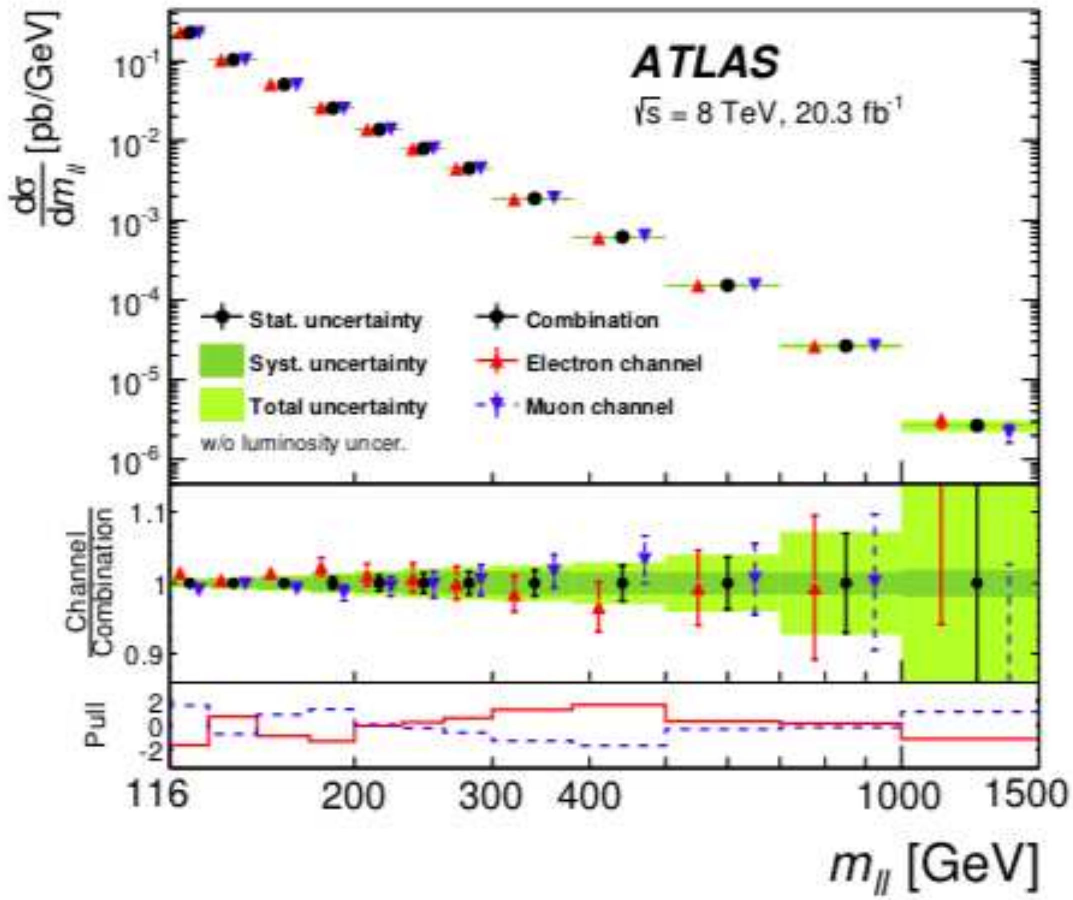
The number of systematic errors= 36

High-mass Drell-Yan process at ATLAS 8 TeV

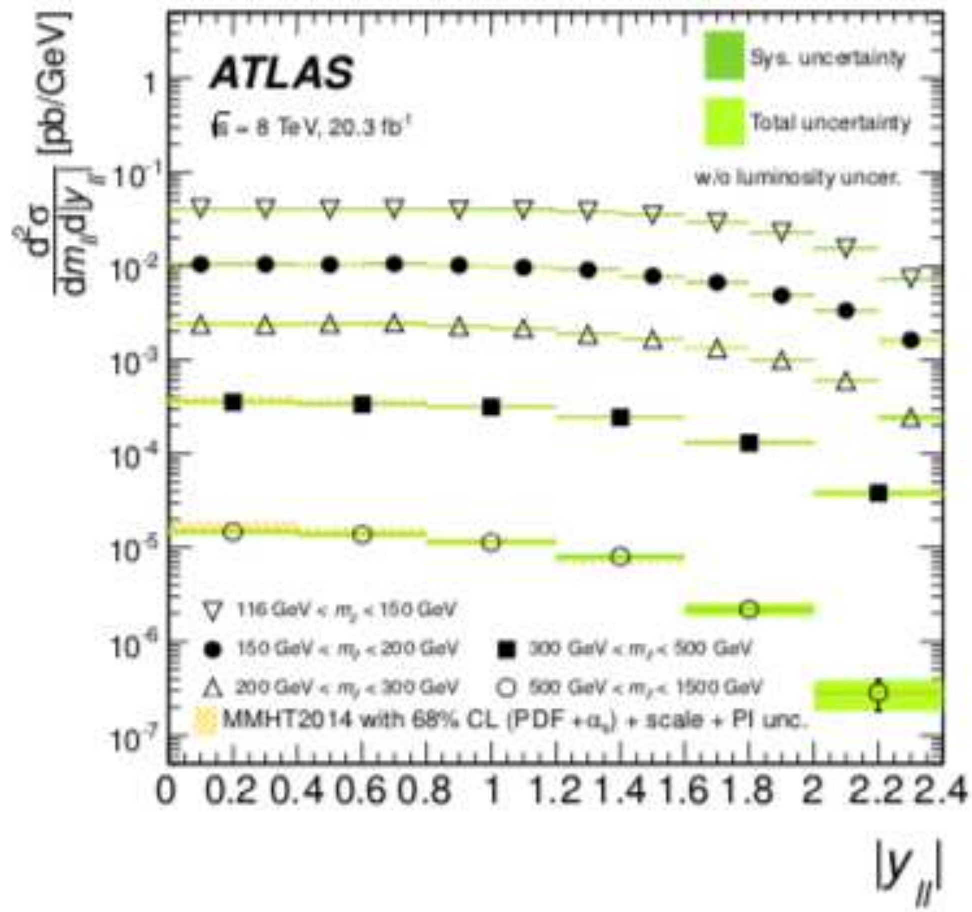


At the LHC, *backgrounds* are important!

$d\sigma / dm_{ll}$

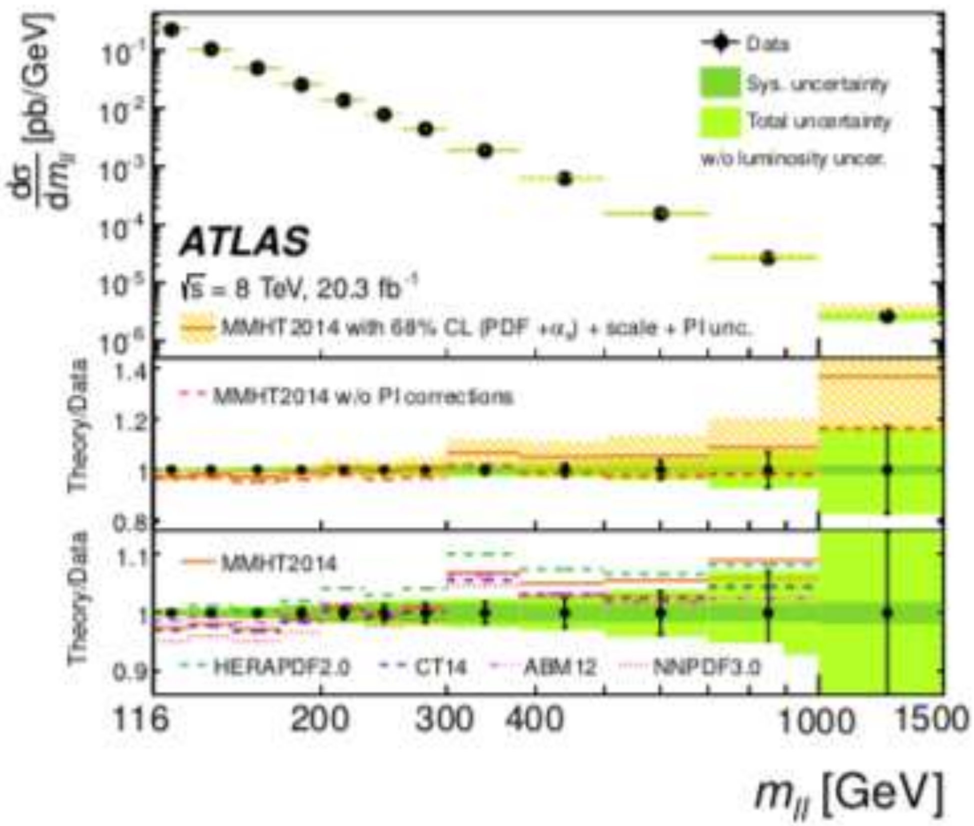

ATLAS
√s = 8 TeV, 20.3 fb⁻¹
Stat. uncertainty
Syst. uncertainty
Total uncertainty
w/o luminosity uncer.
Combination
Electron channel
Muon channel
dσ/dm_ll [pb/GeV]
Channel/Combination
Pull
m_ll [GeV]

$d\sigma / dy_{ll}$

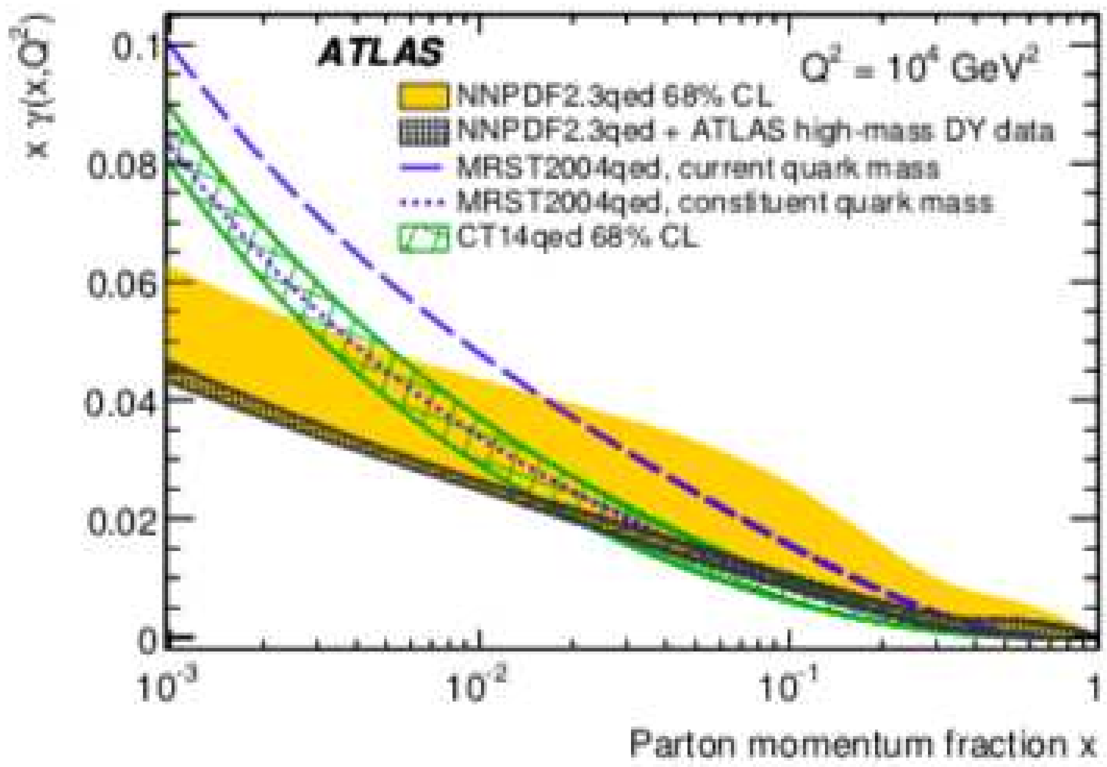


ATLAS
√s = 8 TeV, 20.3 fb⁻¹
Sys. uncertainty
Total uncertainty
w/o luminosity uncer.
116 GeV < m_ll < 150 GeV
150 GeV < m_ll < 200 GeV
300 GeV < m_ll < 500 GeV
200 GeV < m_ll < 300 GeV
500 GeV < m_ll < 1500 GeV
MMHT2014 with 68% CL (PDF + α_s) + scale + PI unc.
d²σ / dm_ll d|y_ll| [pb/GeV]
|y_ll|

The PI contribution (Photon Induced)

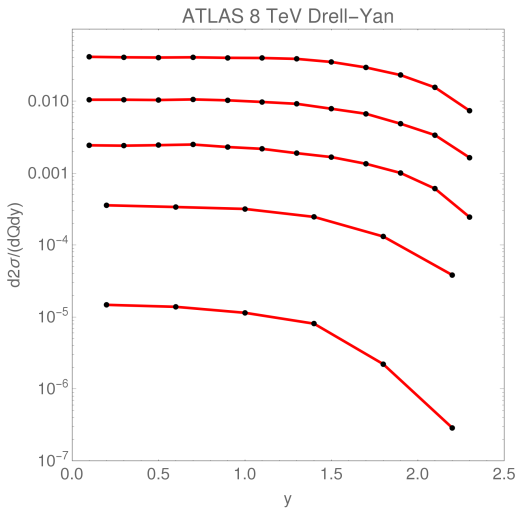


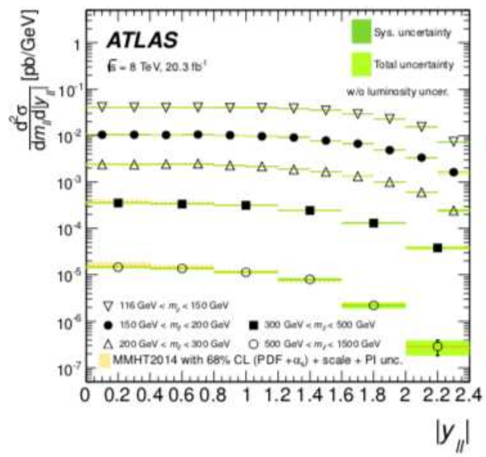
The PI contribution ⇔ photon as a parton x $\gamma(x,\ Q^2)$

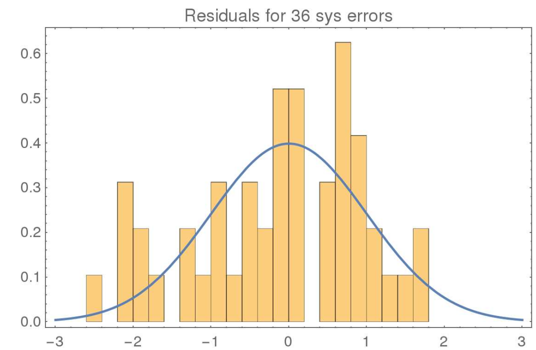


The photon parton distribution function is very small, but it may be important for some processes at highest precision.

ATLAS 8 TEV high-mass Drell-Yan data compared to CT18







$\chi^2/\mathrm{N} = 75.7 \,/\, 48$