Charm, beauty, and truth at hadron colliders

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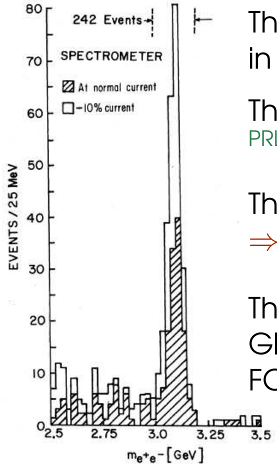
Lecture 1
July 23, 2019

Contents

1. Heavy-quark production: history and applications
2. Massive quark scattering, key ideas
3. Practical example: single-top production
4. Part 2 (John Collins, tomorrow): QCD factorization formalism for heavy quarks
5. Experimental results: Ben Nachman's lecture

Heavy-quark production: history and applications

1974: Physicists discover charm



The first heavy quark, charm was discovered in in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC

The observations were published together:
PRL 33, 1404 (1974); PRL 33, 1406 (1974)

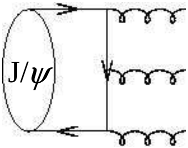
The J/ψ was recognized as a $c\bar{c}$ bound state $\Rightarrow m_c \sim 1.5$ GeV

The existence of a 4th quark confirmed the Glashow-Iliopoulos-Maiani explanation for why FCNC decays ($s \to d\nu\bar{\nu}$) did not occur.

And it loosened the shackles of SU(3)$_{\text{flavor}}$, Gell-Mann's "Eightfold way"

Too narrow a resonance

While the J/ψ was clearly a quark bound state, it had an extremely narrow width of 88 keV.



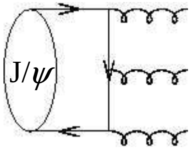
This caused a minor crisis in the fledgling QCD…

After all how could a strongly interacting state be narrow?

$\Gamma_\rho \sim 150$ MeV, $\Gamma_\omega \sim 8.5$ MeV, $\Gamma_\phi \sim 4.3$ MeV, $\Gamma_{J/\psi} \sim 88$ keV

Too narrow a resonance

An explanation was found by Appelquist and Politzer, PRL 34, 43 (75).



Write the width as
$\Gamma({}^3S_1 \to 3\text{ gluons}) = |R(0)|^2 |M(q\bar{q} \to ggg)|^2$

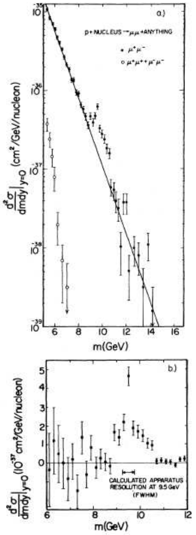
Following the model of positronium, solve the Schroedinger Eqn. for
$R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$, where $a_0 = \frac{1}{\alpha_s m_c/2}$.

$|M(q\bar{q} \to ggg)|^2 \sim \alpha_s^3$ — one power for each gluon

$\Rightarrow \Gamma({}^3S_1 \to 3\text{ gluons}) \sim 0.2\,\alpha_s^6\, m_c \sim 90\text{ keV}; \quad \Rightarrow \quad \alpha_s \approx 0.26$

Homework: Why do we not see $J/\psi \to gg$?

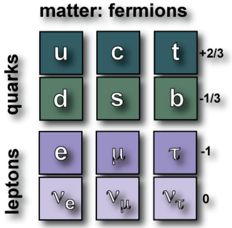
1977: Physicists discover beauty



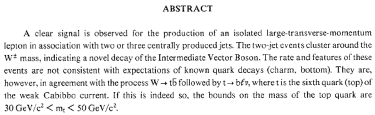
In 1975 the τ was discovered and led to the search for other 3rd-generation particles.

In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron.
(The Upsilon is also very narrow.) PRL 39, 252 (1977)

Once the bottom quark was found, it was clear that a sixth quark was needed to complete the family structure.



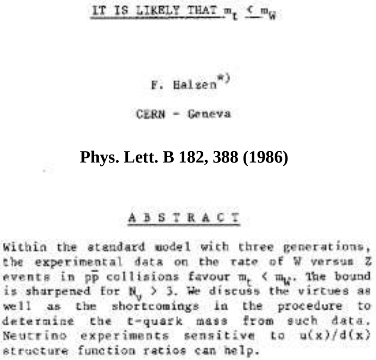
“The top quark was discovered several times!”



ABSTRACT

A clear signal is observed for the production of an isolated large-transverse-momentum lepton in association with two or three centrally produced jets. The two-jet events cluster around the $W^\pm$ mass, indicating a novel decay of the Intermediate Vector Boson. The rate and features of these events are not consistent with expectations of known quark decays (charm, bottom). They are, however, in agreement with the process $W \to t\bar{b}$ followed by $t \to b\ell\bar{\nu}$, where t is the sixth quark (top) of the weak Cabibbo current. If this is indeed so, the bounds on the mass of the top quark are $30\ \mathrm{GeV}/c^2 < m_t < 50\ \mathrm{GeV}/c^2$.

UA1, Phys. Lett. B 147, 493 (1984)



IT IS LIKELY THAT $m_t < m_W$

F. Halzen*)

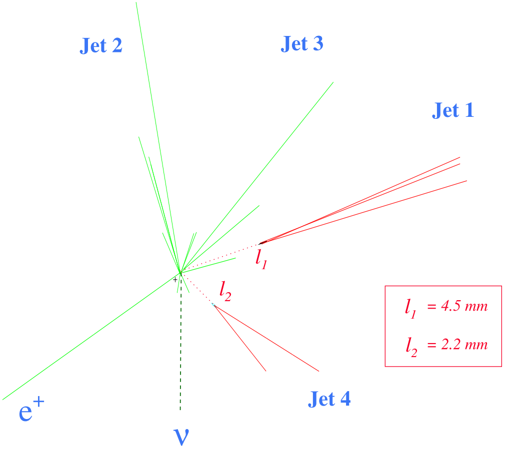
CERN - Geneva

Phys. Lett. B 182, 388 (1986)

ABSTRACT

Within the standard model with three generations, the experimental data on the rate of W versus Z events in $p\bar{p}$ collisions favour $m_t < m_W$. The bound is sharpened for $N_\nu > 3$. We discuss the virtues as well as the shortcomings in the procedure to determine the t-quark mass from such data. Neutrino experiments sensitive to u(x)/d(x) structure function ratios can help.

1992: “This is the top quark!”

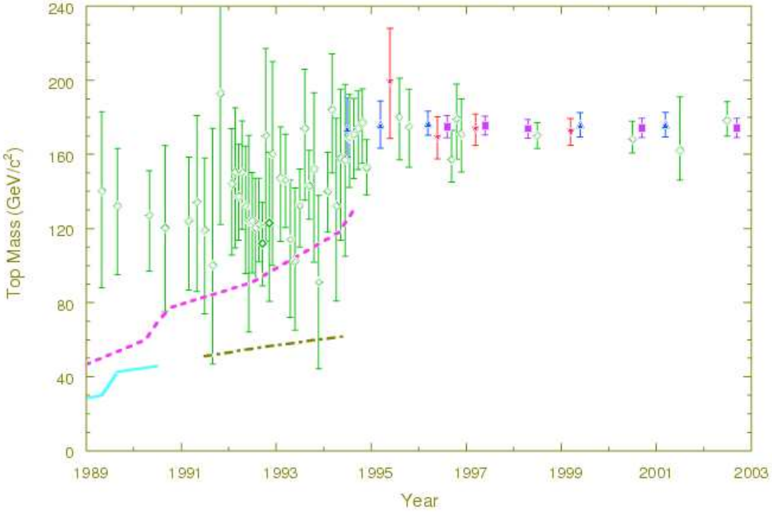


$M_{top}^{Fit} = 170 \pm 10$ GeV/c^2

24 September, 1992
run #40758, event #44414

OR MAYBE NOT

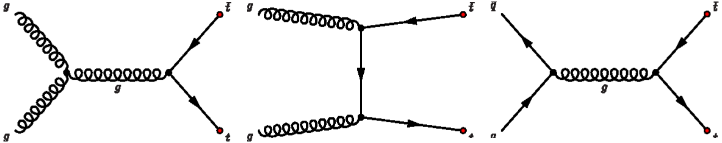
1995: FNAL announces the top quark discovery



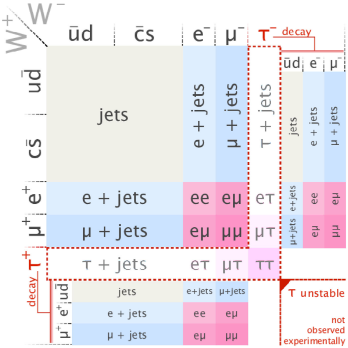
Did LEP predict the mass? Look at Sept. 1992 . . . Quigg

2009 - : The LHC era, heavy-quark production

Heavy quarks are produced and observed in many channels
Millions of $t\bar{t}$ pairs are produced in gg (85%), qg, and $q\bar{q}$ channels...



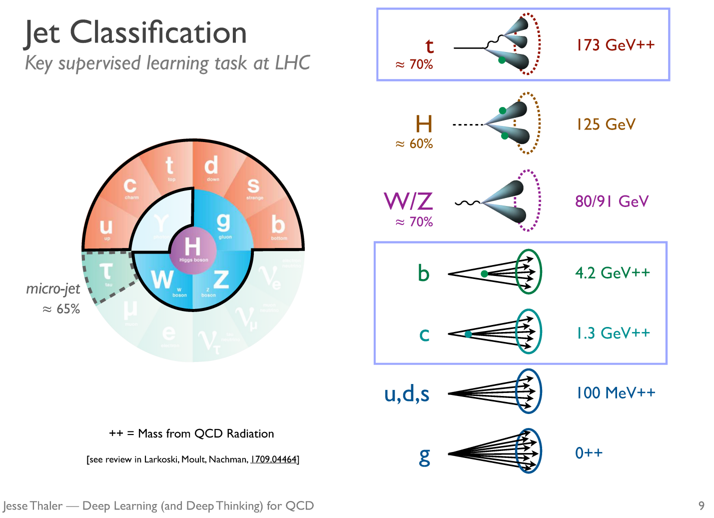
...and decay
hadronically ($\sim 70\%$),
semi-leptonically ($\sim 20\%$),
leptonically ($\sim 10\%$)



W^+ \ W^-	ūd	c̄s	e⁻	μ⁻	τ⁻	τ⁻ decay: ūd	e⁻	μ⁻
ūd	jets		e + jets	μ + jets	τ + jets	jets	e + jets	μ + jets
c̄s								
e⁺	e + jets		ee	eμ	eτ	e+jets	ee	eμ
μ⁺	μ + jets		eμ	μμ	μτ	μ+jets	eμ	μμ
τ⁺	τ + jets		eτ	μτ	ττ			
τ⁺ decay: ūd	jets		e+jets	μ+jets		τ unstable not observed experimentally		
e⁺	e + jets		ee	eμ				
μ⁺	μ + jets		eμ	μμ				

○ full hadronic
🔵 semileptonic
🔴 dileptonic

2009 - : The LHC era, hadronic decays



Jet Classification

Key supervised learning task at LHC

++ = Mass from QCD Radiation

[see review in Larkoski, Moult, Nachman, 1709.04464]

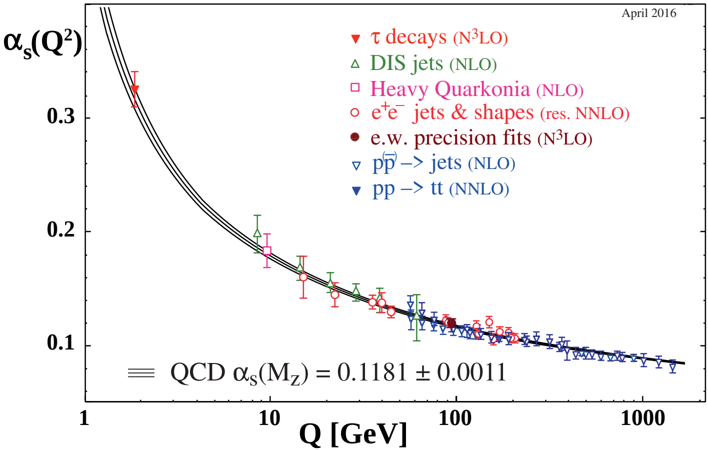
Jesse Thaler — Deep Learning (and Deep Thinking) for QCD 9

t: jet substructure (high $p_T^{t\bar{t}}$)

c, b: displaced decay vertices

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends

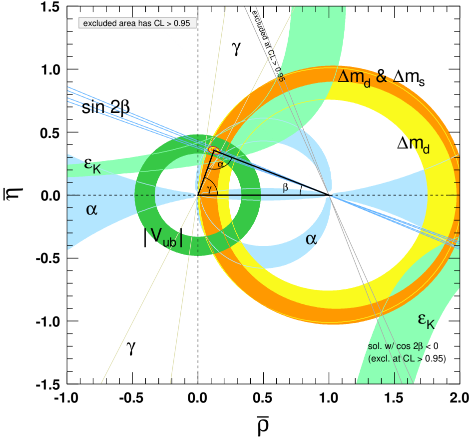


measure the QCD coupling $\alpha_s(\mu)$ at low and high μ

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends

12. CKM quark-mixing matrix



test electroweak quark mixing and CP violation

2009 - : The LHC era, heavy-quark applications

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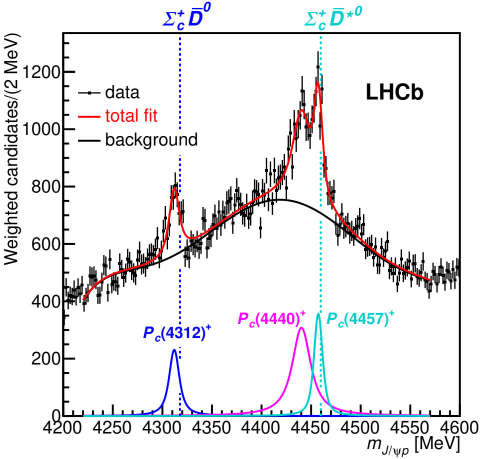
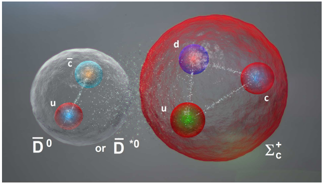


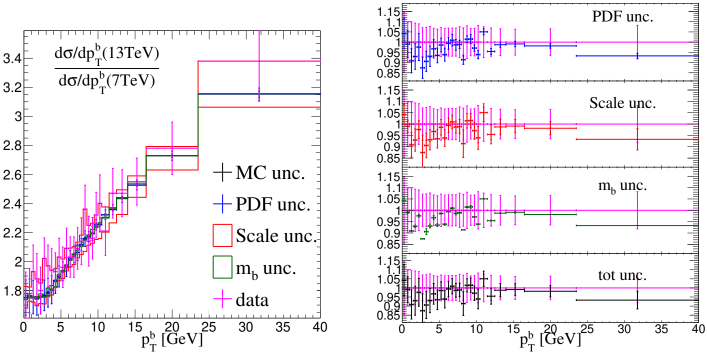


search for new QCD bound states, such as pentaquark "molecules"

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends

The ratio $R(13\text{TeV}/7\text{TeV})$ of LHCb cross sections

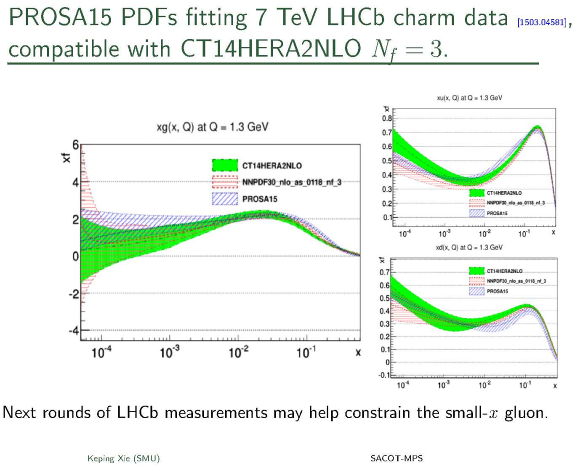


Xie, Campbell, PN, 2019

LHCb c and b meson production can constrain the gluon PDF at $x \sim 10^{-5}$

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends



PROSA15 PDFs fitting 7 TeV LHCb charm data [1503.04581], compatible with CT14HERA2NLO $N_f = 3$.

Next rounds of LHCb measurements may help constrain the small-x gluon.

Keping Xie (SMU) SACOT-MPS 15/16

LHCb c and b meson production can constrain the gluon PDF at $x \sim 10^{-5}$

2009 - : The LHC era, heavy-quark applications

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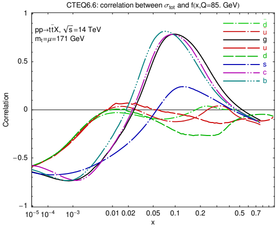
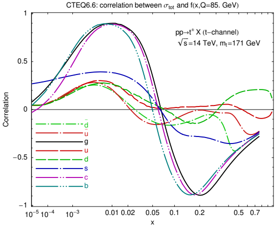
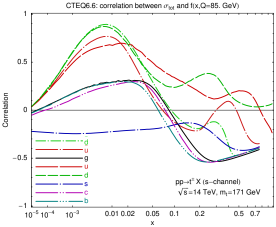


ttbar and single-top production cross sections at the LHC are highly correlated and may constrain gluon, quark PDFs in the x regions relevant for Higgs and EW precision studies

CTEQ6.6, PRD 78 (2008) 013004

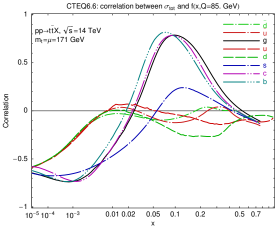




Right before the LHC began operation, it was proposed to constrain the gluon and other PDFs in $t(\bar{t})$ production

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends



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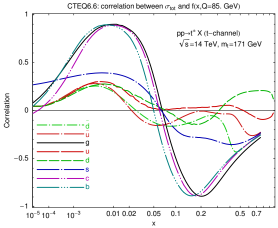
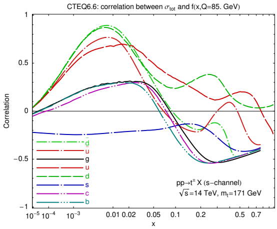
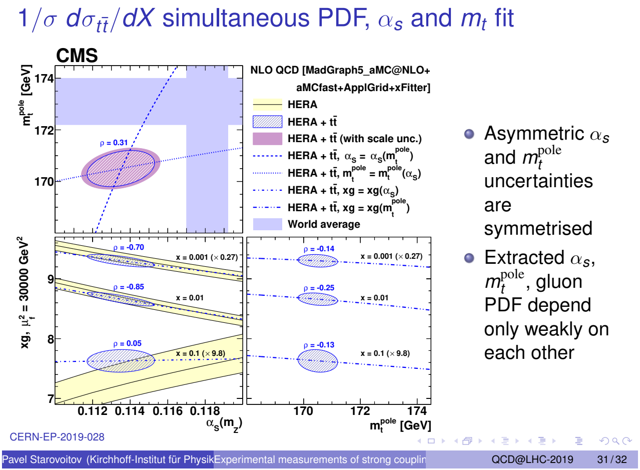




Note that the correlations for g,c, and b PDFs closely track one another

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends



$1/\sigma\ d\sigma_{t\bar{t}}/dX$ simultaneous PDF, α_s and m_t fit

CERN-EP-2019-028

- Asymmetric α_s and $m_t^{\rm pole}$ uncertainties are symmetrised
- Extracted α_s, $m_t^{\rm pole}$, gluon PDF depend only weakly on each other

Pavel Starovoitov (Kirchhoff-Institut für Physik Experimental measurements of strong couplin QCD@LHC-2019 31 / 32

First determinations of **correlated** constraints on α_s, m_t, and gluon PDF have been published

Heavy-quark production: key issues and ideas

Only one statement is correct. Which one? (1 minute)

1. The number N_f of active quark flavors is the number of quark masses satisfying $m_i \leq \mu_F$ at a given factorization scale μ_F.
2. In the $N_f = 4$ factorization scheme, no scattering contributions with b quarks are included.
3. The charm PDF $c(x, \mu_F)$ can be non-zero at $\mu_F < m_c$.
4. The $\overline{MS}$ PDFs are defined by setting heavy quark masses to zero to factorize collinear poles using dimensional regularization.

What is a “heavy quark?”

Standard definition: A heavy quark is a quark with $m_q \gg \Lambda_{\mathrm{QCD}}$.

	Pole mass M	$\overline{MS}$ mass $\overline{m}(\overline{m})$
Charm	~ 1.3–1.7 GeV	1.275 ± 0.025 GeV
Bottom	~ 4.5–5 GeV	4.18 ± 0.03 GeV
Top	173.1 ± 0.6 GeV (?)	$160^{+4.8}_{-4.3}$ GeV

PDG (5/30/17)

Top: TEVEWWG: $174.30 \pm 0.35 \pm 0.54$ GeV, LHC: $172.64 \pm 0.25 \pm 0.55$ GeV

It seems kind of funny to list 2 different masses. . .

Pole Mass: $\sim \frac{1}{\not{p} - M}$

TeV vs. LHC $\sim 3\sigma$

$\overline{MS}$ mass: Related to pole mass by

$$\frac{M}{\overline{m}(\overline{m})} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(-1.0414 \ln(M^2/\overline{m}^2) + 13.4434\right) + \ldots$$

c and b masses are best written in $\overline{MS}$ scheme.

t mass is often quoted in the pole-mass scheme.

Which heavy-quark mass is it?

1. Current mass: An effective parameter of the SM Lagrangian $\mathcal{L} \sim m_q \bar{\psi}_q \psi_q$.

Note that $m_q = 0$ in the unbroken EW Lagrangian, because the bare Dirac masses are inconsistent with $SU(2)_L$ gauge invariance (S. Dawson's lecture).

At experimental scales, effective $m_q = Y_q/(2\sqrt{2}G_F)^{1/2}$ are generated by $SU(2)_{EW} \otimes U(1)_Y \to U(1)_{EM}$ breaking from $q\bar{q}h$ Yukawa couplings Y_q.

For top quark, $Y_1 \approx 1$ in the SM.

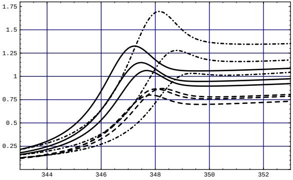
Which heavy-quark mass is it?

2. In perturbation theory, m_q depends on the renormalization scheme. In the $\overline{MS}$ scheme, mass depends on the renorm. scale, $m_q^{bare} \to \overline{m}_q(\mu_R)$. In the on-shell scheme, we define the **pole mass**. The pole mass for heavy quarks is deprecated in precise computations – its value acquires an ambiguity of $\mathcal{O}(\Lambda_{QCD})$ because of QCD renormalon contributions.

3. The kinematic mass reconstructed by the experiments. In many LHC/Tevatron measurements, m_t is a parameter in parton showering programs. It does not have an exact meaning in perturbation theory, has precision no better than $\sim$1 GeV when inferred from LHC differential $t\bar{t}$ distributions.

4. **Auxiliary mass parameters** of order of HQ mass (e.g., the factorization scale). All-order cross section is indepedent of them.

Top mass from $t\bar{t}$ threshold at a e^-e^+ collider



Yakovlev,Groote PRD63, 074012(01)

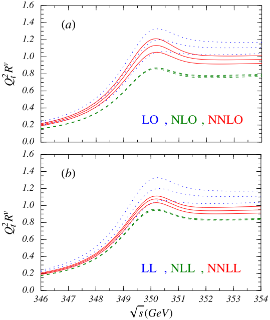
There is a subtle question when you try to make a precision measurement of QCD: **What mass do you use?**

The pole mass is affected by nonperturbative corrections; ill-defined for bound quarks.

Solution: Use the 1S mass (pseudo-bound state)

There are large non-relativistic corrections

$$\sigma_{t\bar{t}} \propto v \sum \left(\frac{\alpha_s}{v}\right) \times \left\{ \begin{array}{c} 1 \\ \sum(\alpha_s \ln v) \end{array} \right\} \times \left\{ \begin{array}{l} \mathrm{LO}(1) + \mathrm{NLO}(\alpha_s, v) + \mathrm{NNLO}(\alpha_s^2, \alpha_s v, v^2) \\ \mathrm{LL} + \mathrm{NLL} + \mathrm{NNLL} \end{array} \right\}$$



Hoang, et al. PRD69, 034009(04)

Normalization changes, but peak stable.

$\delta\sigma_{t\bar{t}}$ is $\pm 6\%$ before ISR/beamstrahlung

$\delta\overline{m}_t(\overline{m}_t) \sim 100$ MeV is attainable

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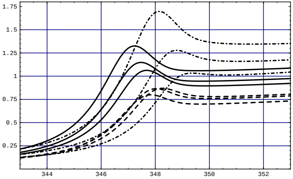

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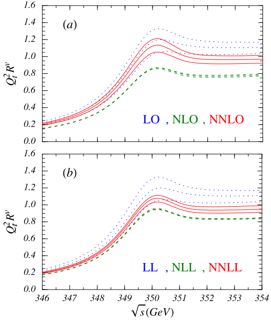


Hoang, et al. PRD69, 034009(04)

A factorized hadronic cross section

$$\sigma_{\rm obs.} = \int f_1(x_1,\mu_F;\{m_q\}) f_2(x_2,\mu_F;\{m_q\})$$
$$\otimes \overline{|M|^2}(...;\alpha_s(\mu_R),\{m_q\}) \otimes d{\rm P.S.} \otimes D(p_i) + O(\Lambda)$$

- f: parton distribution functions for the initial state
- $\overline{|M|^2}$: a short-distance squared matrix element
- $dP.S.$: phase space which you may not want to completely integrate out.
  $\Rightarrow$ Exclusive cross sections (jet counting), angular correlations
- $D(p_i)$: a "measurement function" for the final state, such as a fragmentation function or jet definition
- $O(\Lambda)$ are power-suppressed terms with $\Lambda \sim 1$ GeV

α_s, m_q, f **depend on the renormalization scheme, including** N_f**, the number of active flavors in UV counterterms**

PDFs for heavy flavors

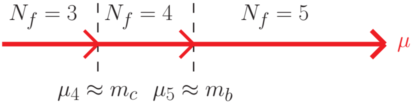
PDFs for heavy partons h can be generated via DGLAP evolution at $\mu \geq m$. At LO, a common boundary condition is $f_{h/p}(x,\mu) = 0$ at $\mu \leq m$.

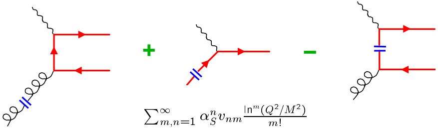
In practice:

- PDFs are usually introduced for c and b quarks
  - starting from $\mathcal{O}(\alpha_s^2)$, an initial condition $f_{c/p}(x,\mu_0) \neq 0$ is generated at $\mu_0 = m_c$ by perturbative matching; also, one can obtain $f_{c/p}(x,\mu_0) \neq 0$ from twist-4 intrinsic charm DIS terms *(arXiv:1707.00657)*
- QCD coupling $\alpha_s(\mu)$ and PDFs are evaluated with 5 active flavors at all $\mu \geq m_b$
- Logarithmic enhancements may exist in collinear t, W, Z production at $\mu \gtrsim 1$ TeV; PDFs for t, W, Z "partons" may be introduced at such μ

General-mass variable-flavor number scheme

- A series of factorization schemes with N_f active quark flavors in $\alpha_s(\mu)$ and $f_{a/p}(x,\mu)$
  - N_f is incremented sequentially at momentum scales $\mu_{N_f} \approx m_{N_f}$
- incorporates essential $m_{c,b}$ dependence near, and away from, heavy-flavor thresholds
- implemented in all latest PDF fits except ABM



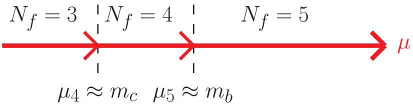


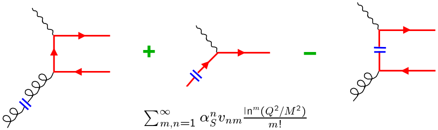
General-mass variable-flavor number scheme

Proved for *inclusive DIS* by J. Collins (1998)

$$F_2(x, Q, m_c) = \sum_a \int_\chi^1 \frac{d\xi}{\xi} C_a(\frac{\chi}{\xi}, \frac{Q}{\mu}, \frac{m_c}{Q}) f_a(\xi, \frac{\mu}{m_c}) + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

- $\lim_{Q\to\infty} C$ exists and is infrared safe
- collinear logarithms $\sum_{k,n=1}^{\infty} \alpha_s^k v_{kn} \ln^n(\mu/m_c)$ are resummed in $f_c(x, \mu/m_c)$
- no terms $\mathcal{O}(m_c/Q)$ in the remainder





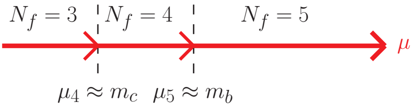
$\sum_{m,n=1}^{\infty} \alpha_S^n v_{nm} \frac{\ln^m(Q^2/M^2)}{m!}$

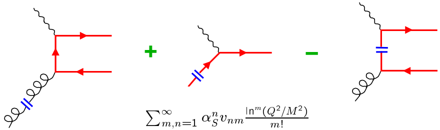
General-mass variable-flavor number scheme

Proved for *inclusive DIS* by J. Collins (1998)

$$F_2(x,Q,m_c)=\sum_a\int_\chi^1\frac{d\xi}{\xi}C_a\Big(\frac{\chi}{\xi},\frac{Q}{\mu},\frac{m_c}{Q}\Big)f_a\Big(\xi,\frac{\mu}{m_c}\Big)+\mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

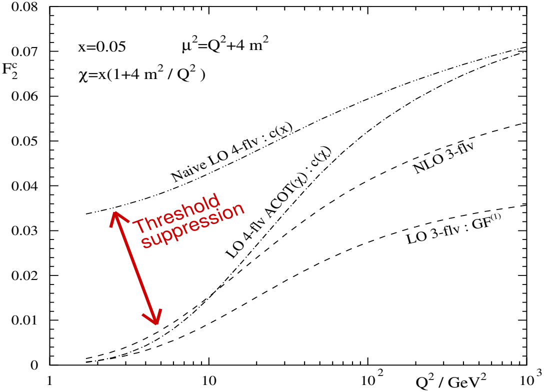
- Implemented in DIS, Drell-Yan process, pp hadroproduction, ...; practical implementation requires
  1. efficient treatment of mass dependence, rescaling of momentum fractions χ in processes with incoming c, b
  2. physically motivated factorization scale to ensure fast PQCD convergence (e.g., $\mu = Q$ in DIS)





$\sum_{m,n=1}^{\infty} \alpha_S^n v_{nm} \frac{\ln^m(Q^2/M^2)}{m!}$

Example: GM-VFN factorization scheme, DIS

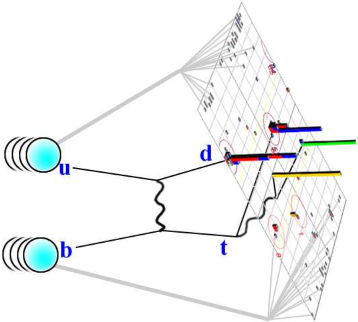


- Charm Wilson coefficient function is suppressed at $Q \to m_c$
- To keep agreement with F_2 data, $u, d, \bar{u}, \bar{d}$ PDF's are enhanced at small x, as compared to the zero-mass (ZM-VFN) scheme

Example: single-top quark production

A heavy quark testbed for QCD: single top

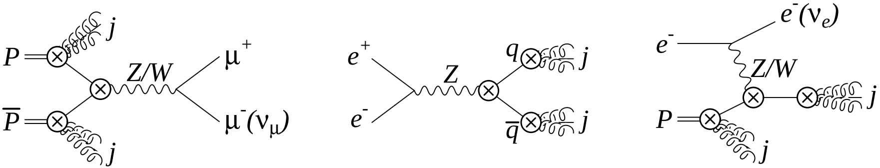
Experimentalist: Single top quark production is the observation of $b\,\ell^{\pm}\,\not{E}_T$ that reconstruct to a top quark mass, plus an extra jet (or two).



Theorist: Single top quark production is a playground in which we refine our understanding of perturbative QCD in the presence of heavy quarks.

Drell-Yan and DIS

The traditional testbed of perturbative QCD have been restricted to Drell-Yan production, e^+e^- to jets, or deeply inelastic scattering (DIS).



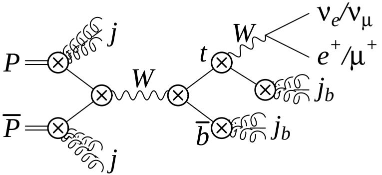
A key property that all three processes share is a complete factorization of QCD radiation between different parts of the diagrams.

- Drell-Yan → Initial-state (IS) QCD radiation only.
- e^+e^- →jets → Final-state (FS) QCD radiation only.
- DIS → Proton structure and fragmentation functions probed.

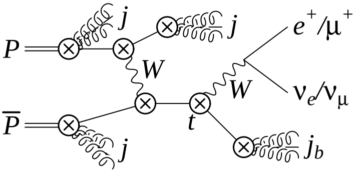
Simple color flow.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*[†] *to DY and DIS!*



Generalized Drell-Yan. IS/FS radiation are independent.



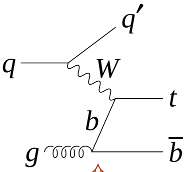
Double-DIS (DDIS) w/ 2 scales: $\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

Color conservation forbids the exchange of just 1 gluon between the independent fermion lines.

[†] Mass terms: m_t,m_b, and m_t/m_b

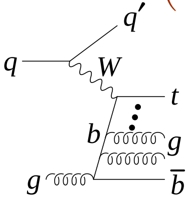
Rethinking the initial state: W-gluon fusion $\to$ t-channel single-top

W-gluon fusion (circa 1996)



$$\Uparrow$$

$$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$$



Each order adds $\frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$

Looks bad for perturbative expansion...

Look at the internal b.

The propagator is

$$\frac{1}{(P_g-P_{\bar b})^2-m_b^2} = \frac{1}{-2P_g\cdot P_{\bar b}}$$

$$P_g = E_g(1,0,0,1),\ P_{\bar b} = (E_b, \vec p_T, p_z)$$

$$P_g\cdot P_{\bar b} = E_g\left(p_z\sqrt{1+\frac{p_T^2+m_b^2}{p_z^2}} - p_z\right)$$

$$\approx E_g p_z\left(\frac{p_T^2+m_b^2}{2p_z^2}\right) \sim (p_T^2+m_b^2)$$

$$\int_{p_{T\,\mathrm{cut}}} \frac{\mathrm{d}p_T^2}{p_T^2+m_b^2} \to \ln\left(\frac{1}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$$

We now have multiple scales entering the problem:

$Q, m_t, m_b, p_{T\,\mathrm{cut}}$.

$m_t \approx 35 m_b$! $\quad \alpha_s \ln \sim .7$-$.8$

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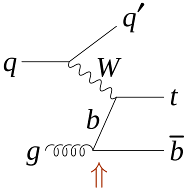

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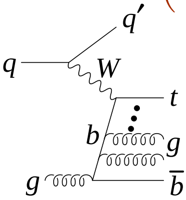

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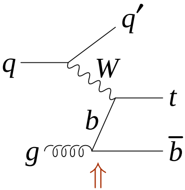

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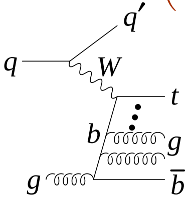

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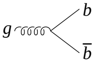
Resummation of large logs and b-quark PDF

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation sums large logs in (almost) collinear singularities in gluon splitting.

$$\frac{db(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{bg} \otimes g + \frac{\alpha_s}{2\pi} \cancel{P_{bb} \otimes b}; \qquad b \ll g$$



$$P_{bg}(z) = \tfrac{1}{2}[z^2 + (1-z)^2]$$

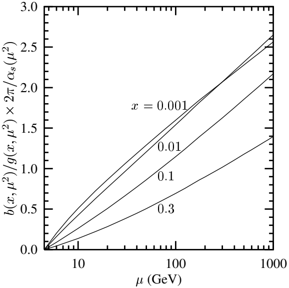
$$b(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}(z) g\left(\frac{x}{z},\mu^2\right)$$

Barnett, Haber, Soper, NPB 306, 697 (88)

Olness, Tung, NPB 308, 813 (88)
Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94)

The procedure is the same for c or t.

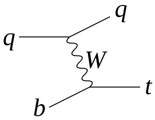
$$b \propto \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g + \mathcal{O}(\alpha_s^2)$$



Stelzer, ZS, Willenbrock,
PRD 56, 5919 (1997)

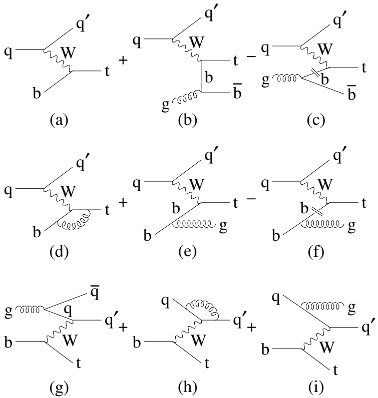
Improved perturbation theory with b-quark PDF

New Leading Order diagram



⇑

$$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$$



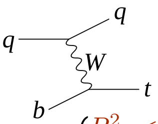
$\frac{1}{\ln(m_t^2/m_b^2)}$

α_s

NLO: Terms that generated large logs are already resummed.
⇒ Must subtract overlap to avoid double-counting (general issue)

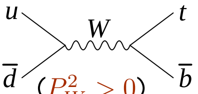
The procedure is repeated at higher α_s orders

Classification of scattering channels



$(P_W^2 < 0)$

t-channel production:
"t-channel"
exchange of a W boson.

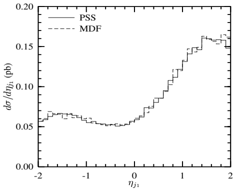


$(P_W^2 > 0)$

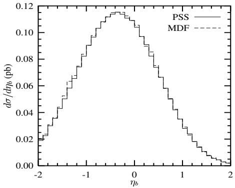
vs.

s-channel production:
"s-channel"
exchange of a W boson.

Classifying processes by analytical structure leads to kinematic insight: Jets from t-channel processes are more forward than those from s-channel.



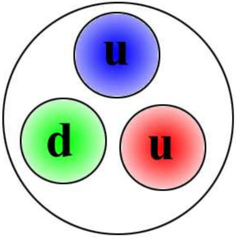
jet from t-channel



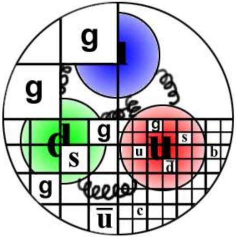
b jet from s-channel

Rethinking the proton

$\mu^2 < 1$ GeV^2



$\mu^2 \gg 1$ GeV^2

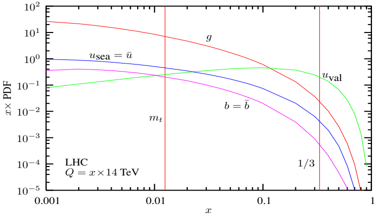


Using DGLAP was NOT just a math trick!

The "valence" uud picture of the proton at $\mu < 1$ GeV is not complete.

Larger energies resolve smaller structures.

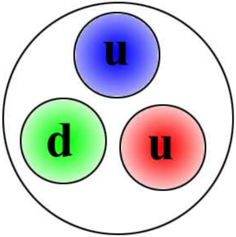
The probability of finding a particle inside the proton is given by a Parton Distribution Function



b and c quarks are full-fledged members of the nucleon structure. **What about t?**

Rethinking the proton

$\mu^2 < 1$ GeV^2



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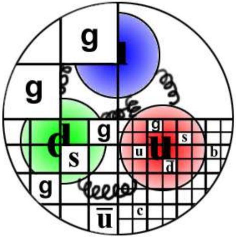

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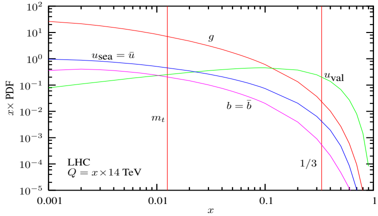

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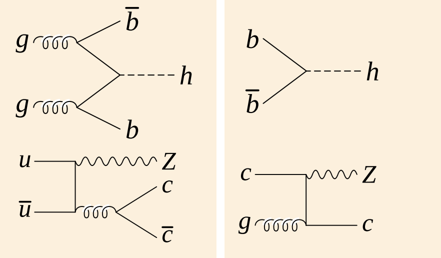
Logarithms in other heavy-flavor processes

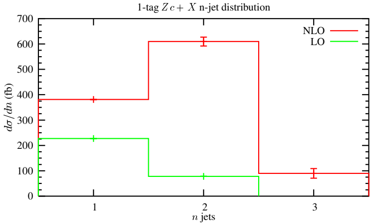
Leading α_s order:

$\mu^2 \approx m_q^2$	$\mu^2 \gg m_q^2$



Why is this important?

Zc at Tevatron



Parton luminosity and large logs modify the counting of powers of α_s!

Starting with a c/b gives us:

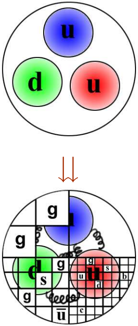
$b\bar{b} \to h$	**Largest** SUSY Higgs cross section
Zb/Zc	Enhanced rates for BSM bkgds.
Zbj/Zcj	Higgs background
Wbj	Largest single-top background

etc.

Summary Day 1

“Heavy quarks” (c, b, and t) are interesting because their mass adds a new scale to any problem.

$$\sigma \sim \alpha_s \ln\left(\frac{\mu^2}{p^2_{T\,\mathrm{cut}}+m_Q^2}\right)$$



$\Rightarrow$ Far more complex PQCD calculations than in the massless case

$\Rightarrow$ Perturbative convergence is improved by working in the appropriate heavy-quark factorization scheme

c/b PDFs at NNLO (M. Guzzi’s lecture)

Homework: Why do we not see $J/\psi \to gg$?

Homework: Which statement in the quiz was correct?