Heavy Quark Theory

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- What is a heavy quark? Why study them especially?
- What theoretical methods are used?
- What is the meaning of 3-flavor, 4-flavor (. . .) coupling and parton densities? Why?
- What are they needed for?

Material today continues Zack Sullivan's lecture.

[*Warning:* I am only selectively examining the basics of a big subject.]

What's a heavy quark?

$[m_c(\sim 1.3\,\mathrm{GeV}),\ m_b(\sim 4.2\,\mathrm{GeV}),\ m_t(\sim 173\,\mathrm{GeV})]$

- $\alpha_s(M)$ is in perturbative region (defining property)
- Hence certain kinds of system perturbative calculation ($\Longrightarrow$ predictions) can be made that are not possible for light quarks and gluons pair production.
- Variations from basic QCD-improved parton framework.
- Decoupling of quarks of mass much heavier than scale Q of process.

  [Leads to simplifications in regions with 3, 4, 5, . . . active quark flavors.]

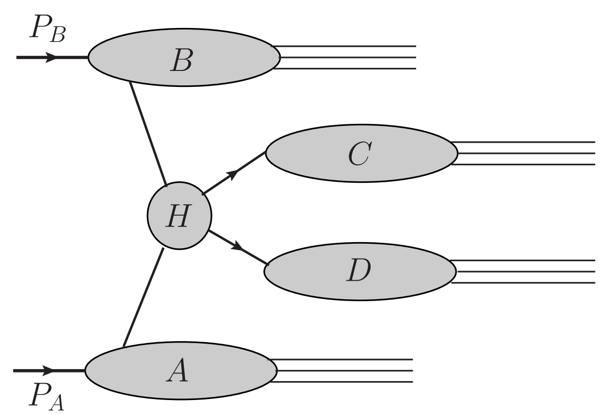
  [Enables application of theory to processes at some scale without worrying about particles, especially undiscovered ones and much higher scales.]

Overview of regions of process scale w.r.t. quark mass

Given process or subprocess with scale Q and a quark of mass M, what happens in different ranges of Q?

quark mass v. scale Q	effect	can be "parton"
$Q \ll M$	Decouples (. . .)	No
$Q \sim M$	Must preserve M	Sort-of-no
$Q \gg M$	$M \to 0$ useful	Yes

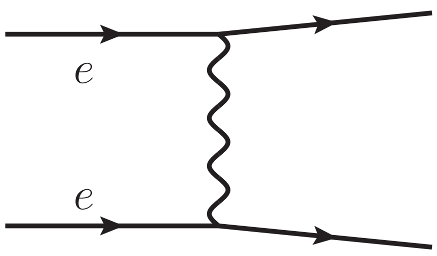
Factorization structure with hard scale $Q \sim p_T$ e.g.,



$$d\sigma_{\text{had}} = \sum \int \text{(pdf(s)) (ff(s))}\ d\hat{\sigma}_{\text{partonic, hard}}\, d(\text{partonic variables})$$

Elementary example of decoupling

ee scattering with γ and Z exchange:



$$\text{EM: } \frac{e^2}{q^2} \qquad \text{WI: } \frac{e^2 \times \text{few}}{q^2 - m_Z^2}$$

When $|q^2| \ll m_Z^2$, there is a power suppression:

$$\left|\frac{\text{WI}}{\text{EM}}\right| \sim \frac{|q^2|}{m_Z^2}$$

This applies always at low s always. Also, at low angles even at high energies.

- Hence, we *expect* that to leading power one can drop the heavy field(s).
- And we can analyze lower scale phenomena and theory without being required to know about undiscovered heavier particles/fields.
- For QCD we have effective field theories $\widehat{\mathrm{QCD}}_3$ (u, d, s), $\widehat{\mathrm{QCD}}_4$ (u, d, s, c), etc.

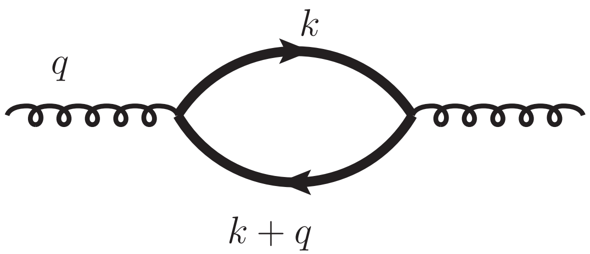
Is it really true . . .

- That effective field theory (EFT) $\widehat{\text{QCD}}_{n \text{ active flavors}}$ is obtained simply by dropping the $6 - n$ inactive flavors?
- That there is just one characteristic scale for a given process?
- That use of EFT is the best method?

Answers, with basic reasons:

- No: There are UV divergences in QFT: All scales to infinity matter. So something fancier is needed.
- No: See elastic scattering example.
- No: EFT alone is too limited . . . .

Unsuppressed effects when $M^2 \gg Q^2$ ($\overline{\mathrm{MS}}$ renormalization)



$$\propto g^2 \int \frac{\mathrm{tr}\, \gamma^\mu (\not{k} + M) \gamma^\nu (\not{q} + \not{k} + M)}{(k^2 - M^2)\ [(k+q)^2 - M^2]} \frac{\mathrm{d}^4 k}{(2\pi)^4} + \text{c.t.}$$

$$\propto (q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{\alpha_s}{\pi} \int_0^1 x(1-x) \ln \frac{M^2 - q^2 x(1-x)}{\mu^2} \, \mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu)\ \frac{\alpha_s}{6\pi} \ln \frac{M^2}{\mu^2} + \text{power-suppressed}$$

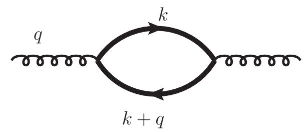
when $|q^2| \ll M^2$. This is *not* suppressed when $M^2 \gg |q^2|$.

Add in light-quark graph. Mass m, with $m^2 \ll |q^2|$:

$$(q^2 g^{\mu\nu} - q^\mu q^\nu)\ \frac{\alpha_s}{6\pi} \left[\ln \frac{q^2}{\mu^2} + \text{constant} \right]$$

So no single choice of $\overline{\mathrm{MS}}$ μ eliminates large logarithms for sum of both heavy and light-quark graphs when $m^2 \ll |q^2| \ll M^2$.

Unsuppressed term v. renormalization counterterm

- Unsuppressed part of 

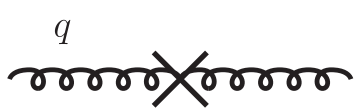
is $\quad (q^2 g^{\mu\nu} - q^\mu q^\nu)\, \dfrac{\alpha_s}{6\pi} \ln \dfrac{M^2}{\mu^2}$

- Insight: Same momentum dependence as renormalization counterterm.

- Relevant piece of renormalization theory:
  - In Lagrangian of QCD with bare fields, there's a term $-\frac{1}{4}(\partial_\mu A_\nu^{(0)} - \partial_\nu A_\mu^{(0)})^2$
  - Rewrite from bare field to renormalized field by $A_\mu^{(0)} = Z_3^{1/2} A_\mu$, and get

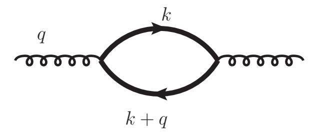
$$\underbrace{-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2}_{\text{in } \textit{free } \mathcal{L}} \qquad \underbrace{-\frac{1}{4}(Z_3 - 1)(\partial_\mu A_\nu - \partial_\nu A_\mu)^2}_{\text{in } \textit{interaction } \mathcal{L}}$$

  - One-loop renormalization implemented by counterterm graph:



$= (q^2 g^{\mu\nu} - q^\mu q^\nu) \times (q\text{-independent coefficient})$

General result: Decoupling theorem

Hence unsuppressed part of 

corresponds to where UV renormalization needed. It can be removed by change of counterterm, and hence by change of renormalization prescription.

This completely generalizes to the decoupling theorem (Appelquist & Carazzone)

> Let Q be the maximum external momentum scale of the processes considered, and let the full theory have a field/particle of much larger mass M. Then to leading power in M/Q, equivalent results are obtained from an EFT obtained by
>
> - Deleting the large mass fields.
> - Adjusting the parameters of the theory. ("Matching")

Relationship of couplings:

$$\frac{\alpha_s^{(3)}}{4\pi} = \frac{\alpha_s^{(4)}}{4\pi} \left[1 + \frac{\alpha_s^{(4)}}{6\pi} \ln \frac{M^2}{\mu^2} + \left(\frac{\alpha_s^{(4)}}{\pi} \right)^2 \times (\ldots) + \ldots \right]$$

N.B. One-loop term is (indirectly) obtained from gluon self energy . . . .

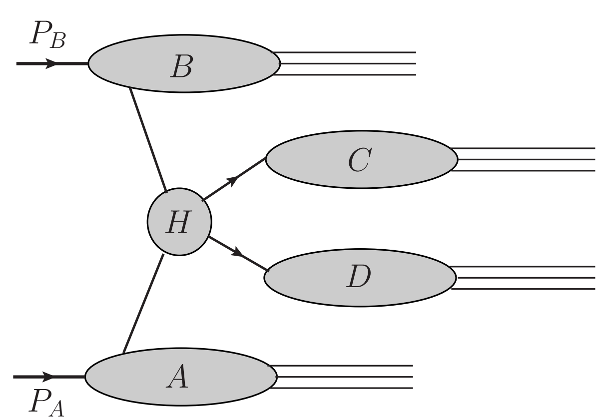
Simple EFT view is not good enough

Simple method:

Going up in scale of process, successively use 3-, 4-, . . . flavor EFT versions of QCD.

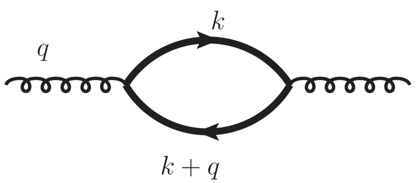
But:

- Multiple scales in process. E.g., in jet production, both p_T of jet (e.g., 100s of GeV or more) and intrinsic phenomena in beams at $0.3\,\mathrm{GeV}$.
- Shouldn't treat bottom quark as incoming parton in DIS at $4\,\mathrm{GeV}$.
- But can treat bottom quark as incoming parton in jet production at $1000\,\mathrm{GeV}$.



First step: CWZ (Collins-Wilczek-Zee) for renormalization, coupling

Stay in full theory, but for "inactive" quarks, use zero-momentum subtraction:



$$+\,\text{c.t.} \;\propto\; (q^2 g^{\mu\nu} - q^\mu q^\nu)\frac{\alpha_s}{\pi}\int_0^1 x(1-x)\ln\frac{M^2 - q^2 x(1-x)}{M^2}\,\mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu)\,\frac{\alpha_s}{6\pi}\,O\!\left(\frac{q^2}{M^2}\right) \qquad \text{when } |q^2| \ll M^2$$

Use $\overline{\mathrm{MS}}$ for everything else.

Key properties:

- Single theory QCD_6
- "Manifest decoupling"
- Automatically preserves gauge-invariance of QCD
- RG and DGLAP equations are same (mass-independent) as in the EFT approach.

Statement of CWZ

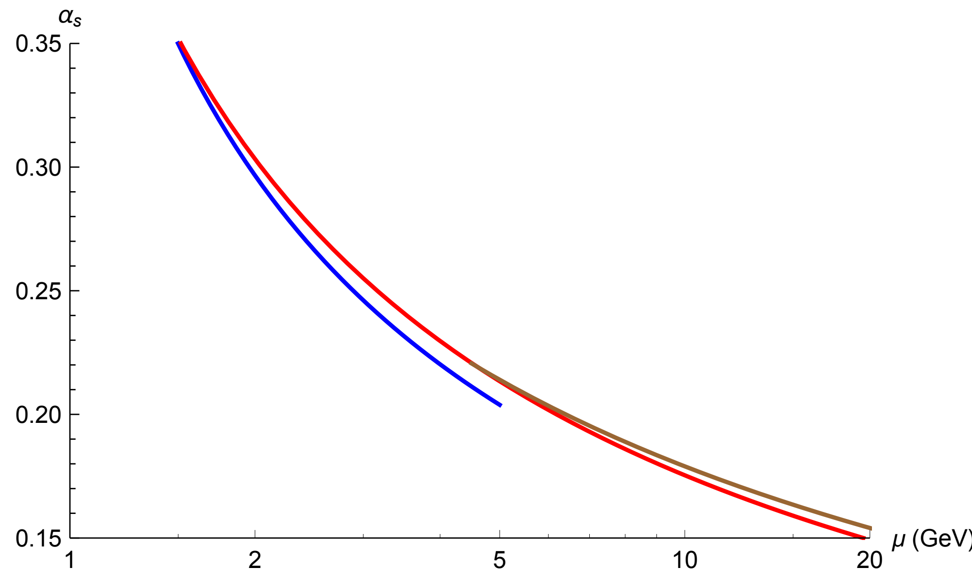
Technical definition:

- Keep all (known or relevant) quarks in theory
- Define a sequence of subschemes with 3, 4, 5, etc “active” flavors. $[(u, d, s)$, (u, d, s, c), etc]
- $\overline{\mathrm{MS}}$ for active flavors, zero-momentum subtraction for graphs with inactive flavors.
- Obtain relations of coupling, etc between subschemes by matching

Adjust choice of $\#$ of active flavors by the following principles:

- At scale Q, quarks with $M \ll Q$ are active.
- Quarks with $M \gg Q$ are inactive.
- Overlapping ranges of usefulness for $m \sim Q$.

What are 3-, 4-, 5-, . . . flavor versions of α_s? (Pdfs later)



RGE:

$$\frac{\mathrm{d}\alpha_s/(4\pi)}{\mathrm{d}\ln\mu^2} = \beta\Big(\frac{\alpha_s}{4\pi}, n_{\mathrm{act}}\Big) = -\left(11 - \frac{2}{3}n_{\mathrm{act}}\right)\Big(\frac{\alpha_s}{4\pi}\Big)^2 - \dots$$

Matching, from calculation of relevant graphs:

$$\frac{\alpha_s^{(3)}}{4\pi} = \frac{\alpha_s^{(4)}}{4\pi}\left[1 + \frac{\alpha_s^{(4)}}{6\pi}\ln\frac{M^2}{\mu^2} + \left(\frac{\alpha_s^{(4)}}{\pi}\right)^2 \times (\dots) + \dots\right]$$

Λ_{QCD} for different numbers of active flavors (PDG values)

$$\begin{array}{ll} \Lambda^{(3)}_{\overline{\mathrm{MS}}} & 332 \pm 17\,\mathrm{MeV} \\ \Lambda^{(4)}_{\overline{\mathrm{MS}}} & 292 \pm 16\,\mathrm{MeV} \\ \Lambda^{(5)}_{\overline{\mathrm{MS}}} & 210 \pm 14\,\mathrm{MeV} \\ \Lambda^{(6)}_{\overline{\mathrm{MS}}} & 89 \pm 6\,\mathrm{MeV} \end{array}$$

It is $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ that is related to scale of non-perturbative physics.

What about pdfs, hard scattering etc?

I'll use charm-quark effects in *fully inclusive* DIS as example.

- Important special cases:

  $Q \gg m_c$**:** 4 active flavors, and must include use of $f_{c/p}$, with hard scattering on c-quark parton.

  $Q \lesssim m_c$**:** Use of $f_{c/p}$ seems inappropriate.

- Both regions can occur in a single experiment. So we'll use full theory, QCD$_6$, but view it differently for different cases (cf. CWZ).

- We'll find a way (ACOT) to use a variable number of active flavors, with corresponding pdfs.

- *Warnings*:

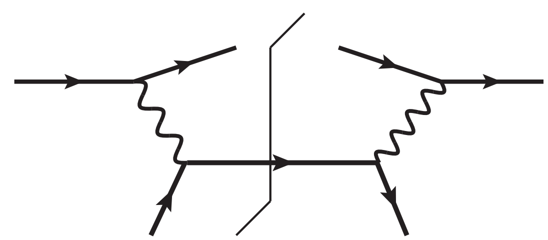
  – Some of the literature on factorization and heavy quarks is conceptually confused (at least)!

  – Q itself is not necessarily exactly the right scale.

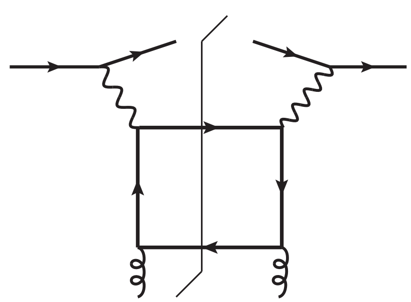
  – Cross section restricted to charm hadrons in final state needs further discussion.

Overview of charm in DIS at $Q \gg$ few GeV, 4 active flavors

- Factorization, pdfs, etc:
  - Standard treatment of factorization says we need c quark as parton, since it can have collinear kinematics.
  - So we include c pdf term

$f_c \otimes$  $\Big|_{\text{on-shell quark}}$

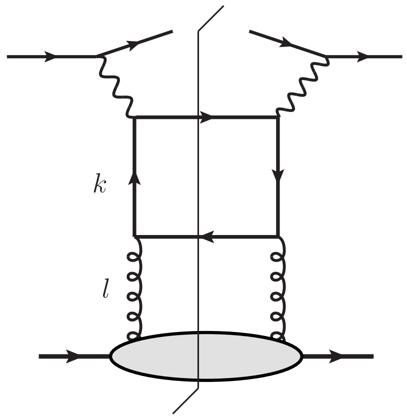
  - Also have subtracted photon-gluon-fusion term, as usual:

$f_g \otimes \Big[$  $+$ etc $-$ subtraction for c-in-g pdf $\Big]_{\text{gluon on-shell}}$

  - [Other subprocesses, NLO, NNLO, . . .]
  - Can keep m_c in hard scattering, for initial g, u, d, s.
- Value of charm density: Perturbative estimate in terms of gluon density (etc).

4 active flavors: m_c and hard scattering on gluon

Photon-gluon fusion term is from where gluon obeys $l_T \ll Q$ in



$+$ etc $-$ subtraction for c-in-g pdf

- Integral over charm k_T in first term with neglect of l_T is

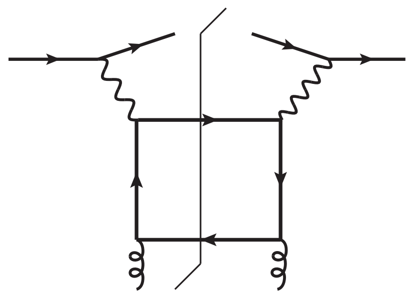
$$\alpha_s k^+/l^+\text{-dependent factor} \times \int \mathrm{d}k_T^2 \begin{cases} \text{smoothly cut-off function} : k_T \gtrsim Q, \\ \dfrac{1}{m_c^2 + k_T^2} : k_T \ll Q \end{cases}$$

- When $m_c \ll k_T \ll Q$, there's a factor $1/k_T^2$, and hence $\ln(Q^2/m_c^2)$ in integral.
- But $k_T \ll Q$ is region of collinear c-quark, already in LO c-induced term (with non-neglected l_T!).
- Hence subtraction needed.

4 active flavors: subtraction in γg hard scattering

Subtraction in γg hard scattering,

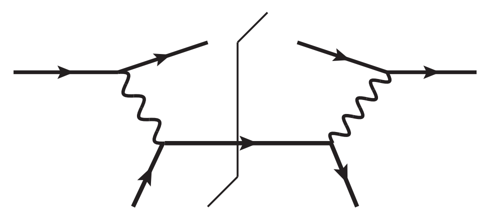
$\Bigg[$



$+$ etc $-$ subtraction for c-in-g pdf $\Bigg]_{\text{gluon on-shell}}$

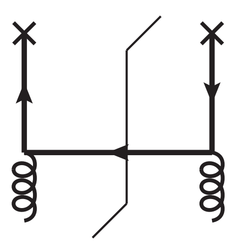
is to stop double counting of contribution included in LO term

$f_c \otimes$

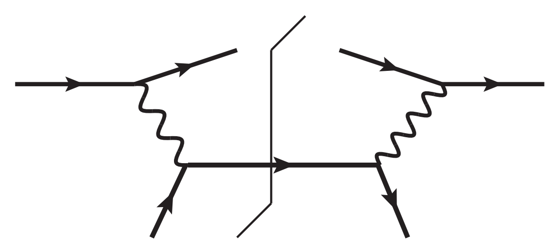


$\Big|_{\text{on-shell quark}}$

Subtraction term in order α_s photon-gluon hard scattering is



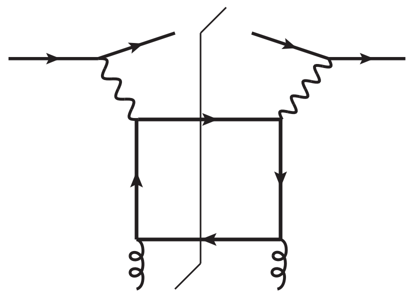
$\Big|_{\text{gluon on-shell}} \otimes$



$\Big|_{\text{on-shell quark}}$

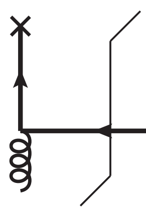
Integral for subtraction term

Subtraction for



$$\Big[\ \cdots\ + \text{ etc } - \text{ subtraction for } c\text{-in-}g \text{ pdf}\Big]_{\text{gluon on-shell}}$$

has factor for one-loop pdf of c in (on-shell) gluon:



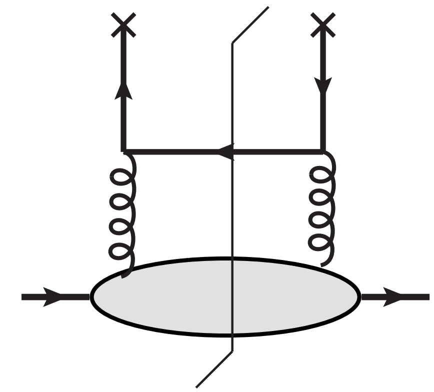
$$\Big|_{\text{gluon on-shell}} \propto \alpha_s \int_0^\infty \mathrm{d}k_T^2 \left[\frac{1-2\xi(1-\xi)}{k_T^2+m_c^2} + \text{convergent}\right] + \overline{\mathrm{MS}} \text{ counterterm}$$

$$\propto \alpha_s[\xi^2+(1-\xi)^2]\ln\frac{M^2}{\mu^2}$$

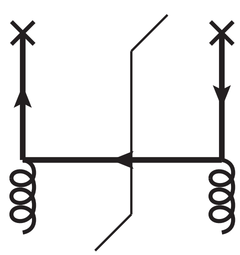
with $\xi = k^+/l^+$.

Heavy-quark pdfs are from perturbative short distance effects

An important Feynman graph for c (etc) pdf in proton:



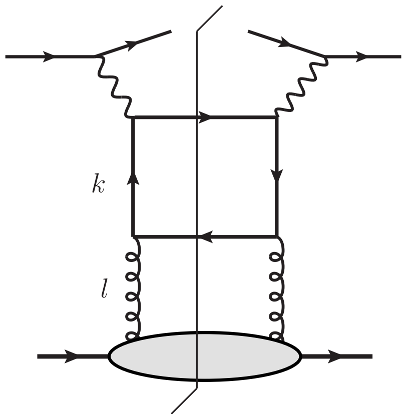
Leading approximation:

- Gluon of low l_T
- Get $f_c = f_g \otimes$  $\Big|_{\text{gluon on-shell}}$

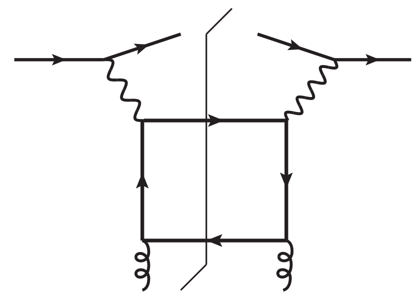
But with pure 4-flavor scheme there are important charm-quark effects inside f_g, including its DGLAP evolution.

ACOT solve this by obtaining 4-flavor pdfs (including f_c) from 3-flavor pdfs.

Charm in DIS at $Q =$ few GeV: 3 active flavors



$+$ etc $= f_g \otimes \Big[$



$+$ etc $\Big]_{\text{gluon on-shell}}$

- Motivation for use of LO scattering on c quark lost. Therefore omit.
- Then charm generated dynamically in hard scattering only
- No gluon-to-$c\bar{c}$ collinear region nor divergence.
- So, there is no subtraction in hard scattering, unlike light-quark case

ACOT: To do this consistently, use 3-flavor CWZ including for pdfs.

In particular, would-be subtraction by c in gluon is zero. Generally $f_{c/p}^{(3)}$ is power suppressed by power of Λ/m_c.

Overall view for factorization of hard process

With $n_{\rm act}$ $(= 3, 4, \ldots)$ active flavors:

- The active flavors:
  - are the $n_{\rm act}$ lightest quarks,
  - have masses (well) below Q
  - have pdfs, which evolve normally.
- The inactive flavors
  - are the heavier quarks
  - are only generated in the hard scattering
- Masses can be preserved in hard scattering

ACOT implementation: Apply CWZ idea to pdfs and factorization, etc

3-flavor	Evolution: u, d, s only Usual 3-flavor DGLAP	c pdf suppressed by $(\Lambda/m_c)^p$, and not used
4-flavor	Evolution: u, d, s, c Usual 4-flavor DGLAP Start $f_{c/p}^{(4)}$ at $\mu \simeq m_c$ from calculated matching to $f_{c/p}^{(3)}$	Usually neglect $f_{c/p}^{(3)}$ in matching. ($Pace$ Brodsky & intrinsic charm).

ETC

Summary

Basics:

- Heavy quarks, i.e., with masses in perturbative region, allow simplifications, and extra perturbative predictions c.w. light quarks.
- Simplest methods involve decoupling theorem and EFTs
- Fancier methods (CWZ/ACOT) allow keeping heavy quarks in the theory, without penalty of large logarithms in calculations
- Get concept of number of "active" partonic quarks
- See the vast literature for a range of views

But we need more work:

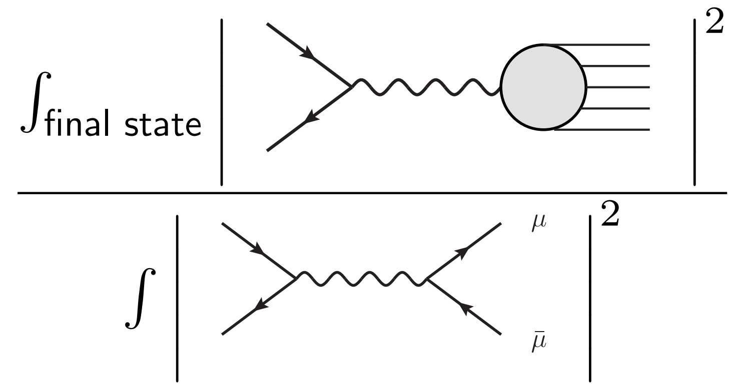
- Interesting processes have lots of different scales. E.g., $\sqrt{s}$, $P_{T,\text{jet}}$, jet width, relative momenta of components of events.
- Measurement of heavy hadrons (e.g., D-meson) in final state messes up rationale of ACOT, when heavy hadron is not strongly relativistic (i.e., not in a jet).

EXTRA SLIDES

Elementary context for decoupling theorem: $e^+e^- \to$ hadrons

Define

$$R = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-, \text{LO, em})} =$$



Perturbatively calculable in powers of $\alpha_s(Q)$. (Cf. Soper's lectures.)

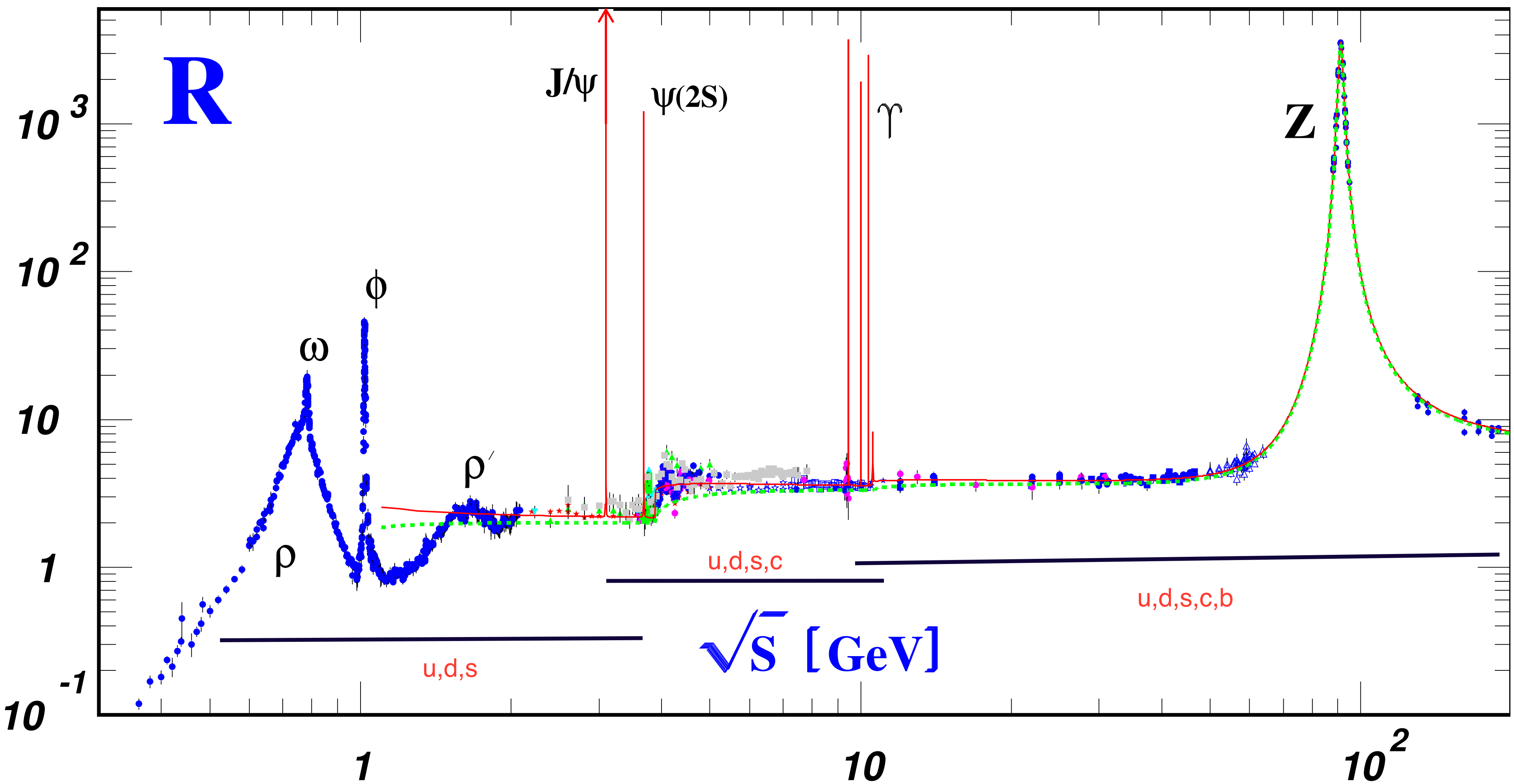
First term is sum of squares of charges of *active* quarks, with factor of number of colors:

$$R = 3 \sum_q e_q^2 + O(\alpha_s(Q))$$

LO values:

$$R_{\text{LO}} = \begin{cases} 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2 & (u, d, s) \\ 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = 3\frac{1}{3} & (u, d, s, c) \\ 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = 3\frac{2}{3} & (u, d, s, c, b) \end{cases}$$

R for $e^+e^- \to$ hadrons



From PDG

Green dotted line: LO prediction, with quark masses. Red solid line: QCD prediction.