SCALAR FIELDS, BLACK HOLES AND SPHERICAL COORDINATES

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SUPERRADIANCE

- Phenomenon of enhancement of radiation
- Appears in several fields of physics (including BH physics)
- Can be used to extract rotational energy from a BH

$$\omega - m\Omega > 0$$

SUPERRADIANT INSTABILITIES

- Massive bosons are unstable near spinning black holes
- The instability leads to an exponential growth in the fields, reaching up to 10% of the mass of the BH
- Looking for signals of these instabilities can be used as an indirect probe of the existence of such bosons

NUMERICAL RELATIVITY

- Einstein's field equations are hard to solve analytically
- It is useful to perform numerical simulations
- NR is the field that concerns itself with numerical solution of Einstein's equations
- Finding suitable evolution equations and initial data is very important

SCALAR FIELD SUPERRADIANCE IN NUMERICAL RELATIVITY

- Write a Lagrangian that couples GR to a scalar Klein-Gordon field
- Use the Euler-Lagrange equations to extract the equations of motion
- Use a 3+1 decomposition to write evolution equations and constraints

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^* \partial_\nu \Phi - \frac{1}{2} \mu_S^2 \Phi^* \Phi - V(\Phi) \right)$$

3+1 DECOMPOSITION OF SPACETIME

- Evolution equations are covariant, meaning that time and space are treated equally
- There is a need for a direction of numerical evolution
- We foliate spacetime into 3D spacelike slices and identify the slices as level sets of a parameter t
- We evolve in t

$$ds^2 = -\left(\alpha^2 - \beta_i\beta^i\right)dt^2 + 2\gamma_{ij}\beta^i dt\, dx^j + \gamma_{ij}dx^i dx^j$$

SPHERICAL COORDINATES IN NUMERICAL RELATIVITY

Cartesian Coordinates

- Regular everywhere (no coordinate singularities)
- Over-resolve angular directions
- Need for mesh refinement infrastructures, which introduce numerical errors

Spherical Coordinates

- Adapted to the (approximate) spherical symmetry of the problems
- Number of cells per unit angle is constant
- Coordinate singularities at the origin and at $\theta = 0, \pi$
- Require much smaller time steps in the evolution

GOALS OF THE WORK

- Writing a 3+1 code in spherical coordinates for the evolution of scalar fields
- Using the code for numerical evolution in curved spacetimes
- Learning how to use the Einstein Toolkit and coupling the evolution code to this infrastructure
- Using the code to study systems of scalar fields coupled to BH spacetimes