SUPERRADIANCE

- Phenomenon of enhancement of radiation
- Appears in several fields of physics (including BH physics)
- Can be used to extract rotational energy from a BH

\[ \omega - m\Omega > 0 \]
SUPERRADIANT INSTABILITIES

• Massive bosons are unstable near spinning black holes
• The instability leads to an exponential growth in the fields, reaching up to 10% of the mass of the BH
• Looking for signals of these instabilities can be used as an indirect probe of the existence of such bosons
Einstein’s field equations are hard to solve analytically

It is useful to perform numerical simulations

NR is the field that concerns itself with numerical solution of Einstein’s equations

Finding suitable evolution equations and initial data is very important
SCALAR FIELD SUPERRADIANCE IN NUMERICAL RELATIVITY

• Write a Lagrangian that couples GR to a scalar Klein-Gordon field

• Use the Euler-Lagrange equations to extract the equations of motion

• Use a 3+1 decomposition to write evolution equations and constraints

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^* \partial_\nu \Phi - \frac{1}{2} \mu_5^2 \Phi^* \Phi - V(\Phi) \right) \]
3+1 DECOMPOSITION OF SPACETIME

- Evolution equations are covariant, meaning that time and space are treated equally.
- There is a need for a direction of numerical evolution.
- We foliate spacetime into 3D spacelike slices and identify the slices as level sets of a parameter $t$.
- We evolve in $t$.

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\gamma_{ij}\beta^i dt dx^j + \gamma_{ij} dx^i dx^j$$
SPHERICAL COORDINATES IN NUMERICAL RELATIVITY

Cartesian Coordinates

• Regular everywhere (no coordinate singularities)
• Over-resolve angular directions
• Need for mesh refinement infrastructures, which introduce numerical errors

Spherical Coordinates

• Adapted to the (approximate) spherical symmetry of the problems
• Number of cells per unit angle is constant
• Coordinate singularities at the origin and at $\theta = 0, \pi$
• Require much smaller time steps in the evolution
GOALS OF THE WORK

• Writing a 3+1 code in spherical coordinates for the evolution of scalar fields
• Using the code for numerical evolution in curved spacetimes
• Learning how to use the Einstein Toolkit and coupling the evolution code to this infrastructure
• Using the code to study systems of scalar fields coupled to BH spacetimes