From transition electromagnetic form factors $\gamma^*\gamma^*\eta_c(1S, 2S)$ to the production of $\eta_c(1S, 2S)$ at the LHC

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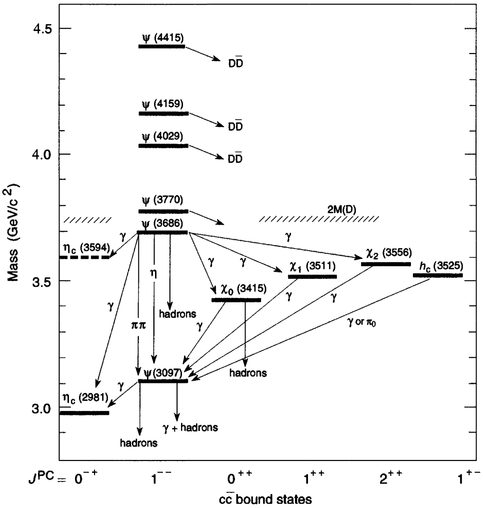
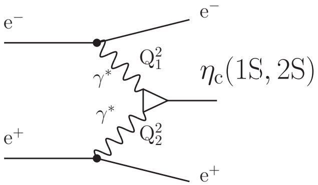
Contents

Introduction



Description of the mechanism $\gamma^*\gamma^* \to \eta_c(1S, 2S)$

Babiarz, Goncalves, Pasechnik, Schäfer and Szczurek,
Phys. Rev. **D100**, 054018 (2019).

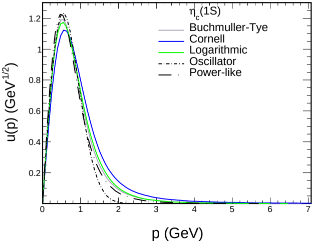


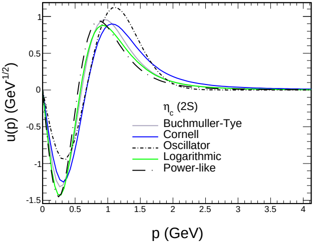
$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)$$

Light-front representation of the transition form factor:

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3}\psi(z,\mathbf{k})$$
$$\left\{\frac{1-z}{(\mathbf{k}-(1-z)\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} + \frac{z}{(\mathbf{k}+z\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2}\right\}.$$

Nonrelativistic quarkonium wave functions





Radial momentum-space wave function for different potentials.

Radial spatial wave function are obtained by solving the Schrödinger equation.

J. Cepila, J. Nemchik, M. Krelina and R. Pasechnik, arXiv:1901.02664 [hep-ph].

$$\frac{\partial^2 u(r)}{\partial r^2} = (V_{\rm eff}(r) - \epsilon)u(r)\,, \qquad u(r) = \sqrt{4\pi}\, r\psi(r)\,,$$

$$\int\limits_0^\infty |u(r)|^2 dr = 1 \quad \Rightarrow \quad \int_0^\infty |u(p)|^2 dp = 1$$

Light-front wave functions

We treat the η_c as a bound state of a charm quark and antiquark, assuming that the dominant contribution comes from the $c\bar{c}$ component in the Fock-state expansion:

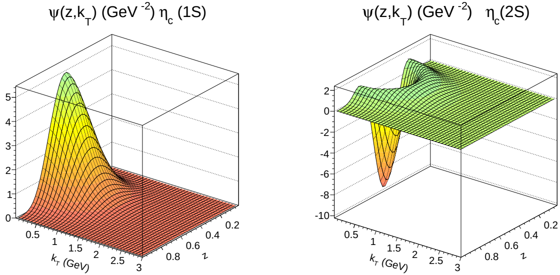
$$|\eta_c; P_+, \boldsymbol{P}\rangle = \sum_{i,j,\lambda,\bar{\lambda}} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \Psi_{\lambda\bar{\lambda}}(z,\boldsymbol{k}) |c_{i\lambda}(zP_+, \boldsymbol{p}_c) \bar{c}^j_{\bar{\lambda}}((1-z)P_+, \boldsymbol{p}_{\bar{c}})\rangle + \dots \tag{1}$$

Here the c-quark and $\bar{c}$-antiquark carry a fraction z and $1-z$ respectively of the η_c's plus-momentum. The light-front helicites of quark and antiquark are denoted by $\lambda, \bar{\lambda}$, and take values ± 1. The transverse momenta of quark and antiquark are

$$\boldsymbol{p}_c = \boldsymbol{k} + z\boldsymbol{P}\,, \quad \boldsymbol{p}_{\bar{c}} = -\boldsymbol{k} + (1-z)\boldsymbol{P}\,. \tag{2}$$

The light-cone representation is obtained by Terentev's prescription valid for weakly bound systems.

Light-front wave functions



Radial light-front wave function for Buchmüller-Tye potential.

Terentev prescription $\Rightarrow \mathbf{p} = \mathbf{k}, \quad p_z = (z - \frac{1}{2})M_{c\bar{c}},$

$$\psi(z, \mathbf{k}) = \frac{\pi}{\sqrt{2M_{c\bar{c}}}} \frac{u(p)}{p} .$$

$F(0,0)$ transition for both on-shell photons

$$F(0,0) = e_c^2 \sqrt{N_c}\, 4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \frac{\psi(z,\mathbf{k})}{\mathbf{k}^2 + m_c^2} \,,$$

$F(0,0)$ is related to the two-photon decay width by the formula:

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{\pi}{4} \alpha_{\rm em}^2 M_{\eta_c}^3 \, |F(0,0)|^2 \,.$$

$F(0,0)$ can be rewrite in the terms of radial momentum space wave function $u(p)$:

$$F(0,0) = e_c^2 \sqrt{2N_c} \, \frac{2m_c}{\pi} \int_0^\infty \frac{dp\, p\, u(p)}{\sqrt{M_{c\bar{c}}^3}(p^2 + m_c^2)} \frac{1}{2\beta} \log\left(\frac{1+\beta}{1-\beta}\right) \,,$$

In the non-relativistic (NR) limit, where $p^2/m_c^2 \ll 1, \beta \ll 1$, and $2m_c = M_{c\bar{c}} = M_{\eta_c}$, we obtain

$$F(0,0) = e_c^2 \sqrt{N_c}\sqrt{2} \frac{4}{\pi\sqrt{M_{\eta_c}^5}} \int_0^\infty dp\, p\, u(p) = e_c^2 \sqrt{N_c} \frac{4\, R(0)}{\sqrt{\pi M_{\eta_c}^5}} \,,$$

where $\beta = \frac{p}{\sqrt{p^2+m_c^2}}$, the velocity v/c of the quark in the $c\bar{c}$ cms-frame and R(0) radial wave function at the origin.

$F(0,0)$ for both on-shell photons

Transition form factor $|F(0,0)|$ for $\eta_c(1S)$ at $Q_1^2 = Q_2^2 = 0$.

potential type	m_c [GeV]	$\lvert F(0,0)\rvert$ $[\mathrm{GeV}^{-1}]$	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.051	2.89	0.2757
logarithmic	1.5	0.052	2.95	0.3373
power-like	1.334	0.059	3.87	0.3074
Cornell	1.84	0.039	1.69	0.3726
Buchmüller-Tye	1.48	0.052	2.95	0.3276
experiment	-	0.067 ± 0.003 [1]	5.1 ± 0.4 [1]	0.335 ± 0.075 [2]

[1] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no.3, 030001 (2018).
[2] K. W. Edwards *et al.* [CLEO Collaboration], Phys. Rev. Lett. **86**, 30 (2001) [hep-ex/0007012].

R(0) and $\gamma\gamma$-width for $\eta_c(1S)$ derived in **the non-relativistic limit**.

potential type	R(0)$[\mathrm{GeV}^{3/2}]$	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = \mathrm{M}_{\eta_c}$	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = 2\mathrm{m_c}$
harmonic oscillator	0.6044	5.1848	5.8815
logarithmic	0.8919	11.290	11.157
power-like	0.7620	8.2412	10.297
Cornell	1.2065	20.660	13.568
Buchmüller-Tye	0.8899	11.240	11.409

$$f_{\eta_c}\varphi(z,\mu_0^2) = \frac{1}{z(1-z)}\frac{\sqrt{N_c}\,4m_c}{16\pi^3}\int d^2\mathbf{k}\,\theta(\mu_0^2-\mathbf{k}^2)\,\psi(z,\mathbf{k}) \text{ and } \int_0^1 \mathrm{dz}\,\varphi(\mathrm{z},\mu_0^2) = 1$$

$F(0,0)$ for both on-shell photons

Transition form factor $|F(0,0)|$ for $\eta_c(2S)$ at $Q_1^2 = Q_2^2 = 0$.

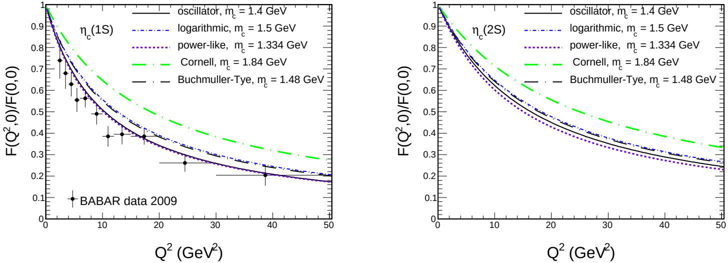
potential type	m_c [GeV]	$\vert F(0,0)\vert$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.03492	2.454	0.2530
logarithmic	1.5	0.02403	1.162	0.1970
power-like	1.334	0.02775	1.549	0.1851
Cornell	1.84	0.02159	0.938	0.2490
Buchmüller-Tye	1.48	0.02687	1.453	0.2149
experiment [1]	-	0.03266 ± 0.01209	2.147 ± 1.589	

[1] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no.3, 030001 (2018).

R(0) and $\gamma\gamma$-width for $\eta_c(2S)$ derived in the **non-relativistic limit.**

potential type	R(0) [GeV$^{3/2}$]	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = \mathrm{M}_{\eta_c}$	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = 2\mathrm{m}_c$
harmonic oscillator	0.7402	5.2284	8.8214
logarithmic	0.6372	3.8745	5.6946
power-like	0.5699	3.0993	5.7594
Cornell	0.9633	8.8550	8.6493
Buchmüller-Tye	0.7185	4.9263	7.4374

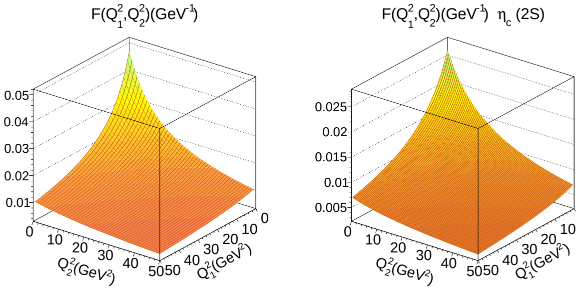
Normalized transition form factor $\tilde{F}(Q^2, 0)$



Normalized transition form factor $\tilde{F}(Q^2, 0)$ as a function of photon virtuality Q^2. The BaBar data are shown for comparison.

J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **81**, 052010 (2010) [arXiv:1002.3000 [hep-ex]].

Transition form factor $F(Q_1^2, Q_2^2)$ for $\gamma^*\gamma^* \to \eta_c(1S, 2S)$



Transition form factor for $\eta_c(1S)$ and $\eta_c(2S)$ for Buchmüller-Tye potential. The $F(Q_1^2, Q_2^2)$ should obey Bose symmetry.

Some comments

In order to estimate the factorization breaking in the transition form factor we will also estimate the normalized form factor, defined by:

$$\tilde{F}(Q_1^2, Q_2^2) = \frac{F(Q_1^2, Q_2^2)}{F(0,0)} \,, \qquad (3)$$

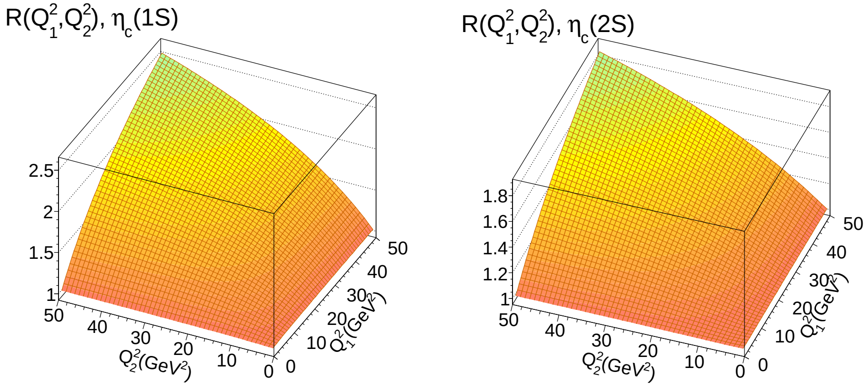
which nicely quantifies the deviation from point-like coupling. A popular model for the transition form factor is based on the vector meson dominance approach and reads:

$$\tilde{F}(Q_1^2, Q_2^2) = \frac{M_{J/\Psi}^2}{Q_1^2 + M_{J/\Psi}^2} \cdot \frac{M_{J/\Psi}^2}{Q_2^2 + M_{J/\Psi}^2} \,. \qquad (4)$$

It features a factorized dependence on the photon virtualities, In our analysis, we will quantify the factorization breaking of the transition form factor by estimating the quantity defined by:

$$R(Q_1^2, Q_2^2) = \frac{\tilde{F}(Q_1^2, Q_2^2)}{\tilde{F}(Q_1^2, 0)\tilde{F}(0, Q_2^2)} \,. \qquad (5)$$

Factorization breaking

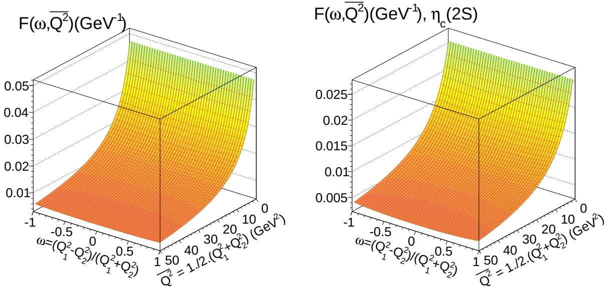


The deviations from the factorization breaking (R) as a function of (Q_1^2, Q_2^2) for Buchmüller-Tye potential;

left panel - $\eta_c(1S)$, right panel - $\eta_c(2S)$).

Clear evidence for the factorization breaking

Transition form factor $F(\omega, \overline{Q^2})$



The $\gamma^*\gamma^* \to \eta_c$ (1S) and $\gamma^*\gamma^* \to \eta_c$ (2S) form factor as a function of $(\omega, \bar{Q}^2)$ for the Buchmüller-Tye potential for illustration.

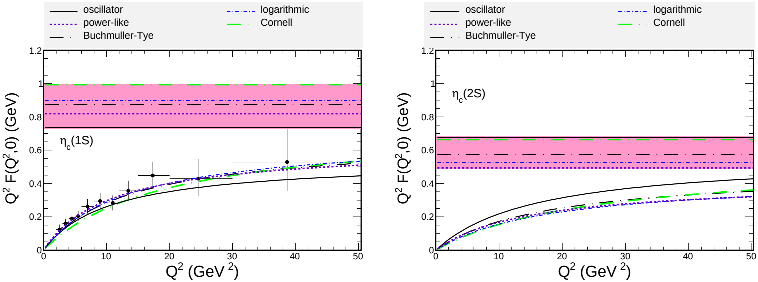
$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} \quad \text{and} \quad \bar{Q}^2 = \frac{Q_1^2 + Q_2^2}{2}.$$

Asymptotic behaviour of $Q^2F(Q^2, 0)$

The rate of approaching of $Q^2F(Q^2)$ to its asymptotic value predicted by Brodsky and Lepage

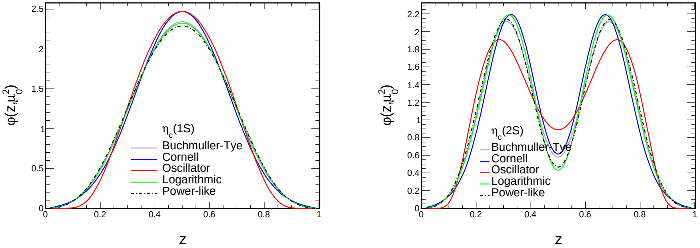
G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

$Q^2F(Q^2) \to \frac{8}{3}f_{\eta_c}$, while $Q^2 \to \infty$



$Q^2F(Q^2, 0)$ as a function of photon virtuality Q^2. The horizontal lines $\frac{8}{3}f_{\eta_c}$ are shown for reference.

Distribution amplitudes and quarkonium wave functions



Distribution amplitudes for different wave functions for η_c (1S) (left panel) and for η_c (2S) (right panel).

$$f_{\eta_c} \varphi(z, \mu_0^2) = \frac{1}{z(1-z)} \frac{\sqrt{N_c}\, 4m_c}{16\pi^3} \int d^2\mathbf{k}\, \theta(\mu_0^2 - \mathbf{k}^2)\, \psi(z, \mathbf{k})$$

$$\int_0^1 dz\, \varphi(z, \mu_0^2) = 1$$

The evolution of the distribution amplitudes

The distribution amplitudes can be expanded in terms of the Gegenbauer $C_n^{3/2}$ polynomials:

$$\varphi(z,\mu^2) = 6z(1-z)\left(1 + a_2(\mu^2)C_2^{3/2}(2z-1) + \ldots\right) ,$$

and then extract the coefficients:

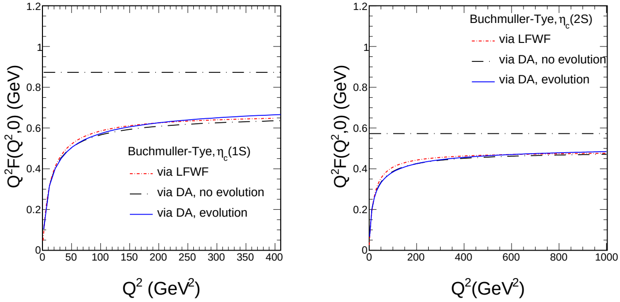
$$a_n(\mu_0) = \frac{2(2n+3)}{3(n+1)(n+2)} \cdot \int_0^1 dz \varphi(z,\mu_0) C_n^{3/2}(2z-1) ,$$

$$a_n(\mu) = a_n(\mu_0) \cdot \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_n/\beta_0} .$$

Extracted coefficients $a_n(\mu_0)$, for the Buchmüller-Tye potential

n	$a_n(\mu_0)$ $\eta_c(1S)$	$a_n(\mu_0)$ $\eta_c(2S)$
2	-0.284	-0.0765
4	0.0635	-0.1627
6	-0.008157	0.128
8	-0.000619	-0.049
10	0.000216	0.0088

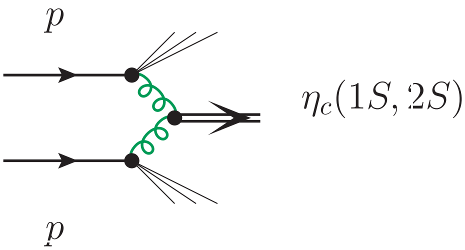
The evolution of the distribution amplitudes



$Q^2F(Q^2)$ for η_c (1S)(left panel) and η_c (2S) (right panel) as a function of photon virtuality. The horizontal line is the limit for $Q^2 \to \infty$, calculated for the Buchmüller-Tye potential.

Inclusive production of η_c quarkonia in proton-proton collisions

The diagram below illustrates the situation adequate for the k_T-factorization calculations used in the present paper.



Rysunek: Generic diagram for the inclusive process of η_c(1S) or η_c(2S) production in *pp* scattering via two gluons fusion.

I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, arXiv:1911.03403

k_t-factorization approach

The inclusive cross section for η_c-production via the $2 \longrightarrow 1$ gluon-gluon fusion mode is obtained from

$$d\sigma = \int \frac{dx_1}{x_1} \int \frac{d^2 \boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{dx_2}{x_2} \int \frac{d^2 \boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2} \mathcal{F}(x_2, \boldsymbol{q}_2^2) \frac{1}{2 x_1 x_2 s} \overline{|\mathcal{M}|}^2 \, d\Phi(2 \to 1). \tag{6}$$

The unintegrated gluon distributions are normalized such, that in the DGLAP-limit

$$\mathcal{F}(x, \boldsymbol{q}^2) = \frac{\partial x g(x, \boldsymbol{q}^2)}{\partial \log \boldsymbol{q}^2} \, . \tag{7}$$

Let us denote the four-momentum of the η_c by P. It can be parametrized as:

$$P = (P_+, P_-, \boldsymbol{P}) = (\frac{m_\perp}{\sqrt{2}} e^y, \frac{m_\perp}{\sqrt{2}} e^{-y}, \boldsymbol{P}) \, , \tag{8}$$

k_t-factorization approach

The phase-space element is

$$d\Phi(2 \to 1) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - P) \frac{d^4 P}{(2\pi)^3} \delta(P^2 - m^2) \quad (9)$$

The gluon four momenta are written as

$$q_1 = (q_{1+}, 0, \boldsymbol{q}_1), \; q_2 = (0, q_{2-}, \boldsymbol{q}_2), \quad (10)$$

with

$$q_{1+} = x_1 \sqrt{\frac{s}{2}}, \; q_{2-} = x_2 \sqrt{\frac{s}{2}}. \quad (11)$$

We can then calculate the phase-space element as

$$d\Phi(2 \to 1) = 2\pi \delta(q_{1_+} - P_+)\delta(q_{2-} - P_-)\delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P})\, dP_+ dP_- d^2\boldsymbol{P}\, \delta(2P_+P_- - \boldsymbol{P}^2 - m^2). \quad (12)$$

This gives

$$\begin{aligned} d\Phi(2 \to 1) &= 2\pi \frac{2}{s} \delta(x_1 - \frac{m_\perp}{\sqrt{s}} e^{y})\delta(x_2 - \frac{m_\perp}{\sqrt{s}} e^{-y})\delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P}) \frac{dP_+}{2P_+} d^2\boldsymbol{P} \\ &= \frac{2\pi}{s} \delta(x_1 - \frac{m_\perp}{\sqrt{s}} e^{y})\delta(x_2 - \frac{m_\perp}{\sqrt{s}} e^{-y})\delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P}) dy d^2\boldsymbol{P}. \end{aligned} \quad (13)$$

k_t-factorization approach

We therefore obtain for the inclusive cross section

$$\frac{d\sigma}{dy d^2\boldsymbol{P}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2} \mathcal{F}(x_2, \boldsymbol{q}_2^2)\, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P})\, \frac{\pi}{(x_1 x_2 s)^2} \overline{|\mathcal{M}|}^2, \qquad (14)$$

where the momentum fractions $x_{1,2}$ of gluons are

$$x_1 = \frac{m_\perp}{\sqrt{s}} e^{y}\,,\; x_2 = \frac{m_\perp}{\sqrt{s}} e^{-y}. \qquad (15)$$

The off-shell color singlet matrix element is written in terms of the Feynman amplitude as (we restore the color-indices):

$$\mathcal{M}^{ab} = \frac{q_{1\perp}^{\mu} q_{2\perp}^{\nu}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|} \mathcal{M}^{ab}_{\mu\nu} = \frac{q_{1+} q_{2-}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^{+}_{\mu} n^{-}_{\nu} \mathcal{M}^{ab}_{\mu\nu} = \frac{x_1 x_2 s}{2|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^{+}_{\mu} n^{-}_{\nu} \mathcal{M}^{ab}_{\mu\nu}\,. \qquad (16)$$

Then, we obtain for the cross section

$$\frac{d\sigma}{dy d^2\boldsymbol{P}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^4} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^4} \mathcal{F}(x_2, \boldsymbol{q}_2^2)\, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P})\, \frac{\pi}{4} \overline{|n^{+}_{\mu} n^{-}_{\mu} \mathcal{M}_{\mu\nu}|}^2, \qquad (17)$$

k_t-factorization approach

The CS matrix element squared averaged over color is

$$\overline{|n^+_\mu n^-_\mu \mathcal{M}_{\mu\nu}|}^2 = \frac{1}{(N_c^2-1)^2} \sum_{a,b} |n^+_\mu n^-_\mu \mathcal{M}^{ab}_{\mu\nu}| \,. \qquad (18)$$

The matrix element has the form

$$\begin{aligned} n^+_\mu n^-_\mu \mathcal{M}^{ab}_{\mu\nu} &= 4\pi\alpha_S(-i)[\boldsymbol{q}_1, \boldsymbol{q}_2]\frac{[t^a t^b]}{\sqrt{N_c}}\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \\ &= 4\pi\alpha_S(-i)\frac{1}{2}\delta^{ab}\frac{1}{\sqrt{N_c}}[\boldsymbol{q}_1, \boldsymbol{q}_2]\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \qquad (19) \end{aligned}$$

It is related to the $\gamma^*\gamma^*\eta_c$ transition formfactor through the relation

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \,. \qquad (20)$$

The vector product $[\boldsymbol{q}_1, \boldsymbol{q}_2]$ is defined as

$$[\boldsymbol{q}_1, \boldsymbol{q}_2] = q_1^x q_2^y - q_1^y q_2^x = |\boldsymbol{q}_1||\boldsymbol{q}_2| \sin(\phi_1 - \phi_2) \,. \qquad (21)$$

k_t-factorization approach

Then, the averaged matrix element squared becomes

$$\overline{|n_\mu^+ n_\mu^- \mathcal{M}_{\mu\nu}|}^2 = 16\pi^2\alpha_S^2 \frac{1}{4}\frac{1}{N_c}|[\boldsymbol{q}_1, \boldsymbol{q}_2]\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)|^2 \frac{1}{(N_c^2-1)^2}\sum_{a,b}\delta^{ab}\delta^{ab}$$
$$= 4\pi^2\alpha_S^2 \frac{1}{N_c(N_c^2-1)}|[\boldsymbol{q}_1, \boldsymbol{q}_2]\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)|^2 \quad (22)$$

This leads to our final result:

$$\frac{d\sigma}{dyd^2\boldsymbol{P}} = \int \frac{d^2\boldsymbol{q}_1}{\pi\boldsymbol{q}_1^4}\mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi\boldsymbol{q}_2^4}\mathcal{F}(x_2, \boldsymbol{q}_2^2)\, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{P})\, \frac{\pi^3\alpha_S^2}{N_c(N_c^2-1)}|[\boldsymbol{q}_1, \boldsymbol{q}_2]\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)|^2 .$$

In real calculation we take $\mu_F^2 = m_T^2$ and for renormalization scale(s)

$$\alpha_s^2 \to \alpha_s(max(m_t^2, q_{t,1}^2))\alpha_s(max(m_t^2, q_{t,2}^2)) . \quad (23)$$

Normalization of the $g^* g^* \eta_c(1S, 2S)$ form factors

From the proportionality of the $g^* g^* \eta_c$ and $\gamma^* \gamma^* \eta_c$ vertices to the leading order (LO), we obtain, that at LO:

$$\Gamma_{\rm LO}(\eta_c \to gg) = \frac{N_c^2 - 1}{4N_c^2} \frac{1}{e_c^4} \left(\frac{\alpha_s}{\alpha_{\rm em}}\right)^2 \Gamma_{\rm LO}(\eta_c \to \gamma\gamma) \,, \tag{24}$$

where the LO $\gamma\gamma$ width is related to the transition form factor for vanishing virtualities through

$$\Gamma_{\rm LO}(\eta_c \to \gamma\gamma) = \frac{\pi}{4} \alpha_{\rm em}^2 M_{\eta_c}^3 |F(0,0)|^2 \,. \tag{25}$$

At NLO, the expressions for the widths read (see Lansberg et al.)

$$\begin{aligned} \Gamma(\eta_c \to \gamma\gamma) &= \Gamma_{\rm LO}(\eta_c \to \gamma\gamma) \left(1 - \frac{20 - \pi^2}{3} \frac{\alpha_s}{\pi}\right) \,, \\ \Gamma(\eta_c \to gg) &= \Gamma_{\rm LO}(\eta_c \to gg) \left(1 + 4.8 \, \frac{\alpha_s}{\pi}\right) . \end{aligned} \tag{26}$$

Normalization and decay widths

In order to control the model uncertainty on the normalization, one may want to adjust its value $F(0,0)$ to the measured decay width. Here we face the ambiguity of fitting either to the hadronic or to the $\gamma\gamma$ width. As there are no other known radiative decays besides $\gamma\gamma$, one may try to identify the gg-width with the total (hadronic) width.

In Tables on next pages, we show the values of $|F(0,0)|$ obtained in three different ways.

In the first table we show the result extracted from the total decay width. Here $\alpha_s = 0.26$, which is appropriate to our choice of the renormalization scale in the production amplitudes.

In the second table we extract $|F(0,0)|$ from the radiative decay width in two different ways. The first result is obtained based on LO relation using the experimental value for $\Gamma(\eta_c \to \gamma\gamma)$ on the left hand side, while the second one uses the NLO relation.

Decay widths

Tablica: Total decay widths as well as $|F(0,0)|$ obtained from Γ_{tot} using the next-to-leading order approximation.

	Experimental values Γ_{tot} (MeV)	Derived from Eq.(26) $|F(0,0)|_{gg}[GeV^{-1}]$
$\eta_c(1S)$	31.9±0.7	0.119±0.001
$\eta_c(2S)$	11.3±3.2±2.9	0.053±0.010

Decay widths

Tablica: Radiative decay widths as well as $|F(0,0)|$ obtained from $\Gamma_{\gamma\gamma}$ using leading order and next-to-leading order approximation.

	Experimental values $\Gamma_{\gamma\gamma}$(keV)	Derived from Eq.(25) $\|F(0,0)\|[GeV^{-1}]$	Derived from Eq.(26) $\|F(0,0)\|_{\gamma\gamma}[GeV^{-1}]$
$\eta_c(1S)$	5.0 $\pm$0.4	0.067$\pm$0.003	0.079$\pm$0.003
$\eta_c(2S)$	1.9 $\pm$1.3 $\cdot 10^{-4} \cdot \Gamma_{\eta_c(2S)}$	0.033$\pm$0.012	0.038$\pm$0.014

Extracting $F(0,0)$, a comment

We observe a substantial difference between the two different extractions of $|F(0,0)|$. While in the $\eta_c(2S)$ case, the error bars are too large to claim an inconsistency, the situation for the $\eta_c(1S)$ is not satisfactory.
This may hint at an insufficiency of the potential model treatment of the η_c. Possible solutions:
admixture of light hadron states (Shifman),
a mixing with a pseudoscalar glueball (Kochelev),
nonperturbative instanton effects in the hadronic decay (Zetocha et al.).

Unintegrated gluon distributions

We use a few different UGDs which are available from the literature, e.g. from the TMDLib package (Hautmann et al.) or the CASCADE Monte Carlo code (Jung et al.).

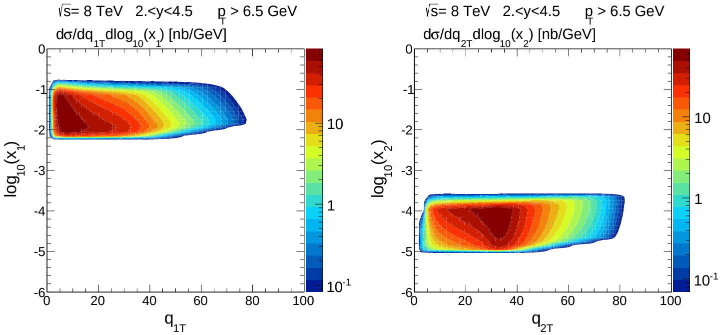
1. Firstly we use a glue constructed according to the prescription initiated in (Kimber et al.) and later updated in (Martin et al.), which we label below as "KMR". It uses as an input the collinear gluon distribution from Harland-Lang et al.
2. Secondly, we employ two UGDs obtained by Kutak. There are two versions of this UGD. Both introduce a hard scale dependence via a Sudakov form factor into solutions of a small-x evolution equation. The first version uses the solution of a linear, BFKL evolution with a resummation of subleading terms and is denoted by "Kutak (linear)". The second UGD, denoted as "Kutak (nonlinear)" uses instead a nonlinear evolution equation of Balitsky-Kovchegov type. Both of the Kutak's UGDs can be applied only in the small-x regime, $x < 0.01$.
3. The third type of UGD has been obtained by Hautmann and Jung from a description of precise HERA data on deep inelastic structure function by a solution of the CCFM evolution equations. We use "Set 2".

KMR UGDF

For the case of the KMR UGD, it has recently been shown (Maciula, Szczurek), that it includes effectively higher order corrections of the collinear factorization approach. In this sense should give, within our approach, a result similar to that found recently in the NLO approach (Feng, Lansberg et al.) at not too small transverse momenta.

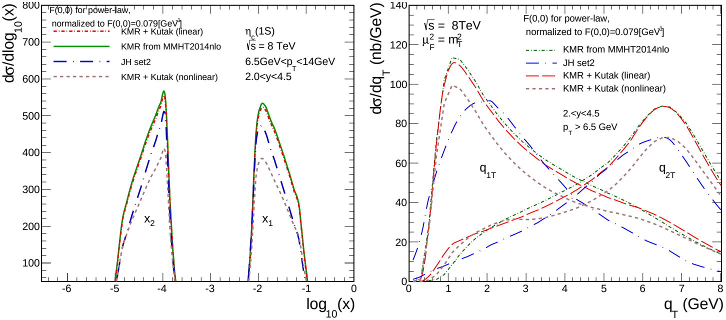
In our approach we can go to very small transverse momenta close to $p_T = 0$.

Results for the LHC



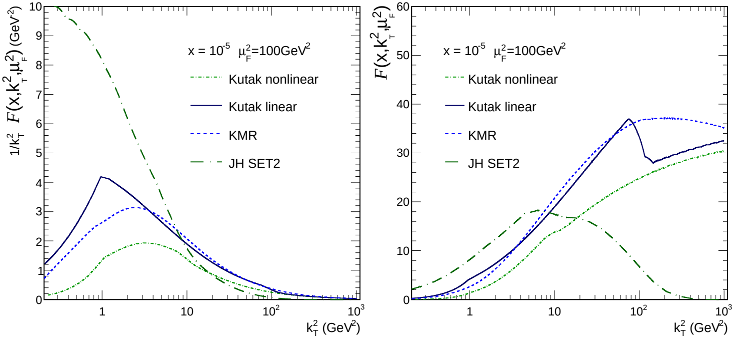
Rysunek: Two-dimensional distributions in (x_1, q_{1T}) (left panel) and in (x_2, q_{2T}) (right panel) for $\eta_c(1S)$ production for $\sqrt{s} = 8$ TeV. In this calculation the KMR UGD was used for illustration.

Results for the LHC



Rysunek: Distributions in $\log_{10}(x_1)$ or $\log_{10}(x_2)$ (left panel) and distributions in q_{1T} or q_{2T} (right panel) for the LHCb kinematics. Here the different UGDs were used in our calculations. Here we show an example for $\sqrt{s} = 8$ TeV.

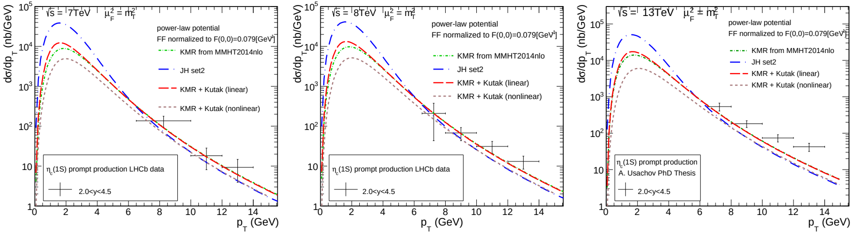
Results for the LHC



Rysunek: Unintegrated gluon densities for typical scale $\mu^2 = 100$ GeV^2 for $\eta_c(1S)$ production in proton-proton scattering at LHCb kinematics.

UGDs are quite different but ...

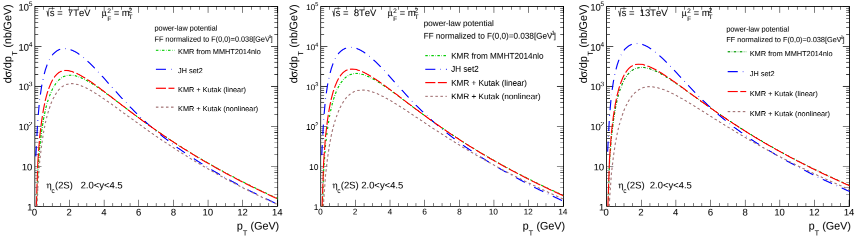
Results for the LHC



Rysunek: Differential cross section as a function of transverse momentum for prompt $\eta_c(1S)$ production compared with the LHCb data (Aaij et al.) for $\sqrt{s} = 7, 8\,\mathrm{TeV}$ and preliminary experimental data (Usachov PhD) for $\sqrt{s}$ = 13 TeV. Different UGDs were used. Here we used the $g^*g^* \to \eta_c(1S)$ form factor calculated from the power-law potential.

$F(0,0)$ extracted from $\Gamma_{\eta_c(1S)}$ at NLO accuracy

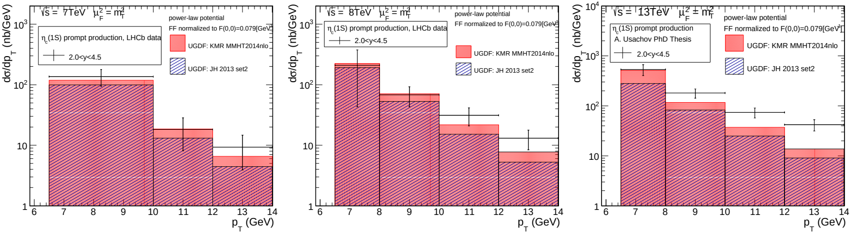
Results for the LHC, $\eta_c(2S)$



Rysunek: Differential cross section as a function of transverse momentum for prompt $\eta_c(2S)$ production for $\sqrt{s} = 7, 8, 13\,\mathrm{TeV}$.

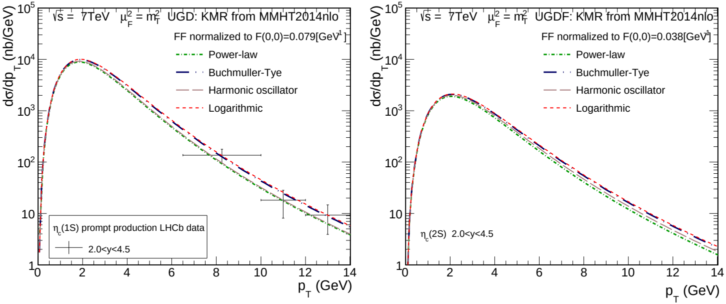
$F(0,0)$ extracted from $\Gamma_{\eta_c(2S)}$ at NLO accuracy

Results for the LHC, another representation



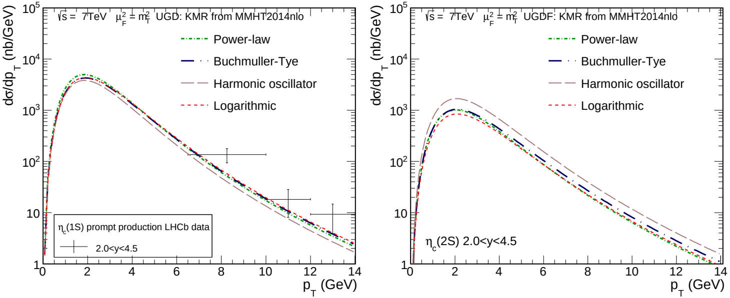
Rysunek: Differential cross section as a function of transverse momentum for prompt $\eta_c(2S)$ production for $\sqrt{s} = 7, 8, 13\,\mathrm{TeV}$.

Results for the LHC, different form factors



Rysunek: Transverse momentum distributions calculated with different form factors obtained from different potential models of quarkonium wave function and one common normalization of $|F(0,0)|$.

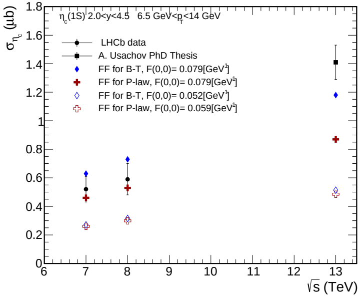
Results for the LHC



Rysunek: Distributions calculated with several different form factors obtained from different potential models of quarkonium.

Different F(0,0).

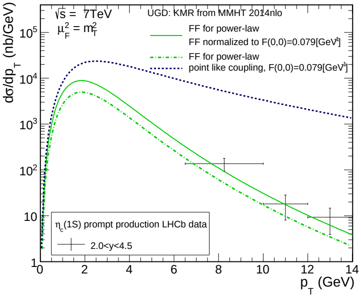
Results for the LHC, integrated cross section



Rysunek: The integrated cross section computed within LHCb range of p_T and y with our transition form factors, compared to experimental values. Here red crosses represent values for Buchmüller-Tye potential (B-T) and deltoids for Power-law potential (P-law).

Somewhat faster grow for experimental data.

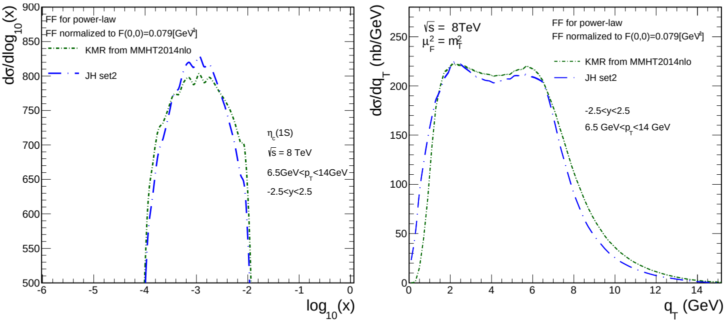
Results for the LHC, effect of form factor



Rysunek: Comparison of results for two different transition form factor, computed with the KMR unintegrated gluon distribution. We also show result when the (q_{1T}^2, q_{2T}^2) dependence of the transition form factor is neglected.

Is the form factor included in collinear calculations ?
Not always.

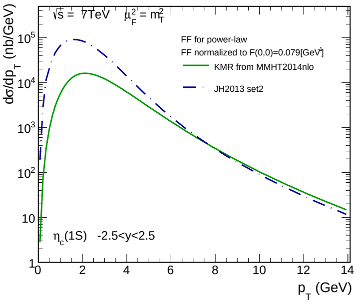
Results for the ATLAS/CMS kinematics



Rysunek: Distribution in $\log_{10}(x_1)$ or $\log_{10}(x_2)$ (left panel) and distribution in q_{1T} or q_{2T} (right panel) for ATLAS or CMS conditions.

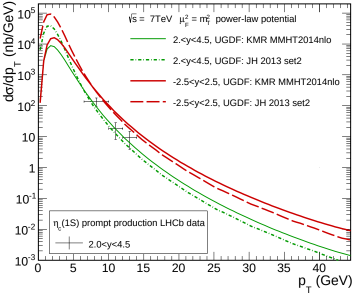
Not so small x_1, x_2 as for LHCb.

Results for the ATLAS/CMS kinematics



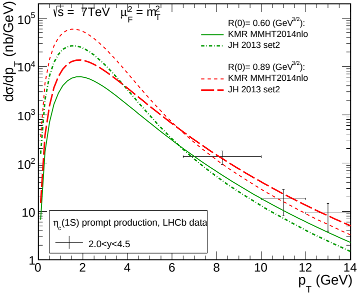
Rysunek: Transverse momentum distribution of prompt $\eta_c(1S)$ for $-2.5 < y < 2.5$ and $\sqrt{s} = 7$ TeV.

Broader range of transverse momenta



Rysunek: Transverse momentum distribution of prompt $\eta_c(1S)$ for $\sqrt{s} = 7$ TeV.

Broader range of transverse momenta



Rysunek: Transverse momentum distribution of prompt $\eta_c(1S)$ for $\sqrt{s} = 7$ TeV.

Unphysically large R(0) necessary in NRQCD approach.

Conclusions, $\gamma^*\gamma^* \to \eta_c$

- The transition form factor for different wave functions obtained as a solution of the Schrödinger equation for the $c\bar{c}$ system, for different phenomenological $c\bar{c}$ potentials from the literature, has been calculated.
- We have studied the transition form factors for $\gamma^*\gamma^* \to \eta_c$ (1S,2S) for two space-like virtual photons, which can be accessed experimentally in future measurements of the cross section for the $e^+e^- \to e^+e^-\eta_c$ process in the double-tag mode.
- The transition form factor for only one off-shell photon as a function of its virtuality was studied and compared to the BaBar data for the $\eta_c(1S)$ case.
- Predictions for $\eta_c(2S)$ have been presented.
- Dependence of the transition form factor on the virtuality was studied and delayed convergence of the form factor to its asymptotic value $\frac{8}{3} f_{\eta_c}$ as predicted by the standard hard scattering formalism, was presented.
- It seems that nonrelativistic approach may be too approximate.
- There is practically no dependence on the asymmetry parameter ω, which could be verified experimentally at Belle 2.

Conclusions, $pp \to \eta_c$

- k_t-factorization approach with modern UGDs lead to good description of the LHCb data for $pp \to \eta_c(1S) \to p\bar{p}$ for $\sqrt{s} = 7, 8$ TeV and somewhat worse for $\sqrt{s} = 13$ TeV (a PhD thesis).
  Some room for color octet.
  Feed down is small (Baranov).
- Range of x_1, x_2 and q_{1T}, q_{2T} was discussed.
  For the LHCb kinematics very small longitudinal momentum fractions are probed. Transverse momenta not too small.
- We do not see an obvious sign of the onset of saturation.
  LHCb cross section grows even faster than our result without saturation.
  But gluon transverse momenta are not small.
- Predictions for $\eta_c(2S)$ has been presented.
  Strong deviations could signal large CO contribution.
- We have also discussed uncertainties related to $g^* g^* \to \eta_c$ form factor. They are somewhat smaller than those related to UGDs. No uncertainties due to renormalization/factorization scales were discussed.