On the universality of the MC factorization scheme

S. JADACH

Institute of Nuclear Physics PAN, Kraków, Poland

Partly supported by the grant of **the Polish National Science Center grant 2016/23/B/ST2/03927**

XXV Cracow EPIPHANY Conference, January 7-10th, 2020

On the meaning of “universality” of PDFs

- Monte Carlo factorization scheme (FS) is a variant of MS-bar system (including new definition of the PDFs for initial hadrons). It is therefore trivially universal, that is process independent.
- The question of its universality is formulated differently: As the basic role of MC FS is to simplify drastically NLO corrections, the question is now whether the same single variant of the MC FS is able to achieve the same maximal simplification of the NLO corrections for all processes with one or two initial hadrons and any number of the final hadrons?
- The answer is positive and the proof is elaborated within the Catani-Seymour subtraction methodology.
- MC FS is mandatory in KrkNLO matching NLO and parton shower – a much simpler alternative of POWHEG and/or MC NLO
- However, the use of MC FS simplifies NLO calculations for any method and arbitrary process.

How MC FS simplifies NLO Catani-Seymour master formula?

Thanks to use on PDFs in the (physical) MC factorization scheme
and the use of the new modified soft-collinear counterterms,
the Catani-Seymour NLO master formula

$$\begin{aligned}
\sigma^{NLO}(p) = \sigma^{B}(p) + \\
+ \int_m \left[d\sigma^V(p) + d\sigma^B(p) \otimes \mathbf{I}\right]_{\varepsilon=0} + \int dz \int_m \left[d\sigma^B(zp) \otimes (\mathbf{P}+\mathbf{K})(z)\right]_{\varepsilon=0} \\
+ \int\limits_{m+1} \left[d\sigma^R(p)_{\varepsilon=0} - \Big(\sum_{dipoles} d\sigma^B(p) \otimes dV_{dipole}\Big)_{\varepsilon=0}\right],
\end{aligned} \tag{1}$$

turns into a much simpler one

$$\begin{aligned}
\sigma^{NLO}(p) = \sigma^{B}(p) + \int_m \left[d\sigma^V(p) + d\sigma^B(p)\ I(\varepsilon)\right]_{\varepsilon=0} \\
+ \int\limits_{m+1} \left[d\sigma^R(p)_{\varepsilon=0} - \Big(\sum_{dipoles} d\sigma^B(p) \otimes dV_{dipole}\Big)_{\varepsilon=0}\right]
\end{aligned} \tag{2}$$

for ANY process with one or two initial hadrons
and any number m of final coloured partons.

Consequently, KrkNLO matching scheme with parton shower
(much simpler alternative of POWHEG or MC@NLO)
applies not only to DY-like processes but to ANY process.

DY example of NLO for CS with PDFs in the MC scheme

JHEP 1510 (2015) 052 [arXiv:1503.06849] (gluonstrahlung channel only):

Including measurement functions $J^F_{\rm LO} = J_{\rm LO}(x_F z, x_B)$, $J^B_{\rm LO} = J_{\rm LO}(x_F, x_B z)$, $J_{NLO}(x_F, x_B, z, k^T)$, the NLO x-section with CS dipole subtractions reads:

$$\sigma^{\overline{MS}}_{\rm NLO}[J] = \int dx_F dx_B dz\; dx\; \delta_{x=zx_F x_B} \Big\{$$
$$\delta_{1=z}(1+\Delta_{VS})\; d^2\sigma^{\rm LO}(sx,\hat\theta)\; J_{\rm LO} + \mathcal{G}(z)(J^F_{\rm LO}+J^B_{\rm LO})\; d^2\sigma^{\rm LO}(szx,\hat\theta)$$
$$+\Big(d^5\rho_1^{\rm NLO}\; J_{\rm NLO} - (d^3\rho_1^F J^F_{\rm LO} + d^3\rho_1^B J^B_{\rm LO}))\, d^2\sigma^{\rm LO}(\hat s,\hat\theta)\Big)\delta_{1-z=\alpha+\beta}\Big\} D^{\overline{MS}}{}_q(sx,x_F) D^{\overline{MS}}{}_{\bar q}(sx,x_B).$$

The dipole for real gluon emission in $d=4$ using Sudakov parametrization:

$$d^3\rho_1^F(s_1) = \frac{\alpha_s}{2\pi} H^{qq}(\alpha,\beta,\varepsilon)\Big|_{\varepsilon=0} = \frac{\alpha_s}{2\pi}\frac{d\beta_1 d\alpha_1}{\beta_1}\frac{d\phi_1}{2\pi} P_{qq}(1-\alpha_1-\beta_1) \quad \text{and } \rho_1^B \text{ defined similarly.}$$

In the KrkNLO matching, <u>the absence of $\mathcal{G}(z)$</u> allows for single multiplicative MC weight:

$$W^{\rm MC}_{\rm NLO}(k)\big|_{qq\; chan.} = (1+\Delta^{\rm MC}_{VS})\; \frac{d^5\rho_1^{\rm NLO}(k)}{(d^3\rho_1^F + d^3\rho_1^B)\; d^2\sigma^{\rm LO}(\hat s,\hat\theta)}.$$

NB. the finite virtual+soft corrections ($q\bar q$ channel) is:

$$\Delta^{\rm MC}_{VS} = \Delta^{virt.}_{q\bar q}(\varepsilon) + \frac{\alpha_s}{2\pi}\frac{\Gamma(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\frac{\hat s}{4\pi\mu^2}\right)^{\varepsilon}\int_0^1 dz\; z\widetilde{\mathcal{V}}^{q\leftarrow q}(z,\varepsilon) = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{4}+\frac{2}{3}\pi^2\right)$$

Last but not least $\hat s = \mu^2$ was instrumental!

DY example of NLO for CS with PDFs in the MC scheme

JHEP 1510 (2015) 052 [arXiv:1503.06849] (gluonstrahlung channel only):

Including measurement functions $J^F_{\rm LO} = J_{\rm LO}(x_F z, x_B)$, $J^B_{\rm LO} = J_{\rm LO}(x_F, x_B z)$, $J_{NLO}(x_F, x_B, z, k^T)$, the NLO x-section with CS dipole subtractions reads:

$$
\begin{aligned}
\sigma^{\rm MC}_{\rm NLO}[J] = \int dx_F\, dx_B\, dz\; dx\; \delta_{x=zx_F x_B} \Big\{ \\
&\delta_{1=z}(1+\Delta^{\rm MC}_{VS})\; d^2\sigma^{\rm LO}(sx,\hat\theta)\; J_{\rm LO} \\
&+ \Big(d^5\rho_1^{\rm NLO}\; J_{\rm NLO} - \big(d^3\rho_1^F J^F_{\rm LO} + d^3\rho_1^B J^B_{\rm LO}\big)\big) d^2\sigma^{\rm LO}(\hat s,\hat\theta) \Big) \delta_{1-z=\alpha+\beta} \Big\} D^{\rm MC}{}_q(sx, x_F) D^{\rm MC}{}_{\bar q}(sx, x_B).
\end{aligned}
$$

The dipole for real gluon emission in $d=4$ using Sudakov parametrization:

$$d^3\rho_1^F(s_1) = \frac{\alpha_s}{2\pi} H^{qq}(\alpha,\beta,\varepsilon)\big|_{\varepsilon=0} = \frac{\alpha_s}{2\pi}\frac{d\beta_1 d\alpha_1}{\beta_1}\frac{d\phi_1}{2\pi} P_{qq}(1-\alpha_1-\beta_1) \quad \text{and } \rho_1^B \text{ defined similarly.}$$

In the KrkNLO matching, <u>the absence of $\mathcal{G}(z)$</u> allows for single multiplicative MC weight:

$$W^{\rm MC}_{\rm NLO}(k)\big|_{qq\ chan.} = (1+\Delta^{\rm MC}_{VS})\; \frac{d^5\rho_1^{\rm NLO}(k)}{(d^3\rho_1^F + d^3\rho_1^B)\; d^2\sigma^{\rm LO}(\hat s,\hat\theta)}.$$

NB. the finite virtual+soft corrections ($q\bar q$ channel) is:

$$\Delta^{\rm MC}_{VS} = \Delta^{virt.}_{q\bar q}(\varepsilon) + \frac{\alpha_s}{2\pi}\frac{\Gamma(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\frac{\hat s}{4\pi\mu^2}\right)^{\varepsilon} \int_0^1 dz\; z\widetilde{\mathcal{V}}^{q\leftarrow q}(z,\varepsilon) = \frac{C_F\alpha_s}{\pi}\left(\frac14 + \frac23\pi^2\right)$$

Last but not least $\hat s = \mu^2$ was instrumental!

Explicit transformation of LO PDFs from $\overline{MS}$ to MC FS

At every $Q^2 = \mu^2$ the following "rotation" in the x and flavour space:

$$\begin{bmatrix} q(x,Q^2) \\ \bar{q}(x,Q^2) \\ G(x,Q^2) \end{bmatrix}_{\rm MC} = \begin{bmatrix} q \\ \bar{q} \\ G \end{bmatrix}_{\overline{MS}} + \frac{\alpha_s}{2\pi} \int dzdy \begin{bmatrix} \mathbb{K}^{\rm MC}_{qq}(z) & 0 & \mathbb{K}^{\rm MC}_{qG}(z) \\ 0 & \mathbb{K}^{\rm MC}_{\bar{q}\bar{q}}(z) & \mathbb{K}^{\rm MC}_{\bar{q}G}(z) \\ \mathbb{K}^{\rm MC}_{Gq}(z) & \mathbb{K}^{\rm MC}_{G\bar{q}}(z) & \mathbb{K}^{\rm MC}_{GG}(z) \end{bmatrix} \begin{bmatrix} q(y,Q^2) \\ \bar{q}(y,Q^2) \\ G(y,Q^2) \end{bmatrix}_{\overline{MS}} \delta(x-yz)$$

where

$$\mathbb{K}^{\rm MC}_{Gq}(z) = C_F \left\{ \frac{1+(1-z)^2}{z} \ln\frac{(1-z)^2}{z} + z \right\},$$

$$\mathbb{K}^{\rm MC}_{GG}(z) = C_A \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ + 2\left[\frac{1}{z} - 2 + z(1-z)\right] \ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36}\frac{T_f}{C_A}\right) \right\},$$

$$\mathbb{K}^{\rm MC}_{qq}(z) = C_F \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z)\ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} + 1 - z - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{17}{4}\right) \right\},$$

$$\mathbb{K}^{\rm MC}_{qG}(z) = T_R \left\{ \left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z) \right\}.$$

All virtual parts $\sim \delta(1-z)$ are adjusted using momentum sum rules:

$$\sum_b \int dz \;\; z\, \mathbb{K}^{\rm MC}_{ba}(z) = 0$$

From *Eur. Phys. J. C76 (2016) 649* [arXiv:1606.00355].

KrkNLO method and PDFs in MC factorization scheme

(A) 1-st idea of the KrkNLO for DY process and MC FS:
Acta Phys.Polon. B42 (2011) 2433 , [arXiv:1111.5368] Ustron 2011 Proc.

(B) KrkNLO scheme for DY and DIS, PDFs in the MC factorization scheme:
Phys.Rev. D87 (2013) 3, 034029 , [arXiv:1103.5015].

(C) Implementation for DY process of top of SHERPA and HERWIG in
JHEP 1510 (2015) 052 [arXiv:1503.06849],
comparisons with NLO and NNLO (fixed order), MC@NLO and POWHEG.

(D) PDFs in Monte Carlo factorization scheme, DY and Higgs production
Eur. Phys. J. C76 (2016) 649 [arXiv:1606.00355].

(E) MC simulations of Higgs-boson production at the LHC with the KrkNLO method
Eur.Phys.J. C77 (2017) 164 , [arXiv:1607.06799],

KrkNLO team: W. Płaczek, M. Sapeta, A. Siódmok, M. Skrzypek and S.J.

Are PDFs in the MC factorization scheme universal?

i.e. are they process independent?

In the MC scheme one subtract from NLO distribution in d dimensions the following soft-collinear counterterm:

$$\Lambda^{\rm MC}_{J\leftarrow I}(\varepsilon,z) = \frac{\alpha_s}{2\pi}\frac{(4\pi)^{-\varepsilon}}{\Gamma(1+\varepsilon)}\left(\frac{1}{\varepsilon}P_{JI}(z) + \mathbb{K}_{J\leftarrow I}(z)\right), \quad J,I = q,\bar{q},G.$$

($d = 4+2\varepsilon$) instead of

$$\Lambda^{\overline{MS}}_{J\leftarrow I}(\varepsilon,z) = \frac{\alpha_s}{2\pi}\frac{(4\pi)^{-\varepsilon}}{\Gamma(1+\varepsilon)}\frac{1}{\varepsilon}P_{JI}(z), \quad J,I = q,\bar{q},G.$$

This implies the relation between PDFs in $\overline{MS}$ and MC scheme:

$$D^{\rm MC}_J(\mu^2,x) = D^{\overline{MS}}_J(\mu^2,x) + \sum_I \int dx_0\, \frac{dz}{z}\, \frac{\alpha_s}{2\pi}\mathbb{K}_{J\leftarrow I}(z)\; D^{\overline{MS}}_I(\mu^2,x_0/z)$$

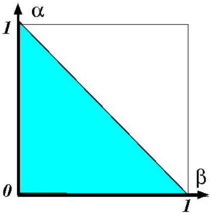
Is $\mathbb{K}$-transformation on PDFs “universal”? Process independent?

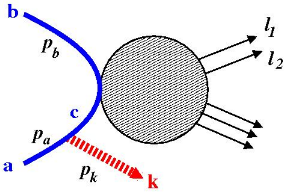
Is $\mathbb{K}$-transformation on PDFs "universal"?

- The freedom of the $\mathbb{K}$-transformation on PDFs is known for ages.
- $\mathbb{K}$ was adjusted semi-empirically in KrkNLO in Refs.(C,D) such that for $pp \to Z/\gamma$ and $pp \to \mathrm{Higgs}$ process the "collinear remnant" terms $\sim \delta(k_T)$ in the NLO calculations have disappeared (DY scheme?)
- Is it possible that the same $\mathbb{K}$ does the same for other processes?
- To answer this question systematically we derive $\mathbb{K}$ from subtraction terms of the NLO calculations, i.e. "dipoles" of the Catani-Seymour.
- This was already done in early papers on KrkNLO method Ref.(B), albeit only for gluonstrahlung in DY and DIS, for "dipoles" of our own.
- Warm up exercise:
  do we get our $\mathbb{K}_{qq}$ directly from the Initial-Initial dipole of Catani-Seymour paper Nucl.Phys. B485 (1997) 291?
  next slide ...

Do we get $\mathbb{K}_{qq}(z)$ from $\mathbb{JJ}$ CS dipole of Nucl.Phys. B485 (1997) ?

Start with kinematics of DY in Sudakov parametrization...





$$p_k = \alpha p_a + \beta p_b + p_k^T, \; \alpha = \frac{p_k p_b}{p_a p_b}, \beta = \frac{p_k p_a}{p_a p_b},$$

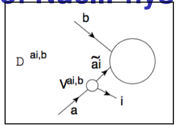
$$\alpha + \beta \leq 1 \;\; |p_k^T|^2 = 2 p_a p_b \alpha \beta.$$

Some auxiliary variables:

$$s = 2 p_a p_b, \; \hat{s} = Q^2 = (p_a + p_b - p_k)^2 = (1 - \alpha - \beta) s = sz, \quad z = 1 - \alpha - \beta.$$

Do we get $\mathbb{K}_{qq}(z)$ from $\mathbb{JJ}$ CS dipole of Nucl.Phys.B485 (1997)??

Yes, but not easily...

The initial-emitter initial-spectator $\mathcal{D}^{ai,b}$ dipole 

in CS ($d = 4 - 2\varepsilon$):

$$\boxed{\frac{\alpha_S}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{2p_a p_b}\right)^{\epsilon} \tilde{\mathcal{V}}^{a,ai}(x;\epsilon) \equiv \int [dp_i(p_a,p_b,x)]\, \frac{1}{2p_a p_i}\frac{n_s(\widetilde{ai})}{n_s(a)} < \boldsymbol{V}^{ai,b}(x_{i,ab}) > .}\,, \quad (5.152)$$

In our notation: $x = x_{i,ab} = 1 - \alpha - \beta, \quad \bar{v}_i = \beta$ and from **direct evaluation** one gets:

$$\tilde{\mathcal{V}}^{q,qG}(z,\varepsilon)|_{z\neq 1} = \frac{1}{\varepsilon} P_{qq}(z) + 2C_F(1+z^2)\frac{\ln(1-z)}{1-z} - C_F(1-z).$$

The same result in eq. (5.155-156) of CS paper looks mysteriously complicated:

$$\tilde{\mathcal{V}}^{a,b}(x;\epsilon) = \mathcal{V}^{a,b}(x;\epsilon) + \delta^{ab}\boldsymbol{T}_a^2\left[\left(\frac{2}{1-x}\ln\frac{1}{1-x}\right)_+ + \frac{2}{1-x}\ln(2-x)\right] + \widetilde{K}^{ab}(x) + \mathcal{O}(\epsilon)\ , \quad (5.155)$$

In fact $\sim \ln(2-x)$ term is in reality absent – it cancels out with another one in $\mathcal{V}^{a,b}(x,\varepsilon)$.
The term $\sim \frac{2}{1-x}\ln\frac{1}{1-x}$ cancels with another identical term inside $\mathcal{V}^{a,b}(x,\varepsilon)$.
$\tilde{K}$ corrects for the unlucky definition of $\mathcal{V}^{a,b}$ for DIS in CS paper, where $m_+ = \alpha/(\alpha+\beta)$ is applied only to soft part of DIS dipole, while in the DY it is applied to the entire dipole.

More details: $\mathbb{K}_{ba}(z)$ from CS initial-initial $\mathcal{II}$ dipoles

Let us recalculate $\mathcal{II}$ dipoles from the scratch, because in CS paper they are obscured by the unlucky choice of the $\mathcal{IF}$ dipoles (DIS/ISR) as a baseline objects.

Our compact elegant definition of all nine $\mathcal{II}$ dipoles, $K, I = q, \bar{q}, G$:

$$\tilde{\mathcal{V}}^{K\leftarrow I}(z,\varepsilon) = \int d\alpha d\beta\; \delta_{1-z=\alpha+\beta}\; H(\alpha,\beta,\varepsilon) = \int d\alpha d\beta\; \delta_{1-z=\alpha+\beta}\; (\alpha\beta)^{\varepsilon}\; z^{-\varepsilon}\, \frac{P^*_{K\leftarrow I}(\alpha,\beta)}{\beta}$$

$$= \delta_{z=1}\; \delta_{KI} \sum_{J=G,q,\bar{q}} \int_0^1 dz\; z\tilde{\mathcal{V}}^{J\leftarrow I}(z,\varepsilon) + \delta_{z=1} \frac{1}{\varepsilon}\; P_{KI}(z) + \mathcal{G}_{K\leftarrow I}(z),$$

$$\boxed{\mathbb{K}_{KI}(z) = \mathcal{G}_{K\leftarrow I}(z) = \delta_{z=1}\; \mathcal{G}^0_{KI} + \frac{1}{z}\Big[zP'_{KI}(z) + \ln\frac{(1-z)^2}{z}\; zP_{KI}(z)\Big]_+},$$

where $\mathcal{G}^0_{KI}$ are from momentum sum rules. Agrees with CS for DY.
Denoting $\bar{P}_{KI}(z) \equiv (1-z)P_{KI}(z)$ we are using CS choice of the "soft partition function":

$$P^*_{K\leftarrow K} = \frac{\bar{P}_{KK}(1-\alpha-\beta,\varepsilon)}{(\alpha+\beta)\beta}, \quad P^*_{K\leftarrow I} = \frac{P_{KI}(1-\alpha-\beta,\varepsilon)}{\beta},\; K \neq I.$$

NB. The same result is obtained with sharp "soft partition function" of paper (B):

$$P^*_{K\leftarrow K} = \frac{\bar{P}_{KK}(1-\alpha-\beta,\varepsilon)}{\alpha\beta}\theta_{\alpha>\beta}.$$

All $P_{KI}(z)$ kernels are here standard DGLAP splitting kernels.

What about $\mathcal{FI}$ and $\mathcal{FF}$ dipoles?

$\mathbb{K}$-matrix from $\mathcal{IF}$, $\mathcal{FI}$, $\mathcal{FF}$ original and modified CS dipoles

Point by point overview:

(1) $\mathbb{K}$ matrix and $\mathcal{FF}$ dipoles (final emiter and final spectator) are unrelated. Hence $\mathcal{G}_{ab}(z)|_{z\neq 1} = 0$. Factor $\overline{\mathcal{V}}_{ab}(z,\varepsilon)$ decouples kinematically from PDFs. Only $\overline{\mathcal{V}}_{ab}(\varepsilon) = \int_0^1 dz\, \overline{\mathcal{V}}_{ab}(z,\varepsilon)$ matter (get combined with virt. corrs.)

(2) In CS paper, $\mathcal{V}_{ab}(z,\varepsilon)$ for $\mathcal{FI}$ dipoles (final emiter and initial spectator as in DIS) couples kinematicaly with PDFs and LO part through $\mathcal{G}_{ab}(z) \neq 0$.

(3) However, we have modified kinematic mapping in $\mathcal{FI}$ dipoles such that they kinematicaly decouple from PDFs, $\mathcal{G}_{ab}(z)|_{z\neq 1} = 0$, as for $\mathcal{FF}$. Next slide.

(4) It remains to check whether $\mathbb{K}$-matrix from $\mathcal{IF}$ dipoles is the same as from $\mathcal{II}$.

(5) Not true for original $\mathcal{IF}$ dipoles of CS, however...

(6) Easy to modify *diagonal* $\mathcal{IF}$ dipoles such that $\mathbb{K}_{aa}(z)$ are the same. Next slide.

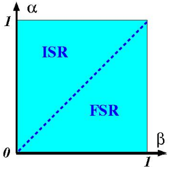
(7) For *nondiagonal* $\mathcal{IF}$ dipoles $a \neq b$ ($G \leftrightarrow q$) a workaround is found. Next slide.

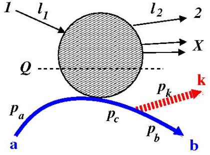
(8) Finally, it is possible to eliminate ALL collinear remanats $\mathcal{G}_{ab}(z)|_{z\neq 1}$ for ALL dipoles using common $\mathbb{K}$-rotation of PDFs from MS-bar to MC FS.

(9) Last problem: collinear remnant terms $\sim \ln \frac{2p_i \cdot p_j}{\mu^2} P_{ab}(z)$ coupled with PDFs survive for more than two "legs"?? It looks that a recipee for zeroing them was found:)

New kinematic mapping in $\mathcal{FI}$ dipoles (initial spectator & final emitter)

This is the most important point!





$$p_k = \bar{\alpha}\, p_a + \bar{\beta}\, p_b + p_k^T, \quad \bar{\alpha} = \frac{p_k \cdot p_b}{p_a \cdot p_b}, \bar{\beta} = \frac{p_k \cdot p_a}{p_a \cdot p_b},$$

$$\alpha = \frac{\bar{\alpha}}{1+\bar{\beta}} = \frac{p_k p_b}{p_a(p_k+p_b)}, \ \beta = \frac{\bar{\beta}}{1+\bar{\beta}} = \frac{p_k p_a}{p_a(p_k+p_b)},$$

$$\bar{\alpha} = \frac{\alpha}{1-\beta}, \ \bar{\beta} = \frac{\beta}{1-\beta}, \ \max(\alpha,\beta) \le 1,$$

$$Q = p_b + p_k - p_a, \quad |Q^2| = 2 p_a p_b \frac{1-\alpha}{1-\beta},$$

$$d\sigma^a_{bk} = d\Phi_{4+2\varepsilon}(p_k)\, \frac{1}{2p_b p_k} 8\pi\mu^{-2\varepsilon} \alpha_s P^*_{b\leftarrow c}(\alpha,\beta)\, \frac{p_a \tilde{p}_b}{p_a(\tilde{p}_b - p_k)} \left\{ \frac{1}{s} d\Phi(l_1' + \tilde{p}_a; \tilde{p}_b, l_2', \ldots)\, |\mathcal{M}(l_1', \tilde{p}_a; \tilde{p}_b, l_2', \ldots)|^2 \right\}$$

$$= \frac{\alpha_s}{2\pi} \left(\frac{Q^2}{4\pi\mu^2}\right)^{\varepsilon} \frac{1}{\Gamma(1+\varepsilon)}\, \frac{d\Omega^{n-3}(p_k^T)}{\Omega^{n-3}}\, H_{bc}(\alpha,\beta,\varepsilon) \left\{ d\sigma^{LO}(l_1', \tilde{p}_a; \tilde{p}_b, l_2', \ldots) \right\},$$

$$H_{bc}(\alpha,\beta,\varepsilon) = \left(\frac{\alpha\beta(1-\beta)}{(1-\alpha)}\right)^{\varepsilon} \frac{P^*_{b\leftarrow c}(\alpha,\beta,\varepsilon)}{\alpha}, \qquad \tilde{p}_a = (1-\alpha) p_a, \quad \tilde{p}_b = Q - \tilde{p}_a.$$

$$P^*_{b\leftarrow c}(\alpha,\beta,\varepsilon)\big|_{\alpha\to 0} = P_{bc}(1-\beta,\varepsilon),$$ NEXT SLIDE

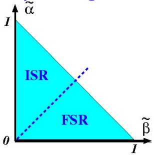
The essential difference with the original CS is an **additional active boost B_x** (tested in MC): $l_1' = B_x l_1,\ l_2' = B_x l_2,\ X' = B_x X$, in the plane perpendicular to Q, i.e. $B_x Q = Q$, with hypervelocity η adjusted such that: $2 l_1' \cdot \tilde{p}_a = (B_x(\eta) l_1) \cdot \tilde{p}_a = 2 l_1 \cdot p_a = s$.

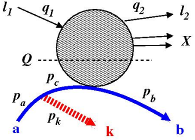
The resulting LO part $\{d\sigma^{LO}(l_1', \tilde{p}_a; \tilde{p}_b, l_2', \ldots)\}$ does not depend on α and β anymore and to complete NLO calculations one needs to know only (as in $\mathcal{FF}$ case):

$$\mathcal{V}_{b\leftarrow c}(\varepsilon) = \int d\alpha d\beta\, H_{bc}(\alpha,\beta,\varepsilon).$$

Modified diagonal $\mathcal{IF}$ dipoles, (initial emitter & final spectator)

Exploiting freedom in $K^*_{c\leftarrow a}(\alpha,\beta)$ to get the same $\mathbb{K}_{ca}(z)$ as for $\mathcal{II}$.





$$d\sigma_b^{ak}(p_b,p_k,l_1,l_2,\dots) = \frac{\alpha_s}{2\pi}\left(\frac{Q^2}{4\pi\mu^2}\right)^{\varepsilon}\frac{1}{\Gamma(1+\varepsilon)}$$
$$\times H^{a\leftarrow c}(\alpha,\beta)\, d\alpha d\beta\, d\omega^{n-3}(p_k^T)\left\{d\sigma^{LO}(l_1,\tilde{p}_a;\tilde{p}_b,l_2,\dots)\right\}_{d=4+2\varepsilon},$$
$$H^{a\leftarrow c}(\alpha,\beta) = (\alpha\beta(1-\beta))^{\varepsilon}\, x_B^{-\varepsilon}\,\frac{P^*_{c\leftarrow a}(\alpha,\beta)}{\beta},\quad d\omega^{n-3} = \frac{d\Omega^{n-3}}{\Omega^{n-3}},$$

- $P^*_{a\leftarrow a}(\alpha,\beta)$ for $\mathcal{IF}$ and $\mathcal{FI}$ dipoles have to build together the correct soft limit.
- The CS choices for $\mathcal{IF}$, e.g. $P^*_{q\leftarrow q} = C_F\left[\frac{2}{\alpha+\beta} - (2-\alpha) + \varepsilon\alpha\right]$, are not good.
- The following general construction for diagonal $\mathcal{IF}$ and $\mathcal{FI}$ splittings was examined:

  $\mathcal{IF}$: $\quad P^*_{a\leftarrow a}(\alpha,\beta) = m_+(\alpha,\beta)\frac{1}{\alpha}[(1-z)P_{aa}(z)]\big|_{z=z(\alpha,\beta)},$

  $\mathcal{FI}$: $\quad P^*_{a\leftarrow a}(\alpha,\beta) = m_-(\alpha,\beta)\frac{1}{\alpha}[(1-z)P_{aa}(z)]\big|_{z=z(\alpha,\beta)},$

  with several choices of soft partition functions:
  $$m_+^{(a)}(\alpha,\beta) = \theta_{\beta<\alpha},\quad m_+^{(b)}(\alpha,\beta) = \frac{\alpha}{\alpha+\beta},\quad m_+^{(c)}(\alpha,\beta) = \frac{\alpha-\alpha\beta}{\alpha+\beta-\alpha\beta},\quad m_- = 1-m_+.$$
  and several choices of z-variable:
  $$z_A(\alpha,\beta) = 1-\max(\alpha,\beta),\quad z_B(\alpha,\beta) = 1-\alpha,\quad z_C(\alpha,\beta) = (1-\alpha)(1-\beta).$$
- The corresponding radiator functions for $\mathcal{IF}$ were calculated:
  $$\tilde{\mathcal{V}}^{c\leftarrow a}(z,\varepsilon) = \int d\alpha d\beta\; H^{a\leftarrow c}(\alpha,\beta)\;\delta(z - z_X(\alpha.\beta)),\; X = A, B, C.$$
- Good choices (compatible with $\mathcal{II}$) were found, for instance: *Aa*, *Ac*, *Ca* and *Cc*. The choice $z_B = 1-\alpha$ (Bjorken) used by CS is not good!

Problem and workaround for non-diagonal $\mathcal{IF}$ dipoles

- Non-diagonal dipoles, $a \neq b$, are not IR-divergent, hence $m_\pm$ not really needed:
  $P^{*}_{c\leftarrow a}(\alpha,\beta) = P_{ca}\big(z(\alpha,\beta)\big)$ in principle is OK.
- However, we get slightly different $\widetilde{\mathcal{V}}^{c\leftarrow a}(z,\varepsilon)$ than for $\mathcal{II}$ for ALL choices of $z = z(\alpha,\beta)$. The difference traced back to upper phase space limit: $\max(\alpha,\beta) \leq 1$ versus $\alpha+\beta \leq 1$.
- The simplest workaround is to split $\mathcal{IF}$ non-diag. dipoles into two parts:
  $$P^{*+}_{c\leftarrow a}(\alpha,\beta) = m^{(i)}_{+}(\alpha,\beta) P_{ca}(z)\big|_{z=z(\alpha,\beta)}, \quad c \neq a,$$
  $$P^{*-}_{c\leftarrow a}(\alpha,\beta) = m^{(i)}_{-}(\alpha,\beta) P_{ca}(z)\big|_{z=z(\alpha,\beta)},$$
  and treat $P^{*-}_{c\leftarrow a}$ as extra (non-singular) dipoles in the $\mathcal{FI}$ class (decoupled from PDFs).
- This above solution works for $m^{(a)}_{\pm}$ and $m^{(c)}_{\pm}$ and looks like an affordable complication.

Summarizing, it was shown that by means of judicious adjustment of CS dipoles one may define single set of PDFs in the MC scheme, for all dipoles in NLO calculation with $\mathcal{G}_{ba}(z) = 0,\ z \neq 1$, for all processes, with arbitrary number of initial/final state legs,

However, this is not the end of the story...

Collinear remnants of CS scheme in general case

Total cross-section in CS for m partons schematically (hh scattering):

$$\sigma = \int_m d\sigma^{Born} + \left[\int_m d\sigma^{Virt.} + \int_{m+1} d\sigma^{A} + \int_{m+1} d\sigma^{Ct} \right] + \int_{m+1} \left[d\sigma^{Real}_{\varepsilon=0} - d\sigma^{A}_{\varepsilon=0} \right]$$

2-nd term [...] for $h(p_1) h'(p_2) \to a(p_a) + b(p_b) \to 1 + 2 + \ldots m$, eq.(10.30) in CS:

$$\sigma^{Virt.+A+Ct}_{ab} = \sum_{a'} \int dx_a dx_b dx\; f_a(x_a)\; f_b(x_b)\; \langle\, (\mathbf{K} + \mathbf{P})^{aa'}(x)\; d\sigma^{Born}_{a',b}(xp_a, p_b)\, \rangle_{color}$$

$$+ \sum_{b'} \int dx_a dx_b dx\; f_a(x_a)\; f_b(x_b)\; \langle\, (\mathbf{K} + \mathbf{P})^{bb'}(x)\; d\sigma^{Born}_{a,b'}(p_a, xp_b)\, \rangle_{color}, \text{ where}$$

$$\boldsymbol{K}^{a,a'}(x) = \frac{\alpha_S}{2\pi} \Big\{ \overline{K}^{aa'}(x) - K^{aa'}_{\rm F.S}(x) + \delta^{aa'} \sum_i \boldsymbol{T}_i \cdot \boldsymbol{T}_a \frac{\gamma_i}{\boldsymbol{T}_i^2} \left[\left(\frac{1}{1-x} \right)_+ + \delta(1-x) \right] \Big\} - \frac{\alpha_S}{2\pi} \boldsymbol{T}_b \cdot \boldsymbol{T}_{a'} \frac{1}{\boldsymbol{T}_{a'}^2} \widetilde{K}^{aa'}(x)$$

$$\widetilde{K}^{ab}(x) = P^{ab}_{\rm reg}(x)\; \ln(1-x) + \delta^{ab} \boldsymbol{T}_a^2 \left[\left(\frac{2}{1-x} \ln(1-x) \right)_+ - \frac{\pi^2}{3} \delta(1-x) \right]$$

$$K^{aa'}_{F.S.} \equiv 0$$

$$\overline{K}^{qg}(x) = P^{qg}(x) \ln\frac{1-x}{x} + C_F\, x \;, \tag{8.32}$$

$$\overline{K}^{gq}(x) = P^{gq}(x) \ln\frac{1-x}{x} + T_R\, 2x(1-x) \;, \tag{8.33}$$

$$\overline{K}^{qq}(x) = C_F \left[\left(\frac{2}{1-x} \ln\frac{1-x}{x} \right)_+ - (1+x) \ln\frac{1-x}{x} + (1-x) \right] - \delta(1-x) \left(5 - \pi^2\right) C_F \;, \tag{8.34}$$

$$\overline{K}^{gg}(x) = 2C_A \left[\left(\frac{1}{1-x} \ln\frac{1-x}{x} \right)_+ + \left(\frac{1-x}{x} - 1 + x(1-x) \right) \ln\frac{1-x}{x} \right] - \delta(1-x) \left[\left(\frac{50}{9} - \pi^2 \right) C_A - \frac{16}{9} T_R N_f \right] . \tag{8.35}$$

With our dipoles and PDFs in the MC FS we are getting $\boxed{\mathbf{K}^{a,a'} = 0}$!!!

This is for ANY process, with h+h beams or lepton+h beams (DIS)!

Last problem: What about P? See next slide...

The remaining collinear remnant P due to multiscales in NLO

Last problem

P-matrix is a quite primitive object (CS eq.10.25):

$$\boldsymbol{P}^{a,a'}(p_1,\dots,p_m,p_b;xp_a,x;\mu_F^2)$$
$$= \frac{\alpha_S}{2\pi} P^{aa'}(x) \frac{1}{\boldsymbol{T}_{a'}^2} \left[\sum_i \boldsymbol{T}_i \cdot \boldsymbol{T}_{a'} \ \ln \frac{\mu_F^2}{2xp_a \cdot p_i} + \boldsymbol{T}_b \cdot \boldsymbol{T}_{a'} \ \ln \frac{\mu_F^2}{2xp_a \cdot p_b} \right] . \quad (10.25)$$

- It originates from normalization factors like $\left(\frac{xs_{ai}}{\mu_F^2}\right)^{\varepsilon} \times \frac{1}{\varepsilon} P_{aa'}$, $s_{ai} = 2p_a \cdot p_i$.
- For $hh \to Z\gamma, H, WW,..$ and lepton-hadron DIS, only 2nd term is present. It is easily eliminated with $\mu_F^2 = 2xp_a \cdot p_b$ or $\mu_F^2 = Q^2$, getting $\mathbf{P} = 0$.
- The problematic 1-st term is from $\sum_i$ over $\mathcal{IF}$-dipoles with different s_{ai}.
- Is there some choice of μ_F^2 in PDFs eliminating at once the entire 1-st term for all processes with more than two coloured "legs"?
- See next slide...

Zeroing collinear remnant P

(work in progress)

$$\sigma_{ab}^{col.rem.} = \int dx_a dx_b\; f_b(\mu_F, x_b)\; f_a(\mu_F, x_a)\; \Big\{ d\sigma^{Born}_{a,b}(p_a, p_b) +$$

$$+\sum_{a'} \int dx\; \Big\langle\; \frac{\alpha_S}{2\pi} P_{aa'}(x) \Big[\sum_i \frac{T_i \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2x s_{ai}} + \frac{T_b \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2x s_{ab}} \Big] d\sigma^{Born}_{a',b}(x p_a, p_b) \;\Big\rangle_{color} + \dots \Big\}$$

Using colour conservation $\langle\; T_{a'} + T_b + \sum_i T_i \;\rangle_{color} = 0$ and evolution equations for $f_a(\mu, x)$ we obtain the following identity:

$$\sigma_{ab}^{col.rem.} = \int dx_a dx_b\; f_b(\mu_F, x_b)\; f_a(\mu_1, x_a)\; \Big\{ d\sigma^{Born}_{a,b}(p_a, p_b) + \sum_{a'} \int dx\; \frac{\alpha_S}{2\pi} P_{aa'}(x)$$

$$\times \Big\langle\; \Big[\sum_i \frac{T_i \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2x s_{ai}} + \frac{T_b \cdot T_{a'}}{T_{a'}^2} \ln \frac{\mu_F^2}{2x s_{ab}} + \ln \frac{\mu_1^2}{\mu_F^2} \Big] d\sigma^{Born}_{a',b}(x x_a p_1, x_b p_2) \;\Big\rangle_{color} + \dots \Big\}$$

μ_F^2 is local dummy parameter in [...] (colour conservation!), hence we substitute $\mu_F^2 = 2x s_{ab}$, and solve for μ_1 the following equation:

$$\sum_{a'} \int_0^1 dz P_{aa'}(z) \sum_i \ln \frac{s_{ab}}{s_{ai}} \Big\langle \frac{T_i \cdot T_{a'}}{T_{a'}^2} d\sigma^{Born}_{a',b}(z p_a, p_b) \Big\rangle_{c.} + \sum_{a'} \int_0^1 dz P_{aa'}(z) d\sigma^{Born}_{a',b}(z p_a, p_b) \ln \frac{\mu_1^2}{2z s_{ab}} \equiv 0.$$

New scale μ_1 can be calculated numerically (1-dim. integral over z) at each point of the Born phase space, $h_1 + h_2 \to p_a + p_b \to 1 + 2 + \dots m$, or even analytically in some simple cases.

Issues already explored but not covered in this talk:

due to its limited scope...

- Fine details of new modified dipoles, soft-coll. counterterms in $d = 4 + 2\varepsilon$ dimensions, including new kinematic mappings.
- Compatibility of CS scheme with LO parton shower MC. (Correct soft limit and and positivity).
- Explicit x-check calculations of NLO corrections using modified CS dipoles for DIS (DY shown partly on slide 3).

Other important issues to be studied:

- More explicit examples of NLO calculations: $pp \to Z + jet,\ 2Jet, \ldots$.
- Extending KrkNLO to more processes.
- Does MC FS extend to “NLO PDFs” $\otimes$ “NNLO Hard process”?

Summary

- PDFs in the MC scheme are formally and practically as universal (process independent) as in the $\overline{MS}$ scheme thanks to universality of the newly modified CS dipoles and/or related soft-collinear counterterms. NEW!
- Substantial simplification of the classic Catani-Seymour NLO calculation scheme is achieved. NEW!
- KrkNLO method with PDFs in the MC factorization scheme (implementing NLO corrections with single multiplicative MC weight) is NOT limited to processes with two coloured legs (DY, DIS)! NEW!
- All presented results are preliminary and unpublished!

Useful discussions with co-authors of the KrkNLO project
W. Płaczek, M. Sapeta, A. Siódmok, and M. Skrzypek are acknowledged.

"MC Factorization Scheme" is still temporary name, we are looking for some better name in the next publication. What about KRK FS?

Preliminary version of this presentation was given at PSR 2017 Conf. in Cambridge.