

Higgs decays into b-quarks...

Wojciech Bizoń

Institute for Theoretical Particle Physics - Karlsruhe Institute of Technology

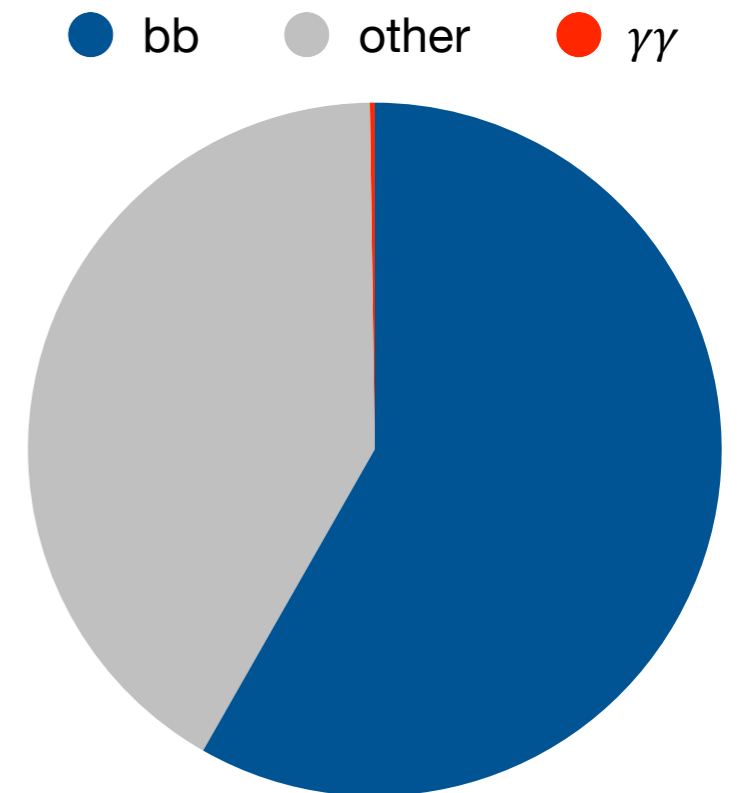
Epiphany 2020, Kraków, 7.01.2020

Outline

- Higgs decay into b-quarks
- NNLO+PS using MiNLO
- NNLO with massive b-quarks
- Summary

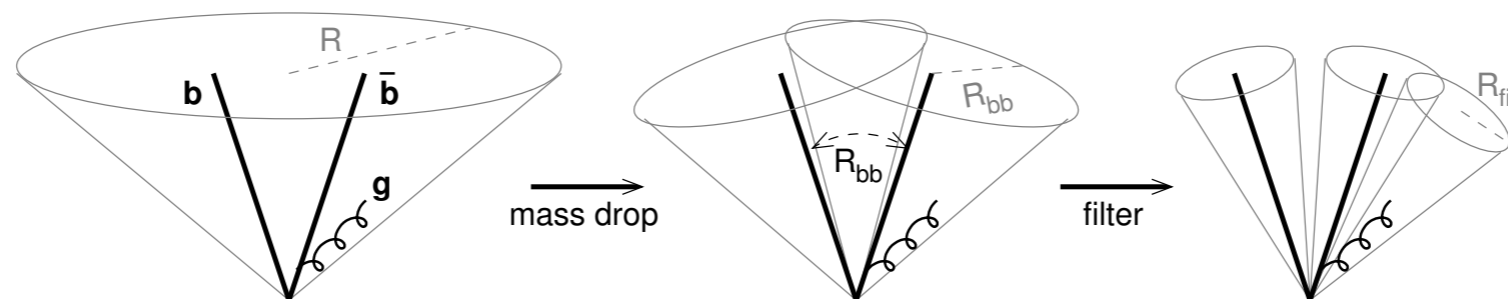
Higgs decay to b-quarks

- ▶ most common decay channel of the Higgs boson ($M_H=125$ GeV)
- ▶ very challenging due to overwhelming QCD backgrounds
- ▶ observed by both **ATLAS & CMS**
[1808.08238, 1808.08242]
- ▶ important for exotic searches which benefit from the large Hbb branching ratio
- ▶ NLO: solved a long time ago [Phys.Rev.D22(1980),715], ...
NNLO with $m_b=0$: multiple results in the literature [1110.2368], [1501.07226], [1712.06954], [1907.05836]
NNLO with finite m_b : this talk... [1805.06658] and [1911.11524]
N3LO: first results appearing [1904.08960]



Finding b-jets from Higgs decays

- ▶ b-quarks confined inside B-hadrons
- ▶ b-jets classified as decay products of the Higgs boson based on the jet properties (M_{bb} , ΔR_{bb} , ...)
- ▶ sophisticated techniques developed to improve signal-to-background ratio
eg. *boosted region* [Butterworth, Davison, Rubin, Salam, 0802.2470] and references thereto



- ▶ precise theoretical description might be helpful for sharpening various tools needed for precision Higgs phenomenology

NNLO+PS using MiNLO

[WB, E.Re, G.Zanderighi, 1912.09982]

NNLO+PS: why and how?

► what do we want?

Goal 1: transition from limited multiplicity to a realistic picture of an event with $O(100)$ particles in a final state

Solution: a parton shower algorithm based on knowledge of soft/collinear QCD evolution

Goal 2: improve accuracy of event generators

Solution: include as much information as possible from higher-order perturbative QCD

► plan of action:

1. construct an NLO-accurate **Hbbg** event generator in POWHEG
(NLO description of exclusive **bb+1jet** observables)
2. upgrade the **Hbbg** event generator with MiNLO
(improve accuracy of the Sudakov form factor responsible for damping soft/collinear emissions,
as a result all **bb+0jet** and **bb+1jet** observables are described with NLO accuracy)
3. perform reweighting of events such that **bb+0jet** observables are NNLO accurate

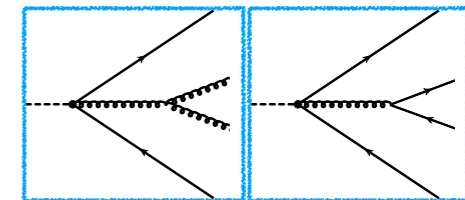
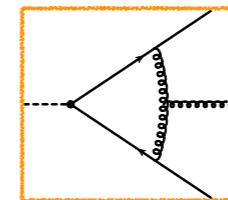
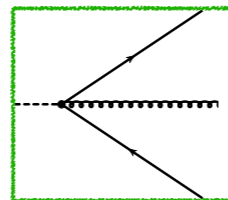
MiNLO: building a generator

► for simplicity we work in the Higgs rest frame

► the main POWHEG function that is being integrated is the B-bar function which contains all necessary ingredients

$$\bar{B}(\Phi_{bbg}) = \alpha_s(q_t^2) \Delta^2(y_3) \left[\underbrace{B_{H \rightarrow bbg}}_{\text{Sudakov Form Factor}} \cdot \left(1 - 2\Delta^{(1)}(y_3) \right) + \underbrace{V_{H \rightarrow bbg}}_{\text{Virtual}} + \int d\Phi_r \underbrace{R_{H \rightarrow bbg}}_{\text{Real}} \right]$$

Sudakov Form Factor



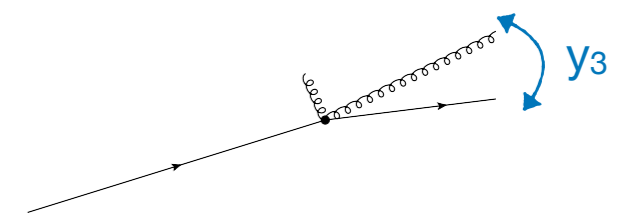
► MiNLO:

- ☞ Sudakov FF needs to contain terms A_1, A_2, B_1, B_2
- ☞ strong coupling should be evaluated at scale q_t

► y_3 = three-jet resolution parameter

- ☞ separates between two- and three-jet configurations within a clustering algorithm
- ☞ we use C/A algorithm for simplicity:

$$v_{ij} = 2(1 - \cos \theta_{ij}) \quad y_{ij} = v_{ij} \cdot \frac{\min(E_i, E_j)^2}{M_H^2}$$



- ☞ configurations with small y_3 parameter are suppressed by vanishing Sudakov FF
- ☞ NNLL resummation performed in [Banfi, McAslan, Monni, Zanderighi, 1607.03111]

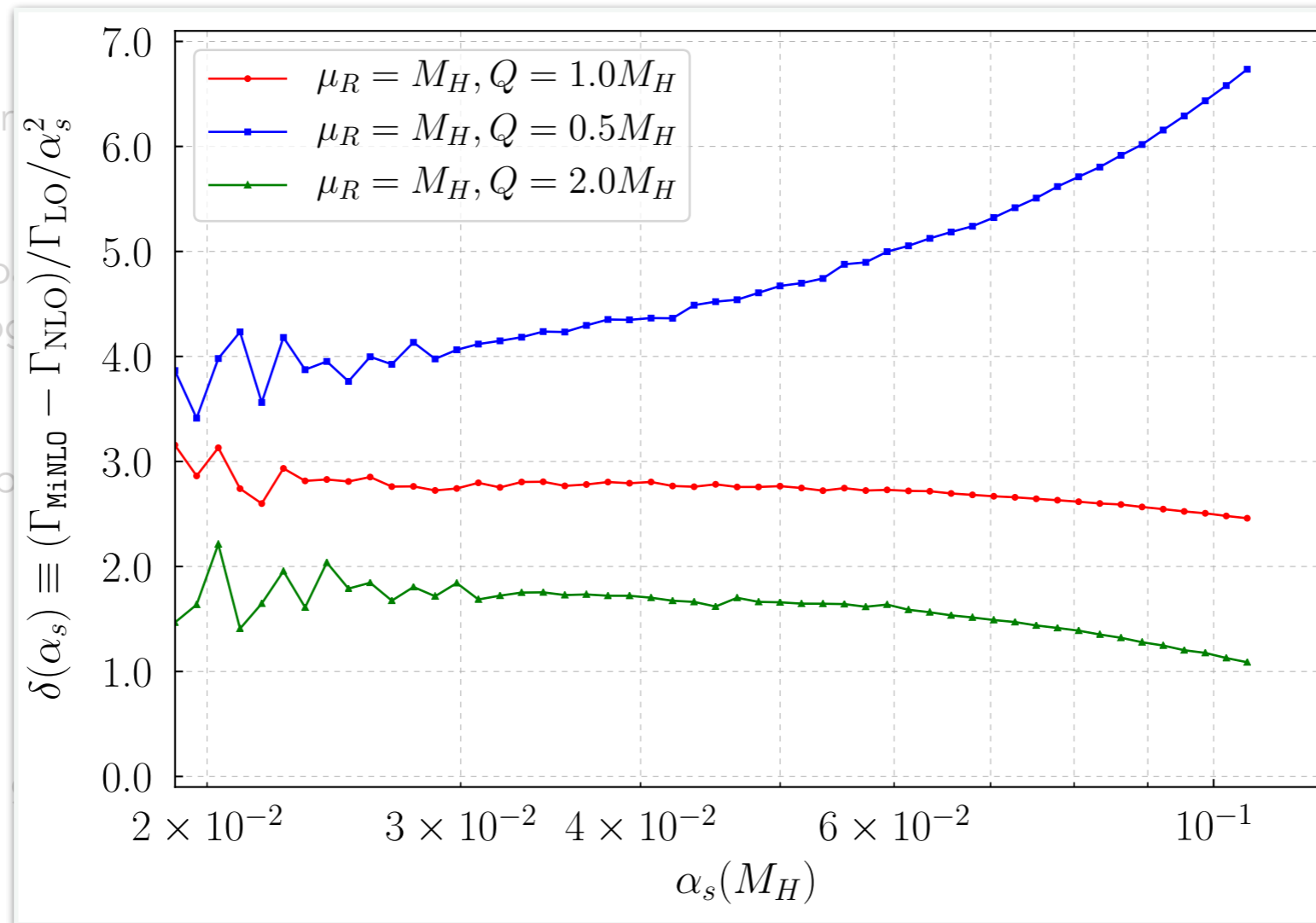
MiNLO: verification

- ▶ **Hbbg_MiNLO** generator constructed within POWHEG framework, upgraded with MiNLO
- ▶ Higgs decay \rightarrow no partons in initial state \rightarrow no PDFs required \rightarrow we can perform small- α_s limit to verify an NLO accuracy of the **Hbbg_MiNLO** generator
- ▶ we consider the Hbb decay width and compare it with the analytical result

$$\delta(\alpha_s) \equiv \frac{1}{\alpha_s^2} \cdot \frac{\Gamma_{H \rightarrow b\bar{b}}^{\text{MiNLO}} - \Gamma_{H \rightarrow b\bar{b}}^{\text{NLO}}}{\Gamma_{H \rightarrow b\bar{b}}^{\text{LO}}}$$

- ▶ if the **Hbbg_MiNLO** generator yields a correct NLO decay width, the difference should approach a constant in the small- α_s limit
- ▶ challenge: for small values of α_s the Sudakov peak moves to lower and lower y_3 values (calculation in quadruple precision required)
- ▶ all three curves approach constant value in the $\alpha_s \rightarrow 0$ limit
☞ the difference between MiNLO and NLO results is at most $O(\alpha_s^2)$

MiNLO: verification



▶ Hbbg_MiNLO gener

▶ Higgs decay \rightarrow no p
accuracy of the Hbbg

▶ we consider the Hbb

▶ if the Hbbg_MiNLO
small- α_s limit

to verify an NLO

each a constant in the

▶ challenge: for small values of α_s the Sudakov peak moves to lower and lower y_3 values
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NNLO+PS: reweighting

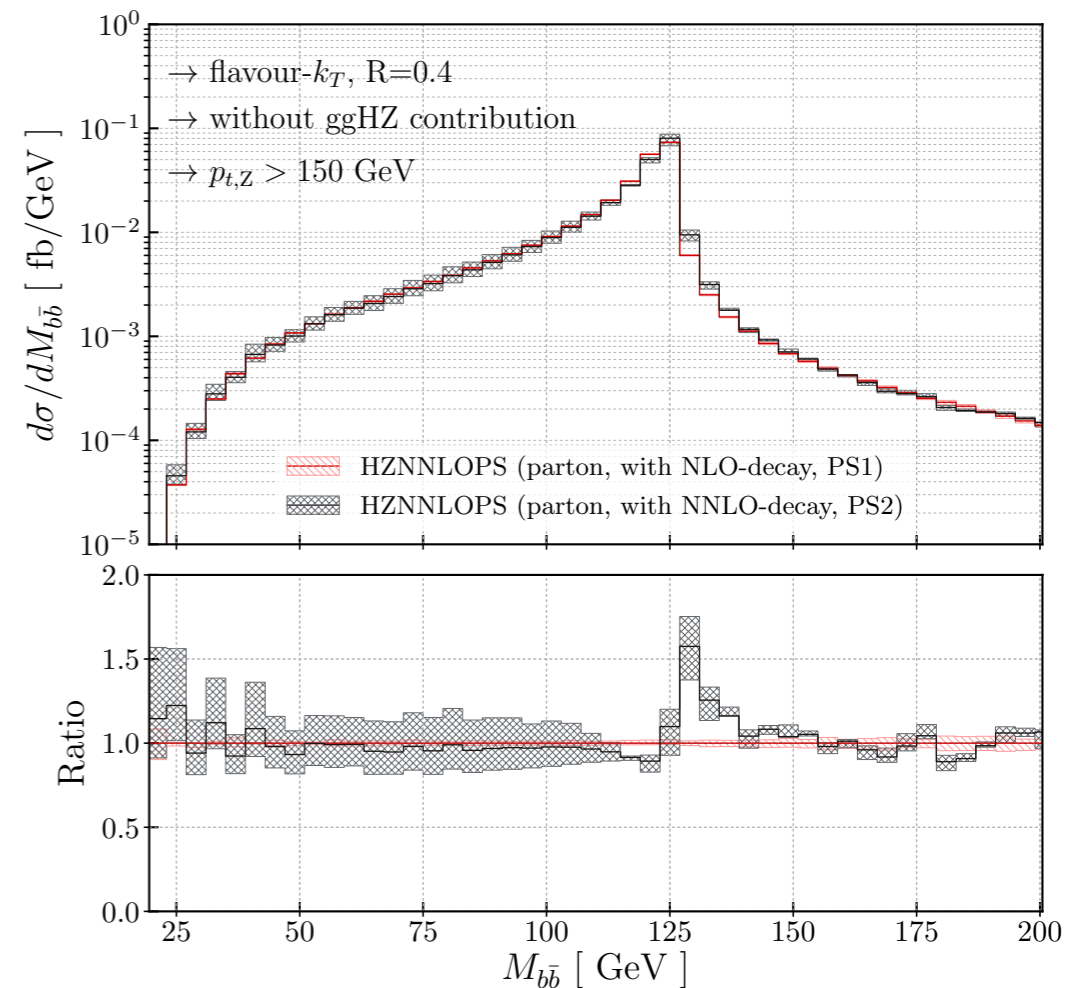
- ▶ the `Hbbg_MiNLO` results can be further improved to reproduce the exact NNLO decay width
 - ☞ this can be achieved by reweighting the `Hbbg_MiNLO` events by a factor

$$\mathcal{W} = \frac{\Gamma_{H \rightarrow b\bar{b}}^{\text{NNLO}}}{\Gamma_{H \rightarrow b\bar{b}}^{\text{MiNLO}}}$$

- ▶ the resulting events can be easily interfaced with any Higgs production mode
- ▶ the amalgamated events can be showered with a kt-ordered parton shower
 - ☞ there are two veto scales (`scalup_prod` and `scalup_dec`)
 - ☞ veto is separately applied to production and decay evolution

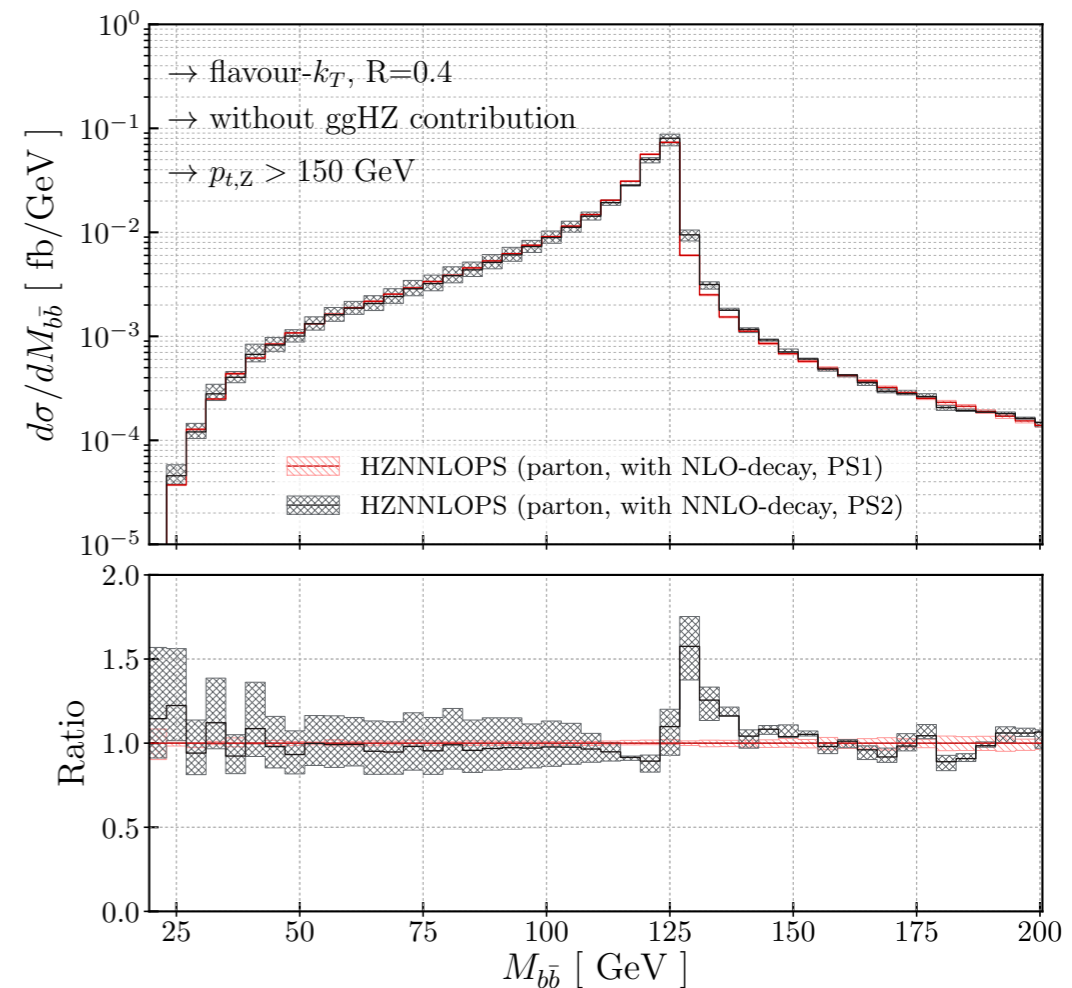
NNLO+PS: example

- ▶ example: decay events interfaced with associated Higgs production
 - ☞ $pp \rightarrow HZ \rightarrow (bb)(e+e-) @13\text{TeV}$
- ▶ results compared to the ones presented in [\[Astill,WB,Re,Zanderighi, 1804.08141 \]](#)
 - ☞ production described at NNLO
 - ☞ decay described at NLO(red) / NNLO(black)
- ▶ parton shower interface:
 - ☞ PS1: whole production/decay event generated by POWHEG,
 - single veto scale for production/decay
 - ☞ PS2: production/decay separately generated,
 - two veto scales (`scalup_prod` and `scalup_dec`)
- ▶ better scale uncertainty estimation by NNLO decay events (left of the Higgs boson peak)
- ▶ changes around the peak: related to b-quark mass
 - ☞ NLO-decay: massive b-quarks
 - ☞ NNLO-decay: massless b-quarks



NNLO+PS: example

- ▶ example: decay events interfaced with associated Higgs production
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☞ `h_bbg` process available in POWHEG-BOX-V2

☞ 60M ready to use decay events available at <https://cernbox.cern.ch/index.php/s/v1qxYyICeL66InO>

NNLO with massive b-quarks

[A.Behring, WB, 1911.11524]

Overview

- ▶ need of high-precision Standard Model predictions requires consideration of many effects: **masses of b-quarks** [Bernreuther, Chen, Si, 1805.06658] and [Behring, WB, 1911.11524]

- ▶ structure of an NNLO calculation:

- ☞ **double-virtual corrections (VV)**: two-loop $H \rightarrow bb$ amplitudes, **explicit $1/\epsilon$ poles** ← VV obtained using heavy-quark form factor [Bernreuther et al., hep-ph/0508254] [Ablinger et al., 1712.09889]
- ☞ **real-virtual corrections (RV)**: one-loop $H \rightarrow bbg$ amplitudes, **explicit $1/\epsilon$ poles**, **divergences in soft limit**
- ☞ **double-real corrections (RR)**: tree-level $H \rightarrow bb\{gg,qq\}$ amplitudes, **divergences in soft/collinear limits**

- ▶ **soft/collinear divergences** regulated using **nested soft-collinear subtraction scheme**

- ▶ additionally:

- ☞ **four b-quarks contribution**: tree-level $H \rightarrow bbbb$, UV&IR finite
- ☞ **top-quark mediated contribution**: two-loop $H \rightarrow bb(y_t)$ and one-loop $H \rightarrow bbg(y_t)$, UV&IR finite



approx. tot. dec: [Chetyrkin, Kwiatkowski, hep-ph/9505358] & [Larin, Ritbergen, Vermaseren, hep-ph/9506465]
exact & differential: [Primo, Sasso, Somogyi, Tramontano, 1812.07811]

Soft/collinear divergences

- ▶ singularities of QCD amplitudes in soft/collinear limits are **generic and independent of a hard process**
- ▶ soft/collinear divergences in **RR** and **RV** contributions:
 - ☞ RV, single-soft: $H \rightarrow bb(g)$
 - ☞ RR, single-soft: $H \rightarrow bbg(g)$
 - ☞ RR, single-coll: $H \rightarrow bb[gg]$
 - ☞ RR, double-soft: $H \rightarrow bb(gg)$ & $H \rightarrow bb(qq)$

▶ nested soft-collinear subtractions

- ☞ the real-virtual case $H(q_1) \rightarrow b(q_2) + b(q_3) + g(q_4)$ @one-loop is regulated then as

$$2\text{Re}\langle \mathcal{M}_{bbg}^{(0)} | \mathcal{M}_{bbg}^{(1)} \rangle d\Phi_{bbg} = \left[(I - S_4) + S_4 \right] 2\text{Re}\langle \mathcal{M}_{bbg}^{(0)} | \mathcal{M}_{bbg}^{(1)} \rangle d\Phi_{bbg}$$

- ☞ the double-real case $H(q_1) \rightarrow b(q_2) + b(q_3) + g(q_4) + g(q_5)$ is regulated then as

$$F_{LM}(bbgg) = (I - S_5)(I - S_{45})(I - C_{45})F_{LM}(bbgg) \quad \leftarrow \text{fully-regulated term}$$

$$+ \left[S_5(I - S_{45})(I - C_{45}) + (I - S_5)(I - S_{45})C_{45} + S_5(I - S_{45})C_{45} \right] F_{LM}(bbg)[dg_5]$$

$$+ S_{45}F_{LM}(bb)[dg_4][dg_5] \quad \leftarrow \text{double-unresolved term}$$

← single-unresolved term

with $F_{LM}(bbX) = d\Phi_{bbX} |\mathcal{M}_{bbX}^{\text{tree}}|^2 \mathcal{F}_{\text{obs}}(bbX)$

the $[dg_i]$ being the usual real-emission phase-space and **S, C** denoting the soft/collinear operators

- ▶ single- and double-unresolved terms feature $1/\epsilon$ poles that appear after integration over the unresolved phase space ☞ **next slide**

Integrated subtraction terms: RR

- ▶ RR, single-unresolved subtraction terms:

- ☞ computed long time ago as part of the FKS subtraction scheme at NLO

- compendium of formulae: [[Alioli,Nason,Oleari,Re,1002.2581](#)]

- ▶ RR, double-unresolved subtraction terms:

- ☞ factorisation formula

$$S_{45} |\mathcal{M}_{bbgg}^{\text{tree}}|^2 = g_s^4 \text{DSoft}_{gg}^{(0)}(q_2, q_3; q_4, q_5) |\mathcal{M}_{bb}^{\text{tree}}|^2$$

- ☞ split phase space into energy ordered configurations: $1 = \Theta(E_4 - E_5) + \Theta(E_5 - E_4)$

- ☞ in the double-soft limit momenta q_2 and q_3 are in back-to-back configuration

- ☞ the integrated double-soft subtraction term is (integrated numerically)

$$\begin{aligned} \text{DSoft}_{gg,\text{int}}^{(0)}(q_2, q_3) &= \int [dg_4][dg_5] \text{DSoft}_{gg}^{(0)}(q_2, q_3; q_4, q_5) \\ &= \frac{1}{(4\pi)^2} \left(\frac{\mu_R}{E_{\text{max}}} \right)^{4\epsilon} \left[\frac{C_{gg}^{(-3)}}{\epsilon^3} + \frac{C_{gg}^{(-2)}}{\epsilon^2} + \frac{C_{gg}^{(-1)}}{\epsilon} + C_{gg}^{(0)} \right] \end{aligned}$$

with $E_{\text{max}} = (M_H^2 - 4m_b^2)/2$

Integrated subtraction terms: RV

- ▶ soft-limit of one-loop amplitudes with massive quarks studied previously
[Mitov,Moch, hep-ph/0612149] and [Bierenbaum,Czakon,Mitov, 1107.4384]

eikonal-factors

$$S_{ij,k} = \frac{(q_i \cdot q_j)}{(q_i \cdot q_k)(q_j \cdot q_k)}$$

$$S_4 2\text{Re}\langle \mathcal{M}_{bbg}^{(0)} | \mathcal{M}_{bbg}^{(1)} \rangle = -g_s^2 C_F \left(S_{22,4}^{(0)} = 2S_{23,4}^{(0)} + S_{33,4}^{(0)} \right) \\ \times \left[2\text{Re}\langle \mathcal{M}_{bb}^{(0)} | \mathcal{M}_{bb}^{(1)} \rangle + \left(\mathcal{R}_{23,4}^{(1)} + \mathcal{Z}_{\alpha_s}^{(1)} + \mathcal{Z}_A^{(1)} \right) |\mathcal{M}_{bb}^{(0)}|^2 \right]$$

one-loop soft function
renormalisation constants

- ▶ in the soft limit momenta q_2 and q_3 are in back-to-back configuration
- ▶ **one-loop soft function:** the integration over soft-gluon phase space reduces to
 - ☞ an energy integral (dE_4): overall scaling only → trivial integration
 - ☞ angular integral ($d(\cos\theta)$):
 - ✓ (a) numerically very simple
 - ✓ (b) performed also analytically, result in terms of HPLs [using Mathematica, HarmonicSums]

Pole cancellation

- ▶ integrated subtraction terms allow to trade soft/collinear divergences for $1/\epsilon$ poles
- ▶ since singularities of QCD amplitudes in soft/collinear limits are generic and independent of a hard process, pole cancellation can be demonstrated without referring to exact form of matrix elements
- ▶ single-unresolved configuration (RR + RV)

$$d\Gamma_{RR}^{\text{SU}}(bbgg + bbqq) + d\Gamma_{RV}^{\text{SU}}(bbg) = \left[\frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \dots \right] (I - S_4) F_{LM}(bbg)$$

☞ **cancellation shown analytically**

$$F_{LM}(bbg) = d\Phi_{bbg} |\mathcal{M}_{bbg}^{(0)}|^2 \mathcal{F}_{\text{obs}}(bbg)$$

- ▶ fully-unresolved configuration (RR + RV + VV)
- (a) part proportional to one-loop $H \rightarrow bb$ matrix element

$$d\Gamma_{RV}^{\text{FR}}(bbg) + d\Gamma_{VV}^{\text{FR}}(bb) = \left[\frac{\#}{\epsilon} + \dots \right] F_{LV}^{\text{fin}}(bb)$$

☞ **cancellation shown analytically**

$$F_{LV}^{\text{fin}}(bb) = d\Phi_{bb} 2\text{Re}\langle \mathcal{M}_{bb}^{(0)} | \mathcal{M}_{bb}^{(1)} \rangle \mathcal{F}_{\text{obs}}(bb)$$

- (b) part proportional to tree-level $H \rightarrow bb$ matrix element

$$d\Gamma_{RR+RV+VV}^{\text{DU}} = \left[\frac{\#}{\epsilon^3} + \frac{\#}{\epsilon^2} + \frac{\#}{\epsilon} + \dots \right] F_{LM}(bb)$$

☞ **cancellation shown numerically** (since integrated double-soft function computed only numerically)

Results: verification

- ▶ approximate analytical result in heavy-scalar decay into quark-antiquark pair
[Harlander, Steinhauser, hep-ph/9704436]

- ▶ expand the NNLO width as

$$\Gamma_{H \rightarrow b\bar{b}}^{\text{NNLO}} = \Gamma_{H \rightarrow b\bar{b}}^{\text{LO}} \cdot \left[1 + \left(\frac{\alpha_s}{\pi} \right) \gamma_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \gamma_2 \right]$$

with further breakdown into colour structures

$$\gamma_1 = C_F \cdot \gamma_1^{C_F}$$

$$\gamma_2 = C_F^2 \cdot \gamma_2^{C_F^2} + C_F C_A \cdot \gamma_2^{C_F C_A} + C_F T_F n_l \cdot \gamma_2^{C_F T_F n_l} + C_F T_F \cdot \gamma_2^{C_F T_F}$$

- ▶ compare our result to analytical prediction (MH=125.09 GeV, mb=4.78 GeV)

	$\gamma_1^{C_F}$	$\gamma_2^{C_F^2}$	$\gamma_2^{C_F C_A}$	$\gamma_2^{C_F T_F n_l}$	$\gamma_2^{C_F T_F}$
HS '97	-7.446648	+19.4192	-53.5558	+18.6286	+14.7946
Our result	-7.446648(7)	+19.4199(10)	-53.5557(20)	+18.6283(2)	+14.7945(1)

- ▶ very good agreement down to a few digits (thanks to $mb/MH \ll 1$)

Summary

- ▶ $H \rightarrow bb$ decay process is an important channel that is already being explored at the LHC, measurements during the HL-LHC programme will offer many more $H \rightarrow bb$ events
- ▶ full exploitation of such data will benefit from better theoretical understanding
- ▶ NNLO QCD calculation of the $H \rightarrow bb$ decay successfully combined with parton shower (massless b-quarks)
☞ tool available in the POWHEG repository of processes (`h_bbg` process)
- ▶ calculation of b-quark mass effects at NNLO QCD completed within the nested soft-collinear subtraction scheme
- ▶ good position to perform thorough phenomenological studies...

Summary

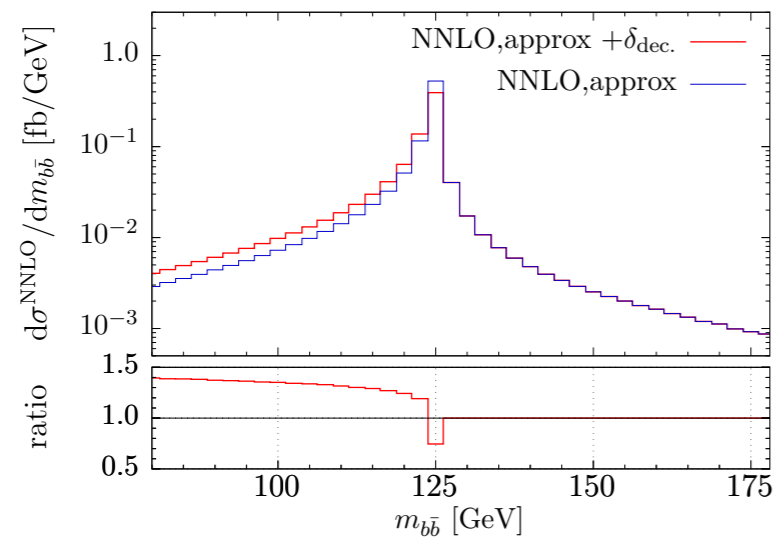
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THANK YOU!

Backup

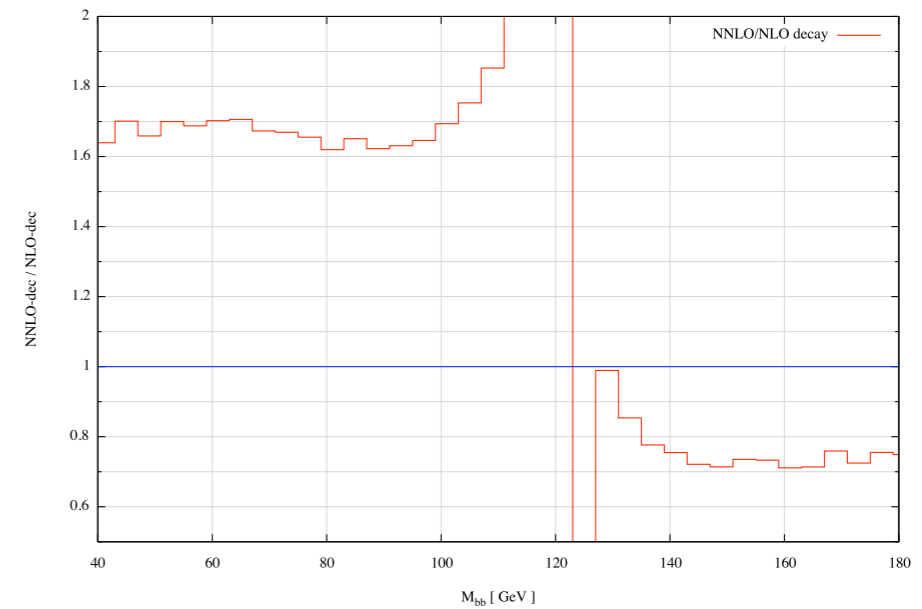
NNLO+PS: M_{bb}

- ▶ invariant mass of the b-jets (M_{bb})
- ▶ NNLO/NLO K-factor agrees with fixed-order predictions



Plot for HW production

[Caola,Luisoni,Melnikov,Rontsch, 1712.06954]



NNLO: Pole cancellation (1)

► single-unresolved

$$2M_H \langle d\Gamma_{\text{RV}}^{\text{FR}}(b\bar{b}g) \rangle = \frac{1}{\epsilon} \left[\left(\frac{\alpha_s}{4\pi} \right) 4C_F \left[1 + \frac{1+\beta^2}{2\beta} \log \left(\frac{1-\beta}{1+\beta} \right) \right] \langle F_{LV}^{\text{fin}}(b\bar{b}) \rangle \right] + \mathcal{O}(\epsilon^0), \quad (4.22)$$

and the explicit expansion of the double-virtual contribution, Eq. (4.20), yields

$$2M_H \langle d\Gamma_{\text{VV}}^{\text{FR}}(b\bar{b}) \rangle = -\frac{1}{\epsilon} \left[\left(\frac{\alpha_s}{4\pi} \right) 4C_F \left[1 + \frac{1+\beta^2}{2\beta} \log \left(\frac{1-\beta}{1+\beta} \right) \right] \langle F_{LV}^{\text{fin}}(b\bar{b}) \rangle \right], \quad (4.23)$$

► fully-unresolved (proportional to one-loop $H \rightarrow b\bar{b}$ matrix element)

$$\begin{aligned} & 2M_H \langle d\Gamma_{\text{RR}}^{\text{SU}}(b\bar{b}gg + b\bar{b}q\bar{q}) \rangle \\ &= \left\langle \left(\frac{\alpha_s}{4\pi} \right) \left[\frac{2C_A}{\epsilon^2} + \frac{1}{\epsilon} \left[4C_F + \beta_0(n_l) + \frac{C_A - 2C_F}{v_{23}} \log \left(\frac{1+v_{23}}{1-v_{23}} \right) \right. \right. \right. \right. \\ & \quad \left. \left. \left. + 2C_A \log \left(\frac{m_b \mu_R}{2(q_2 \cdot q_4)} \right) + 2C_A \log \left(\frac{m_b \mu_R}{2(q_3 \cdot q_4)} \right) \right] \right] (I - S_4) F_{LM}(b\bar{b}g) \right\rangle + \mathcal{O}(\epsilon^0). \end{aligned} \quad (4.24)$$

A similar expansion holds for the real-virtual single-unresolved contribution, Eq. (4.14),

$$\begin{aligned} & 2M_H \langle d\Gamma_{\text{RV}}^{\text{SU}}(b\bar{b}g) \rangle \\ &= \left\langle \left(\frac{\alpha_s}{4\pi} \right) \left[-\frac{2C_A}{\epsilon^2} - \frac{1}{\epsilon} \left[4C_F + \beta_0(n_l) + \frac{C_A - 2C_F}{v_{23}} \log \left(\frac{1+v_{23}}{1-v_{23}} \right) \right. \right. \right. \right. \\ & \quad \left. \left. \left. + 2C_A \log \left(\frac{m_b \mu_R}{2(q_2 \cdot q_4)} \right) + 2C_A \log \left(\frac{m_b \mu_R}{2(q_3 \cdot q_4)} \right) \right] \right] (I - S_4) F_{LM}(b\bar{b}g) \right\rangle. \end{aligned} \quad (4.25)$$

NNLO: Pole cancellation (2)

- ▶ fully-unresolved (proportional to one-loop $H \rightarrow b\bar{b}$ matrix element)
- ☞ cancellation of at least 7-digits

	$\mathcal{C}_{C_F C_A}^{\text{DU},(-3)}$	$\mathcal{C}_{C_F C_A}^{\text{DU},(-2)}$	$\mathcal{C}_{C_F^2}^{\text{DU},(-2)}$	$\mathcal{C}_{C_F T_F n_l}^{\text{DU},(-2)}$	$\mathcal{C}_{C_F C_A}^{\text{DU},(-1)}$	$\mathcal{C}_{C_F^2}^{\text{DU},(-1)}$	$\mathcal{C}_{C_F T_F n_l}^{\text{DU},(-1)}$	$\mathcal{C}_{C_F T_F}^{\text{DU},(-1)}$
RR	-22.11	-279.75	+244.32	+14.74	-1777.55	+2672.58	+185.77	0
RV	+22.11	+320.28	-488.64	-29.47	+1732.44	-2672.58	-161.20	+257.20
VV	0	-40.53	+244.32	+14.74	+45.11	0	-24.56	-257.20
Sum	10^{-13}	10^{-10}	10^{-8}	10^{-11}	10^{-6}	10^{-6}	10^{-5}	0
Rel. canc.	10^{-14}	10^{-13}	10^{-11}	10^{-13}	10^{-10}	10^{-9}	10^{-7}	0

Table 1: Numerical values of the pole coefficients of the double-unresolved term as defined in Eq. (4.27). The numerical values correspond to $m_b = 4.78$ GeV, $M_H = 125.09$ GeV and the renormalisation scale is $\mu_R = 3M_H$. Each column corresponds to a particular colour structure of a given ϵ pole. The three rows correspond to the double-real, real-virtual, and double-virtual contributions. In the last two rows, we report the absolute and relative level of cancellation after adding up RR + RV + VV contributions. The last row is normalised to the largest value of each column.

NNLO: Results

- Expansion coefficients and $H \rightarrow b\bar{b}$ decay width in the $\overline{\text{MS}}$ scheme

μ_R	$\frac{1}{2}M_H$	M_H	$2M_H$
$\bar{\gamma}_1^{b\bar{b}}$ (our res.)	+3.023597(10)	+5.796203(15)	+8.568783(11)
$\bar{\gamma}_1^{b\bar{b}}$ (Ref. [20])	+3.024	+5.798	+8.569
$\bar{\gamma}_1^{b\bar{b}}$ (Ref. [71], $m_b = 0$)	+2.8941	+5.6667	+8.4393
$\bar{\gamma}_2^{b\bar{b}}$ (our res., w/o $y_b y_t$)	-3.2466(31)	+30.4376(33)	+79.1755(38)
$\bar{\gamma}_2^{b\bar{b}}$ (our res., with $y_b y_t$)	+3.7123(31)	+37.3965(33)	+86.1345(38)
$\bar{\gamma}_2^{b\bar{b}}$ (Ref. [20], with $y_b y_t$)	+3.685	+37.371	+86.112
$\bar{\gamma}_2^{b\bar{b}}$ (Ref. [71], $m_b = 0$)	-3.8368	+29.1467	+77.1844
$\bar{\Gamma}_{\text{LO}}^{b\bar{b}}$ [MeV]	+2.17005	+1.92702	+1.73274
$\bar{\Gamma}_{\text{NLO}}^{b\bar{b}}$ [MeV]	+2.43161	+2.32781	+2.21731
$\bar{\Gamma}_{\text{NNLO}}^{b\bar{b}}$ [MeV] (w/o $y_b y_t$)	+2.42041(1)	+2.40333(1)	+2.36344(1)
$\bar{\Gamma}_{\text{NNLO}}^{b\bar{b}}$ [MeV] (with $y_b y_t$)	+2.44441(1)	+2.42059(1)	+2.37628(1)

Table 3: The results for the LO, NLO and NNLO total decay width. The total width is calculated using our results for the expansion coefficients, $\bar{\gamma}_1^{b\bar{b}}$ and $\bar{\gamma}_2^{b\bar{b}}$. For comparison we include corresponding results from Ref. [20]. We also provide results in the limit of massless b -quarks from Ref. [71], which do not contain the $y_b y_t$ contribution. The uncertainties quoted for our results correspond to errors from numerical integration.