Phase-space generation improvements for Belle II and LHC-b $\tau$ decay generation

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Outline

1. Introduction
2. Event generation in TAUOLA
3. Phase-space generator structure
4. New phase-space enhancement
5. Summary
• \( \tau \) lepton has multiple decay channels, both leptonic and hadronic.

• Many more are expected, but not yet detected due to very low branching ratios predicted by theory.

• Searches aimed at those rare channels may expose New Physics phenomena.

• Lepton Flavour Violating \( \tau \) decays are of interest for many physicists.

• Phase-space generation improvements described in this presentation are meant to facilitate models for rare decays.
$	au$ decay is simulated from the phase space Jacobian ($J$) and the matrix element ($\mathcal{M}$) being separate blocks. The decay width is obtained using the canonical calculation:

$$d\Gamma = W \prod_{i}^{N} dx_i,$$

(1)

where $\{x_i\}$ denote set of random numbers and weight:

$$W = \frac{|\mathcal{M}|^2 * J}{2M_{\tau}},$$

(2)

where $M_{\tau}$ is the $\tau$ lepton mass.

If weight is bigger than the random number multiplied by the maximum weight, event is accepted, otherwise it is rejected.
Phase-space generator structure

Phase-spaces are usually parametrized using Lorentz invariant quantities and angles of outgoing particles. In TAUOLA, the square of the invariant masses of systems of particles with descending number of decay products are used as the Lorentz invariant quantities.

If no significant features are expected in the invariant mass squared \( s \), the relevant coordinate of phase space is translated into a random number in \([0,1]\) range using:

\[
s = s_{\text{min}} + (s_{\text{max}} - s_{\text{min}}) \cdot x.
\] (3)

Let us call this Eq. a flat type coordinate presampler.
If the matrix element contains a resonance in the variable $s$, we introduce a change of variable:

$$\alpha_{\text{min}} = \arctan \frac{s_{\text{min}} - M_R^2}{\Gamma_R M_R},$$

$$\alpha_{\text{max}} = \arctan \frac{s_{\text{max}} - M_R^2}{\Gamma_R M_R},$$

$$\alpha = \alpha_{\text{min}} + (\alpha_{\text{max}} - \alpha_{\text{min}}) \cdot x,$$

$$s = M_R^2 + \Gamma_R M_R \tan \alpha,$$

where $M_R$ and $\Gamma_R$ are parameters describing the resonance present in the matrix element. Let us call this Eq. a resonant type phase-space presampler.
Whenever a change of variable is performed, the Jacobian of a such change enters into the phase space formula. In our case we have multiple channels in presampler, which with necessitates multiple change of variables. This can be taken into account by introducing harmonically averaged Jacobian ($J_{total}$) from $n$ different channels in the presampler:

$$\frac{1}{J_{total}} = \sum_{i=1}^{n} \frac{P_i}{J_i},$$

(5)

where $P_i$ is the probability of the $i^{th}$ presampler channel, and $J_i$ is the Jacobian corresponding to change of variable performed for the $i^{th}$ channel.
• Let us recall that: “the square of the invariant masses of systems of particles with descending number of decay products are used as the Lorentz invariant quantities”.

• Above statement implies that inside Monte Carlo proper order of particles has to be maintained.

• Presampler parameters have to be set accordingly to features of matrix element used for generation.

• As long as parameters are within reasonable range of model prediction they affect only efficiency of generation.
• Modern experiments like Belle II and LHC-b allow us to better investigate phenomena like rare $\tau$ decays.

• An example of such rare decay is $\tau^- \rightarrow \bar{\nu}_\mu \mu^- e^- e^+ \nu_\tau$, which also is a source of background for LFV decays.

• Facilitating models for such decays requires making sure that Monte Carlo generator can handle structures like:

$$\mathcal{M} = \frac{1}{k^2},$$

where $k = P_{e^-} + P_{e^+}$. 

(6)
New phase-space enhacement

Facilitating models for decay channels like $\tau^- \rightarrow \bar{\nu}_\mu \mu^- e^- e^+ \nu_\tau$ boils down to making sure that proper phase-space enhancements are present. In this case it required adding and proper parametrization of resonant type phase-space presampler in the mass of $e^+ e^-$, that is in the mass of pair of particles in the final state for tau decays into 5 particles.

<table>
<thead>
<tr>
<th>Presampler param.</th>
<th>Presampler performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>Efficiency</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0005</td>
</tr>
<tr>
<td>0.2</td>
<td>0.036</td>
</tr>
<tr>
<td>0.4</td>
<td>0.066</td>
</tr>
<tr>
<td>0.6</td>
<td>0.099</td>
</tr>
<tr>
<td>0.8</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Table 1: Probability of new phase space channel and resulting efficiency of generation for $\tau^- \rightarrow \bar{\nu}_\mu \mu^- e^- e^+ \nu_\tau$. M= 1/k^2.
New phase-space enhancement

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>theoretical BR</th>
<th>Monte Carlo BR</th>
<th>experimental BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^- \rightarrow \bar{\nu}<em>\mu \mu^- e^- e^+ \nu</em>\tau$</td>
<td>$(4.21 \pm 0.01) \times 10^{-5}$</td>
<td>$(1.538 \pm 0.0005) \times 10^{-5}$</td>
<td>$&lt; 3.2 \times 10^{-5}$ (at 90% C.L.)</td>
</tr>
<tr>
<td>$\tau^- \rightarrow \bar{\nu}<em>e e^- e^- e^+ \nu</em>\tau$</td>
<td>$(1.984 \pm 0.004) \times 10^{-5}$</td>
<td>$(2.871 \pm 0.0004) \times 10^{-5}$</td>
<td>$2.7^{+1.5+0.4+0.10-1.1-0.4-0.310^{-5}}$</td>
</tr>
</tbody>
</table>

• New phase-space enhancements were added to facilitate rare decay channels.

• Enhancements were tested with physical model implemented into Monte Carlo providing good agreement with other predictions.