Fully Bayesian Unfolding

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Outline

1.) What is unfolding
2.) Fully Bayesian Unfolding
3.) Regularization
4.) Conclusion

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Unfolding is the process of correcting measured data to its detector efficiency, acceptance and resolution.

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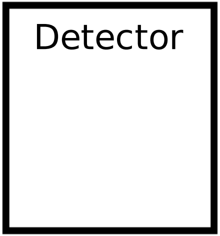

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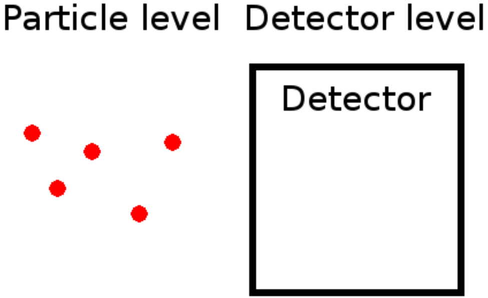

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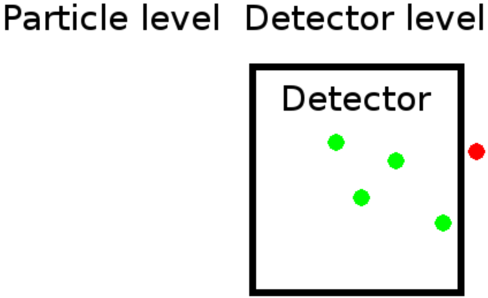

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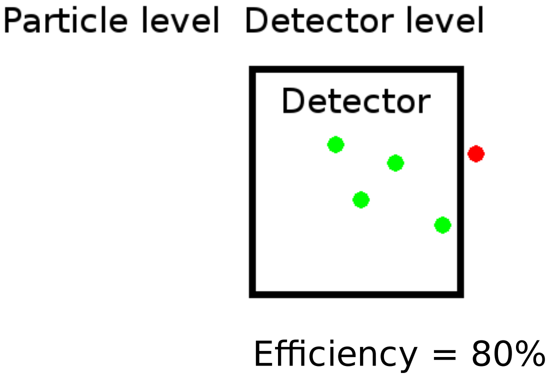

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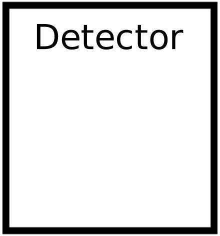

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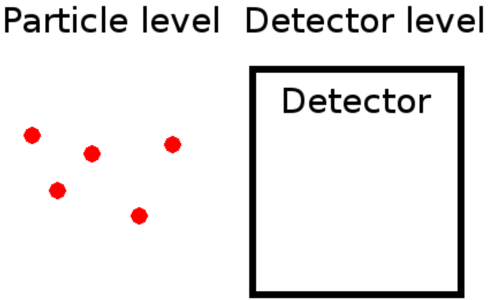

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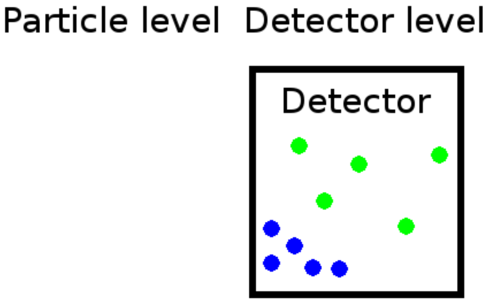

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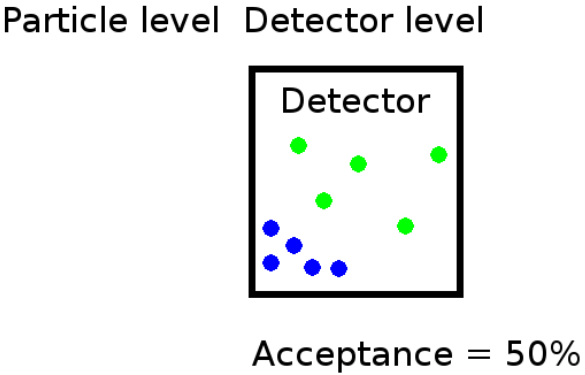

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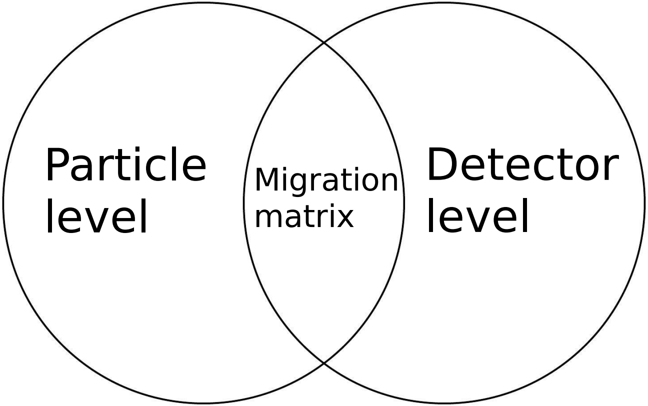

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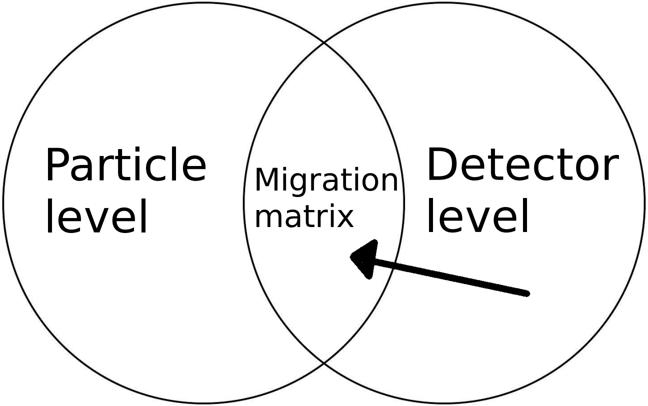

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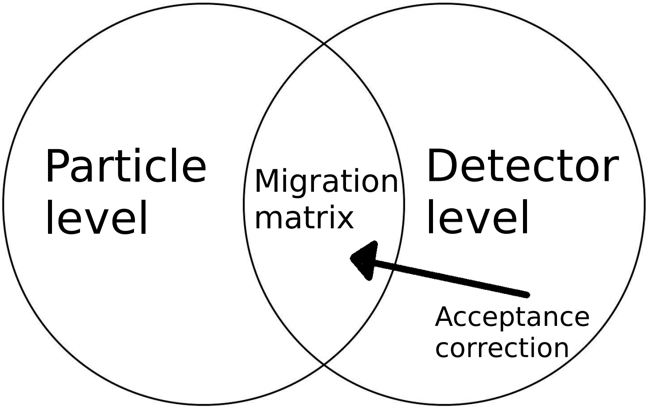

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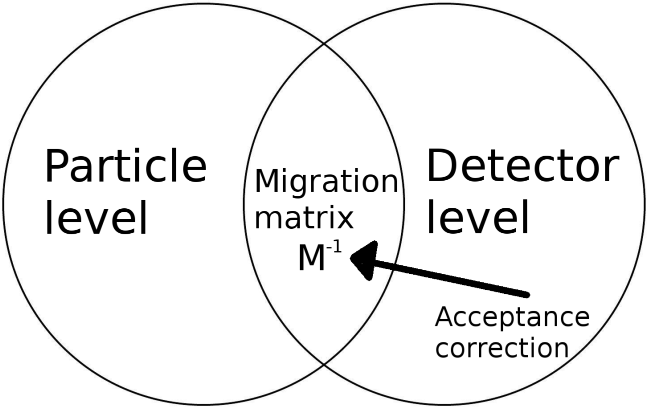

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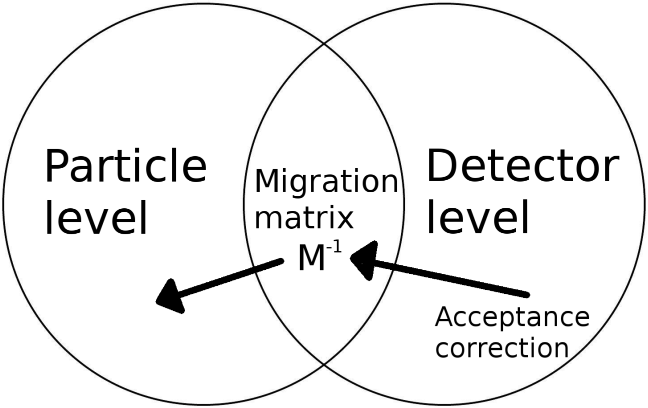

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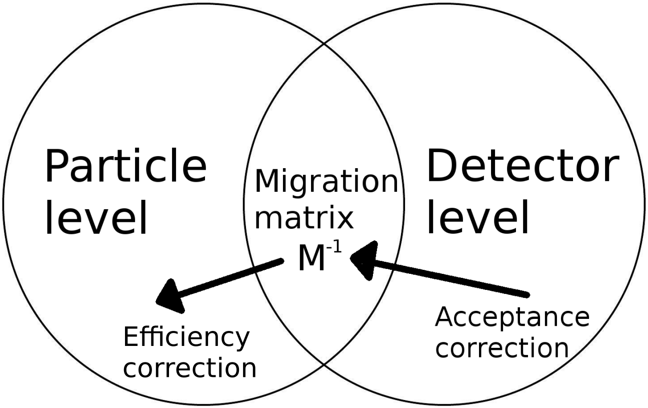

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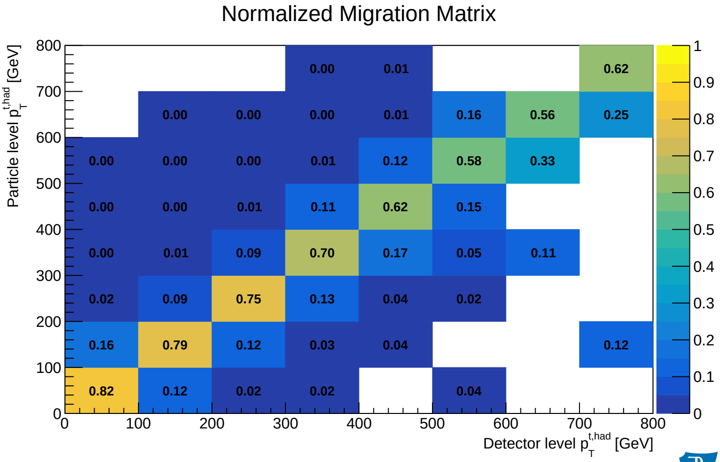

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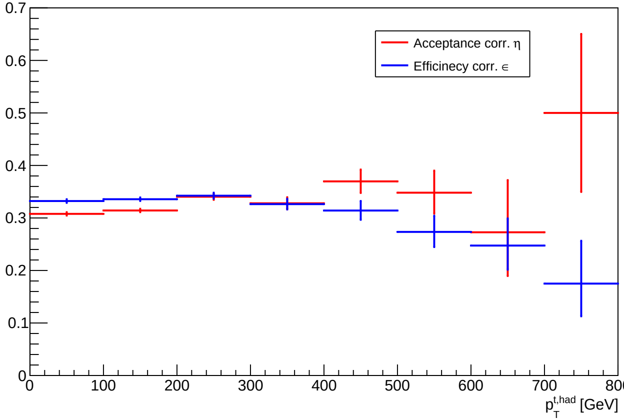

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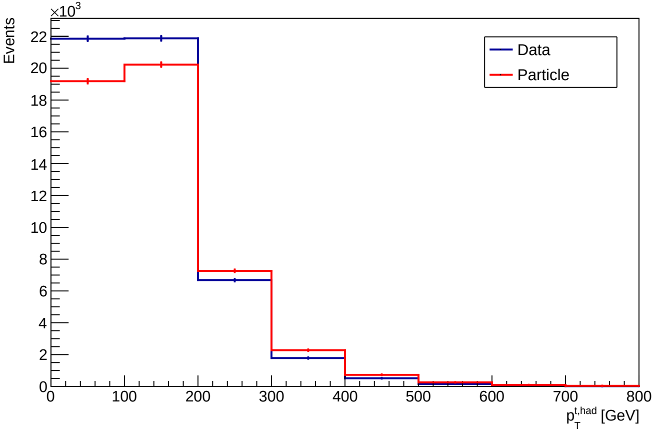

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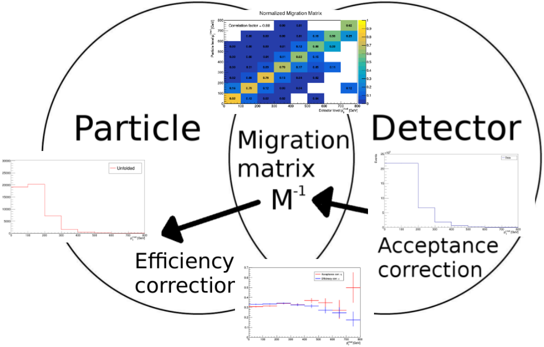
Unfolding is the process of correcting measured data to its detector efficiency, acceptance and resolution.

$$p = \frac{1}{\epsilon} \cdot M^{-1} \cdot \eta \cdot (D - B) \quad (1)$$



Methods of RooUnfold package:

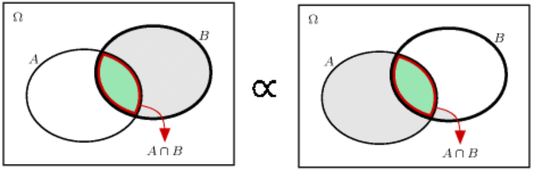
Invert
TUnfold
Svd
Ids
BinByBin
IterativeBayes

Maybe one day FBU - **Fully Bayesian** with regularization
https://arxiv.org/abs/1201.4612

Fully Bayesian Unfolding

Bayesien theorem: $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

$$p(\mathbf{T}|\mathbf{D}, \mathcal{M}) = L(\mathbf{D}|\mathbf{T}, \mathcal{M}) \cdot \frac{\pi(\mathbf{T}, \mathcal{M})}{\text{Norm. Const.}}$$



P(A|B) = P(A∩B)/P(B)

P(B|A) = P(B∩A)/P(A)

Likelihood function $L(T_1, T_2, ..., T_N)$.

$$P(T|D) \quad \propto \quad L(D|T) \cdot \pi(T) =$$

$$= \frac{1}{\epsilon} \left(\prod_{i=1}^{n=bins} \frac{\left(\sum_{j=1}^{n=bins} M_{ij} T_j\right)^{[\alpha_i (D_i - B_i)]}}{[\alpha_i (D_i - B_i)]!} e^{-\left(\sum_{j=1}^{n=bins} M_{ij} T_j\right)} \right) e^{-\tau S(T)}$$

Markov chain Monte Carlo (MCMC)
- general name for sampling algorithms → each step is derived based on previous one, which creates chain.

Gibbs Sampling - Bayesian inference Using Gibbs Sampling
Random Walk Metropolis Hastings
Adaptive Metropolis Hastings
Hamiltonian Monte Carlo
No-U-Turn Sampler - NUTS
https://arxiv.org/abs/1111.4246
Metropolis-adjusted Langevin Algorithm (MALA)
Hessian-Hamiltonian Monte Carlo (H2MC)
Stein Variational Gradient Descent (SVGD)
Nested Sampling with RadFriends (RadFriends-NS)

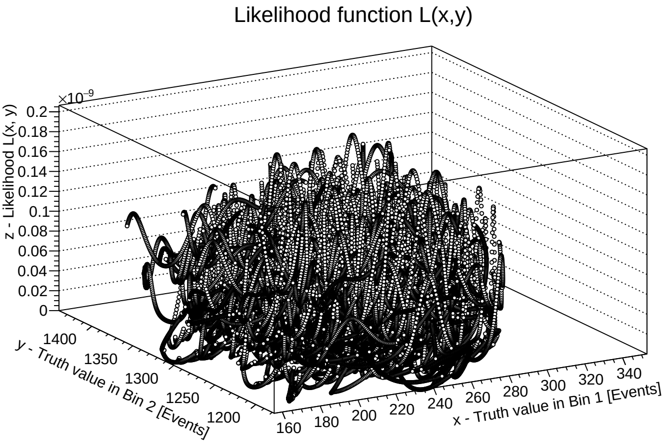
Hamiltonian Monte Carlo (HMC)

- problem of sampling likelihood function is **transformed** to the free particle (frog) motion in the potential given by likelihood function V = L($\mathrm{T}_1, T_2, ..., T_N$).

$$H = \frac{\vec{p}^2}{2m} + V(x, y, z, ...)$$

$$H = \frac{\vec{p}^2}{2m} + L(T_1, T_2, ..., T_N)$$

Motion of the frog in the N-dimensional space



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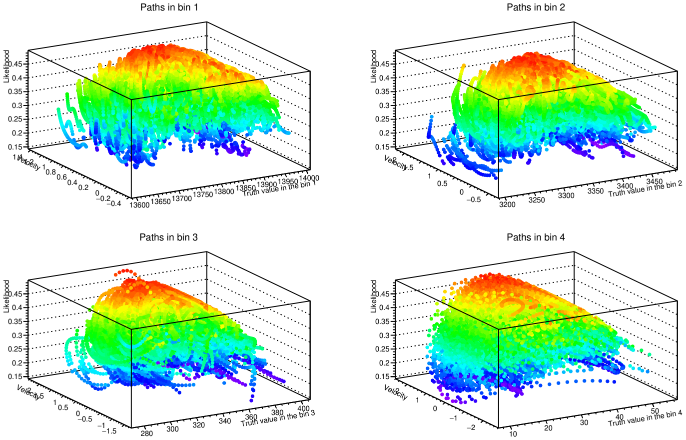
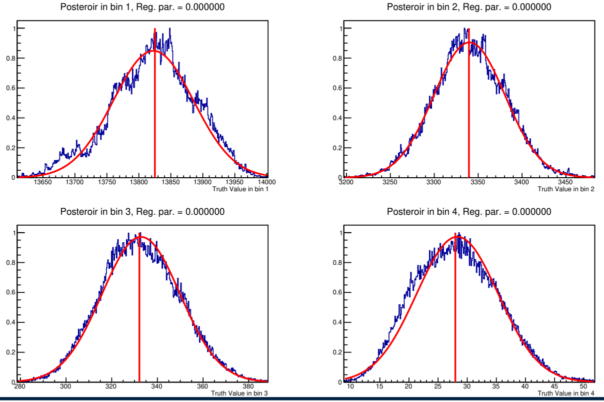


Posteriors

Unfolded spectrum is derived from posteriors which are calculated for each bin i as:

$$p_i(T_i|D) = \int\int P(T|D)dT_1...dT_{i-1}dT_{i+1}...dT_N \qquad (2)$$



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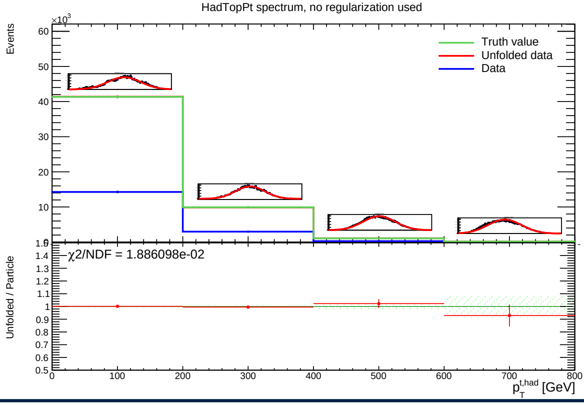
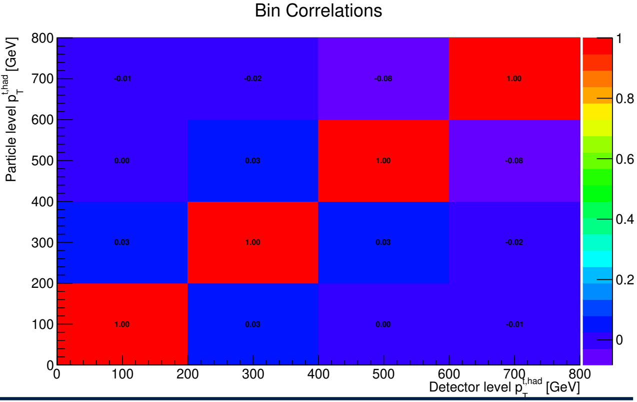
$$p_i(T_i|D) = \int\int P(T|D)dT_1...dT_{i-1}dT_{i+1}...dT_N \qquad (3)$$



Bin Correlations

Unfolded spectrum is derived from posteriors which are calculated for each bin i as:

$$p_i(T_i|D) = \int \int P(T|D) dT_1 ... dT_{i-1} dT_{i+1} ... dT_N \quad (4)$$



$$P(T|D) \quad \propto \quad L(D|T) \cdot \pi(T) =$$

$$= \frac{1}{\epsilon}\left(\prod_{i=1}^{n=bins} \frac{\left(\sum_{j=1}^{n=bins} M_{ij}T_j\right)^{[\alpha_i(D_i - B_i)]}}{[\alpha_i(D_i - B_i)]!} e^{-\left(\sum_{j=1}^{n=bins} M_{ij}T_j\right)}\right) e^{-\tau S(T)}$$

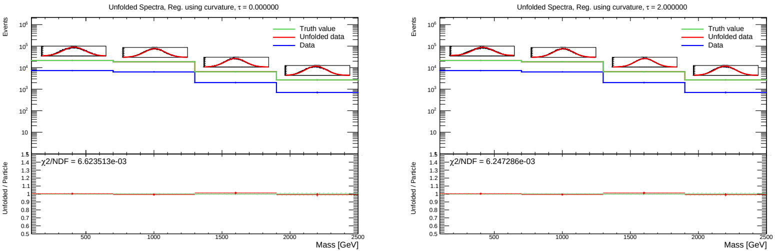
$$S(T) = \sum_{t=2}^{N-1} (\Delta_{t+1,t} - \Delta_{t,t-1})^2 \tag{5}$$

where

$$\Delta_{t_1,t_2} = T_{t_1} - T_{t_2} \tag{6}$$

where T' and T'' are first and second derivatives of the truth pseudo experiment.

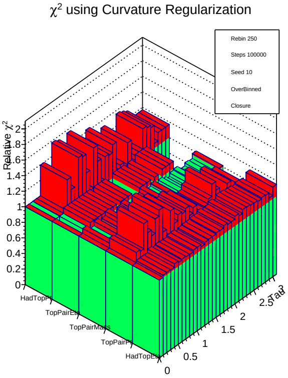
Regularization

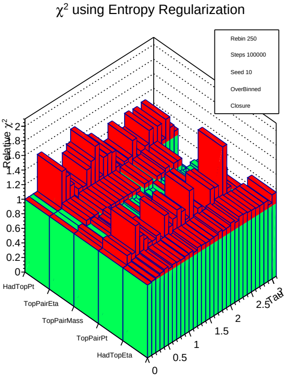


(a) (b)

Figure: **a)** No regularization is applied. **b)** Regularization using curvature with reg. strengt $\tau = 2.0$.

Regularization





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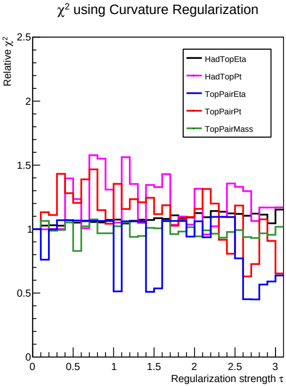

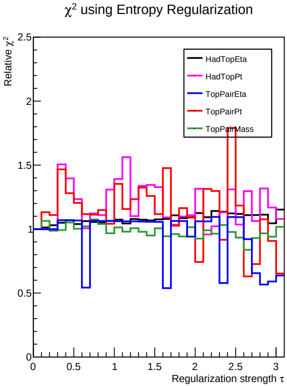

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Conclusion

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Thank to my supervisor Mgr. Jiří Kvita, PhD. , organisers for the opportunity to give this talk and for your attention.

Questions?

$$\epsilon = \frac{P_{\text{particle, proj. from M}}}{P_{\text{level}}};\ \eta = \frac{D_{\text{data, proj. from M}}}{D} \tag{7}$$

Migration matrix

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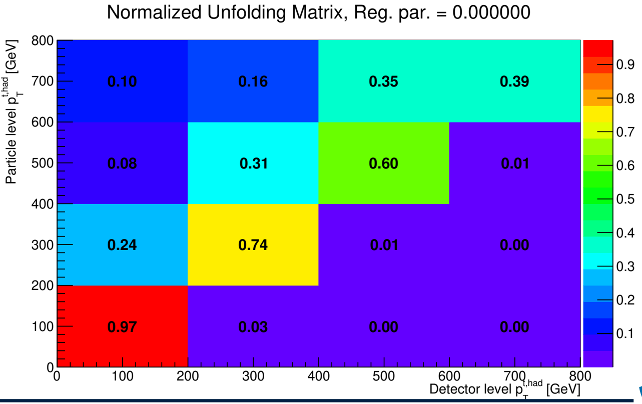
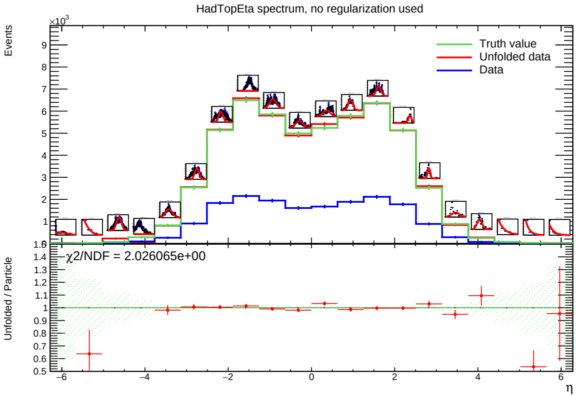
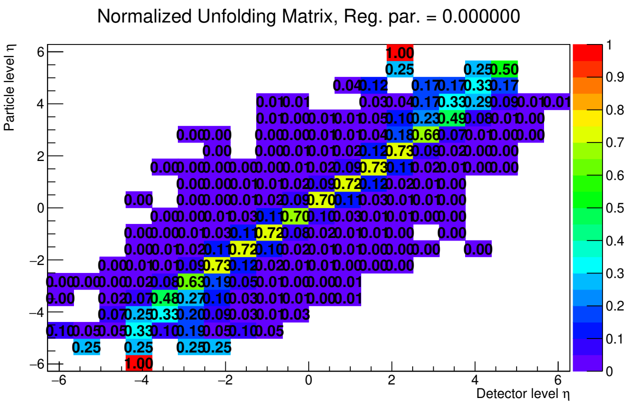
$$p_i(T_i|D) = \int\int P(T|D)dT_1...dT_{i-1}dT_{i+1}...dT_N \qquad (8)$$



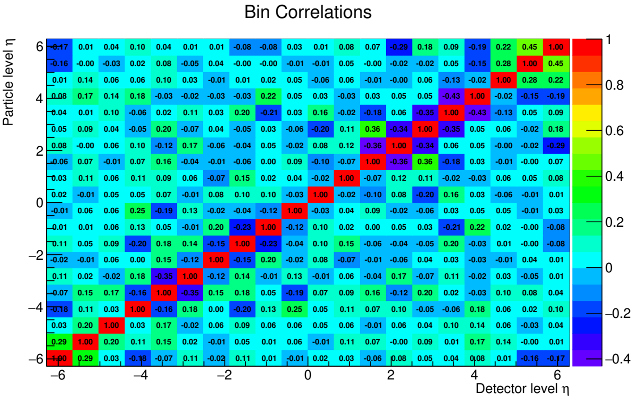
More Bins


HadTopEta spectrum, no regularization used
Events
×10³
Truth value
Unfolded data
Data
Unfolded / Particle
χ2/NDF = 2.026065e+00
η

More Bins


Normalized Unfolding Matrix, Reg. par. = 0.000000
Particle level η
Detector level η
−6
−4
−2
0
2
4
6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

More Bins


Bin Correlations
Particle level η
Detector level η
−6
−4
−2
0
2
4
6
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4