Parton showers and Jet quenching

Edmond Iancu
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Personal motivation

- The phenomenology & theory of Jet quenching:
- One of the most active research fields in relation with heavy ion collisions over the last 25 years ...
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![Lawrence Berkeley Laboratory](image)
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Medium-Induced QCD Cascade: Democratic Branching and Wave Turbulence

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(Received 7 February 2013; published 30 July 2013)

We study the average properties of the gluon cascade generated by an energetic parton propagating through a quark-gluon plasma. We focus on the soft, medium-induced emissions which control the energy transport at large angles with respect to the leading parton. We show that the effect of multiple branchings is important. In contrast with what happens in a usual QCD cascade in vacuum, medium-induced branchings are quasidemocratic, with offspring gluons carrying sizable fractions of the energy of their parent gluon. This results in an efficient mechanism for the transport of energy toward the medium, which is akin to wave turbulence with a scaling spectrum \( \sim 1/\sqrt{\omega} \). We argue that the turbulent flow may be responsible for the excess energy carried by very soft quanta, as revealed by the analysis of the dijet asymmetry observed in Pb-Pb collisions at the LHC.
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... but what about Larry ???
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March, 1986

JETS IN EXPANDING QUARK-GLUON PLASMAS

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- Well, Larry and Jean-Paul wrote the prehistory of jet quenching!

- Besides, $\hat{q}$ is just another name for the saturation momentum!
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Well, Larry and Jean-Paul wrote the prehistory of jet quenching!

Besides, \( \hat{q} \) is just another name for the saturation momentum!

Last but not least, it allows me to complete this picture!
Personal motivation
The phenomenology of jet quenching (very short)

Two types of radiation...

- vacuum-like (bremsstrahlung): parton virtualities
- medium-induced radiation: collisions in the plasma

... and their factorization within perturbative QCD

- probabilistic (Markovian) descriptions

Our original discussion: the leading double-logarithmic approximation


Our new/preliminary results from the Monte-Carlo implementation

*P. Caucal, E.I., and G. Soyez, JHEP 10 (2019) 273*
Jets in hadronic collisions

- Increasing complexity: $e^+e^- \rightarrow pp$, $ep \rightarrow pA$, $eA \rightarrow AA$

- Jets in the vacuum ($e^+e^-, pp$): well understood, nearly from first principles
  - parton showers computed within perturbative QCD
  - A benchmark for understanding medium (nuclear) effects in $pA$, $eA$ and $AA$
Jets in practice

- Experimentally, jets are constructed by grouping together hadrons which propagate at nearby angles: *anti-$k_T$ algorithm* (*Cacciari, Salam, Soyez, 08*)

- The jet opening angle $\theta_0$ (or $R$) and the jet total energy $p_T$ (or $E$, or $p_{jet}^T$)

- Medium effects can refer to
  - global properties of the jet
  - the jet internal structure
  - the outer region
**Nuclear modification factor for jets**

- **LHC**: the jet yield in Pb+Pb collisions normalized by p+p times the average nuclear thickness function $\langle T_{AA} \rangle$

$$R_{AA} \equiv \frac{1}{N_{\text{evt}}} \left. \frac{d^2 N_{\text{jet}}}{dp_T dy} \right|_{AA} \frac{\langle T_{AA} \rangle}{\frac{d^2 \sigma_{\text{jet}}}{dp_T dy} \bigg|_{pp}}$$

- ATLAS, arXiv:1805.05635

- stronger suppression for more central collisions

- Energy loss by the jet: transported at large angles $\theta > R$

- $R_{AA}$ is almost flat at very high $p_T$: energy loss increases with $p_T$
Energy loss

- A jet measured with a given energy $p_T$ has been produced with $p_T + \epsilon$

\[
\frac{d\sigma^{\text{med}}(p_T)}{dp_T} = \int d\epsilon \mathcal{P}(\epsilon) \frac{d\sigma^{\text{vac}}(p_T + \epsilon)}{dp_T}
\]

- $\mathcal{P}(\epsilon)$: probability density for losing $\epsilon$

\[
\frac{d\sigma^{\text{vac}}(p_T)}{dp_T} \propto \frac{1}{p_T^n}, \quad n = 7 \div 10
\]

- Rapidly falling spectrum for the hard process
- Bias towards small values for $\epsilon$

- Even a small $\epsilon$ may imply strong suppression
Jet fragmentation function

- Energy distribution of the hadrons inside the jet

\[ D(\omega) \equiv \omega \frac{dN}{d\omega} = \int_{0}^{R} d\theta \omega \frac{dN}{d\theta d\omega} \]

- \( \omega \equiv p_T \) of a hadron inside the jet

Plot the ratio of FFs in Pb+Pb and p+p: \( R_D(p_T) \)

- enhancement at low energies: \( p_T \ll p_T^{\text{jet}} \) ...
- ... and at relatively high ones: \( p_T \sim p_T^{\text{jet}} \)
- slight suppression at intermediate energies
Huge difference between the energies of the two jets

The missing energy is found in the underlying event:

- many soft ($p_T < 2$ GeV) hadrons propagating at large angles

Very different from the usual jet fragmentation pattern in the vacuum
- The **leading particle (LP)** is produced by a hard scattering.
- It subsequently evolves via **radiation** (branchings) ...
Medium-induced jet evolution

- The **leading particle (LP)** is produced by a hard scattering.
- It subsequently evolves via **radiation** (branchings) ...

... and via **collisions** off the medium constituents.

- Collisions can have several effects:
  - transfer energy and momentum between the jet and the medium
  - trigger additional radiation ("medium-induced")
  - wash out the color coherence (destroy interference pattern)
The leading particle (LP) is produced by a hard scattering.

It subsequently evolves via radiation (branchings) ...

... and via collisions off the medium constituents.

How to simultaneously describe the vacuum-like emissions (virtuality-driven) and the medium-induced ones (as driven by collisions) ?

Does this distinction even make sense ?
The time it takes the daughter partons to lose their mutual coherence

The gluon has been emitted when it has no overlap with its source

\[ \Delta x_\perp \sim \theta \Delta t \gtrsim \lambda_\perp \sim \frac{2}{k_\perp} \]

\[ k_\perp \sim \omega \theta \]

\[ \Delta t \gtrsim t_f = \frac{2\omega}{k_\perp^2} \sim \frac{2}{\omega \theta^2} \]

This argument universally applies to radiation: in vacuum & in the medium
Radiation: Formation time

- The time it takes the daughter partons to lose their mutual coherence
- Radiation in the vacuum: bremsstrahlung
- The emission is triggered by a hard scattering
- \( t_f \) is controlled by the parton virtuality & measured from the hard vertex

\[
t_f \sim \frac{2\omega}{k_{\perp}^2} \sim \frac{2E}{Q^2}
\]

\[
dP_{\text{Brem}} \sim \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2}
\]

- Log enhancement for soft (\( \omega \ll E \)) and collinear (\( \theta \ll 1 \)) gluons
- Radiation in the medium: additional decoherence associated with collisions
Medium-induced radiation

- The formation time is controlled by transverse momentum broadening

\[ k_{\perp}^2 = \hat{q} t_f \quad \text{and} \quad t_f = \frac{2\omega}{k_{\perp}^2} \]

\[ k_{\perp}^2 = \sqrt{2\hat{q}\omega} \quad \text{and} \quad t_f = \sqrt{\frac{2\omega}{\hat{q}}} \]

- Effective only so long as \( t_f < L \) (medium size), hence for \( \omega \leq \omega_c \equiv \frac{1}{2} \hat{q}L^2 \)

- Emissions can occur anywhere in the medium, with a rate \( \propto 1/t_f \)

\[
\frac{d\mathcal{P}}{d\omega \, d^2k \, dt} = \frac{\alpha_s C_R}{\pi} \frac{1}{\omega} \frac{1}{t_f(\omega)} \frac{1}{\sqrt{2\hat{q}\omega}} e^{-\frac{k_{\perp}^2}{\sqrt{2\hat{q}\omega}}}
\]

- no collinear singularity \((k_{\perp} \to 0)\)

- stronger soft singularity than for bremsstrahlung: \( \propto \frac{1}{\omega^{3/2}} \)
Double logarithmic approximation (DLA): strong double ordering

The dominant contribution to the gluon multiplicity at soft energies $\omega \ll E$ and small emission angles $\theta \ll \theta_0$

\[
\frac{d^2 N}{d\omega d\theta^2} \simeq \frac{\bar{\alpha}}{\omega \theta^2} \sum_{n \geq 0} \bar{\alpha}^n \left[ \frac{1}{n!} \left( \frac{\ln E}{\omega} \right)^n \right] \left[ \frac{1}{n!} \left( \frac{\ln \theta_0^2}{\theta^2} \right)^n \right]
\]

Formation times are strongly increasing along the cascade: $t_f \sim 1/(\omega \theta^2)$
Double logarithmic approximation (DLA): strong double ordering

\[ E \gg \omega_1 \gg \omega_2 \gg \cdots \gg \omega \]

\[ \theta_0 \gg \theta_1 \gg \theta_2 \gg \cdots \gg \theta \]

Analytic result at fixed coupling showing exponential growth:

\[ \omega \theta^2 \frac{d^2 N}{d\omega d\theta^2} = \bar{\alpha} I_0 \left(2\sqrt{\bar{\alpha}} \ln \frac{E}{\omega} \ln \frac{\theta^2}{\theta_0^2}\right) \propto \exp \left\{2\sqrt{\bar{\alpha}} \ln \frac{1}{x} \ln \frac{\theta_0^2}{\theta^2}\right\} \]

This growth will be moderated by running coupling and energy conservation.

Beyond DLA: angular ordering still holds due to color coherence.
Angular ordering

- The first splitting \( g \to gg \Rightarrow \) a color-octet antenna with opening angle \( \theta_0 \)
- The next splitting can be sourced by any of the 2 legs: interference

Small-angle emissions \((\theta < \theta_0)\) by the two legs are (quasi)independent
Angular ordering

- The first splitting $g \rightarrow gg \Rightarrow$ a color-octet antenna with opening angle $\theta_0$

- The next splitting can be sourced by any of the 2 legs: interference

- A large-angle emission at $\theta > \theta_0$ “sees” the overall color charge

- It can be formally treated as an emission by the parent gluon
Angular ordering

- The first splitting \( g \rightarrow gg \Rightarrow \) a color-octet antenna with opening angle \( \theta_0 \)
- The next splitting can be sourced by any of the 2 legs: interference

Effectively: independent emissions with angular ordering

Single logarithmic approximation: exact (DGLAP) splitting functions
Jets in the medium: on-shell parton

- An on-shell parton can only radiate due to collisions:

\[ t_f(\omega) = \sqrt{\frac{2\omega}{\hat{q}}}, \quad k_\perp^2 \lesssim Q_s^2 \equiv \hat{q}L, \quad \theta(\omega) \gtrsim \frac{Q_s}{\omega} \]

- BDMPSZ emission spectrum (integrated over \( t \leq L \) and \( k_\perp \))

\[ \omega \frac{dP}{d\omega} \simeq \bar{\alpha} \frac{L}{t_f(\omega)} \simeq \bar{\alpha} \sqrt{\frac{\omega_c}{2\omega}}, \quad \omega < \omega_c = \frac{1}{2} \hat{q}L^2 \]

- Two types of emissions:

- Hard emissions with \( \omega \sim \omega_c \implies \text{small probability of } O(\alpha_s) \)
  - control the average energy loss by the leading parton: \( \Delta E_{LP} \sim \alpha_s \omega_c \)
  - very small emission angle: energy remains inside the jet cone

\[ \theta \sim \theta_c \equiv \frac{Q_s}{\omega_c} = \frac{2}{\sqrt{\hat{q}L^3}} \simeq 0.05 \]

- irrelevant for the physics of jet quenching
An on-shell parton can only radiate due to collisions:

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Two types of emissions:

Soft emissions with \( \omega \sim \omega_{br} \equiv \bar{\alpha}^2 \omega_c \implies \text{probability of } \mathcal{O}(1) \)

- large emission angle: energy goes outside the jet cone

\[ \theta \gtrsim \frac{Q_s}{\omega_{br}} = \frac{1}{\bar{\alpha}^2} \theta_c \simeq 0.5 \]

... which become even larger via multiple branchings

*J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)*
Jets from MIEs

The jet that would be created by an on-shell parton via medium-induced emissions (MIEs), in a typical event.

- either no hard gluon at all, or at most a number of $O(1)$ of gluons with energies $\omega_{br} \ll \omega \lesssim \omega_c$
- a large number of soft gluons with $\omega \leq \omega_{br}$, which leave the jet
- the harder emissions contribute to the jet fragmentation function
- the soft ones to the jet energy loss: $\Delta E_{jet} \sim \omega_{br} = \alpha_s^2 \omega_c$
Jets from MIEs

- The jet that would be created by an on-shell parton via medium-induced emissions (MIEs), in a typical event.

- Interesting differences w.r.t. the usual jets in the vacuum:
  - no angular ordering: color coherence is lost already during formation
  - no strong energy ordering: “democratic branchings”
  - turbulent cascade: energy literally flows outside the jet

- Many soft quanta at large angles outside the jet: di-jet asymmetry ✓
Towards a complete picture

- In realistic situations, the leading parton is **not** on-shell!
  - hard scattering $\Rightarrow$ time-like virtuality $\Rightarrow$ vacuum-like emissions (VLEs)

- **Formation time** offers a criterion to distinguish between VLEs and MIEs:
  - collisions introduce a lower limit on transverse momentum ...
  - ... hence an upper limit on the formation time

\[
\frac{k_{\perp}^2}{\hat{q} t_f} \geq \hat{q} t_f
\]

- The MIEs **saturate** these limits

- VLEs have much shorter formation times: 
  \[
  t_f = \frac{2 \omega}{\hat{q} k_{\perp}^2} \ll \sqrt{\frac{2 \omega}{\hat{q}}}
  \]
Towards a complete picture

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$$k^2_\perp \gtrsim \hat{q}t_f$$

$$t_f = \frac{2\omega}{k^2_\perp} \lesssim \sqrt{\frac{2\omega}{\hat{q}}}$$

- Vacuum-like parton cascades occur faster and can be factorized from the medium-induced ones (Caucal, E. I., A. Mueller, Soyez, PRL 120, 2018)
A phase-space representation for VLEs

- At DLA, emissions can be suggestively depicted as points in the $(\omega, \theta)$ plane
  - $\bar{\theta}$: the maximal angle allowed for the first emission
  - $E$: the energy of the leading parton
- beyond DLA, this picture is generalized by the Lund plane

Evolution stopped by hadronisation: $k_\perp \simeq \omega \theta \gtrsim \Lambda_{QCD}$
VLEs in the presence of the medium

- Two additional lines: \( t_f = L \) and \( t_f = \sqrt{\frac{2\omega}{\hat{q}}} \)

- the 2 lines cross each other at

\[
\omega_c = \frac{1}{2} \hat{q} L^2, \quad \theta_c = \frac{2}{\sqrt{\hat{q} L^3}}
\]

- in-medium VLEs obey \( t_f < \sqrt{\frac{2\omega}{\hat{q}}} \)

- out-of-medium VLEs obey \( t_f > L \)

- Vetoed region: would-be collinear VLEs, that cannot exist in the medium

\( (k_\perp^2 < \sqrt{2\hat{q}\omega}) \)

- Vacuum-like cascades develop first, but within a restricted phase-space
Angular ordering


- Successive VLEs obey **angular ordering**, like usual jets in the vacuum
  - non-trivial: in-medium antennas lose color coherence via rescattering *(Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I., 2010–12)*
  - the VLEs at large angles occur fast enough to be emitted by antennas which are still coherent

\[ \omega \theta^2 = \frac{1}{L} \]
There is a life after formation ...

- The VLEs inside the medium have short formation times $t_f \ll L$
- After formation, gluons propagate in the medium along a distance $\sim L$
- They act as sources for medium-induced radiation
  - jet energy loss increases with the jet $p_T$ and the jet opening $R$
- They can emit (vacuum-like) gluons outside the medium
First emission outside the medium

- First outside emission can violate angular ordering
  - the respective formation time is necessarily large: \( t_f \gtrsim L \)
  - in-medium antennas with \( \theta \gg \theta_c \) have lost their coherence by the time of the emission

- Crucial consequence: re-opening of the angular phase-space
  - enhanced radiation at low energies and large angles

- Subsequent “outside” emissions obey angular-ordering, as usual
Jet fragmentation function at DLA

- No MIEs (no energy loss), just medium constraints on the VLEs:
  
  \[ T(\omega, \theta) \equiv \omega \theta^2 \frac{d^2 N}{d\omega d\theta^2} \quad \Rightarrow \quad D(\omega) \equiv \omega \frac{dN}{d\omega} = \int \frac{d\theta^2}{\theta^2} T(\omega, \theta^2) \]

---

\[ T(\omega, \theta) \]

\[ \frac{\omega \theta^2}{d\omega d\theta^2} \]

\[ D(\omega) \]

\[ \omega \frac{dN}{d\omega} \]

\[ \int \frac{d\theta^2}{\theta^2} T(\omega, \theta^2) \]

---

\[ E=200 \text{ GeV}, \theta qq=0.4, \bar{\alpha}_s=0.3, \hat{q}=2 \text{ GeV}^2/\text{fm}, L=3 \text{ fm} \]

\[ T/T_{\text{vac}} \]

---

\[ \hat{q}=1 \text{ GeV}^2/\text{fm}, L=3 \text{ fm} \]

\[ \hat{q}=2 \text{ GeV}^2/\text{fm}, L=3 \text{ fm} \]

\[ \hat{q}=2 \text{ GeV}^2/\text{fm}, L=4 \text{ fm} \]

---

\[ E=200 \text{ GeV}, \theta qq=0.4, \bar{\alpha}_s=0.3 \]

\[ \text{solid: } \Lambda=100 \text{ MeV} \]

\[ \text{dashed: } \Lambda=200 \text{ MeV} \]
Jet fragmentation function at DLA

- Slight suppression at intermediate energies (from 3 GeV up to $\omega_c$)
  - the phase-space is reduced by the vetoed region

- Significant enhancement at low energy (below 2 GeV)
  - lack of angular ordering for the first emission outside the medium

\[ T/T_{\text{vac}} \]

\[ \omega \text{ [GeV]} \]

- $E=200$ GeV, $\theta_{qq}=0.4$, $\hat{\alpha}_s=0.3$, $\hat{q}=2$ GeV$^2$/fm, $L=3$ fm

- Solid: $\Lambda=100$ MeV
- Dashed: $\Lambda=200$ MeV

$D(\omega)/D_{\text{vac}}(\omega)$

\[ \omega \text{ [GeV]} \]
Monte Carlo results *(Caucal, E.I., Soyez, arXiv:1907.04866)*

- Factorized picture (VLEs $\times$ MIEs), separately Markovian
- Straightforward MC implementation: full DGLAP, running coupling...

![Graph 1](image1)

**RAA: varying uncontrolled parameters**

- $\hat{q}=1.5$ GeV$^2$/fm
- $L=4$ fm
- $\alpha_s=0.24$
- $\theta_{max}=1$ \((0.75,1.5)\), $k_{t,min}=0.25(0.15,0.5)$ GeV
- $\sqrt{s}=5.02$ TeV, 0-10% centrality
- $q^2=1.5$ GeV$^2$/fm
- $L=4$ fm
- $\alpha_s=0.24$

![Graph 2](image2)

**RAA: fixed $\theta_c, \omega_c$, vary $\omega_{br}$**

- $\omega_{br}=2.30$ GeV ($\alpha_s=0.196$)
- $\omega_{br}=3.46$ GeV ($\alpha_s=0.24$)
- $\omega_{br}=5.18$ GeV ($\alpha_s=0.294$)

- The zeroth order test: Can one describe $R_{AA}$?
- Not a fit: just a choice of “reasonable” values for the physical parameters
  - 3 medium parameters: $\hat{q}$, $L$, $\alpha_{s,med}$; 2 kinematical cuts: $k_{t,\text{min}}$, $\theta_{\text{max}}$
Monte Carlo results (Caucal, E.I., Soyez, arXiv:1907.04866)

- Factorized picture (VLEs × MIEs), separately Markovian
- Straightforward MC implementation: full DGLAP, running coupling...

\[ R_{AA}: \text{fixed } \theta_c, \omega_{br}, \text{ vary } \omega_c \]

\[ R_{AA}: \text{fixed } \theta_c, \omega_c, \text{ vary } \omega_{br} \]

- Little sensitivity to the kinematical cuts: “infrared-safe observable”
- Variations in the medium parameters help understanding the physics

\[ R_{AA} \text{ is controlled by the “soft” medium scale } \omega_{br} = \bar{\alpha}^2 \omega_c \sim \alpha_s^2 \hat{q} L^2 \]
Monte Carlo results for $R_{AA}$

- What matters for $R_{AA}$ is the **typical** energy loss by the jet at large angles.
- For an on-shell parton, $\Delta E_{\text{jet}} \sim \alpha_s^2 \omega_c \sim 10$ GeV (indep. of $p_T$).
- The number of partons increases with $p_T^{\text{jet}}$, via VLEs $\Rightarrow \Delta E_{\text{jet}}(p_T^{\text{jet}})$.

This explains why the rise of $R_{AA}$ with $p_T$ is as slow as seen in the data.
Monte Carlo results for $R_{AA}$

- What matters for $R_{AA}$ is the **typical** energy loss by the jet at large angles.
- For an on-shell parton, $\Delta E_{\text{jet}} \sim \alpha_s^2 \omega_c \sim 10 \text{ GeV}$ (indep. of $p_T$).
- The number of partons increases with $p_T^{\text{jet}}$, via VLEs $\Rightarrow \Delta E_{\text{jet}}(p_T^{\text{jet}})$.

![Graph showing average energy loss vs. $p_{T0}$ dependence]

- This explains why the rise of $R_{AA}$ with $p_T$ is as slow as seen in the data.
- Without vacuum-like emissions, $R_{AA}$ would rise much faster with $p_T$.

<table>
<thead>
<tr>
<th>$R_{AA}$: quenching order</th>
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<tbody>
<tr>
<td>ATLAS</td>
</tr>
<tr>
<td>3-step fact. (1.5,4,0.24)</td>
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<tr>
<td>quench 1$^{\text{st}}$ (1.5,4,0.36)</td>
</tr>
<tr>
<td>quench 1$^{\text{st}}$ (2.0,4,0.30)</td>
</tr>
</tbody>
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![Graph showing $R_{AA}$ vs. $p_{T\text{jet}}$]

- Average energy loss $[\text{GeV}]$
- $\hat{q} = 1.5 \text{ GeV/fm}^2$
- $L = 4 \text{ fm}$
- $\alpha_s = 0.24$
- $\theta_{\text{max}} = R$, $k_{\perp,\text{min}} = 0.25 \text{ GeV}$
- $\sqrt{s} = 5.02 \text{ TeV}$, 0-10% centrality
The same “canonical” values for the medium parameters as in the “best” description of $R_{AA}$ (*ATLAS data from arXiv:1805.05424*)

Stronger variability with respect to the “uncontrolled parameters”

- not an IR-safe quantity! (disliked by the theorists)
MC: Fragmentation function *(preliminary)*

- Qualitative & semi-quantitave agreement with the data
- Strong enhancement at low $p_T$: MIEs ... but not directly
  - soft MIEs with $\omega \lesssim \omega_{br} \simeq 3.5$ GeV go outside the jet
Suppression at intermediate $p_T$: energy loss via MIEs (easy !)

Enhancement at large $p_T \sim p_T^{\text{jet}}$: much more subtle

- competition between vetoed region, energy loss, & hard MIEs

![Fragmentation function](image)
Vacuum-like emissions inside the medium can be factorized from the medium-induced radiation via systematic approximations in pQCD.

Medium effects enter already at leading-twist level:
- Reduction in the phase-space for VLEs inside the medium
- Violation of angular ordering by the first emission outside the medium

Angular ordering is preserved for VLEs inside the medium, like in the vacuum.

Probabilistic picture, well suited for Monte-Carlo implementations.

Preliminary MC results, which look promising:
- Qualitative and semi-quantitative agreement with the LHC data for the jet $R_{AA}$, jet fragmentation functions, and jet substructure ($z_g$).

Perspectives: more realistic description of the medium (expansion, hydro ...)
Congratulations for your remarkable achievements so far! Keep going strongly!

See you in 2029! And also in the meantime!