Nuclear Science Computing Center at CCNU

Charm and beauty in deconfined plasma from LQCD

Heng-Tong Ding (丁亨通)

Central China Normal University(华中师范大学)

Symposium on Contemporary QCD Physics and Relativistic Nuclear Collisions
9-11 Nov.@CCNU


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Nuclear IChing

Courtesy from Wei-Tian Deng

PHYSICAL REVIEW D VOLUME 44, NUMBER 11 1 DECEMBER 1991

HIJING: A Monte Carlo model for multiple jet production in *pp*, *p A*, and *A A* collisions

Xin-Nian Wang* and Miklos Gyulassy
Nuclear Science Division, Mailstop 70A-3307, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 29 July 1991)

Combining perturbative-QCD inspired models for multiple jet production with low p_T multistring phenomenology, we develop a Monte Carlo event generator HIJING to study jet and multiparticle production in high energy pp, pA, and AA collisions. The model includes multiple minijet production, nuclear shadowing of parton distribution functions, and a schematic mechanism of jet interactions in dense matter. Glauber geometry for multiple collisions is used to calculate pA and AA collisions. The phenomenological parameters are adjusted to reproduce essential features of pp multiparticle production data for a wide energy range ($\sqrt{s}=5$–2000 GeV). Illustrative tests of the model on $p+A$ and light-ion $B+A$ data at $\sqrt{s}=20$ GeV/nucleon and predictions for Au+Au at energies of the BNL Relativistic Heavy Ion Collider ($\sqrt{s}=200$ GeV/nucleon) are given.

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VOLUME 77, NUMBER 9 PHYSICAL REVIEW LETTERS 26 AUGUST 1996

J/ψ Suppression in Pb-Pb Collisions: A Hint of Quark-Gluon Plasma Production?

Jean-Paul Blaizot* and Jean-Yves Ollitrault*
Service de Physique, CE-Saclay,[†] *91191 Gif-sur-Yvette Cedex, France*
(Received 13 May 1996)

The NA50 Collaboration has recently observed a strong suppression of J/ψ production in Pb-Pb collisions at 158 GeV/nucleon. We show that this recent observation finds a quantitative explanation in a model which relates the suppression mechanism to the local energy density, whose value is higher in Pb-Pb collisions than in any other system studied previously. The sensitivity of the phenomenon to small changes in the energy density could be suggestive of quark-gluon plasma formation. [S0031-9007(96)00831-9]

PACS numbers: 25.75.Dw, 12.38.Mh, 24.85.+p

6 lines of abstract

PHYSICAL REVIEW D VOLUME 31, NUMBER 3 1 FEBRUARY 1985

Photon and dilepton emission from the quark-gluon plasma: Some general considerations

L. D. McLerran
Theory Group, MS-106, Fermilab P.O. Box 500, Batavia, Illinois 60510

T. Toimela
Research Institute for Theoretical Physics, Siltavuorenpenger 20c, 00170 Helsinki, Finland
(Received 14 September 1984)

The emission rates for photons and dileptons from a quark-gluon plasma are related to the thermal expectation value of an electromagnetic current-current correlation function. This correlation function possesses an invariant-tensor decomposition with structure functions entirely analogous to W_1 and W_2 of deep-inelastic scattering of leptons from hadronic targets. The thermal scaling properties of the appropriate structure functions for thermal emission are derived. The thermal structure functions may be computed in a weak-coupling expansion at high plasma temperature. The rates for thermal emission are estimated, and for dileptons, using conservative estimates of the plasma temperature, the thermal-emission process is argued to dominate over the Drell-Yan process for dilepton masses 600 MeV $< M <$ 1—2 GeV. We argue that higher temperatures are entirely possible within the context of the inside-outside cascade model of matter formation, perhaps temperatures as high as 500—800 MeV. If these high temperatures are achieved, the maximum dilepton masses arising from thermal emission are argued to be 5 GeV. Pre-equilibrium emission might dominate over Drell-Yan emission at somewhat higher masses. Signals for thermal emission are presented as the relative magnitude of invariant thermal structure functions, thermal scaling relations, and transverse momenta of thermal dilepton pairs which increase with and are proportional to the dilepton-pair mass. The transverse-mass spectrum is shown to be $dN/dM^2dy\,d^2q_\perp \propto M_\perp{}^{-6}$ and upon integrating over transverse momentum $\propto M^{-4}$, for a high-temperature plasma. The spectrum is power law, not exponential. The dependence of the spectrum of thermal emission upon the existence of a first-phase transition is studied, and the possibility that the spectrum might change its slope as a function of $M_\perp$ or have a sharp break is pointed out. We argue that if there is a first-order phase transition, as beam energy or nuclear baryon number is raised through the threshold for production of a plasma, the rate for photon or dilepton emission might dramatically increase. In the case of a first-order phase transition, in addition to the power-law spectrum of transverse mass, there is an additional contribution of $e^{-M_\perp/T}$, where T is the phase-transition temperature.

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One night missing in Wuhan

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PHYSICAL REVIEW D VOLUME 24, NUMBER 2 15 JULY 1981

Quark liberation at high temperature: A Monte Carlo study of SU(2) gauge theory

Larry D. McLerran
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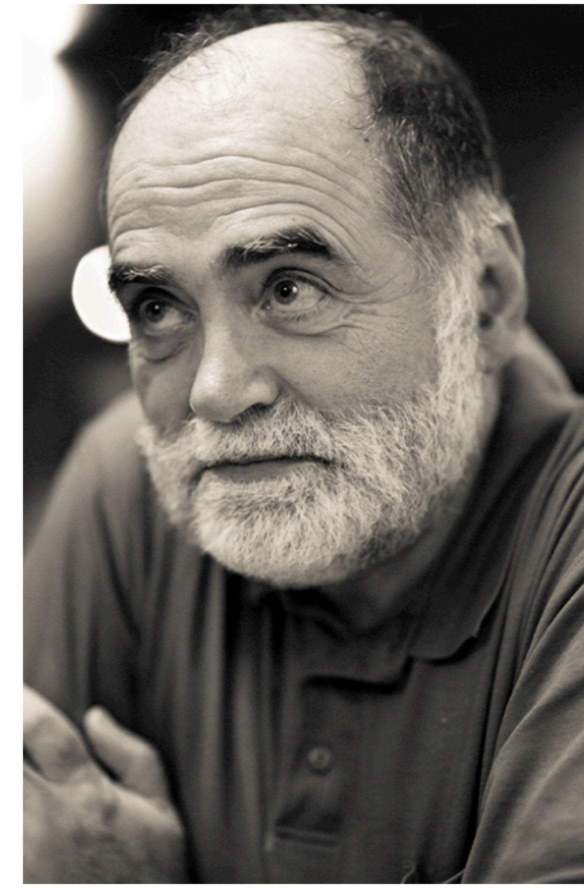
(Received 2 March 1981)

Quark confinement in a finite-temperature SU(N) gauge theory is formulated as the realization of a global Z_N symmetry. Spontaneous breakdown corresponds to a transition to a nonconfining, plasma phase. The free energy of a single quark is an order parameter which probes the phase structure, and it may be calculated in the Euclidean theory in terms of a "Wilson line" running the length of the system along the (periodic) time axis. We present results of a Monte Carlo calculation in the SU(2) lattice theory which confirm the transition at a critical temperature computed in terms of the zero-temperature string tension; data for the quark-antiquark potential are presented as well. We discuss the implications of the finite-temperature transition for efforts to calculate zero-temperature quantities on finite-size lattices. Finally, we note that restoration of Z_N symmetry as the temperature is lowered may be understood as a condensation of instantons and other topological objects.



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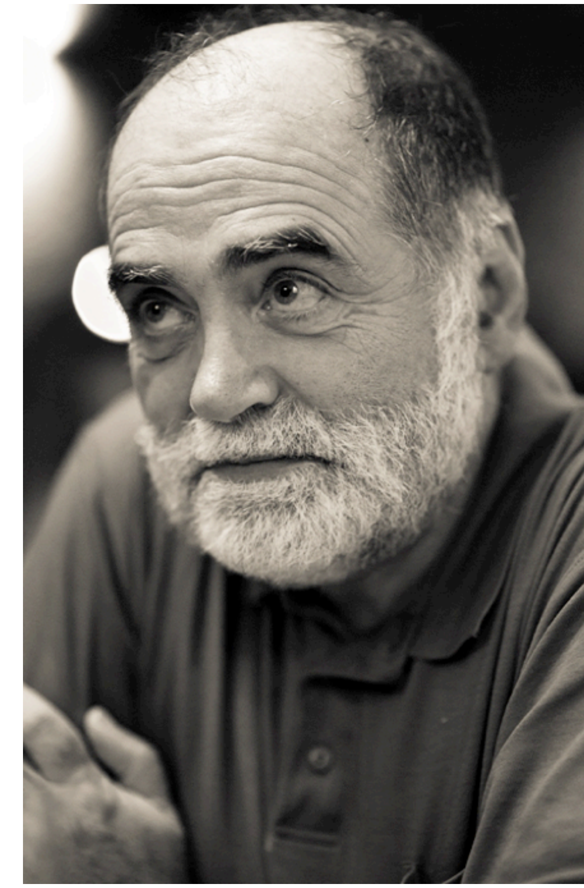

Volume 119B, number 1,2,3 PHYSICS LETTERS 16 December 1982

ORDER PARAMETERS FOR THE CONFINEMENT–DECONFINEMENT PHASE TRANSITION IN SU(N) GAUGE THEORIES WITH QUARKS

Carleton DeTAR [1] and Larry McLERRAN [2]
Research Institute for Theoretical Physics, University of Helsinki, Siltavuorenpenger 20 C, SF 00170 Helsinki 17, Finland

Received 3 August 1982

In the confined phase of an SU(N) gauge theory finite energy bound clusters of n_q quarks and $\bar{n}_q$ antiquarks occur only if the N-ality $(n_q - \bar{n}_q) \bmod N = 0$. In the deconfined phase there is no such restriction. We argue that this feature permits the construction of order parameters in the microcanonical ensemble that vanish in the confined phase and become finite in the deconfined phase.

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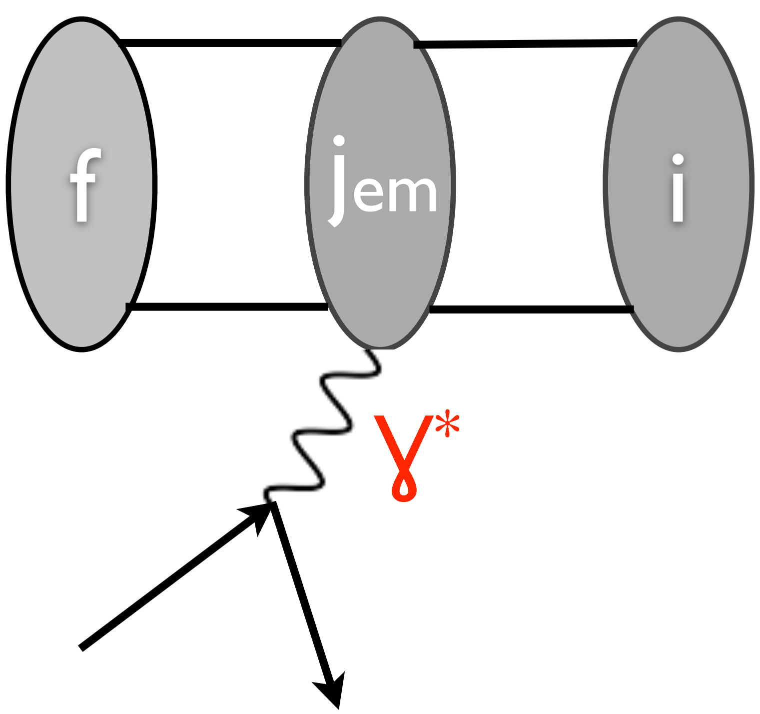

$$\frac{\mathrm{d}R_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = \sum_f Q_f^2 \frac{\alpha_{em}^2}{6\pi^3} \frac{2\rho_T(\omega,\vec{p}) + \rho_L(\omega,\vec{p})}{(\omega^2 - \vec{p}^2)(\exp(\omega/T) - 1)}$$

Above EM emission rate formulae are valid in
leading order of α_{em} and exact to all orders in strong coupling

PHYSICAL REVIEW D **83,** 034504 (2011)

Thermal dilepton rate and electrical conductivity: An analysis of vector current correlation functions in quenched lattice QCD

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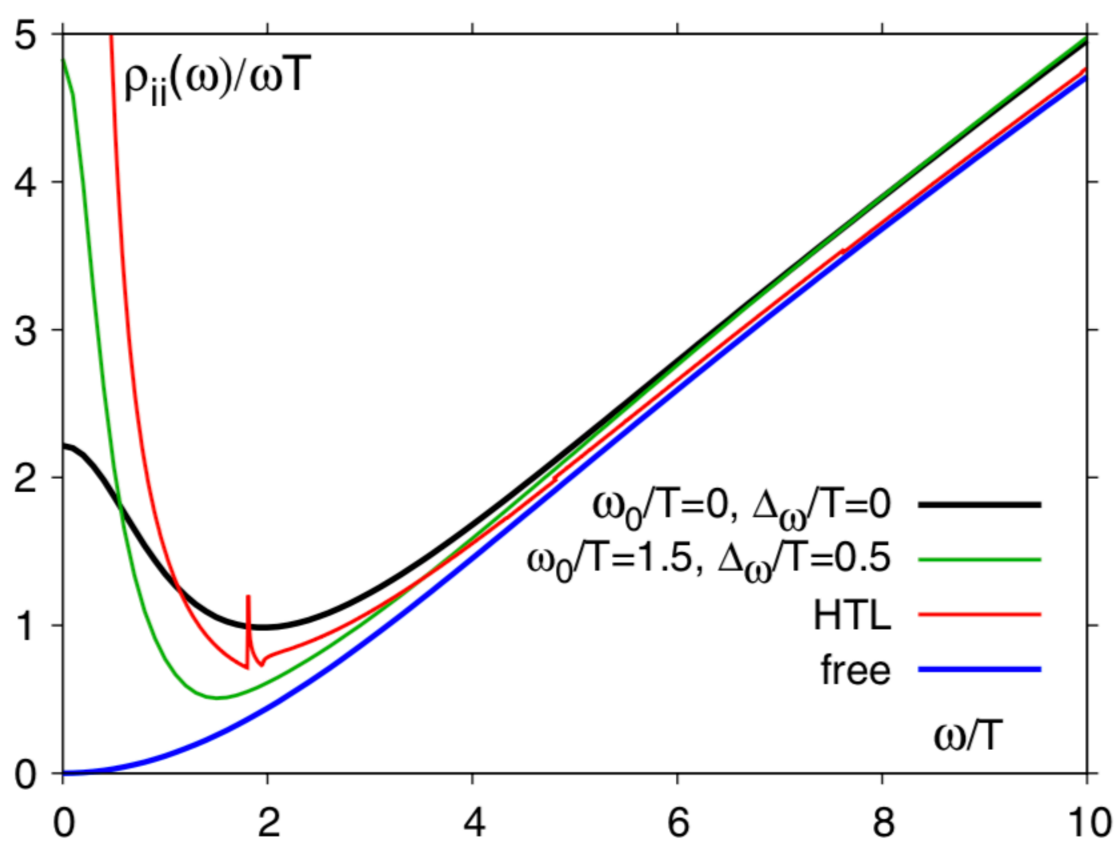
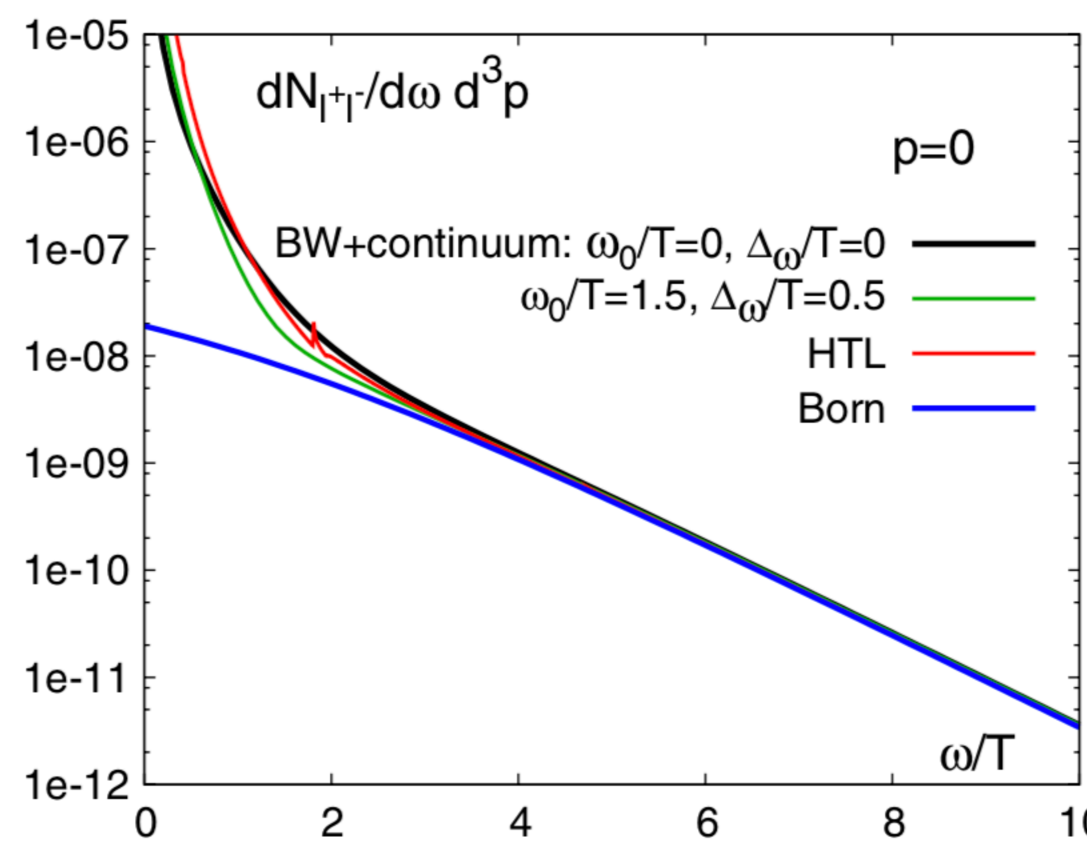
[1]*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[2]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
(Received 24 December 2010; published 23 February 2011)

We calculate the vector current correlation function for light valence quarks in the deconfined phase of QCD. The calculations have been performed in quenched lattice QCD at $T \simeq 1.45T_c$ for four values of the lattice cutoff on lattices up to size $128^3 \times 48$. This allows us to perform a continuum extrapolation of the correlation function in the Euclidean time interval $0.2 \le \tau T \le 0.5$, which extends to the largest temporal separations possible at finite temperature, to better than 1% accuracy. In this interval, at the value of the temperature investigated, we find that the vector correlation function never deviates from the free correlator for massless quarks by more than 9%. We also determine the first two nonvanishing thermal moments of the vector meson spectral function. The second thermal moment deviates by less than 7% from the free value. With these constraints, we then proceed to extract information on the spectral representation of the vector correlator and discuss resulting consequences for the electrical conductivity and the thermal dilepton rate in the plasma phase.





PHYSICAL REVIEW D **94**, 034504 (2016)

Thermal dilepton rates and electrical conductivity of the QGP from the lattice

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[1]*Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*
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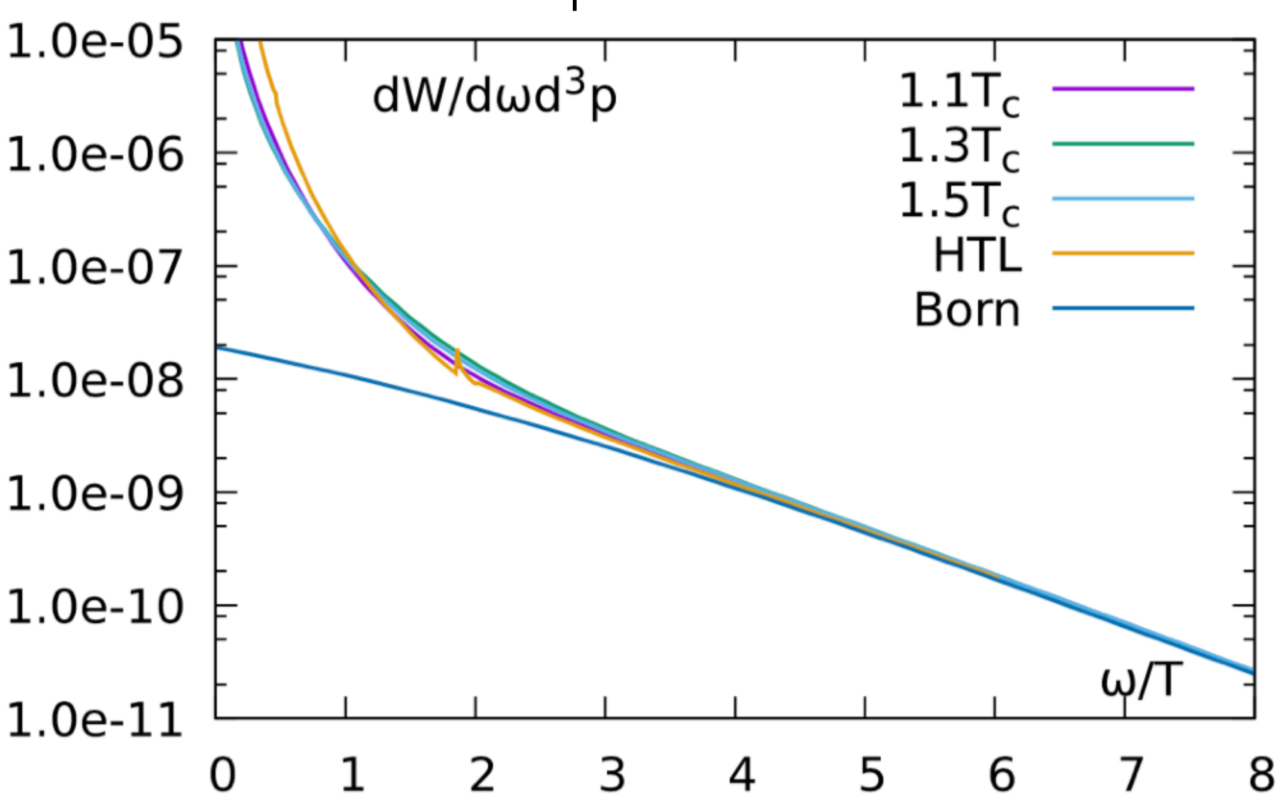
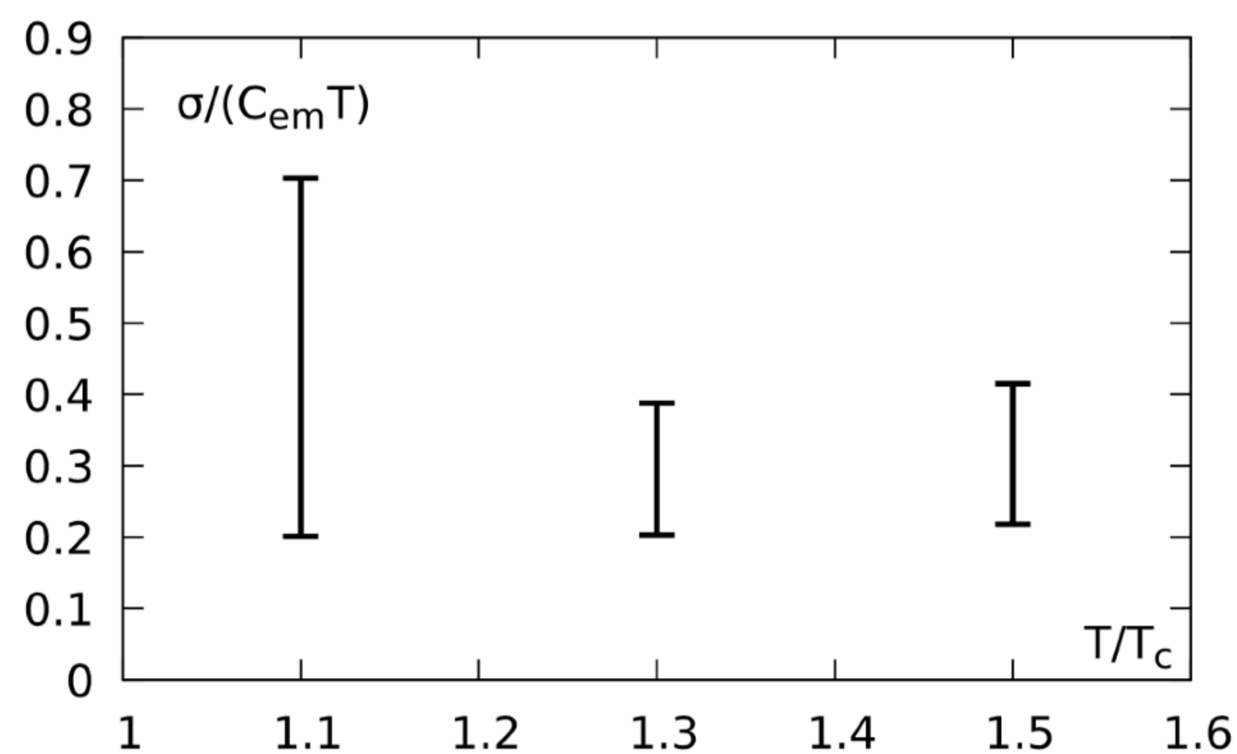
(Received 10 May 2016; published 15 August 2016)

We investigate the temperature dependence of the thermal dilepton rate and the electrical conductivity of the gluon plasma at temperatures of 1.1, 1.3, and $1.5T_c$ in quenched QCD. Making use of nonperturbatively clover-improved Wilson valence quarks allows for a clean extrapolation of the vector meson correlation function to the continuum limit. We found that the vector correlation function divided by T^3 is almost temperature independent in the current temperature window. The spectral functions are obtained by χ^2 fitting of phenomenologically inspired *Ansätze* for the spectral function to the continuum extrapolated correlator data, where the correlations between the data points have been included. Systematic uncertainties arising from varying the *Ansätze* motivated from strong coupling theory as well as perturbation theory are discussed and estimated. We found that the electrical conductivity of the hot medium, related to the slope of the vector spectral function at zero frequency and momentum, is $0.2C_{em} \lesssim \sigma/T \lesssim 0.7C_{em}$ for $T = 1.1T_c$ and $0.2C_{em} \lesssim \sigma/T \lesssim 0.4C_{em}$ for the higher temperatures. The dilepton rates and soft photon rates, resulting from the obtained spectral functions, show no significant temperature dependence, either.

Dilepton rate

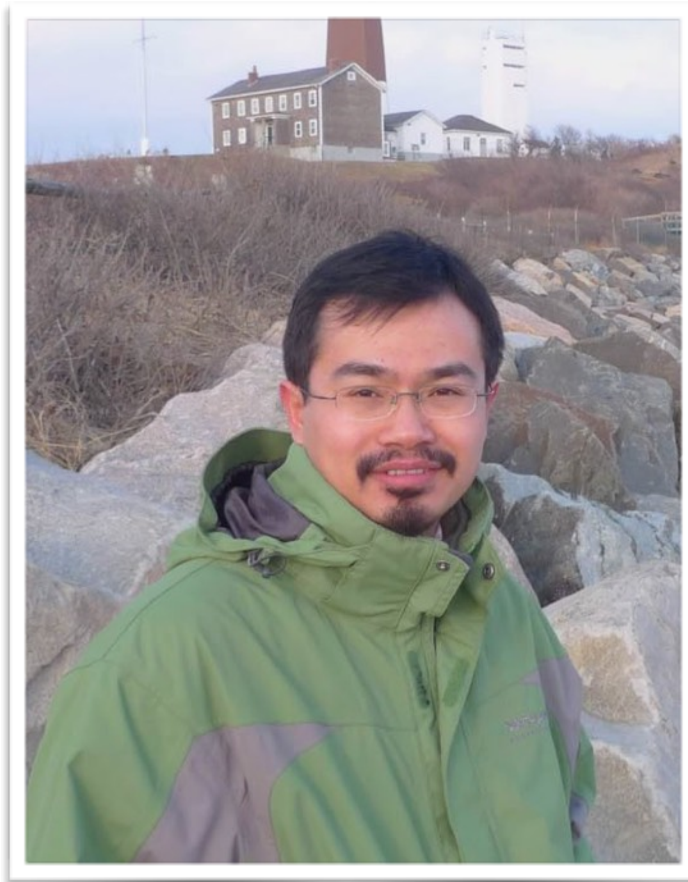


Electrical conductivity



Looking for charm and beauty

Bielefeld, 2010



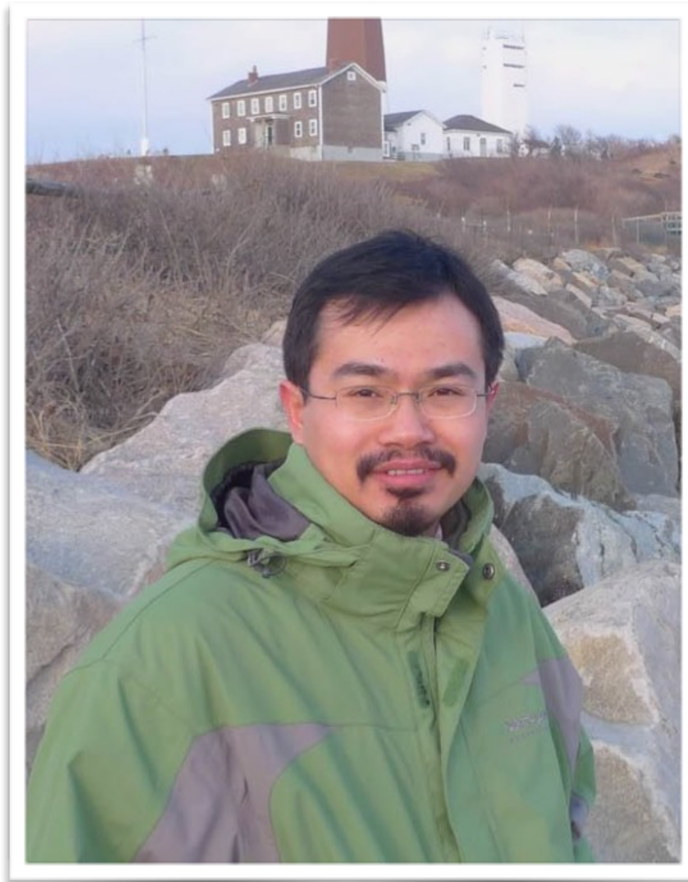
Long Island, 2011

Wuhan, 2012

Larry: What happened to your beard?
HTD: Looking for charm and beauty…

Looking for charm and beauty

Bielefeld, 2010



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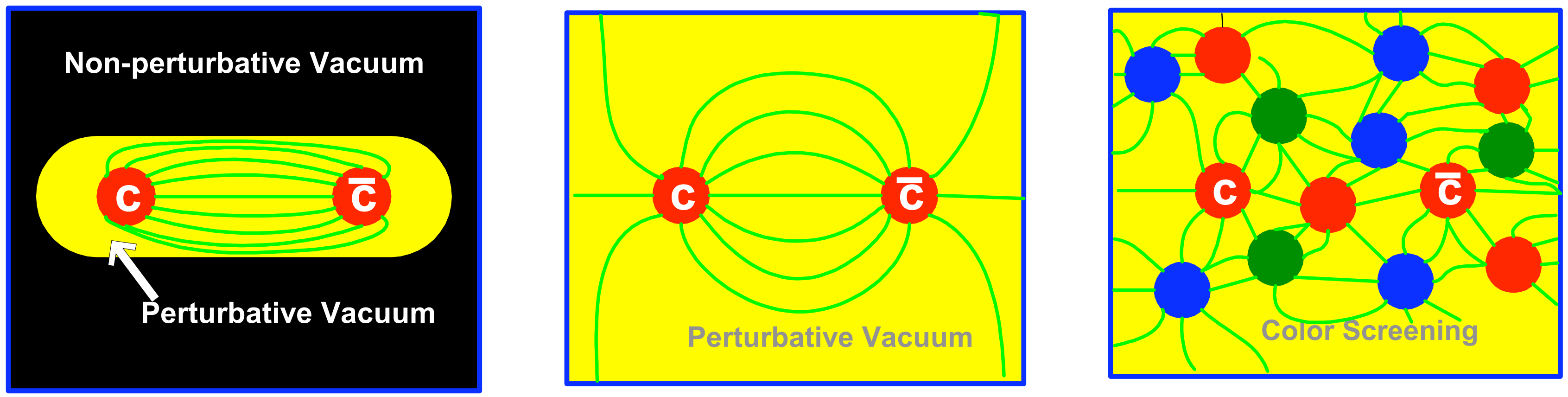
Quarkonia at T>0

★ The suppression of the charmonium production can be a signature of the formation of the Quark Gluon Plasma [Matsui & Satz '86]



$$V(r) = -\frac{\alpha_{eff}}{r} + \sigma\, r \qquad\qquad V(r,T) = -\frac{\alpha_{eff}}{r}\, e^{-r/\lambda_D(T)}$$

- Confining term: string tension becomes weaker with increasing T and vanishes at $T>T_c$
- Color screening due to the presence of the QGP

E(r) of the ccbar system should have a minimum wrt r if a bound state is possible

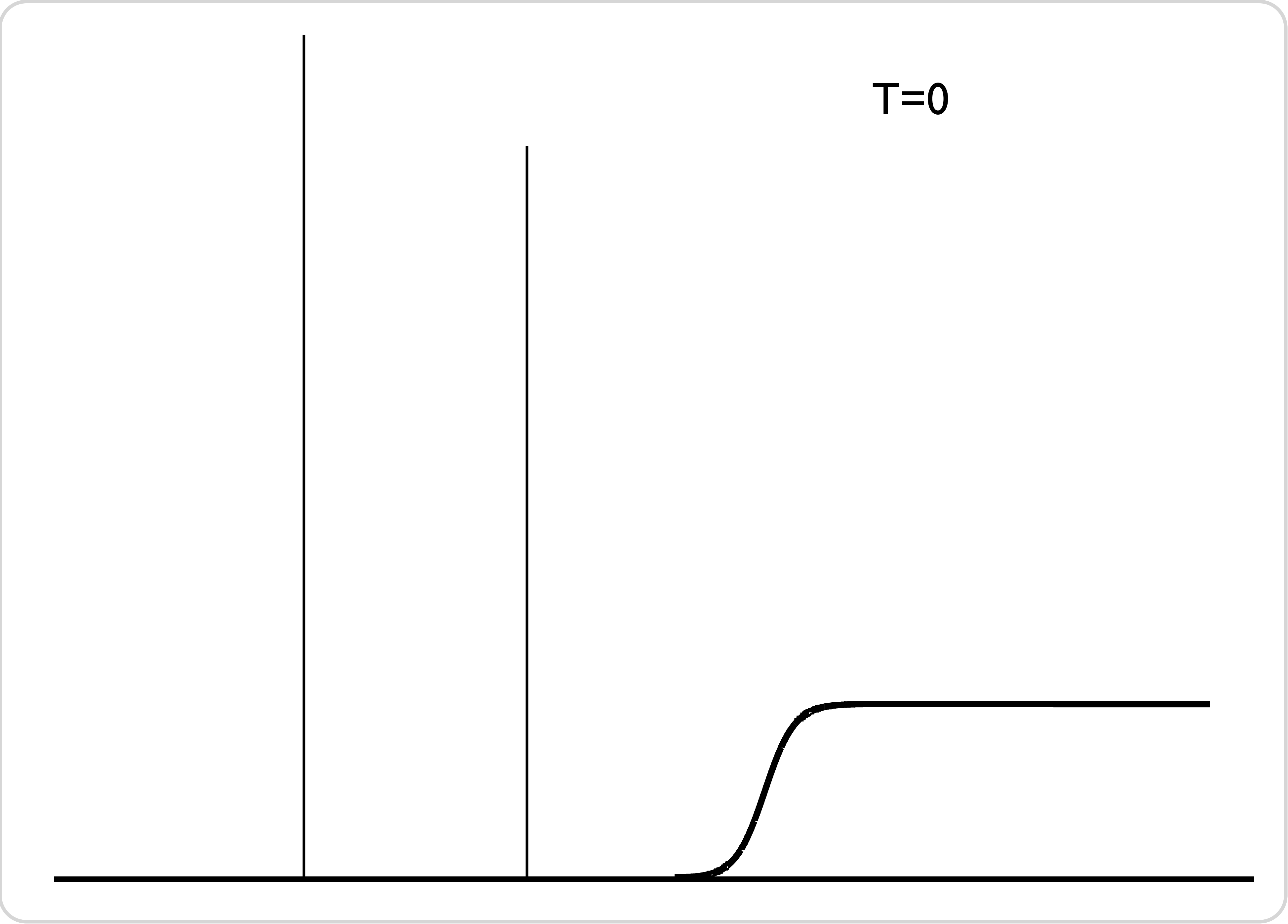
The condition for an extremum of E(r) is

$$x(1+x)\, e^{-x} = \frac{2}{m_c \alpha_{eff} \lambda_D} \quad \text{with} \quad x = r/\lambda_D$$

No solutions → the bound state will be "melted"

QGP thermometers

Hadron Spectral functions



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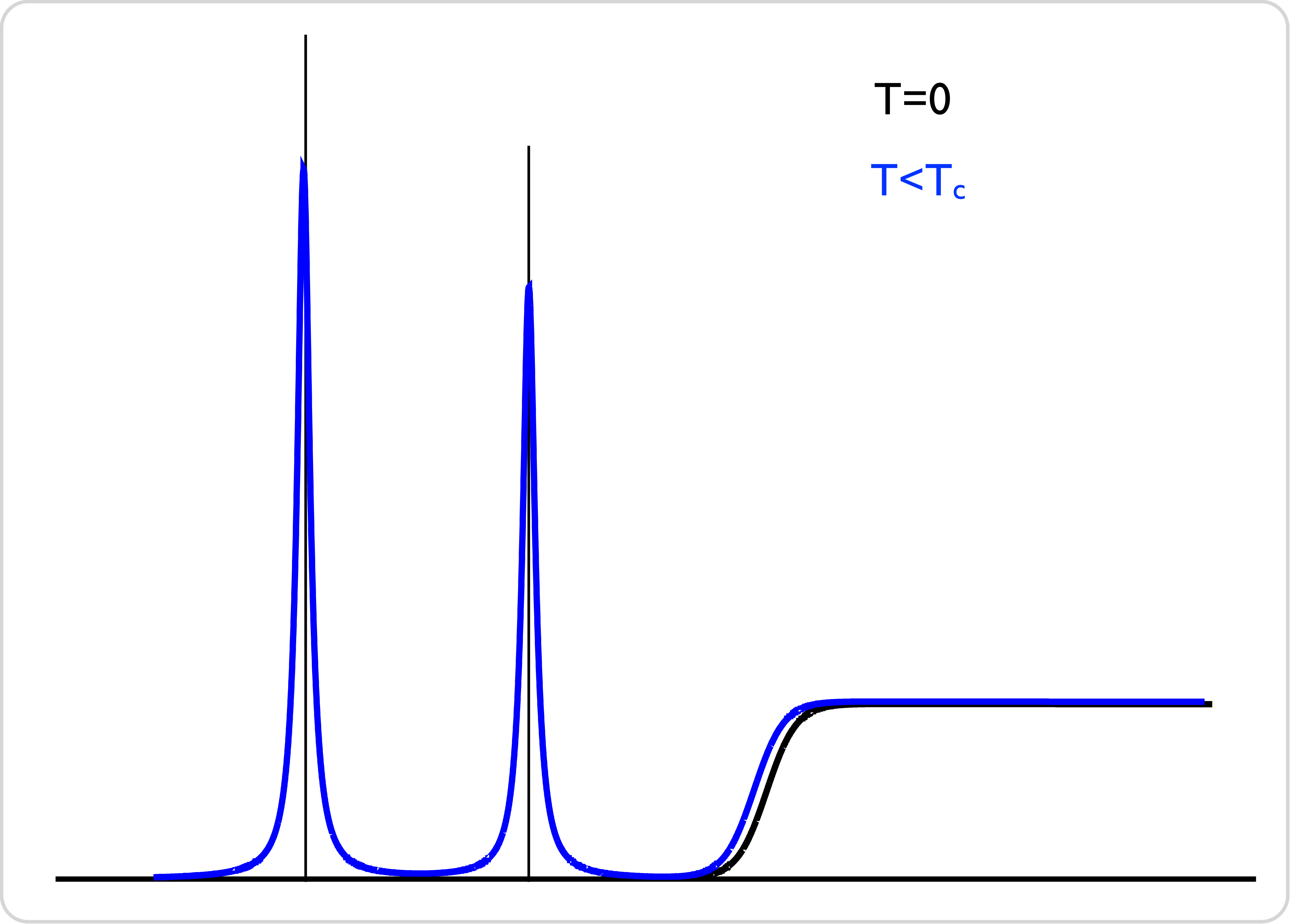

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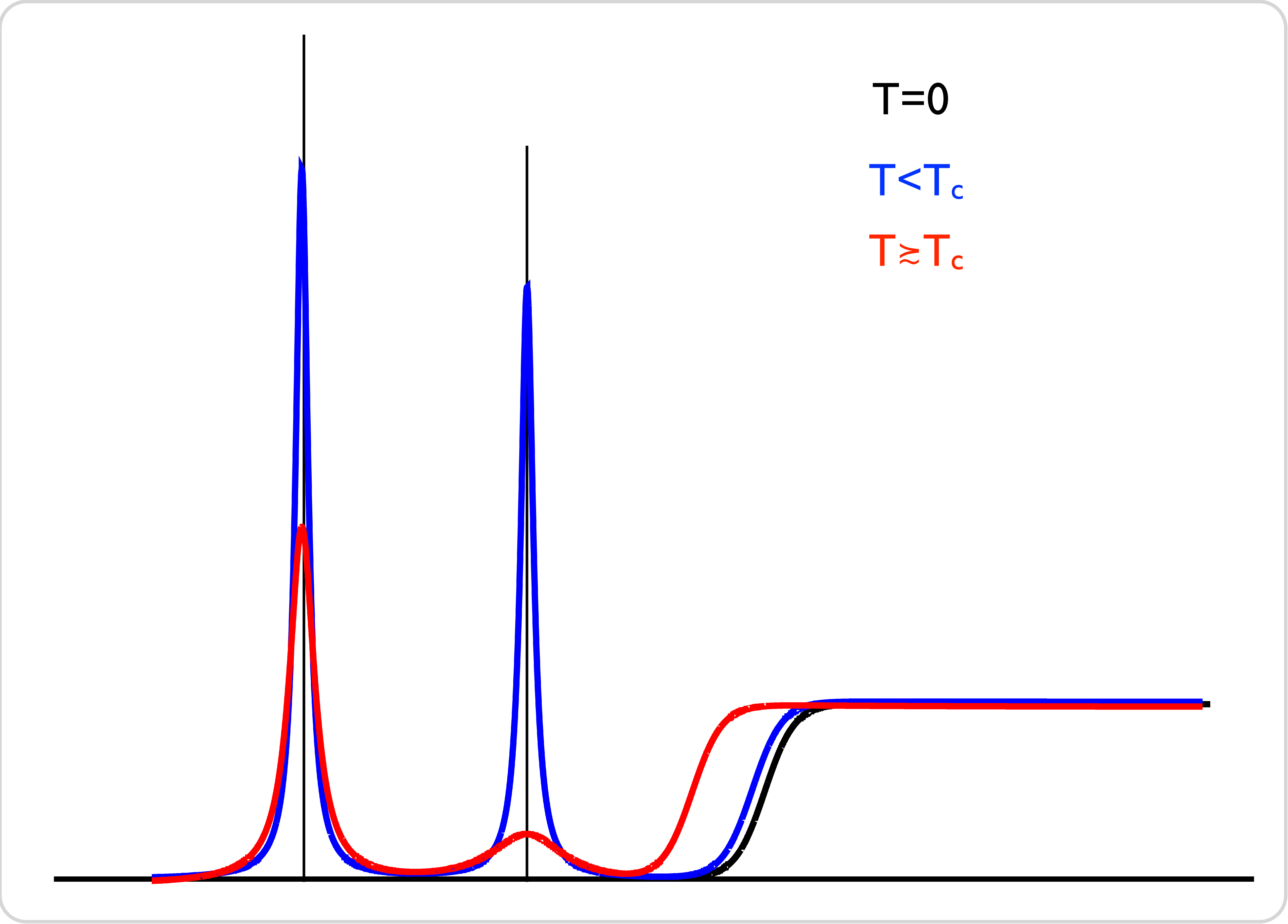

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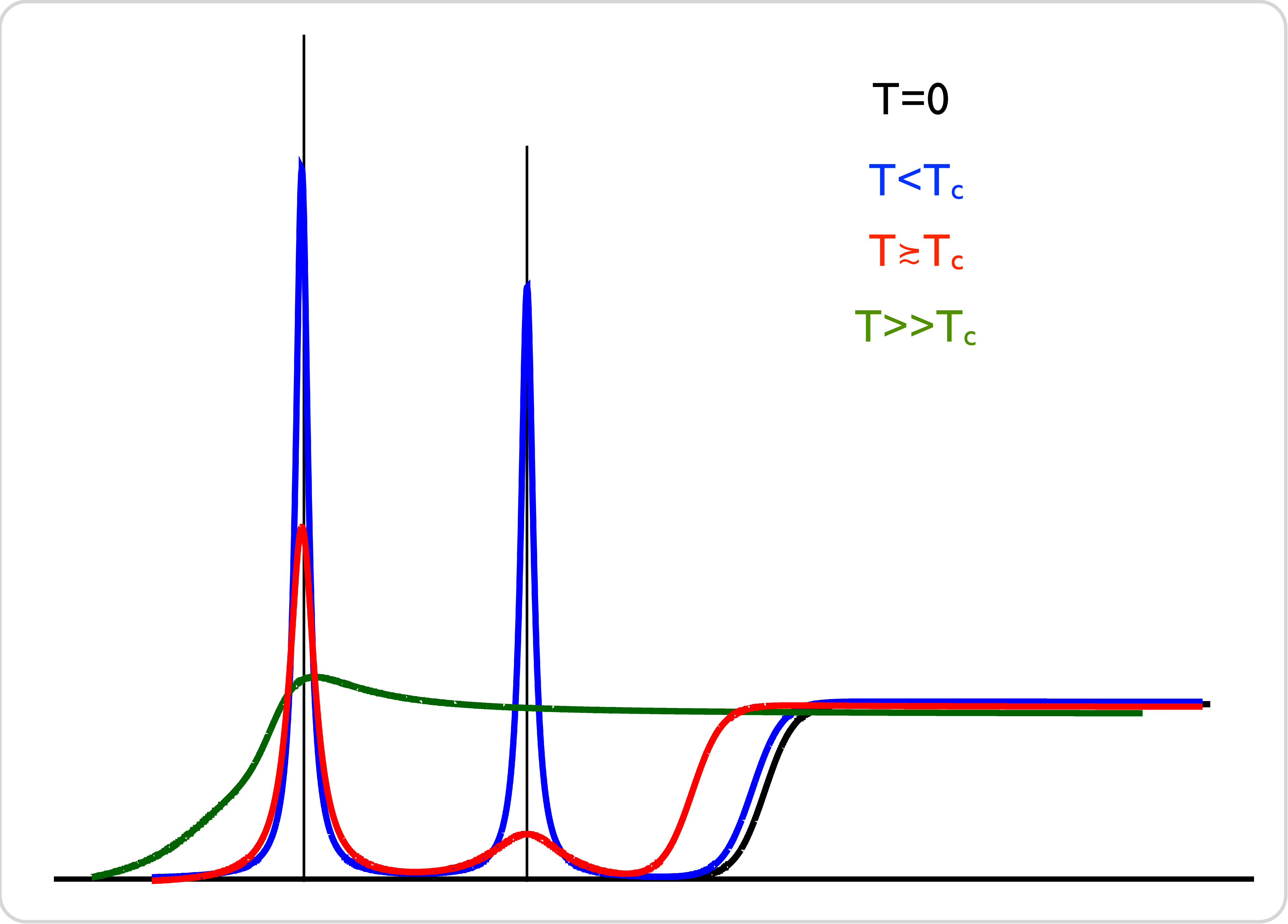
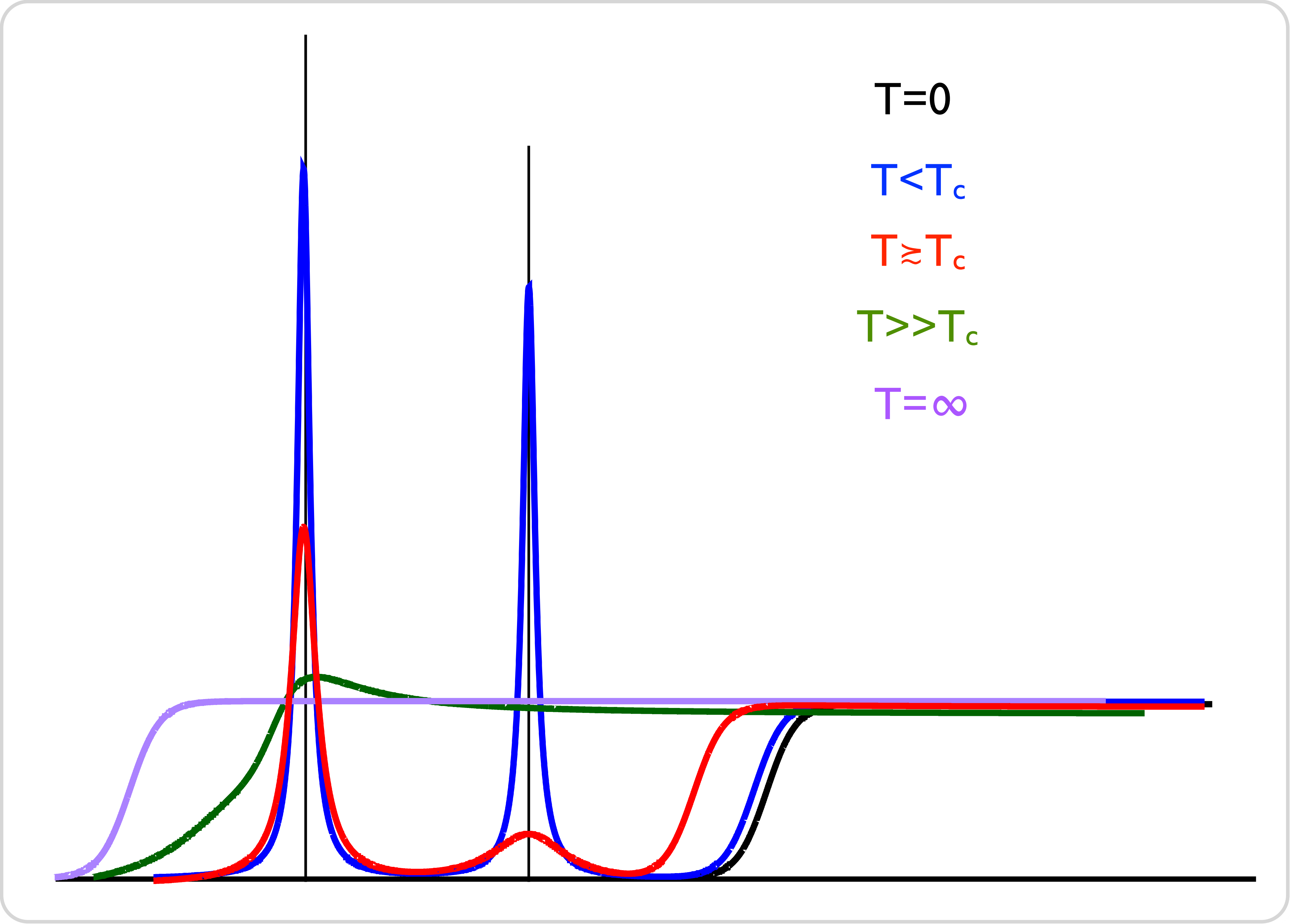


Hadron Spectral functions



A Brownian motion of HQ in the medium

- Langevin Equation

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{p^i}{M},$$

$$\frac{\mathrm{d}p^i}{\mathrm{d}t} = -\,\eta_D\, p^i + \xi^i(t)$$

drag — random force

- Fluctuation-dissipation relation

$$\eta_D = \frac{\kappa}{2MT},$$

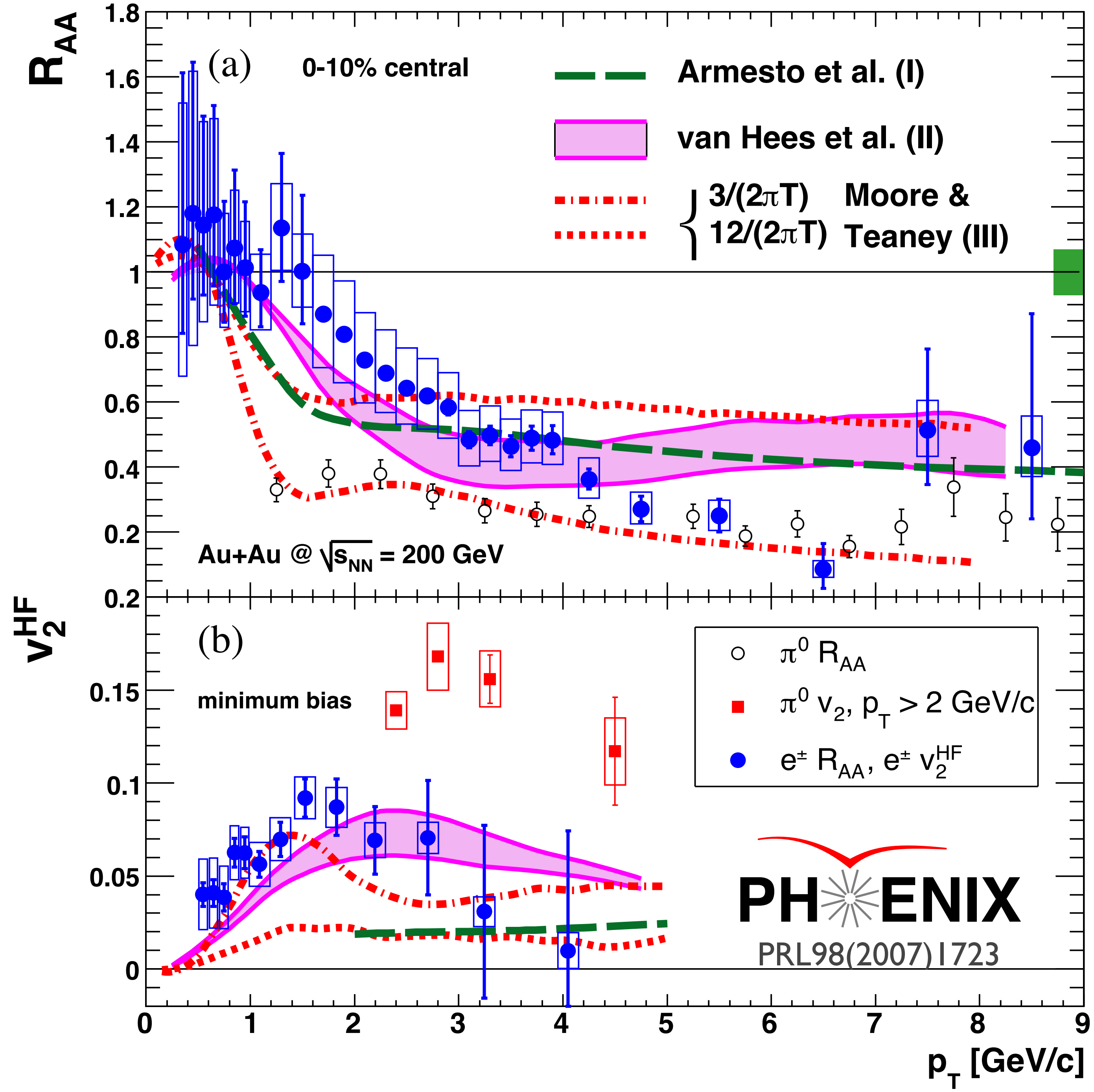
$$\langle \xi^i(t)\xi^j(t') \rangle = \kappa\, \delta^{ij}\, \delta(t-t')$$

- Fick's law of diffusion

$$\partial_t N + D\, \nabla^2 N = 0$$

- Einstein relation

$$D = \frac{T}{M\,\eta_D} = \frac{2T^2}{\kappa}$$

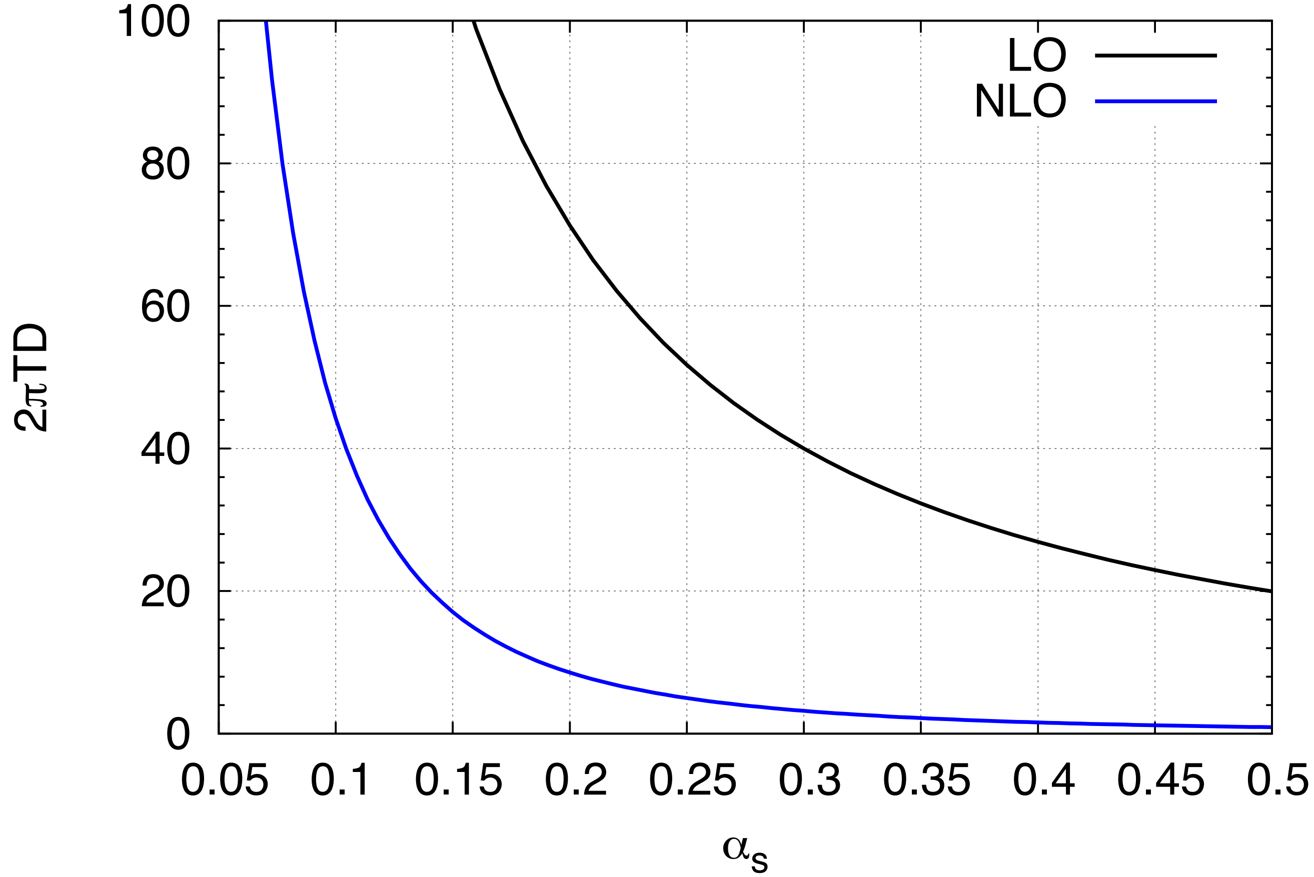


D: diffusion coefficient

3κ: mean squared momentum transfer per time

Heavy Quark Diffusion coefficient

- pQCD calculations



$\alpha_s \sim 0.2$, $g \sim 1.6$

LO:	$2\pi TD \approx 71.2$
NLO:	$2\pi TD \approx 8.4$

Moore & Teaney, PRD71(2005)064904
Caron-Huot & Moore, PRL 100(2008)052301

★ Compute heavy quark diffusion coefficient on the lattice

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \lim_{\vec{p} \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}(\omega, \vec{p})}{\omega}$$

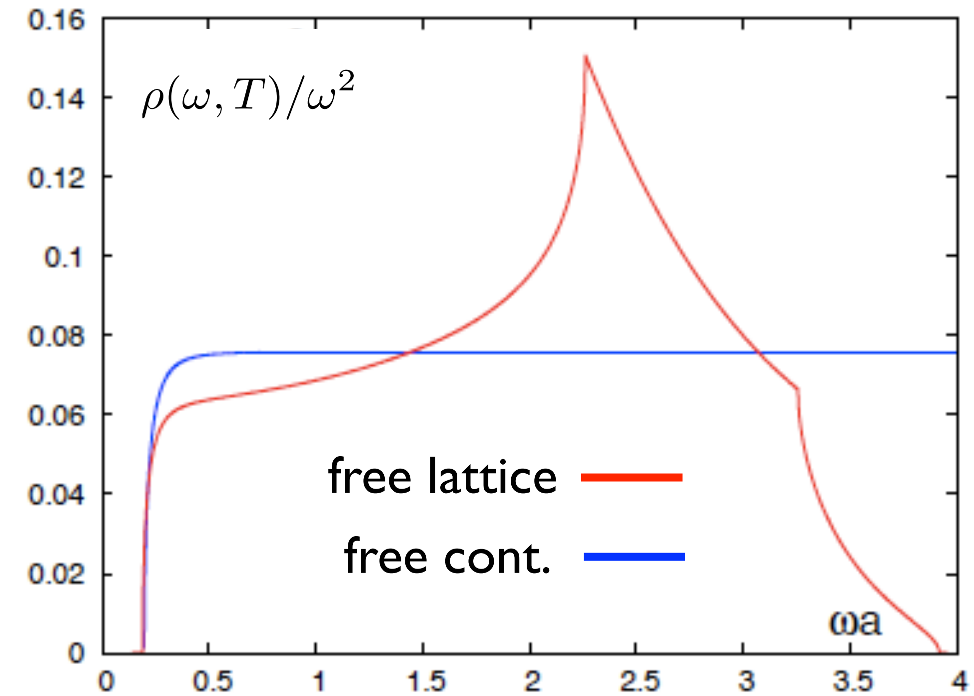
Kubo formula: heavy quark diffusion constant ~ intercept of σ(ω,0)/ω at ω=0

$$\rho_{ii}(\omega, \vec{p}) = \int \mathrm{d}^4x \, e^{i\omega t - i\vec{p}\cdot\vec{x}} \left\langle \left[j_i(t, \vec{x}), j_i(0, \vec{0}) \right] \right\rangle$$

EM current: $j_i = \bar{\psi}\gamma_i\psi$

Prior information of hadron spectral functions (spf)

- Difference seen in free spf obtained on lattice and continuum: continuum extrapolation is important



- low frequency behavior of spf

I: Non-interacting case: [Karsch et al., 03, Aarts et al., '05]

$$\rho(\omega) = N_c \left[\left(a_H^{(1)} + a_H^{(3)}\right) I_1 + \left(a_H^{(2)} - a_H^{(3)}\right) I_2\right] \omega\delta(\omega)$$

- $\omega\delta(\omega)$ term corresponds to a τ independent constant in the correlator, i.e. zero mode contribution [Umeda, '07]
- No zero mode contribution in the PS channel
- Zero mode contribution exists in the Vector, Scalar and Axial Vector channels

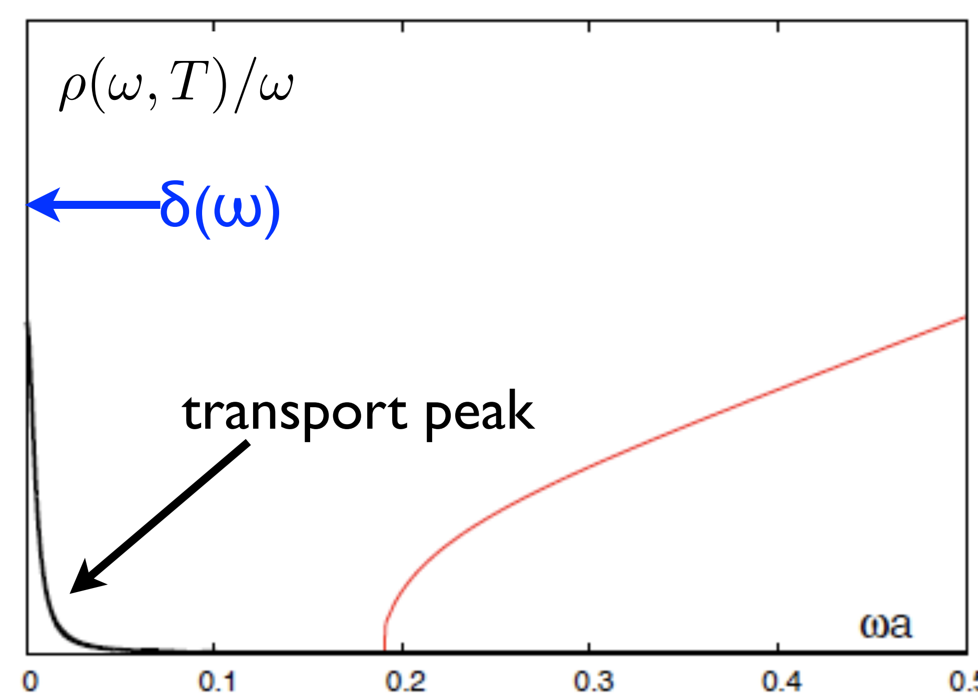
$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \lim_{\vec{p}\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}(\omega,\vec{p})}{\omega}$$

II: Interacting case: [Aarts & Martinez-Resco '02, Petreczky & Teaney '06]

$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \quad \longrightarrow \quad \boxed{D \propto 1/\eta}$$

$\delta(\omega)$ is smeared into a transport peak

spf in vector channel should be linear in ω at $\omega \sim 0$

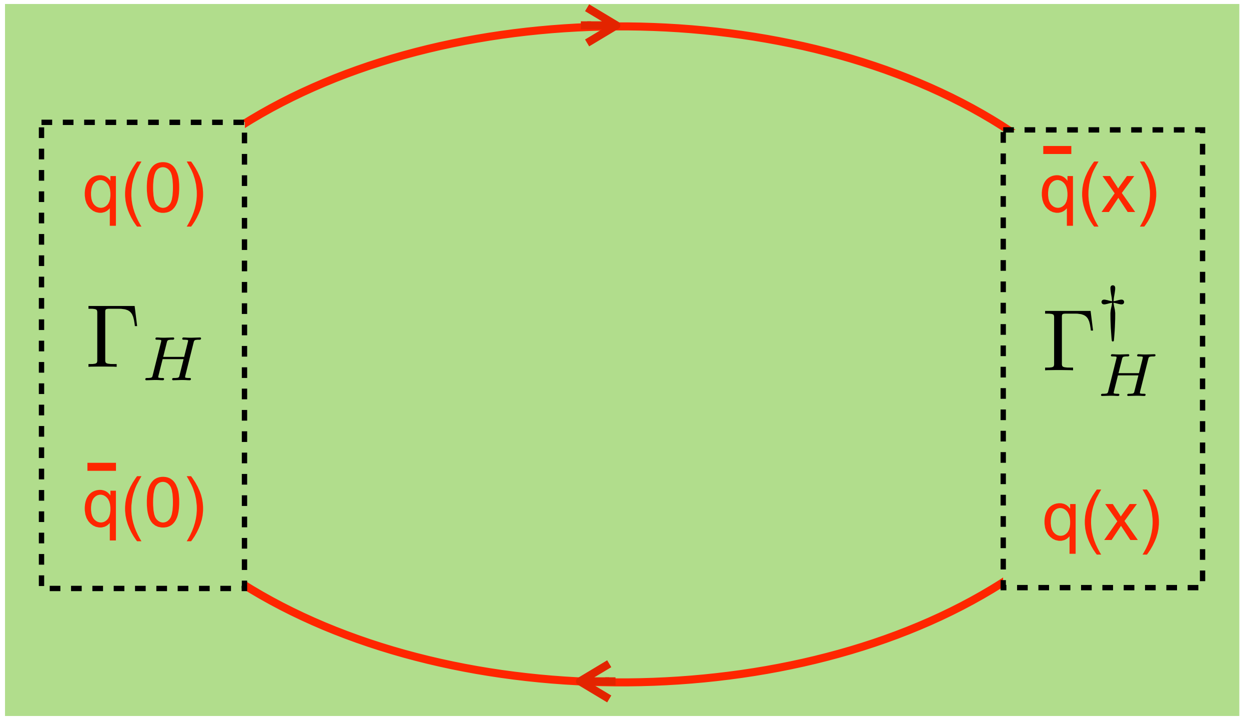


Temporal correlators and spectral functions

$$G(\tau,\vec{p},T) = \sum_{\vec{x}} \exp(-i\vec{p}\cdot\vec{x}) \left\langle J_H(0,\vec{0}) J_H^\dagger(\tau,\vec{x}) \right\rangle$$

$$J_H(\tau,\vec{x}) = \bar{q}(\tau,\vec{x})\,\Gamma_H\, q(\tau,\vec{x})$$

Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	$u\bar{u}$ states	$c\bar{c}$ states	$b\bar{b}$ states
PS	γ_5	$^1S_0$	0^{-+}	π	η_c	η_b
VC	γ_μ	$^3S_1$	1^{--}	ρ	J/ψ	Υ
SC	1	$^3P_0$	0^{++}	a_0	χ_{c0}	χ_{b0}
AV	$\gamma_5\gamma_\mu$	$^3P_1$	1^{++}	a_1	χ_{c1}	χ_{b1}



$$G(\tau,T) = D^+(-i\tau)\,, \qquad \rho(\omega) = 2\mathrm{Im}D^R(\omega) = D^+(\omega) - D^-(\omega)$$

Spectral representation

$$G(\tau,\vec{p},T) = \int \frac{\mathrm{d}\omega}{2\pi}\,\rho(\omega,\vec{p},T)\,K(\tau,\omega,T); \qquad K(\tau,\omega,T) = \frac{\cosh(\omega(\tau-\frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

quenched Lattice QCD simulations

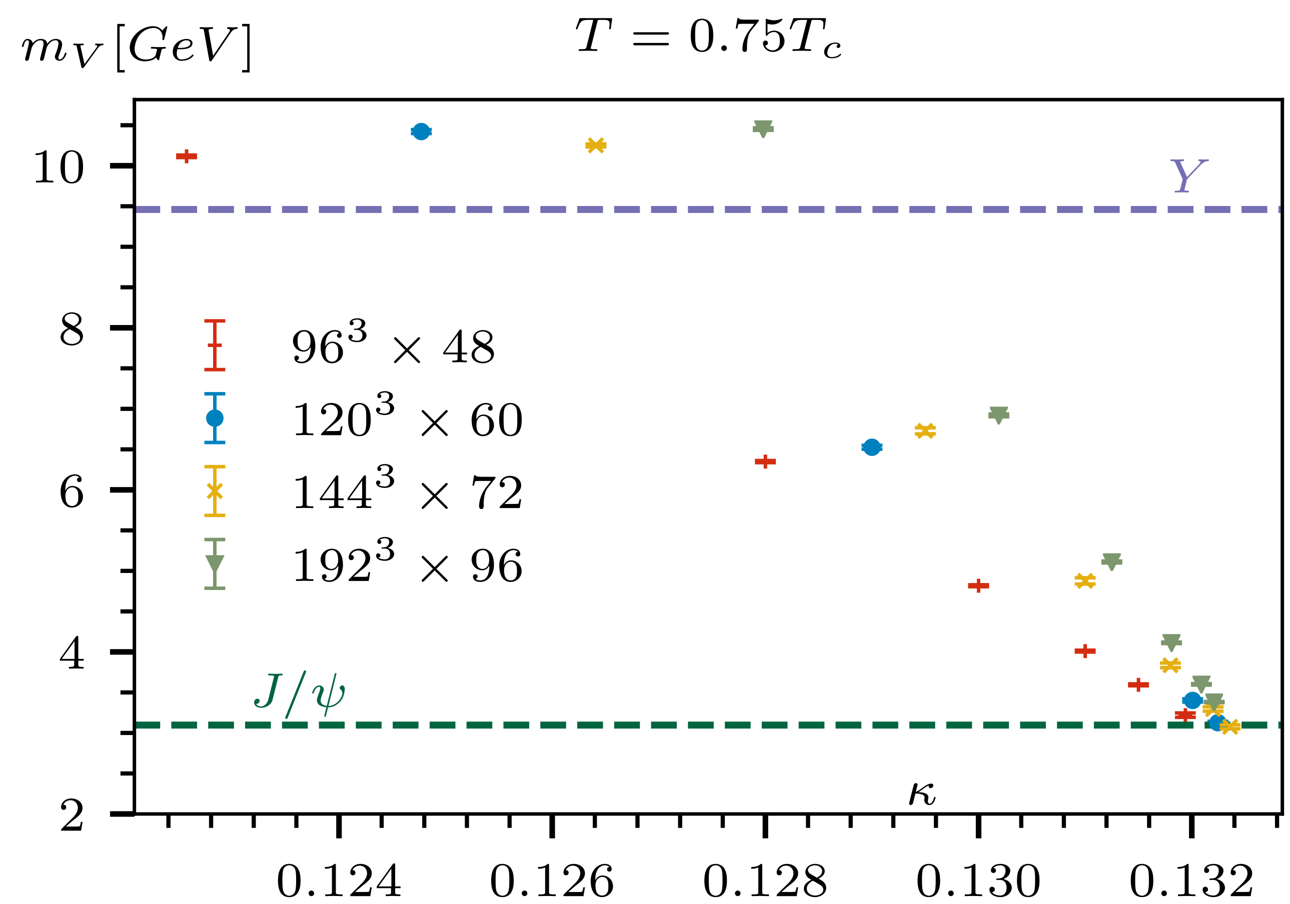
β	r_0/a	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	# confs
7.192	26.6	0.018(11.19)	96	48	0.75	237
				32	1.1	476
				28	1.3	336
				24	1.5	336
				16	2.25	237
7.394	33.8	0.014(14.24)	120	60	0.75	171
				40	1.1	141
				30	1.5	247
				20	2.25	226
7.544	40.4	0.012(17.01)	144	72	0.75	221
				48	1.1	462
				42	1.3	660
				36	1.5	288
				24	2.25	237
7.793	54.1	0.009(22.78)	192	96	0.75	224
				64	1.1	291
				56	1.3	291
				48	1.5	348
				32	2.25	235

- Quenched QCD with Wilson gauge action, and Wilson-clover fermions
- Simulations at 5 different lattice spacings *a*
- Fixed aspect ratio: same physical volume

Continuum extrapolations at 5 different temperatures

Mass tuning to have physical masses of J/Ψ and ϒ

β	κ	m_V [GeV]	β	κ	m_V [GeV]
7.192	0.13194	3.21(1)	7.394	0.132008	3.38(2)
	0.1315	3.59(1)		0.1315	3.94(2)
	0.131	4.01(1)		0.131	4.47(2)
	0.13	4.81(1)		0.129	6.50(2)
	0.128	6.34(1)		0.124772	10.04(1)
	0.12257	10.11(1)			
7.544	0.13236	3.06(2)	7.793	0.13221	3.37(1)
	0.1322	3.28(1)		0.13209	3.59(1)
	0.1318	3.82(2)		0.13181	4.11(1)
	0.131	4.86(2)		0.13125	5.11(1)
	0.1295	6.70(2)		0.13019	6.92(1)
	0.12641	10.23(2)		0.12798	10.42(1)

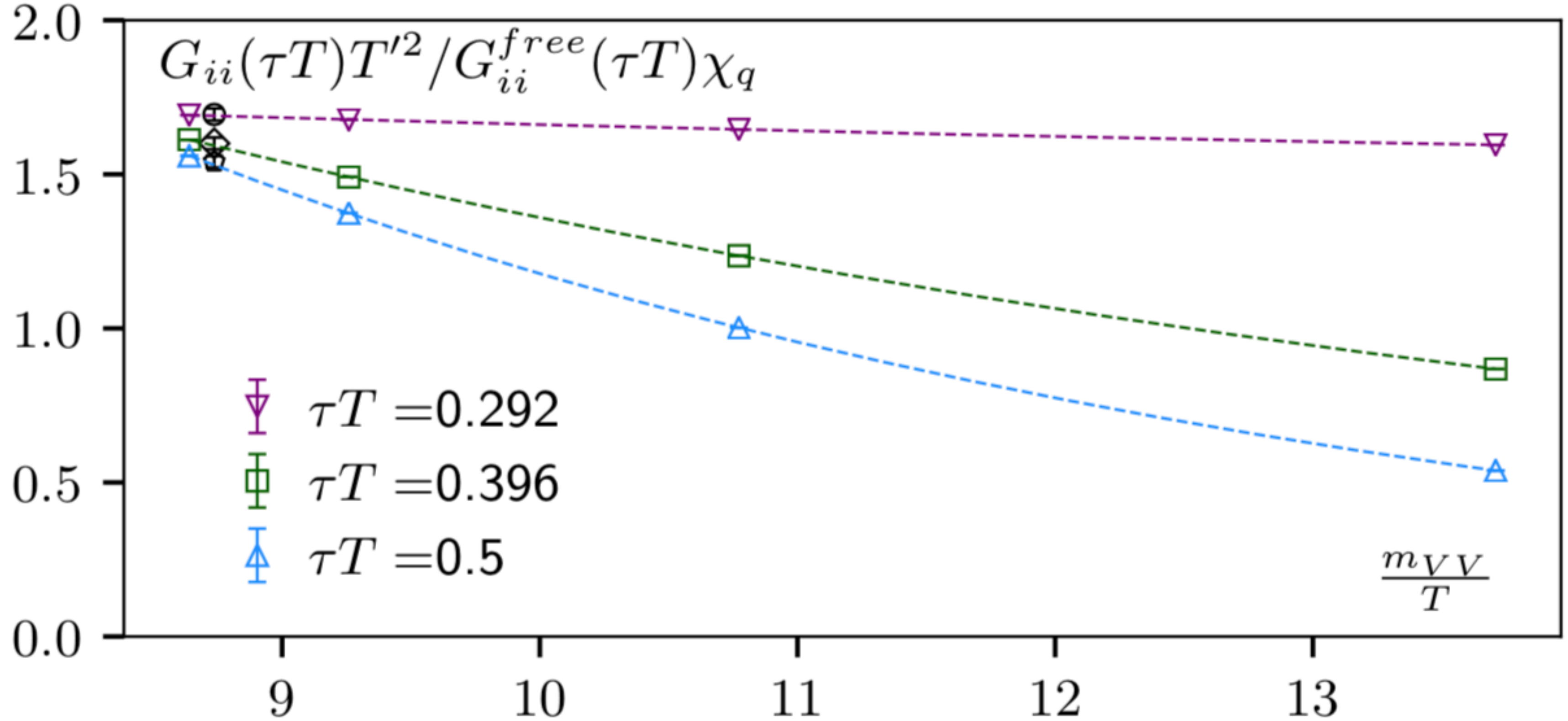


HTD, Kaczmarek, Kruse, Ohno, **H. Sandmeyer**, Lattice 2017, arXiv:1710.08858

Tuning of hoping parameter kappa is non-trivial

correlation functions at each *β* with 5-6 values of kappa corresponding to m_V from ~3 to ~10 GeV

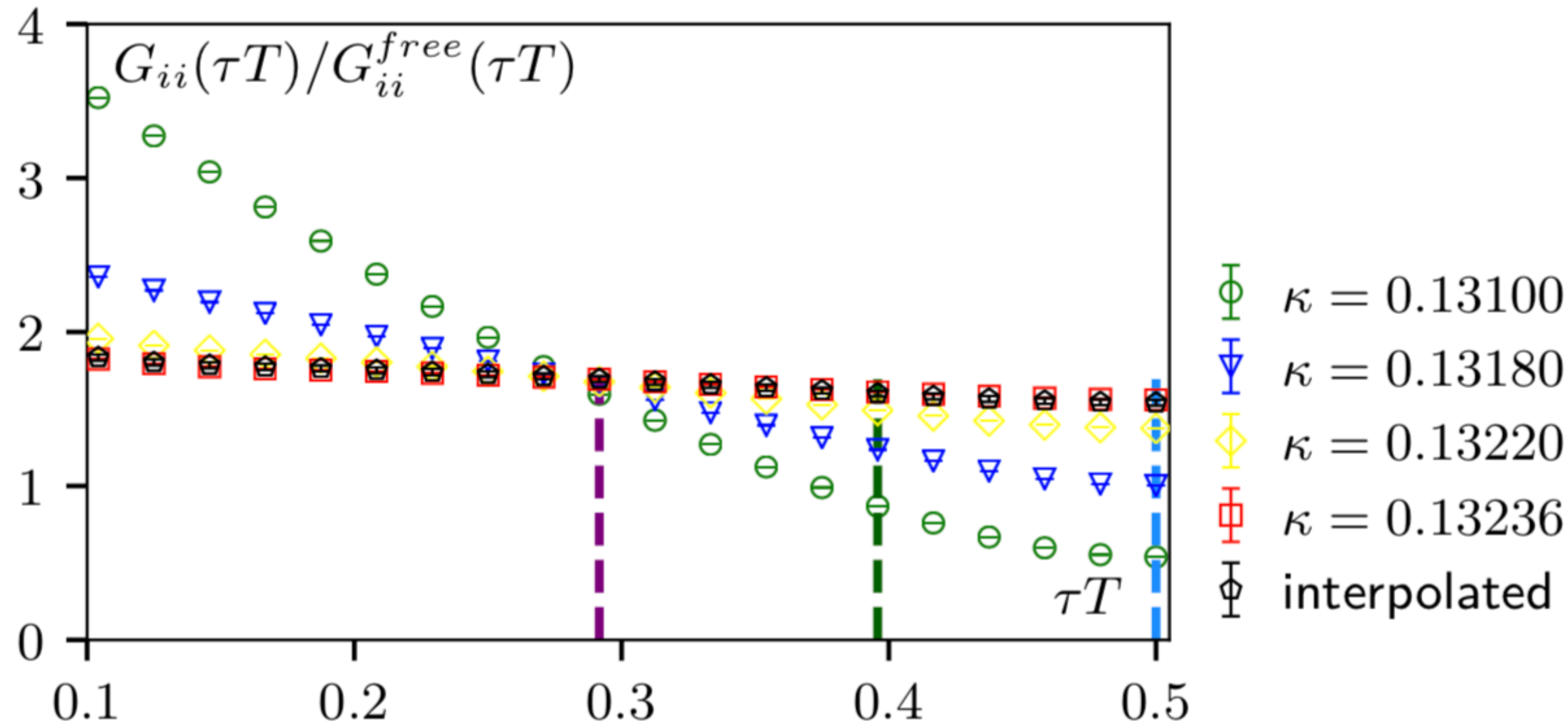
An example: interpolation to the correlator at physical J/Ψ mass



- Choose 4 kappas having m_V closest to mass of J/Ψ
- Interpolation Ansatz: $m_{q\bar{q}} = m_{J/\psi}$

$$\frac{G^{ii}\left(\tau T, \frac{m_{q\bar{q}}}{T}\right) T'^2}{T^3 \chi_q} = \exp\left(p\left(\frac{m_{q\bar{q}}}{T}\right)^2 + q\frac{m_{q\bar{q}}}{T} + r\right)$$

An example: interpolation to the correlator at physical J/Ψ mass



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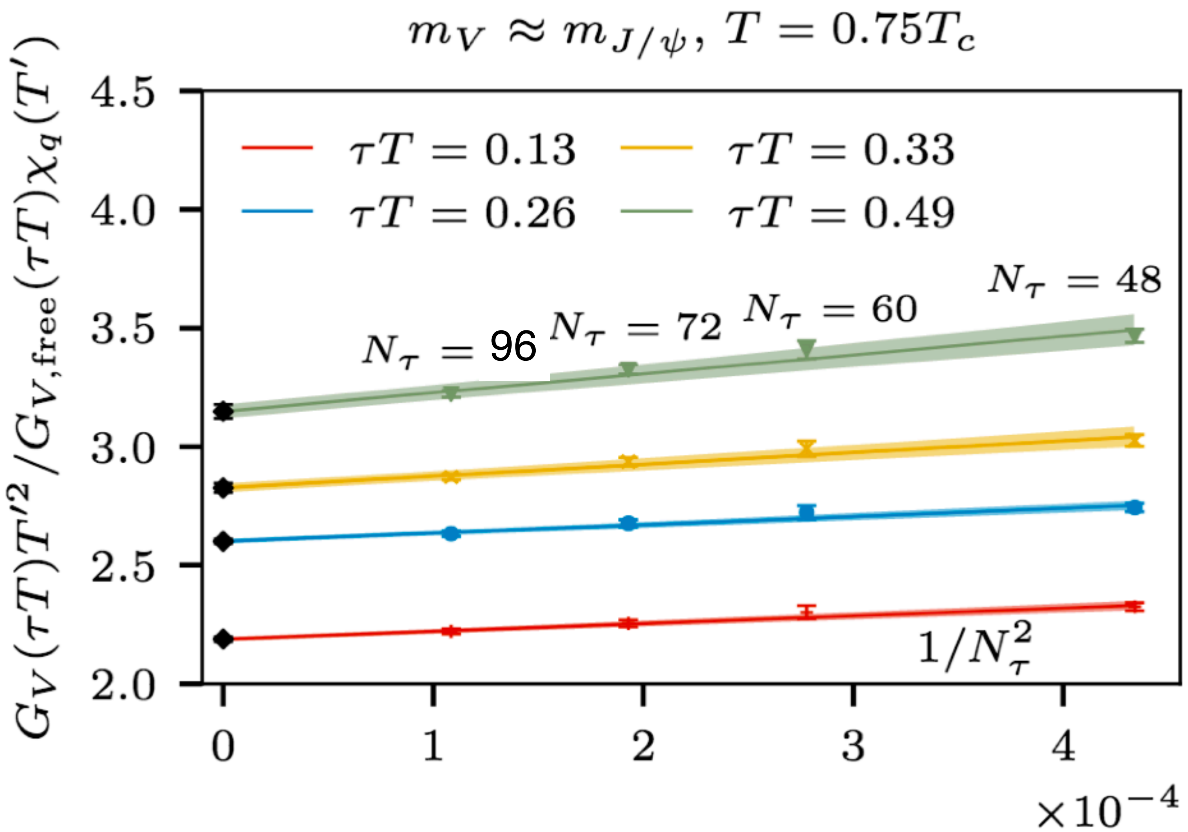
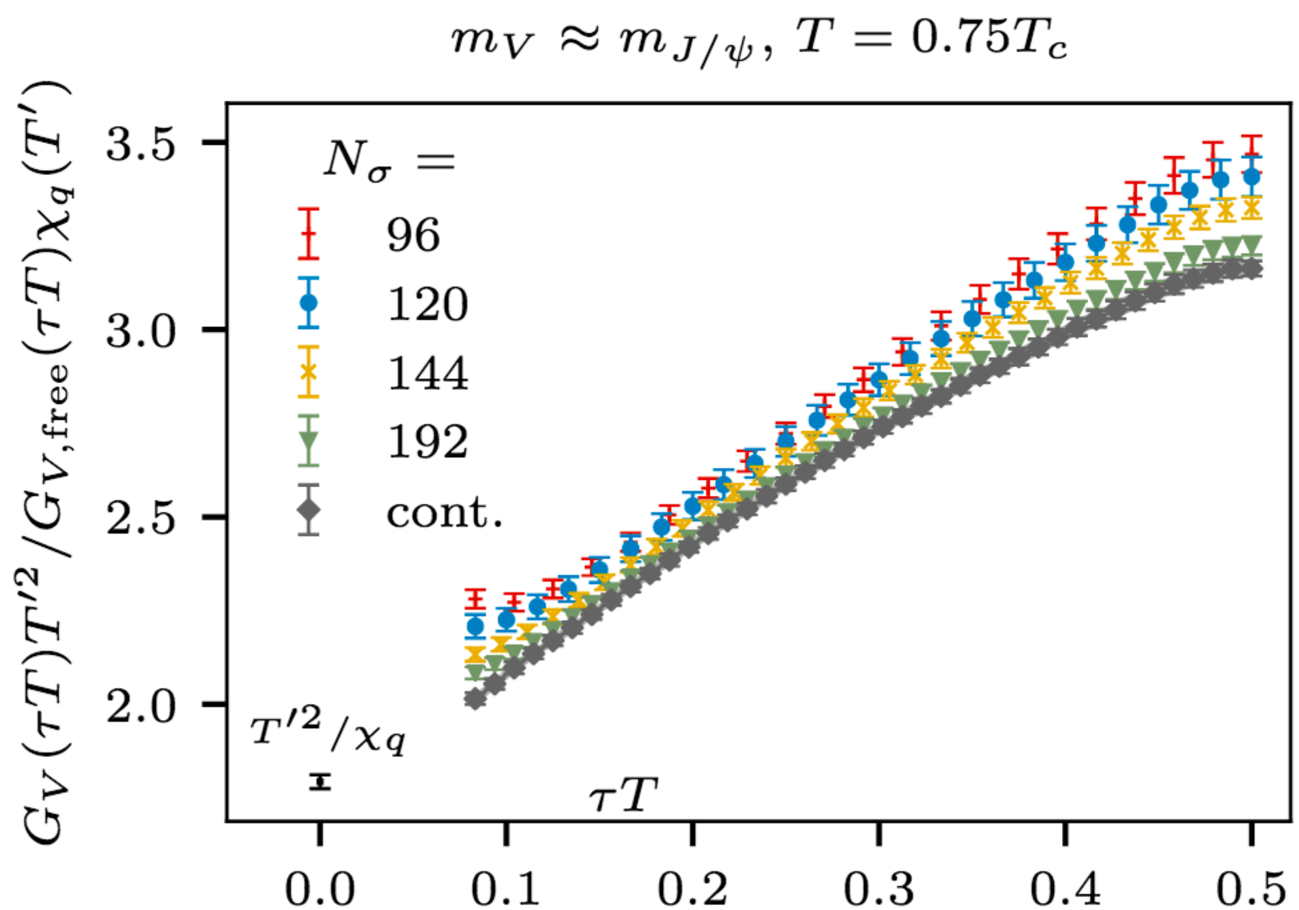
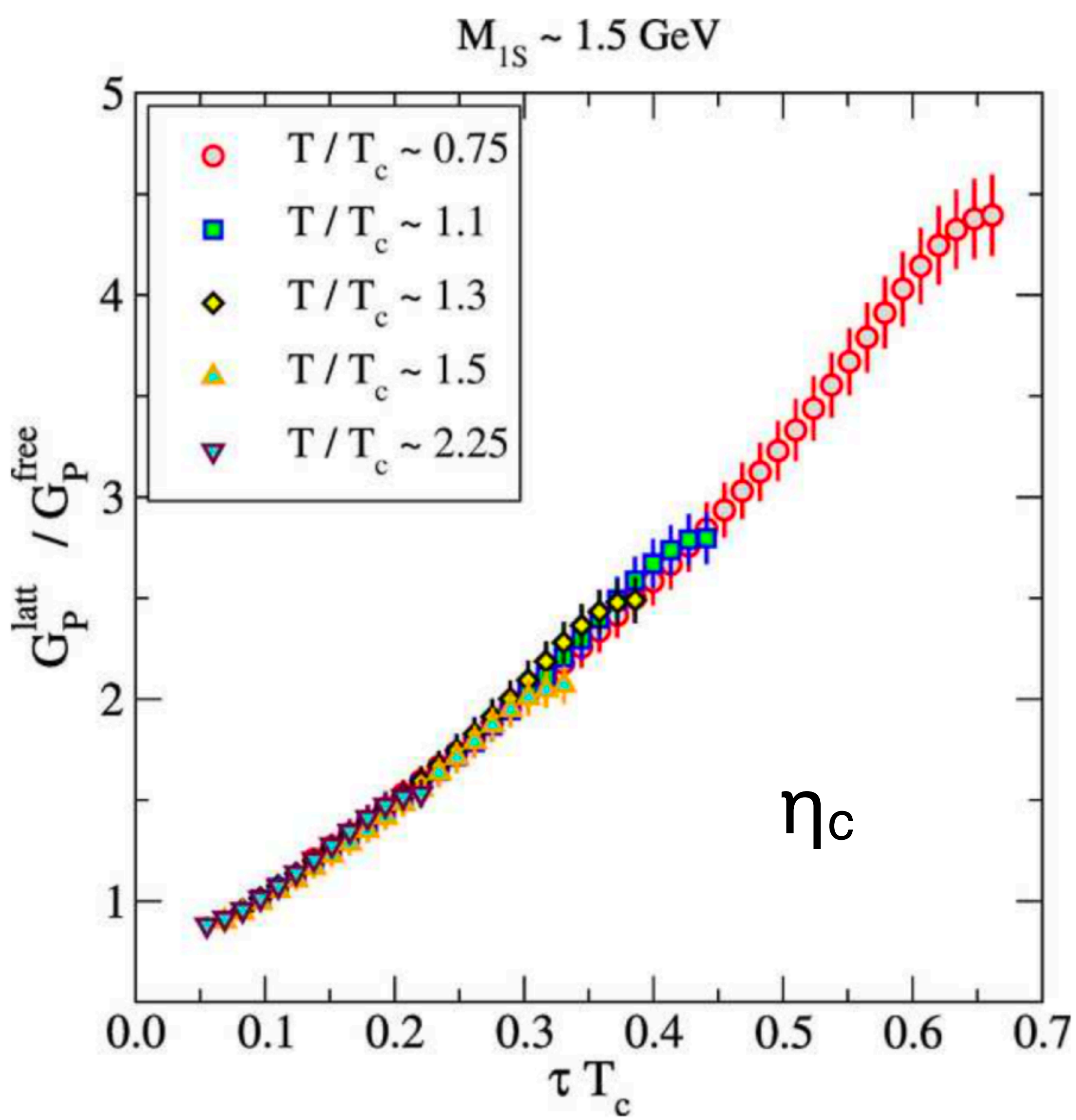
continuum extrapolation of correlation functions

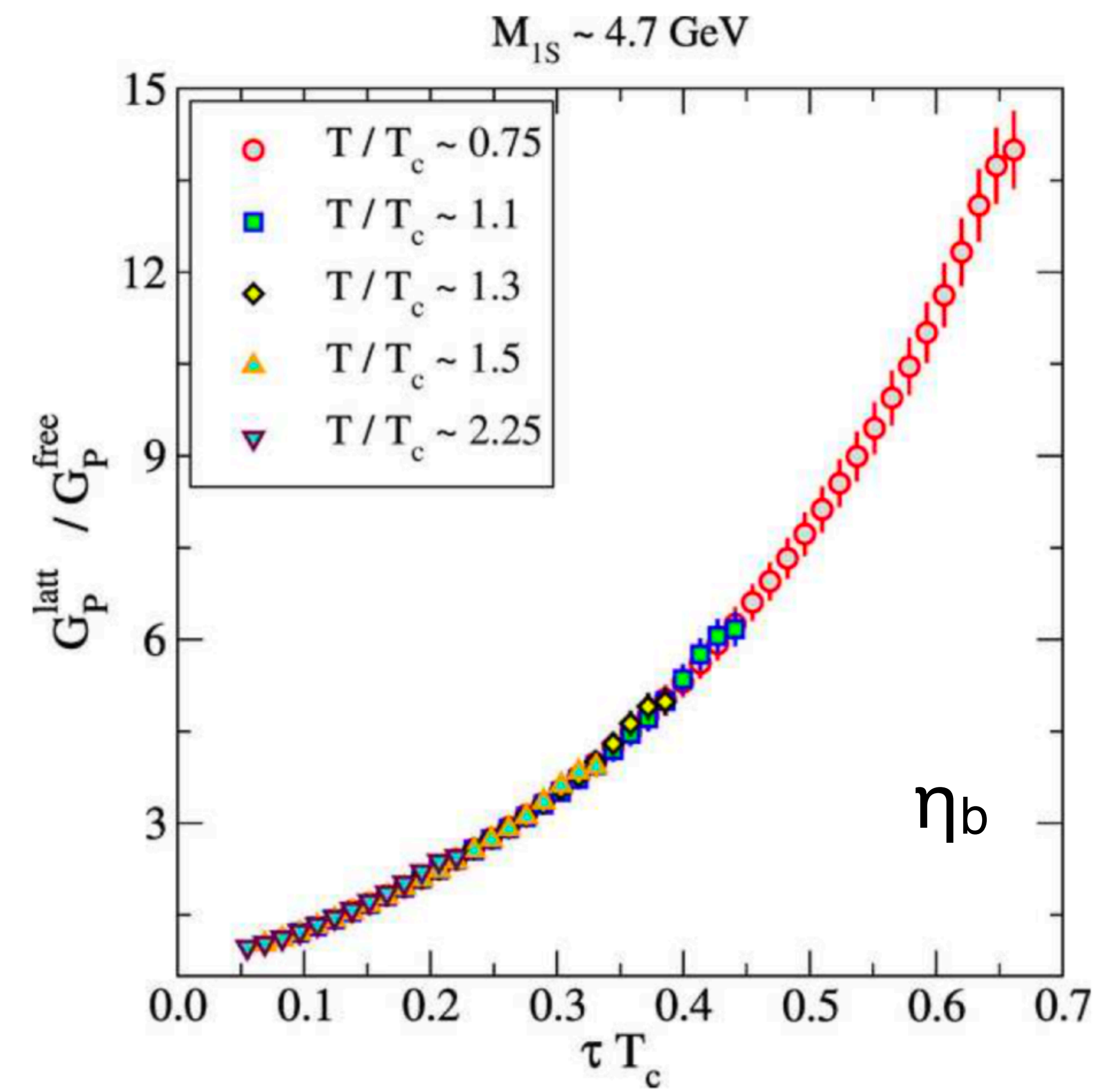
A example at 0.75 Tc in the vector channel





Continuum extrapolated correlators in the pseudo-scalar (PS) channel





No transport contribution is present in the PS channel

Thermal contribution around the threshold

$$\rho_{\rm V}^{\rm NRQCD}(\omega) = \frac{1}{2}\left(1 - e^{-\frac{\omega}{T}}\right) \int_{-\infty}^{\infty} {\rm d}t\, e^{i\omega t}\; C_{>}(t;\mathbf{0},\mathbf{0})$$

Wightman function $C_>$: $$\left\{ i\partial_t - \left[2M + V_T(r) - \frac{\nabla_{\mathbf{r}}^2}{M} \right] \right\} C_{>}^{V}(t;\mathbf{r},\mathbf{r}') = 0\,, \quad t \neq 0$$

For t>0: $$V_T(r) = -\alpha_{\rm s} C_{\rm F}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r} \right] - i\alpha_{\rm s} C_{\rm F} T\, \phi(m_{\rm D} r) + \mathcal{O}(\alpha_{\rm s}^2)$$

Beraudo, **J.-P. Blaizot**, C. Ratti, NPA806(2008)312, Laine et al., JHEP 03(2007)054, Brambilla et al., PRD78(2008)01407

ϕ: real 2->2 scattering of the quark and antiquark off medium particles

Vacuum contribution above the threshold

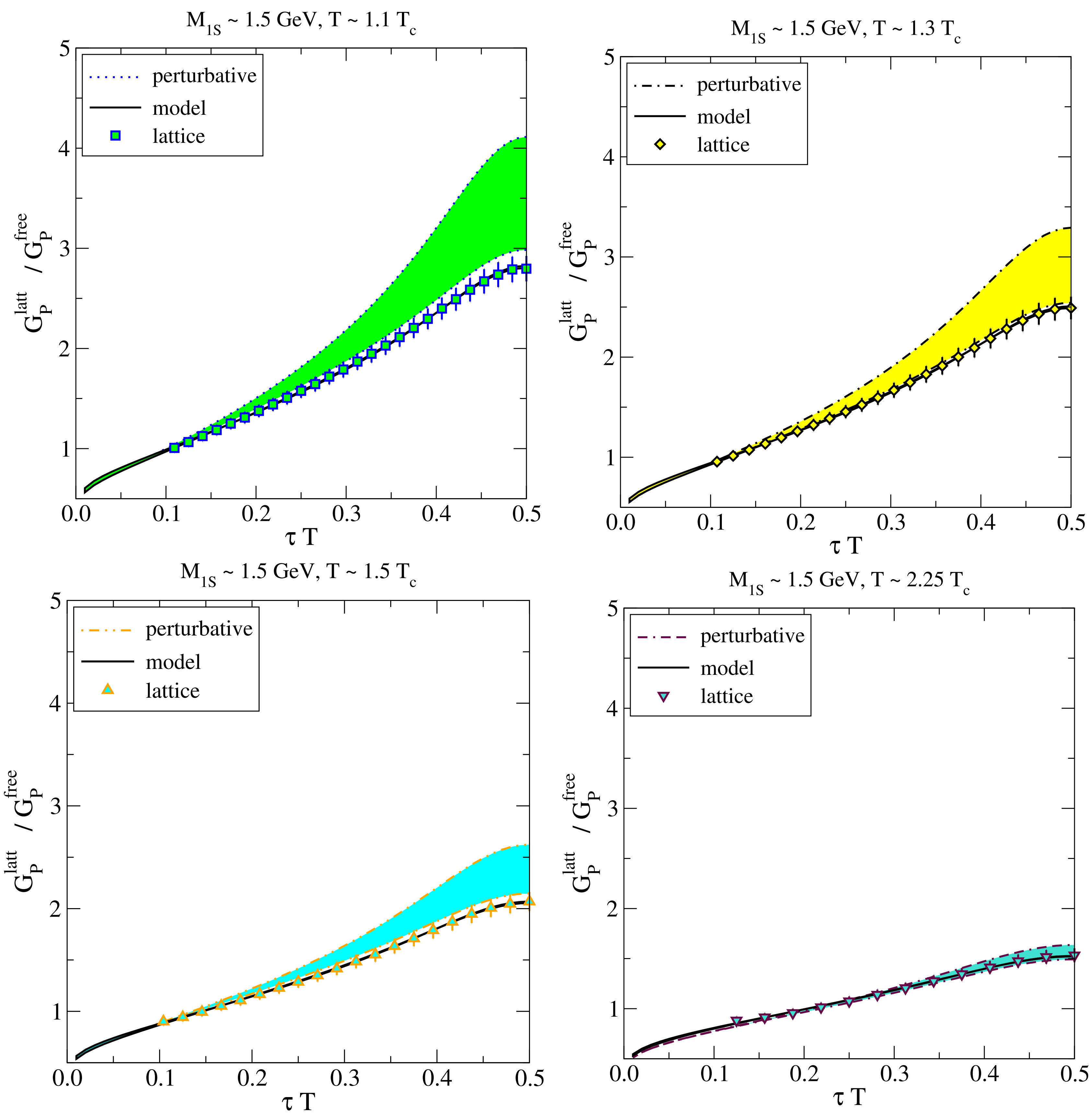
$$\left.\frac{\rho_V(\omega)}{\omega^2}\right|^{\rm vac} = \frac{1}{8\pi}\, R_c^p(\omega)$$

Maier and Marquard, NPB859(2012)42

Harlander and Steinhauser, PRD56(1997)3980

$$\begin{aligned} R_{\rm c}^p(\omega) &= R^{p(0)}(\omega) + \frac{\alpha_{\rm s}(\bar\mu)}{\pi}\, C_{\rm F} R^{p(1)}(\omega) \\ &\quad + \left(\frac{\alpha_{\rm s}(\bar\mu)}{\pi}\right)^2 \left[C_{\rm F}^2\, R_A^{p(2)}(\omega) + C_{\rm F} N_{\rm c}\, R_{NA}^{p(2)}(\omega) + C_{\rm F} T_{\rm F} N_{\rm f}\, R_l^{p(2)}(\omega) \right] + \mathcal{O}(\alpha_{\rm s}^3) \end{aligned}$$

Perturbative QCD v.s. continuum extrapolated Lattice data: for temporal correlators in the PS channel



ρ_P^{pert} : vacuum spf + thermal modification close to the threshold using pNRQCD

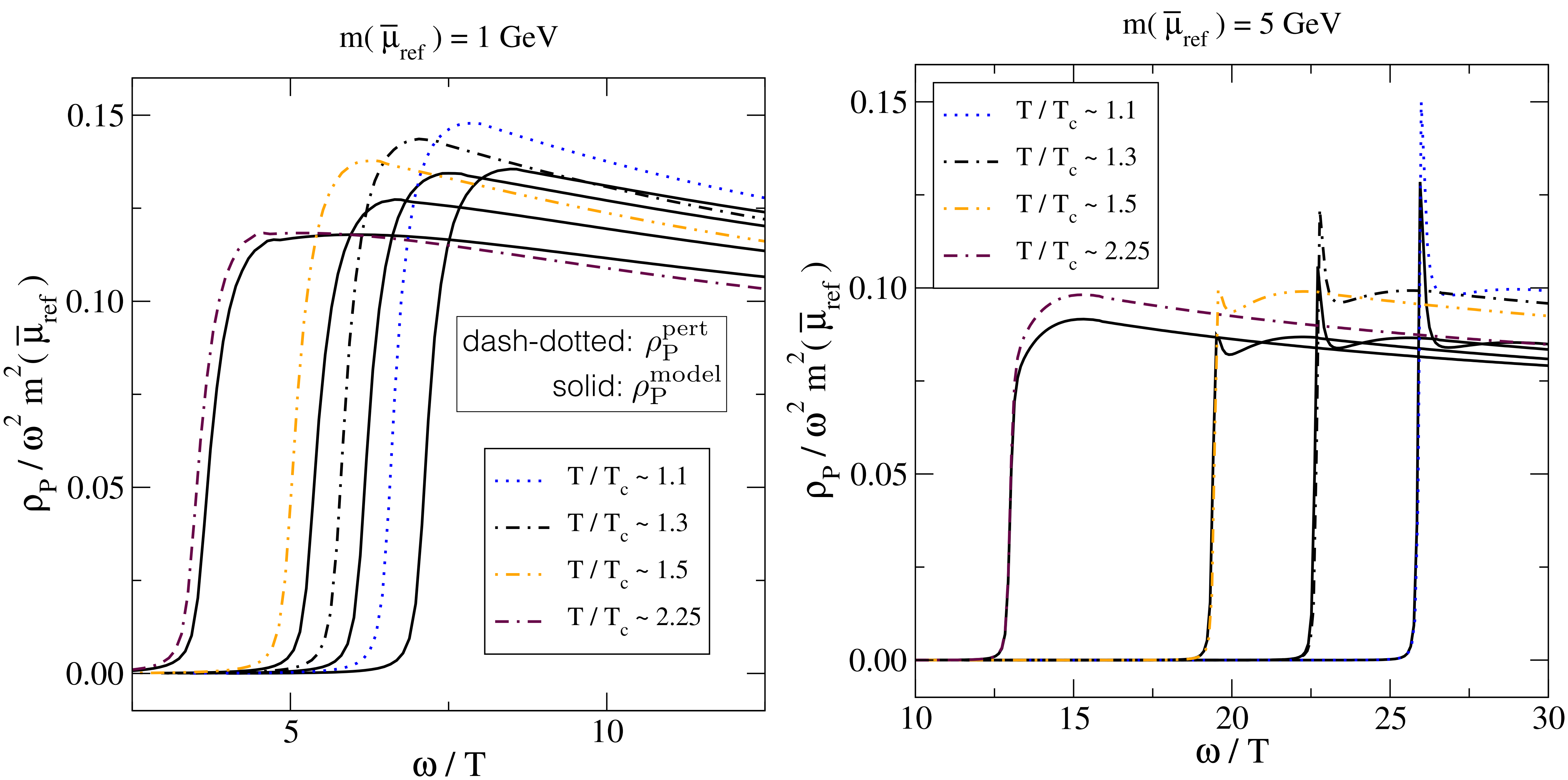
pNRQCD
large r: a leading order complex potential is used
Small r: V replaced by the 2-loop vacuum potential

$$\rho_P^{model} \equiv A\ \rho_P^{pert}(\omega - B)$$

Pretty good fits with the model spf

Y. Burnier, HTD, O. Kaczmarek et al., JHEP 1711 (2017) 206, arXiv:1709.07612

Corresponding spectral functions (η_c & η_b)



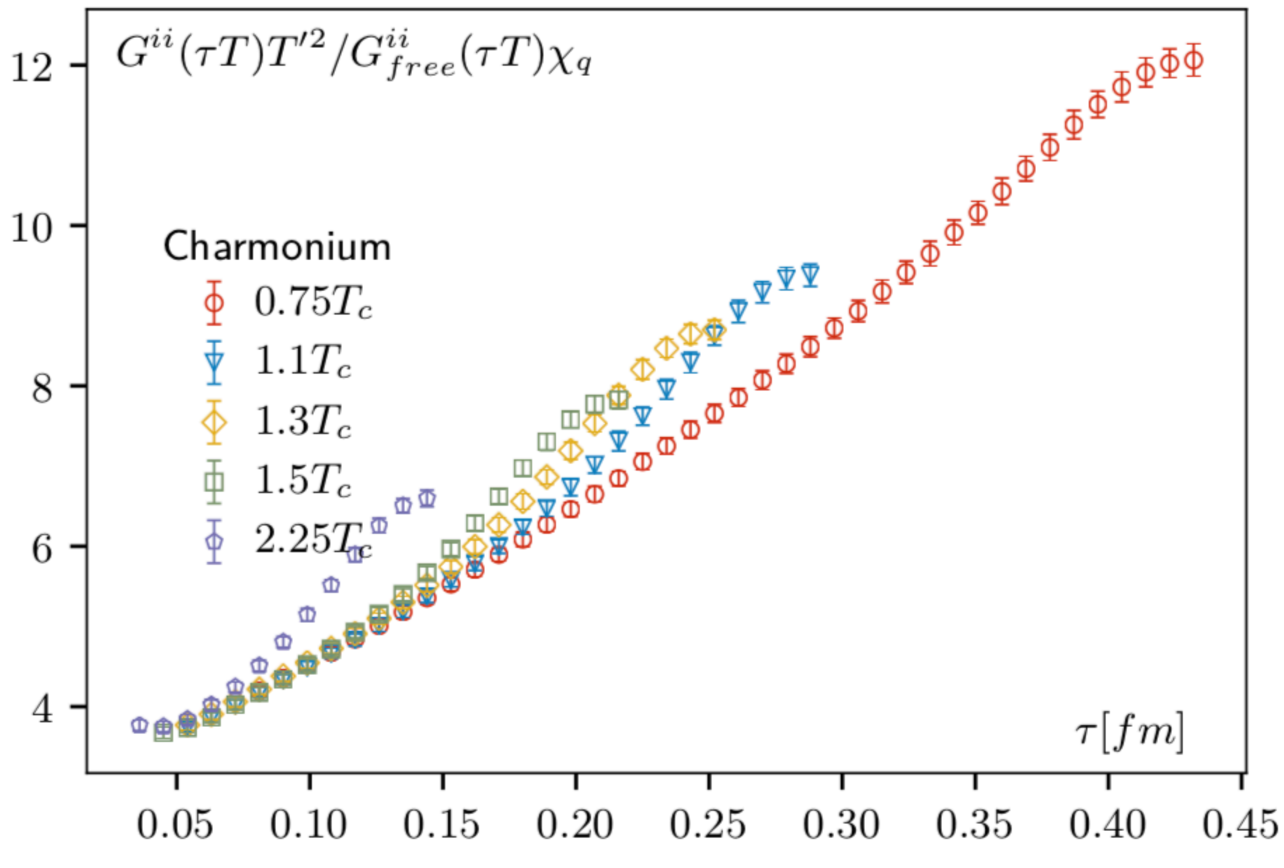
Charmonium: threshold shifts to larger energy, no resonance peaks are needed to fit the lattice data

Bottomonium: a thermally broaden peak maybe present at T≲1.5T_c

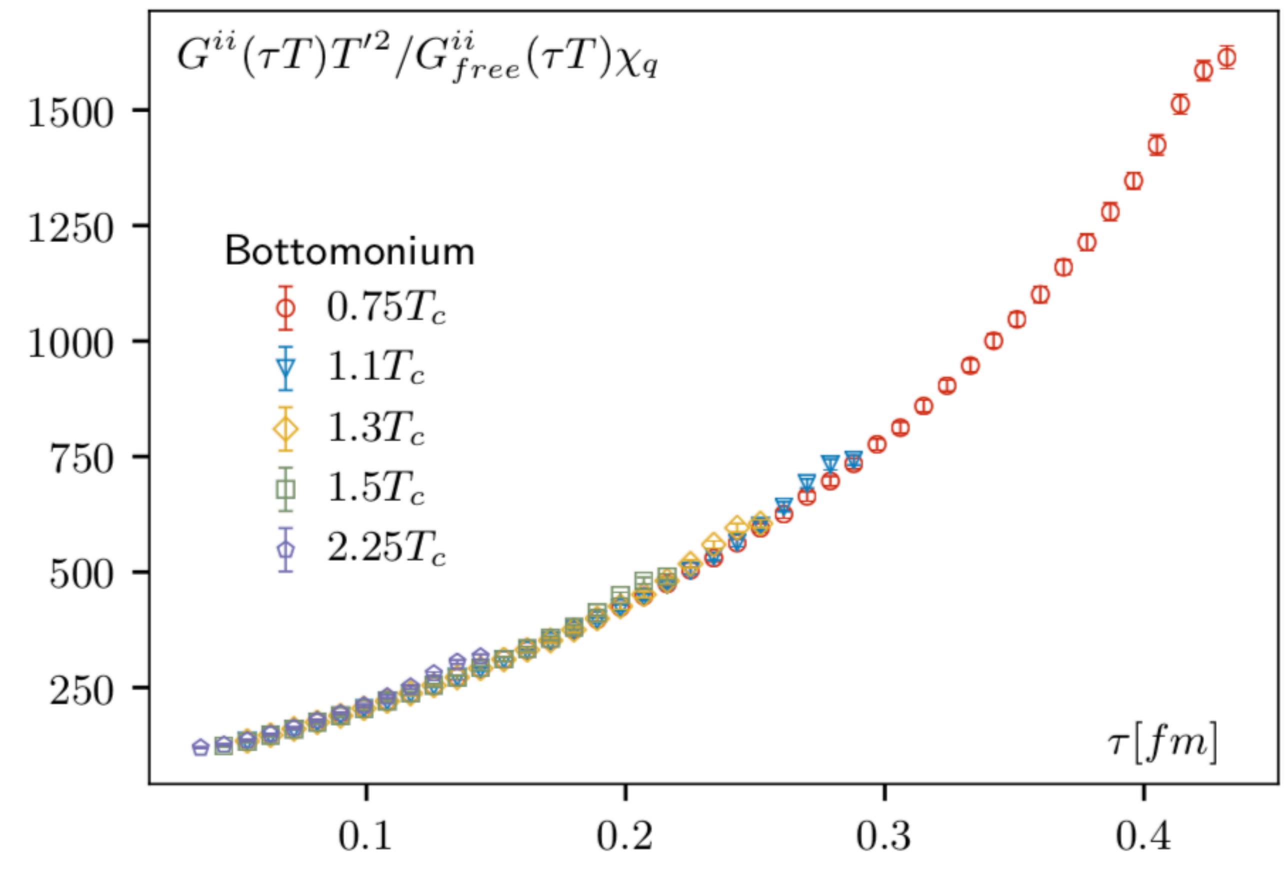
Burnier, HTD, Kaczmarek, Kruse, Laine, Ohno, Sandmeyer, JHEP 1711 (2017) 206

Charmonium and bottomonium correlation functions in the vector channel

J/psi



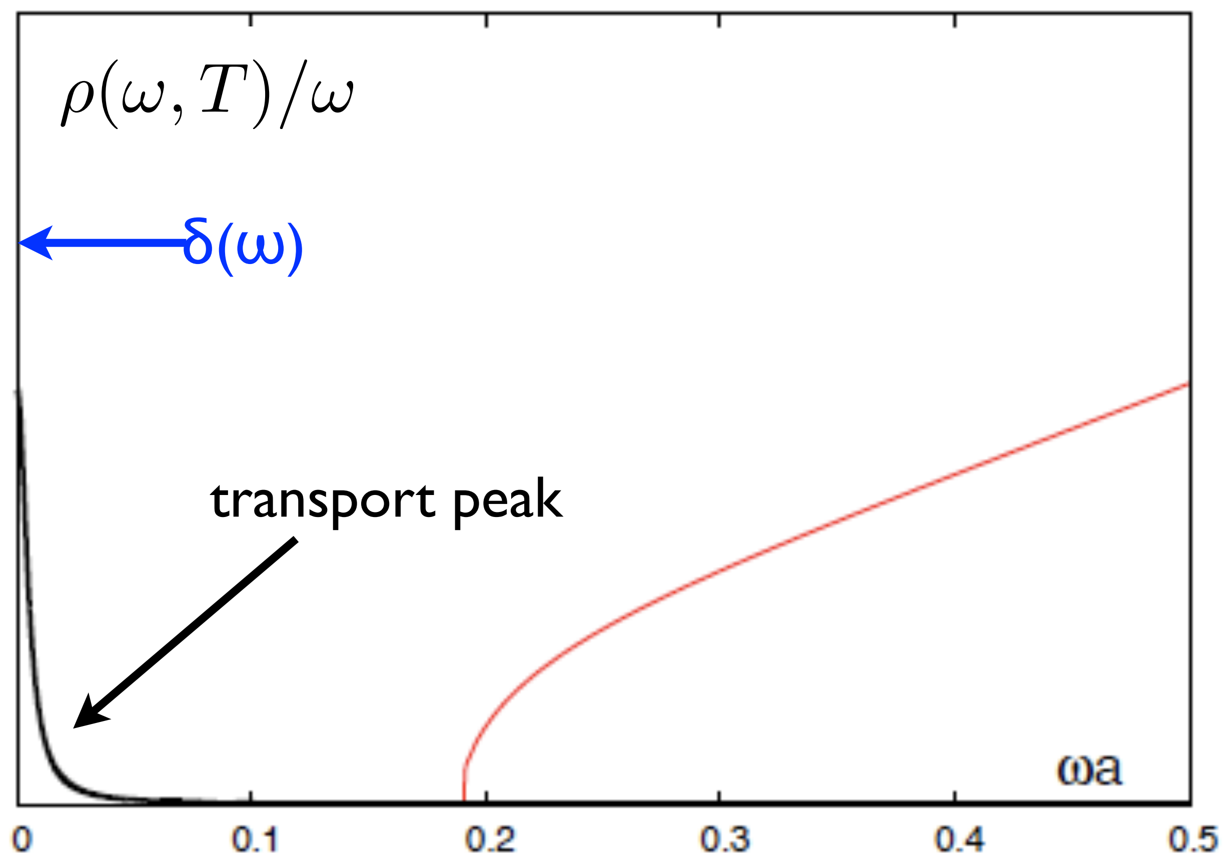
Upsilon



Upsilon is less affected by thermal effects

Additional transport peak present in the Vector channel!

Charmonium and bottomonium correlation functions in the vector channel

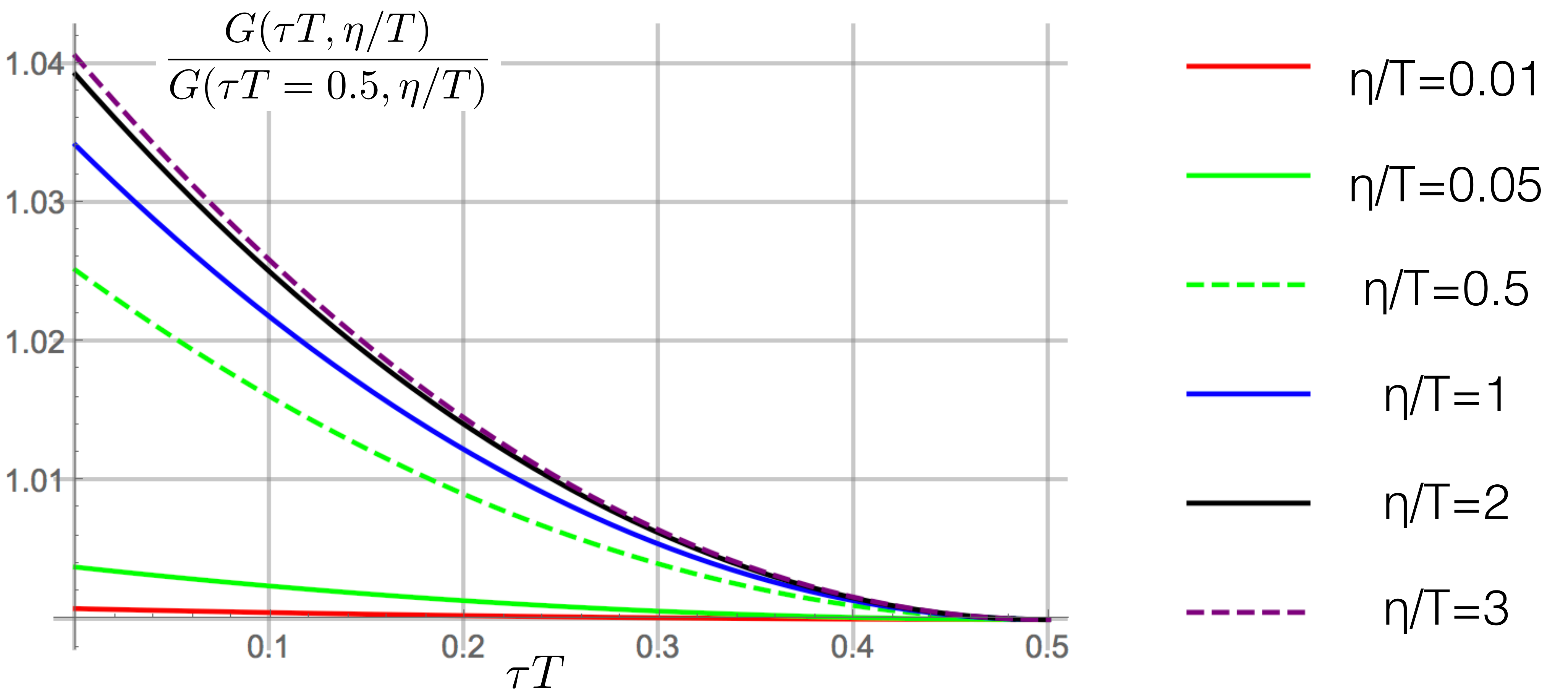


Mixing of resonance peak and transport peak make the extraction of SPF from correlator very hard!

Additional transport peak present in the Vector channel!

Transport contribution to the correlators

model transport peak: $$\rho(\omega) = \chi_q \, \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2} \, , \quad \eta = \frac{T}{MD}$$

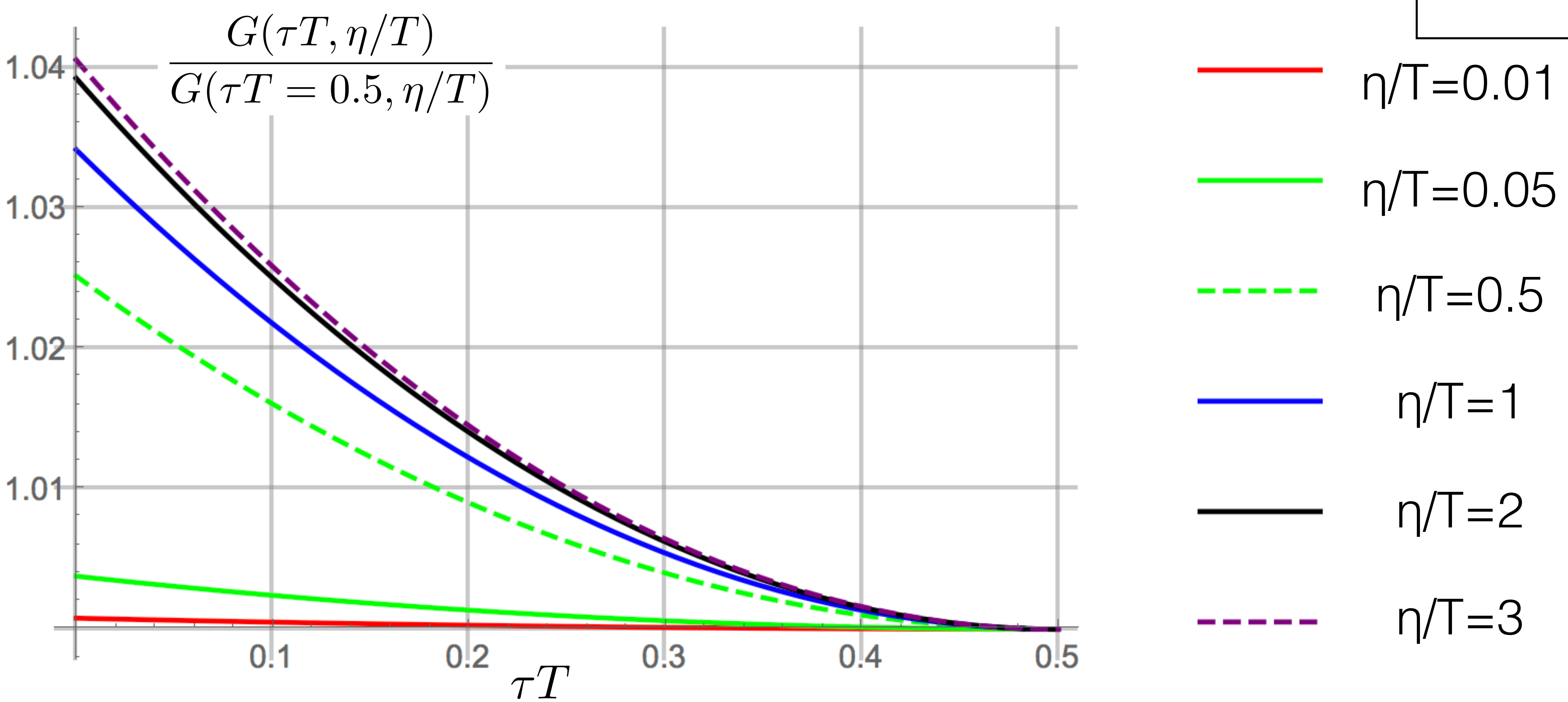


The contribution from transport peak to the correlator is approximately τ-independent constant

Transport contribution to the correlators

model transport peak: $$\rho(\omega) = \chi_q \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}\,, \quad \eta = \frac{T}{MD}$$

At 1.5 T_c, M_c=1.5 GeV
M_c/T=3



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Transport contribution to the correlators

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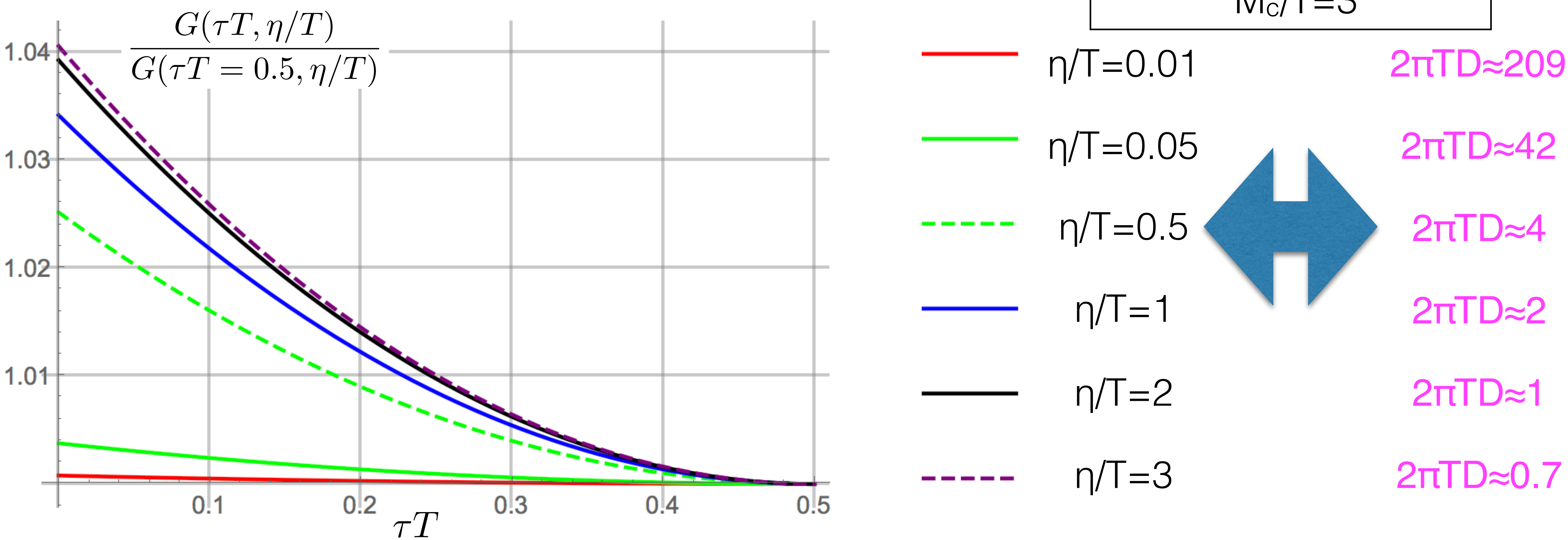
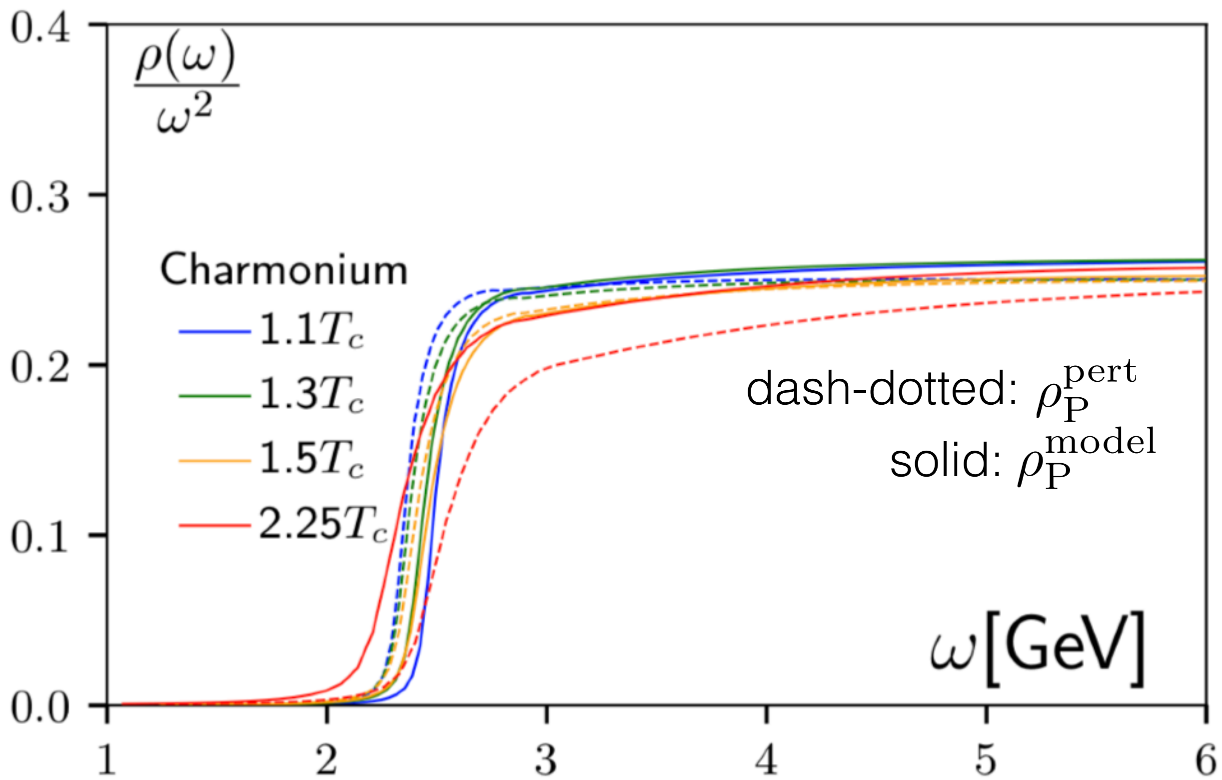
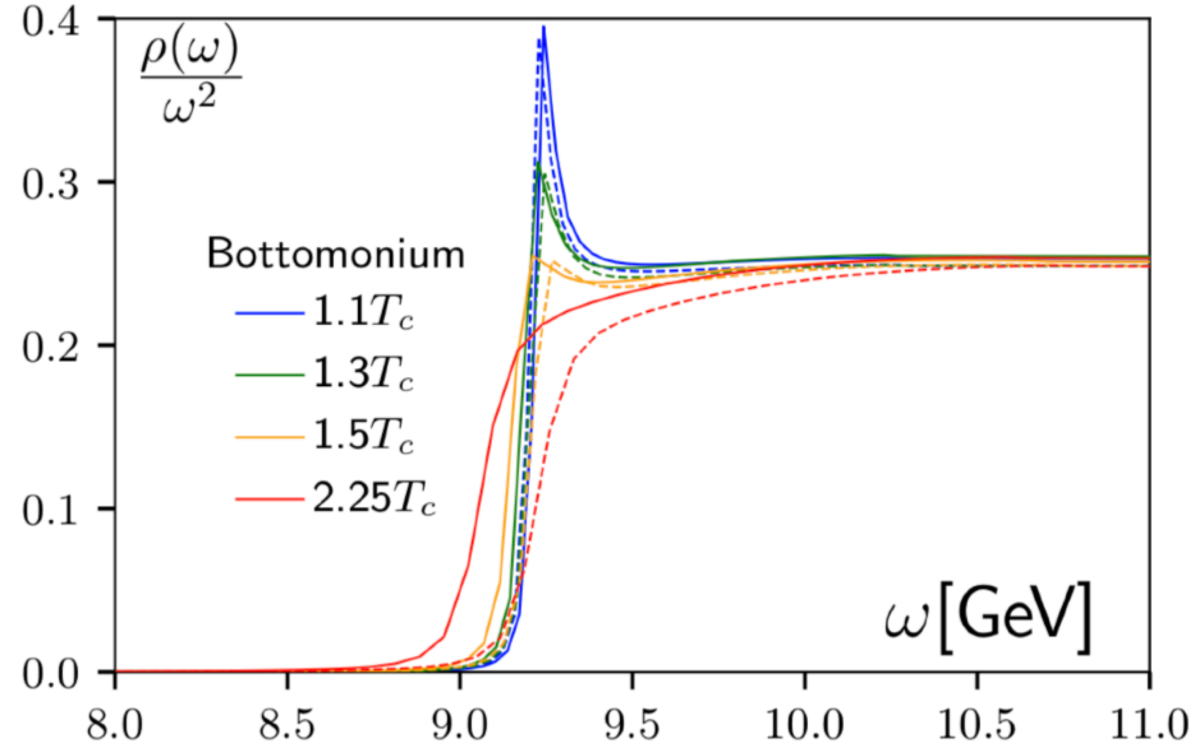


The contribution from transport peak to the correlator is approximately τ-independent constant

Fits to the difference of neighboring vector correlators

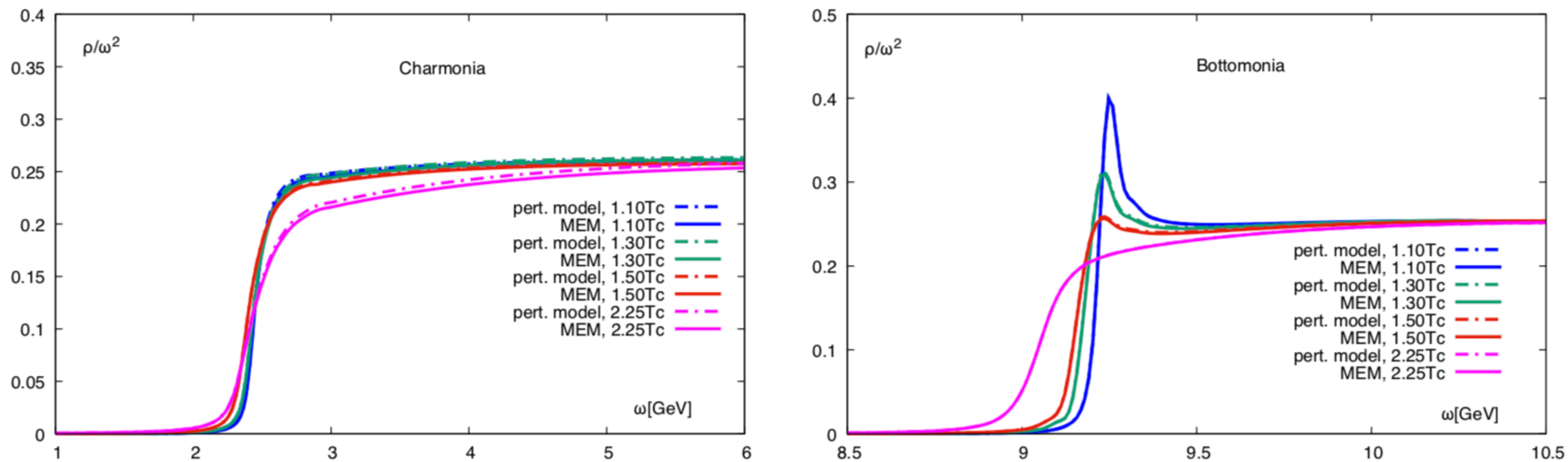
Transport contribution is suppressed in difference of neighboring corolators: $$\frac{\mathrm{d}G(\tau)}{\mathrm{d}\tau} \approx G(\tau) - G(\tau+1)$$





- At T>=1.1T_c : No resonance peaks of J/Ψ is needed
- At T>=1.5T_c: No resonance peaks of ϒ is needed

Cross check with Maximum Entropy Method



Consistent results from Maximum Entropy Method

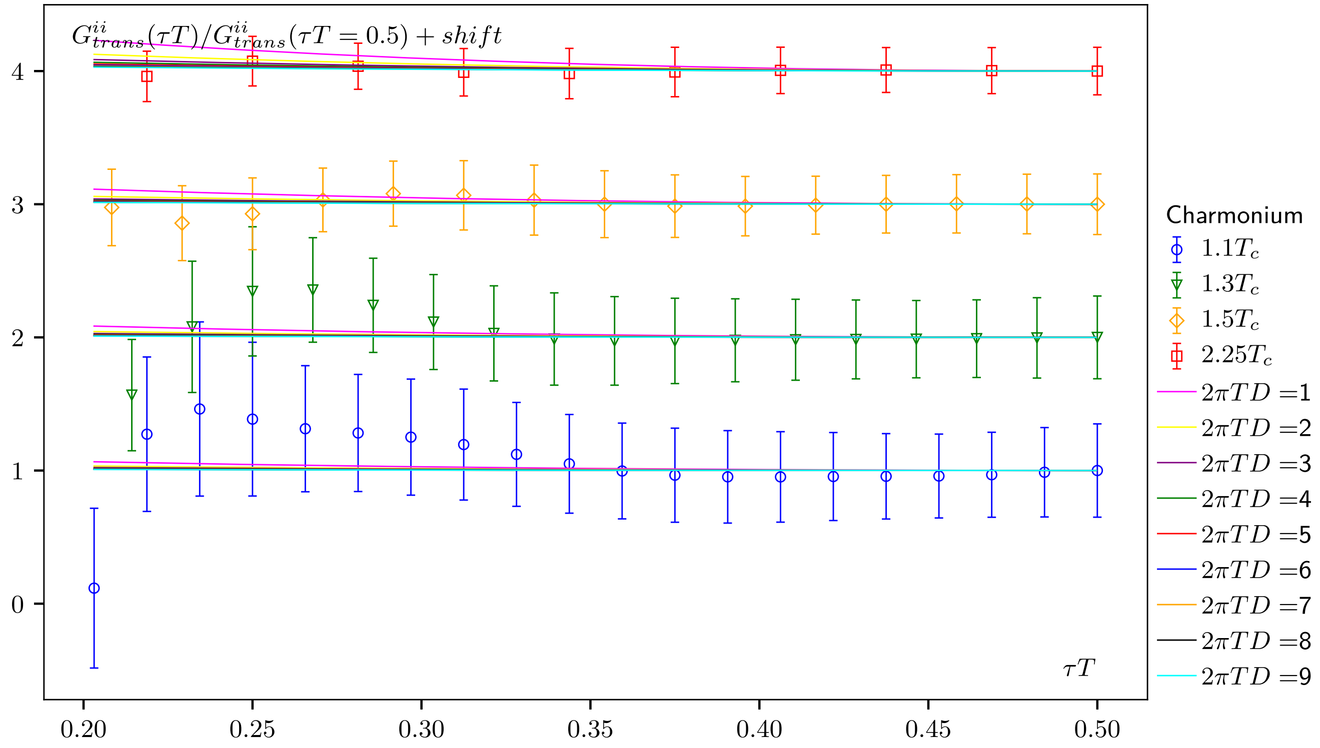
Contribution to correlators from transport peak

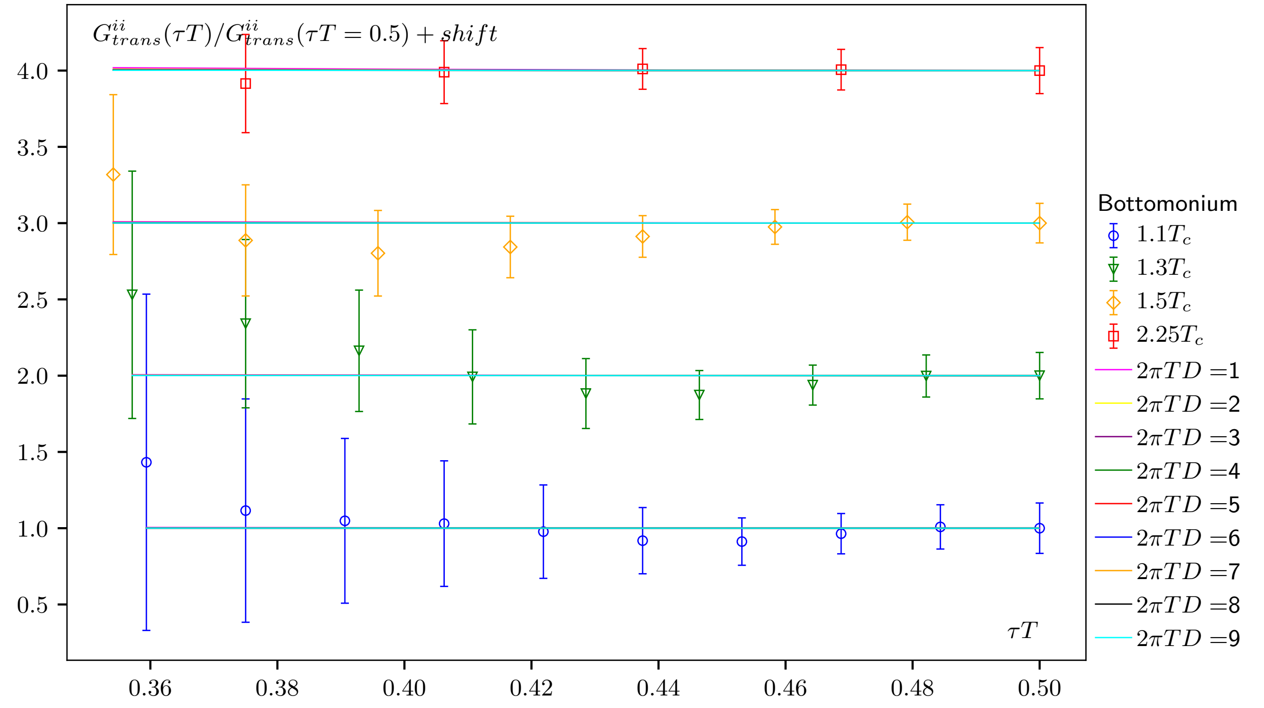
$$G_{trans}(\tau T) = G(\tau T) - G_{fit}(\tau T)$$

contribution from transport peak

original correlator

obtained from SPF without transport peak: Fit to G(τ) -G(τ+1)





It is expected that $G_{trans}(\tau T)$ is almost τ independent

Transport contribution to the correlator at the largest distance

$$G_{trans}(\tau T = 0.5) = \int_0^{\omega_{cut}} \frac{1}{\pi} \frac{1}{\sinh(\omega/2T)} \chi_q \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}$$

Integrand kernel

Lorentzian Ansatz for trans. peak

- Expand the integrand kernel at τT=1/2 about ω=0

$$\frac{\cosh(\omega(1/2T - 1/2T))}{\sinh(\omega/2T)} = \frac{2T}{\omega} - \frac{\omega}{12T} + \frac{7\omega^3}{2880T^3} - \frac{31\omega^5}{483840T^5} + \mathcal{O}(\omega^7)$$

$$\frac{G_{trans}(\tau T = 1/2)}{\chi_q T} = \frac{1}{\pi}\frac{T}{M}\left(f_1 + f_2 + f_3 + f_4 + \mathcal{O}(\omega^7)\right),$$

leading term **f₁** dominates

$$f_1 = 2\tan^{-1}\left(\frac{w_{cut}}{\eta}\right),$$

$$f_2 = \frac{\eta^2}{12T^2}\left(\tan^{-1}\left(\frac{\omega_{cut}}{\eta}\right) - \frac{\omega_{cut}}{\eta}\right),$$

$$f_3 = \frac{7}{8640}\left(\frac{\eta}{T}\right)^4\left(3\tan^{-1}\left(\frac{\omega_{cut}}{\eta}\right) - 3\frac{\omega_{cut}}{\eta} + \left(\frac{\omega_{cut}}{\eta}\right)^3\right),$$

$$f_4 = \frac{31}{7257600}\left(\frac{\eta}{T}\right)^6\left(15\tan^{-1}\left(\frac{\omega_{cut}}{\eta}\right) - 15\frac{\omega_{cut}}{\eta} + 5\left(\frac{\omega_{cut}}{\eta}\right)^3 - 3\left(\frac{\omega_{cut}}{\eta}\right)^5\right)$$

Transport contribution to the correlator at the largest distance

$$G_{trans}(\tau T = 0.5) = \int_0^{\omega_{cut}} \frac{1}{\pi} \frac{1}{\sinh(\omega/2T)} \; \chi_q \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}$$

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Lorentzian Ansatz for trans. peak

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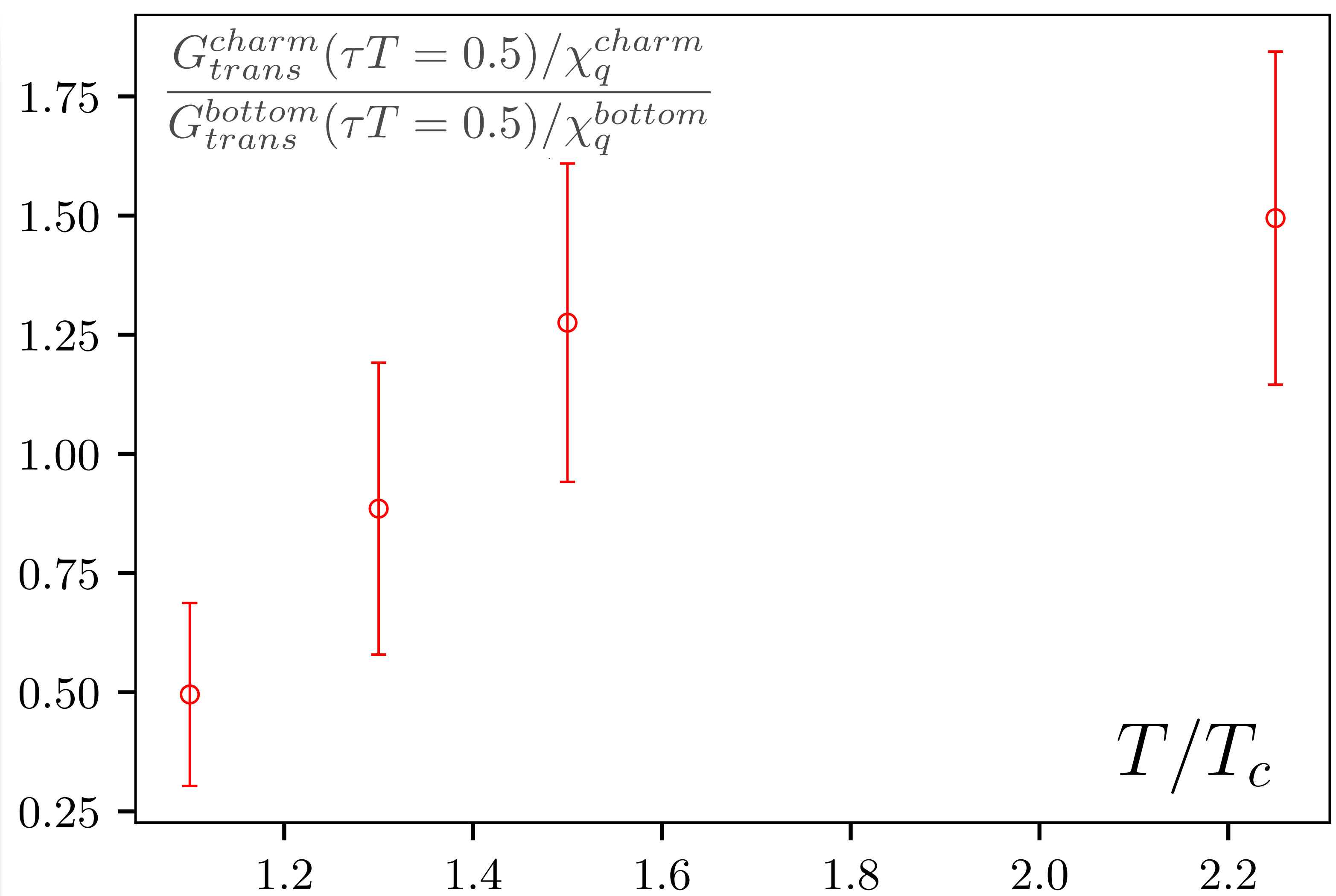
$$\frac{G_{trans}(\tau T = 1/2)}{\chi_q T} = \frac{1}{\pi}\frac{T}{M}\left(f_1 + f_2 + f_3 + f_4 + \mathcal{O}(\omega^7)\right),$$

leading term $\mathbf{f_1}$ dominates

$$\frac{G^{charm}_{trans}/\chi^{charm}_q/T}{G^{bottom}_{trans}/\chi^{bottom}_q/T} \approx \frac{M_{bottom}}{M_{charm}} \frac{\tan^{-1}(T/\eta^{charm})}{\tan^{-1}(T/\eta^{bottom})}$$

Flavor hierarchy of drag coefficients η

$$\frac{G^{charm}_{trans}/\chi^{charm}_q/T}{G^{bottom}_{trans}/\chi^{bottom}_q/T} \approx \frac{M_{bottom}}{M_{charm}} \frac{\tan^{-1}(T/\eta^{charm})}{\tan^{-1}(T/\eta^{bottom})}.$$



$M_{bottom}/M_{charm} \approx 3$

$$\frac{\tan^{-1}(T/\eta^{charm})}{\tan^{-1}(T/\eta^{bottom})} < 1$$

$$\eta^{charm} > \eta^{bottom}$$

Summary & Outlook

1st continuum extrapolation of quarkonia correlators are obtained in quenched QCD

- pNRQCD v.s. Lattice: No resonance peak is needed for η_c J/Ψ at $T>T_c$ while for η_b and ϒ at $T \gtrsim 1.5\, T_c$

- It is suggested that the drag coefficient of charm quark is larger than bottom quark

➡ Further investigations are on the way: quantitative estimate of charm and bottom diffusion coefficient

	charmonium			bottomonium		
T/T_c	A	B/T	χ^2/d.o.f.	A	B/T	χ^2/d.o.f.
1.1	1.04	0.52	0.01	0.85	-0.11	0.02
1.3	1.04	0.37	0.01	0.87	-0.13	0.04
1.5	1.02	0.33	0.02	0.87	-0.11	0.10
2.25	1.06	0.16	0.08	0.93	-0.04	0.28

Table 3. Best fit parameters according to eq. (6.1). The left set corresponds to charmonium, the right to bottomonium. In these fits the errors of the lattice results at different values of τT, which are dominated by systematic uncertainties, have been treated as independent of each other. Therefore the results are somewhat qualitative in nature, and we refrain from citing errors.