Finding oddity in CGC

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~~Finding oddity in CGC~~ Going odd with Larry

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Really odd stuff

I have been lucky to collaborate with Larry on some really odd stuff

- Electroweak Instantons, Axions, and the Cosmological Constant
- The Eccentric Collective BFKL Pomeron
- The Large N Limit with Vanishing Leading Order Condensate for Zero Pion Mass

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...it is easier to understand Larry's ideas than his jokes...

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Today I will be pretty conventional and talk about
Odd Azimuthal Anisotropy of the Glasma for pA Scattering

L. McLerran & V.S., arXiv:1611.09870

Outline

- No introduction – see talks by Raju and Jamal
- CGC was and is odd... but many of us had doubts about it. Why?

  High-energy pA collisions in the color glass condensate approach...
  Jean Paul Blaizot *et al, hep-ph/0402256*

- and why they should not have...

  Non-Abelian Bremsstrahlung and Azimuthal Asymmetries in High Energy
  Miklos Gyulassy *et al, arXiv:1405.7825*

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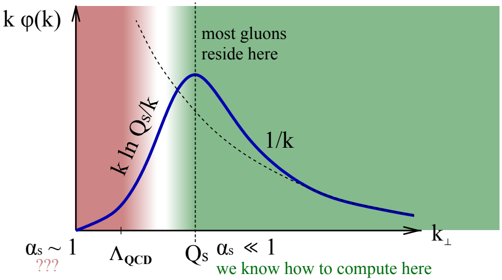
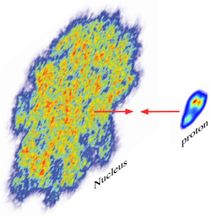
Saturation regime/CGC



- High energy $\rightsquigarrow$ high gluon density
  $\rightsquigarrow$ formation of semi-hard scale, Q_s
- Particle production is dominated by $k_\perp \sim Q_s$
- Weak coupling methods can be applied $\alpha_s(Q_s) \ll 1$
- Still non-perturbative, as fields are strong, $A \sim \frac{1}{g} \rightsquigarrow$ non-linearity is important
- Actual analytical calculations can be rather tricky

What do we know analytically?

Asymmetric collisions, when Q_s of projectile $\neq Q_s$ of target, is the easiest case.



Single inclusive production

- In general

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} f\left(\frac{Q^2_{sp}}{k^2_\perp}, \frac{Q^2_{sA}}{k^2_\perp}\right)$$

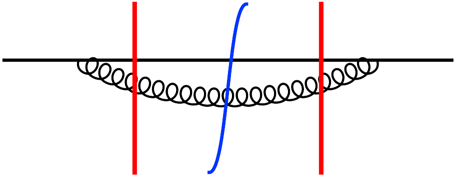
$f\left(\frac{Q^2_{sp}}{k^2_\perp}, \frac{Q^2_{sA}}{k^2_\perp}\right)$ is known only numerically; for large $k_1 \gg Q^2_{sA}$: $\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q^2_{sp}}{k^2_\perp}\frac{Q^2_{sA}}{k^2_\perp} f^{(1,1)}$

A. Krasnitz, R. Venugopalan, arXiv:9809433 *E. Kuraev, L. Lipatov, V. Fadin, '77*

Single inclusive production

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

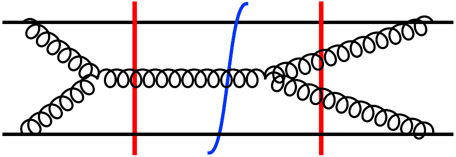
- $f^{(1)}$ is known since '98



⇑ Nuclear shock-wave

Y. V. Kovchegov and A. H. Mueller, arXiv:hep-ph/9802440
A. Dumitru and L. D. McLerran, arXiv:hep-ph/0105268
J.-P. Blaizot, F. Gelis, R. Venugopalan, arXiv:0402256

- $f^{(2)}$: no complete result yet



I. Balitsky, arXiv:hep-ph/0409314
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

Double inclusive production

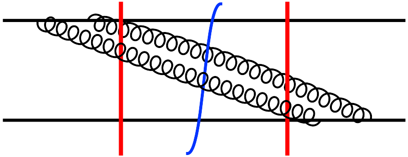
$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \; h^{(1)} \left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6 \; h^{(2)} \left(Q_{sA}\right) + \cdots$$

Momentum dependence is omitted to simplify notation

- Dilute-dilute "Glasma" graph: $\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 Q_{sA}^4 \; h^{(1,1)}$

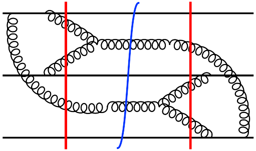
A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858

- $h^{(1)}$ is known since '12 (actually '04) ; invariant under $(k_\perp \to -k_\perp)$



Jean Paul Blaizot et al, hep-ph/0402256
A. Kovner and M. Lublinsky, arXiv:1211.1928
Y. Kovchegov and D. Wertepny, arXiv:1212.1195

- $h^{(2)}$: no complete result yet



L. McLerran and V. S., arXiv:1611.09870
Y. Kovchegov and V. S., arXiv:1802.08166

What do we know analytically?

- From Jean Paul Blaizot et al, hep-ph/0402256 :

$$\frac{d\overline{N}_g}{d^2\boldsymbol{q}_\perp dy} = -\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{k}_{1\perp}}{(2\pi)^2}\frac{d^2\boldsymbol{k}'_{1\perp}}{(2\pi)^2}\frac{C_U(q,\boldsymbol{k}_{1\perp})\cdot C_U(q,\boldsymbol{k}'_{1\perp})}{k_{1\perp}^2 k_{1\perp}'^2}$$
$$\times \left\langle \rho^\dagger_{p,a}(\boldsymbol{k}_{1\perp})\rho_{p,a'}(\boldsymbol{k}'_{1\perp})\right\rangle \left\langle U^\dagger(\boldsymbol{k}_{2\perp})U(\boldsymbol{k}'_{2\perp})\right\rangle_{aa'} .$$

- Double inclusive production:

$$\frac{d^2N}{d^3k d^3p} = \left\langle \frac{dN}{d^3k}\frac{dN}{d^3p}\right\rangle$$

- explicitly even under $\underline{k} \to -\underline{k}$ or $\underline{p} \to -\underline{p}$ and

thus has no odd azimuthal anisotropy

What does presence of odd harmonics mean?

- Double inclusive production

$$\frac{d^2N}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{d^2N}{k_1 dk_1 dy_1 k_2 dk_2 dy_2} \times \left(1 + 2v_2^2\{2\}\cos 2(\phi_1 - \phi_2) + 2v_3^2\{2\}\cos 3(\phi_1 - \phi_2) + \dots\right)$$

- A non-vanishing $v_3^2\{2\}$

$$\int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\Delta\phi\right) = \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\Delta\phi\right) - \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\Delta\phi + \pi\right)$$

$$= \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \left[\frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) - \frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)\right]$$

- Therefore, non-zero $v_3 \rightsquigarrow$

$$\frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) \neq \frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)$$

- Part of the result

$$\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\boldsymbol{B}-\boldsymbol{b}_1)\, T_1(\boldsymbol{B}-\boldsymbol{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2$$
$$\times \left[e^{-i\,\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{y}_2)-i\,\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{y}_1)} + e^{-i\,\boldsymbol{k}_1\cdot(\boldsymbol{x}_1-\boldsymbol{y}_2)+i\,\boldsymbol{k}_2\cdot(\boldsymbol{x}_2-\boldsymbol{y}_1)}\right] \frac{16\,\alpha_s^2}{\pi^2} \frac{C_F}{2N_c} \frac{\boldsymbol{x}_1-\boldsymbol{b}_1}{|\boldsymbol{x}_1-\boldsymbol{b}_1|^2} \cdot \frac{\boldsymbol{y}_2-\boldsymbol{b}_2}{|\boldsymbol{y}_2-\boldsymbol{b}_2|^2} \frac{\boldsymbol{x}_2-\boldsymbol{b}_2}{|\boldsymbol{x}_2-\boldsymbol{b}_2|^2} \cdot \frac{\boldsymbol{y}_1-\boldsymbol{b}_1}{|\boldsymbol{y}_1-\boldsymbol{b}_1|^2}$$
$$\times \Big[Q(\boldsymbol{x}_1,\boldsymbol{y}_1,\boldsymbol{x}_2,\boldsymbol{y}_2) - Q(\boldsymbol{x}_1,\boldsymbol{y}_1,\boldsymbol{x}_2,\boldsymbol{b}_2) - Q(\boldsymbol{x}_1,\boldsymbol{y}_1,\boldsymbol{b}_2,\boldsymbol{y}_2) + S_G(\boldsymbol{x}_1,\boldsymbol{y}_1) - Q(\boldsymbol{x}_1,\boldsymbol{b}_1,\boldsymbol{x}_2,\boldsymbol{y}_2) + Q(\boldsymbol{x}_1,\boldsymbol{b}_1,\boldsymbol{x}_2,\boldsymbol{b}_2)$$
$$+ Q(\boldsymbol{x}_1,\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{y}_2) - S_G(\boldsymbol{x}_1,\boldsymbol{b}_1) - Q(\boldsymbol{b}_1,\boldsymbol{y}_1,\boldsymbol{x}_2,\boldsymbol{y}_2) + Q(\boldsymbol{b}_1,\boldsymbol{y}_1,\boldsymbol{x}_2,\boldsymbol{b}_2) + Q(\boldsymbol{b}_1,\boldsymbol{y}_1,\boldsymbol{b}_2,\boldsymbol{y}_2) - S_G(\boldsymbol{b}_1,\boldsymbol{y}_1) + S_G(\boldsymbol{x}_2,\boldsymbol{y}_2)$$
$$- S_G(\boldsymbol{x}_2,\boldsymbol{b}_2) - S_G(\boldsymbol{b}_2,\boldsymbol{y}_2) + 1\Big].$$

- Why I like this form $\frac{d^2N}{d^3k\,d^3p} = \left\langle \frac{dN}{d^3k}\frac{dN}{d^3p}\right\rangle$

$$\frac{d\overline{N}_g}{d^2\boldsymbol{q}_\perp dy} = -\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{k}_{1\perp}}{(2\pi)^2}\frac{d^2\boldsymbol{k}'_{1\perp}}{(2\pi)^2}\frac{C_U(q,\boldsymbol{k}_{1\perp})\cdot C_U(q,\boldsymbol{k}'_{1\perp})}{k_{1\perp}^2\, k'^2_{1\perp}}$$
$$\times \langle \rho^\dagger_{p,a}(\boldsymbol{k}_{1\perp})\rho_{p,a'}(\boldsymbol{k}'_{1\perp})\rangle \langle U^\dagger(\boldsymbol{k}_{2\perp})U(\boldsymbol{k}'_{2\perp})\rangle_{aa'} \;.$$

$$\frac{d^2 N}{d^3k d^3p} = \left\langle \frac{dN}{d^3k} \frac{dN}{d^3p} \right\rangle$$

$$\frac{dN(\underline{k})}{d^2k dy}\left[\rho_p, \rho_t\right] = \frac{2}{(2\pi)^3} \frac{\delta_{ij}\delta_{lm} + \epsilon_{ij}\epsilon_{lm}}{k^2} \Omega^a_{ij}(\underline{k})[\Omega^a_{lm}(\underline{k})]^\star$$

with

$$\Omega^a_{ij}(\mathbf{x}_\perp) = g\left[\frac{\partial_i}{\partial^2}\rho^b(\mathbf{x}_\perp)\right]\partial_j U^{ab}(\mathbf{x}_\perp)$$

Instead of 8 integrals with oscillating integrand – one Fast Fourier Transform

A conundrum for saturation

Can saturation dynamics account for non-zero odd azimuthal harmonics?

A conundrum for saturation

Dense-dense calculations: non-zero v_3 (Bjorn, Raju, Soeren, . . .)

We note that numerical results of [. . .] do not seem to display the exact symmetry $\underline{k} \to -\underline{k}$, which may be an indication of some subtlety of the numerical procedure of [. . .].

A. Kovner and M. Lublinsky, arXiv:1012.3398

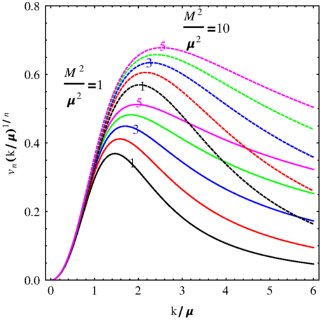
Matt and Yuri: Odd contribution is buried somewhere in multiple rescattering i.e. in high order $h^{(N \gg 1)}$ ⇓

$$\frac{d^2 N}{d^3 k d^3 p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \; h^{(1)}\left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6 \; h^{(2)}\left(Q_{sA}\right) + \cdots$$

This is when Larry got excited about

Non-Abelian bremsstrahlung and azimuthal asymmetries in high energy $p + A$ reactions

M. Gyulassy, P. Levai, I. Vitev, and T. S. Biró
Phys. Rev. D **90**, 054025 – Published 25 September 2014



Larry in 2015: “We need to understand this”

Inspiration from Single Transverse Spin Asymmetry

or why we should have expected CGC to be odd...

- Consider single gluon production

$$\frac{d\sigma}{d^2k} \sim |M(\underline{k})|^2 = \int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, M(\underline{x})\, M^*(\underline{y})$$

- Amplitude may have two contributions

$$M(\underline{x}) = M_1(\underline{x}) + M_3(\underline{x}) + \dots$$

- Asymmetry under $\underline{k} \to -\underline{k}$ would mean that

$$M_1(\underline{x})\, {M_3}^*(\underline{y}) + M_3(\underline{x})\, {M_1}^*(\underline{y}) = -M_1(\underline{y})\, {M_3}^*(\underline{x}) - M_3(\underline{y})\, {M_1}^*(\underline{x})$$

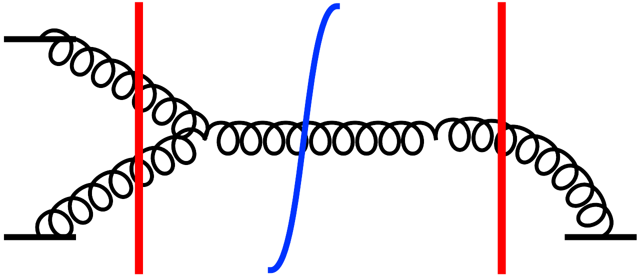
$\rightsquigarrow$ $M_1(\underline{x})\, {M_3}^*(\underline{y})$ is imaginary

$\rightsquigarrow$ Phase difference between M_1 and M_3 in coordinate space

In coordinate space, but not dissimilar from STSA

S. Brodsky, D. S. Hwang, Y. Kovchegov, I. Schmidt, M. Sievert, arXiv:1304.5237

Natural candidate

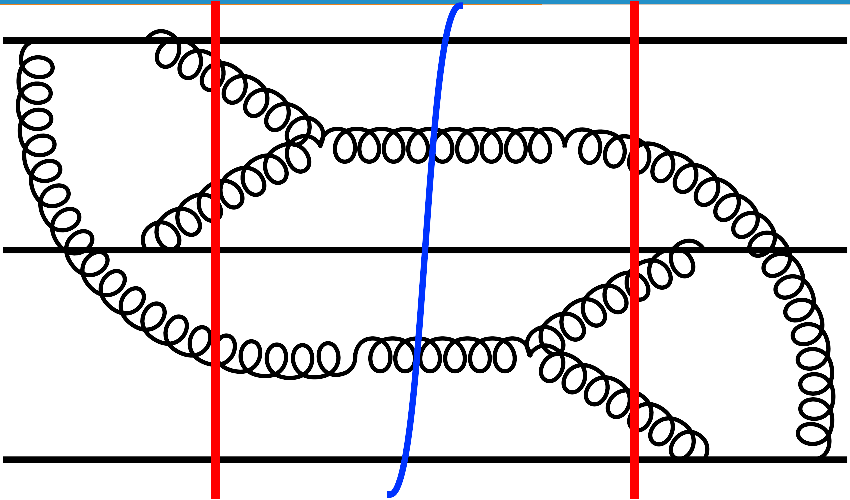


M_3 M_1

- Vanishes for single-inclusive production after performing average with respect to projectile configurations. . .

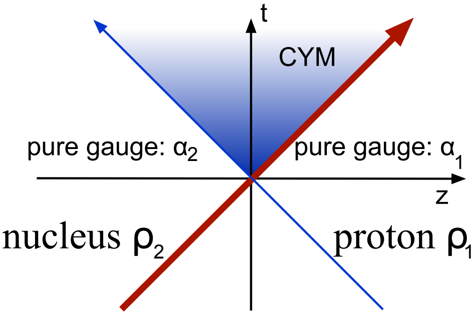
Unless you have an odderon (talk by Yoshitaka)

Double inclusive gluon production



- Non-zero!

Classical Yang-Mills



- Just after collision, $\tau \to 0+$, initial conditions are known

  (Fock-Schwinger gauge $A_\tau = 0$)

  A. Kovner, L. McLerran, H. Weigert, arXiv:9506320

- In forward light-cone $[D_\mu, F^{\mu\nu}] = 0$
- Solve equations perturbatively in ρ_1; then use LSZ

Gluon production

- Leading order and the first saturation correction

$$\frac{dN^{\rm even}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}{\rm Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$

$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

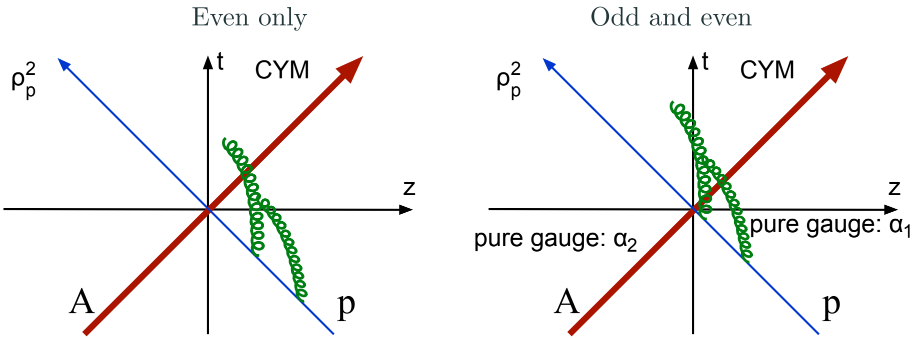
Here $\delta_{ij}\Omega_{ij}=\Omega_{xx}+\Omega_{yy}$ and $\epsilon_{ij}\Omega_{ij}=\Omega_{xy}-\Omega_{yx}$ and

$$\underbrace{\Omega^a_{ij}(\mathbf{x}_\perp) = g\left[\frac{\partial_i}{\partial^2}\overbrace{\rho^b(\mathbf{x}_\perp)}^{\text{val. sour.}}\right]\partial_j\overbrace{U^{ab}(\mathbf{x}_\perp)}^{\text{target W line}}}_{\text{valence sources rotated by the target}}$$

$\frac{dN^{\rm odd}(\underline{k})}{d^2kdy}\left[\rho_p,\rho_T\right]$ is suppressed by extra $\alpha_s\rho_p$

L. McLerran and V. S., arXiv:1611.09870

Beyond classical approximation: a short detour



- In this particular gauge:

  in classical approximation, v_3 requires some degree of final state interaction

- Do we have odd azimuthal component of two parton correlation function

  in hadron wave function?!

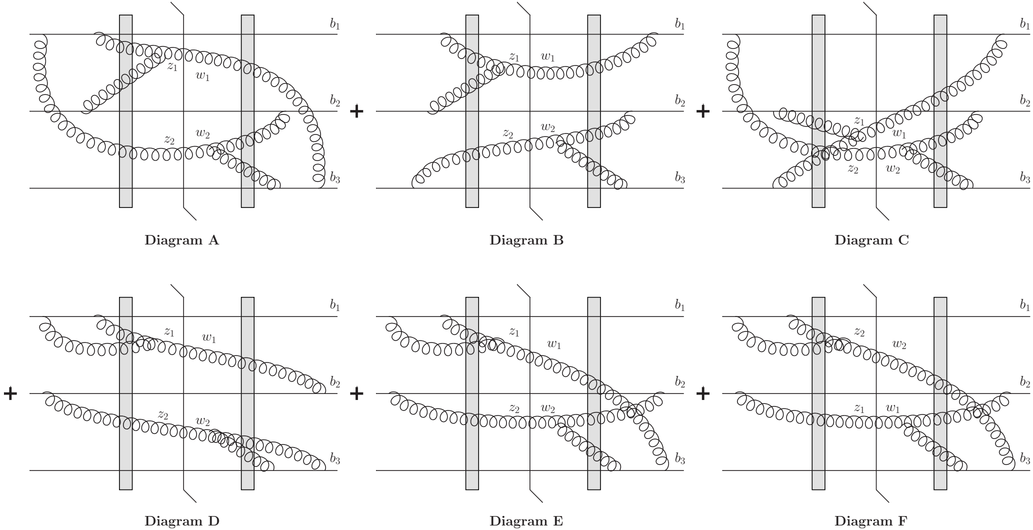
Alternative approach

- This was obtained in Fock-Schwinger gauge $A_\tau = 0$;

  the gauge is singular; defined in coordinate space.

- Motivation to compute in gauge $A^+ = 0$

Yu. Kovchegov and V. S., arXiv:1802.08166



- Reproduces result obtained in Fock-Schwinger gauge!

M3

$$
\begin{aligned}
= & -\frac{g^3}{4\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
& \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] \\
& \times f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{1\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
& + \frac{i\, g^3}{4\pi^3} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[U^{bd}_{\vec{b}_{1\perp}} \left(U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right) \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right. \\
& \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \\
& - \left(U^{bd}_{\vec{x}_\perp} - U^{bd}_{\vec{b}_{1\perp}} \right) U^{ce}_{\vec{b}_{2\perp}} \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right. \\
& \left. \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \right] \\
& - \frac{i\, g^3}{4\pi^2} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
& \times \left[(U^{bd}_{\vec{z}_\perp} - U^{bd}_{\vec{b}_{1\perp}})\, U^{ce}_{\vec{b}_{2\perp}} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\,\Lambda} - U^{bd}_{\vec{b}_{1\perp}} (U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}}) \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}|\,\Lambda} \right] \\
& - \frac{i\, g^3}{4\pi^3} \int d^2x \left[U^{ab}_{x_\perp} - U^{ab}_{z_\perp} \right] f^{bde} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
& \times \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \, \mathrm{Sign}(b_2^- - b_1^-).
\end{aligned}
$$

- In Golec-Biernat–Wusthoff model & Large N_c & at high momentum:

$$\frac{d\sigma_{odd}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b\, [T_1(\underline{B} - \underline{b})]^3\, g^8\, Q_{s0}^6(b)\, \frac{1}{\underline{k}_1^6\, \underline{k}_2^6}$$

$$\times \left\{ \underbrace{\left[\frac{(\underline{k}_1^2 + \underline{k}_2^2 + \underline{k}_1 \cdot \underline{k}_2)^2}{(\underline{k}_1 + \underline{k}_2)^6} - \frac{(\underline{k}_1^2 + \underline{k}_2^2 - \underline{k}_1 \cdot \underline{k}_2)^2}{(\underline{k}_1 - \underline{k}_2)^6} \right]}_{A} + \underbrace{\frac{10\, c^2}{(2\pi)^2} \frac{1}{\Lambda^2} \frac{\underline{k}_1 \cdot \underline{k}_2}{k_1\, k_2}}_{B} \right.$$

$$\left. + \underbrace{\frac{1}{4\pi} \frac{k_1^4}{\Lambda^4} \left[\delta^2(\underline{k}_1 - \underline{k}_2) - \delta^2(\underline{k}_1 + \underline{k}_2) \right]}_{C} \right\}$$

Yu. Kovchegov and V. S., arXiv:1802.08166

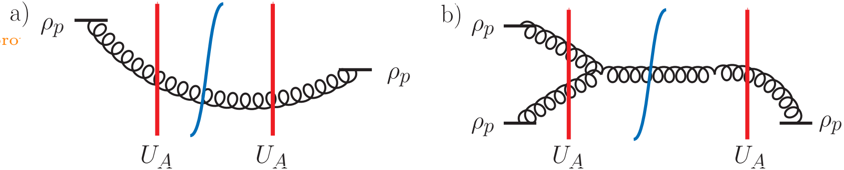
CGC perspective on v_3

- Leading order and the first saturation correction

a) $$\frac{dN^{\text{even}}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

b) $$\frac{dN^{\text{odd}}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}\text{Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{\text{Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$
$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Recall that $\Omega \propto \rho_{\text{pro}}$



- Odd azimuthal harmonics is a sign of emerging coherence in proton wave function:

the first saturation correction!

Multiplicity dependence: scaling argument

- Physical two-particle anisotropy coefficients can be simply expressed as

$$v_n^2\{2\}(N_{\rm ch}) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t \; W[\rho_p] \; W[\rho_t] \; |Q_n \left[\rho_p, \rho_t\right]|^2 \; \delta\left(\frac{dN}{dy}\left[\rho_p, \rho_t\right] - N_{\rm ch}\right)$$

with

$$Q_{2n}\left[\rho_p, \rho_t\right] = \frac{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i2n\phi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p, \rho_t\right]}{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p, \rho_t\right]}, \quad Q_{2n+1}\left[\rho_p, \rho_t\right] = \frac{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i(2n+1)\phi} \frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p, \rho_t\right]}{\int_{p1}^{p2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p, \rho_t\right]}$$

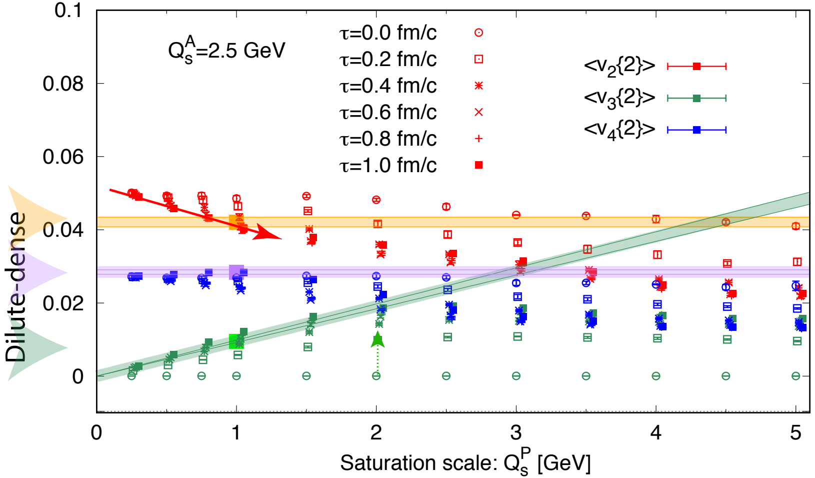
- High multiplicity is driven by fluctuations in ρ_p
- To study multiplicity dependence, rescale $\rho_p \to c\,\rho_p$
- Under this rescaling:

$$\frac{dN}{dy} \to c^2 \frac{dN}{dy}; \qquad v_{2n}^2\{2\} \to v_{2n}^2\{2\}; \qquad v_{2n+1}^2\{2\} \to c^2\; v_{2n+1}^2\{2\}$$

- Therefore in the first approximation: $v_{2n}\{2\}$ is independent of Q_s^P or multiplicity;

$$v_{2n+1}\{2\} \propto Q_s^P \propto \sqrt{\frac{dN}{dy}}$$

Dilute-dense vs Dense-dense



S. Schlichting & V.S., arXiv:1910.12496

Conclusions

- Summary of this story:
  Jean Paul showed that it is hard to get v_3 in CGC
  Miklos's paper motivated to look into it again
  Larry intuited what has to be done

  ... the rest is trivial ...

- Odd azimuthal harmonics are an inherent property of particle production in the saturation framework

- Dilute-dense vs Dense-dense: in a good agreement in the region of validity of dilute-dense expansion