The Hierarchy Problem and the Motivation for Future Colliders

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Summary

Nothing in this talk is really new.

I'll review some arguments that may be well-known to many of us—but which I find are not necessarily well-known to students, some of whom are being taught that there is no motivation to search for BSM physics.

- **The hierarchy problem** motivates new energy-frontier colliders. It's one of the strongest reasons to study the Higgs in particular, rather than high-energy processes in general. We should talk more clearly about its importance.

- I want to highlight some of the recent progress on **electric dipole moments** because they probe rather generic new physics interacting with the Higgs and electroweak gauge bosons. This should be on our radar because it may qualitatively change how we think about the case for future colliders in 5 to 10 years.

About the Hierarchy Problem

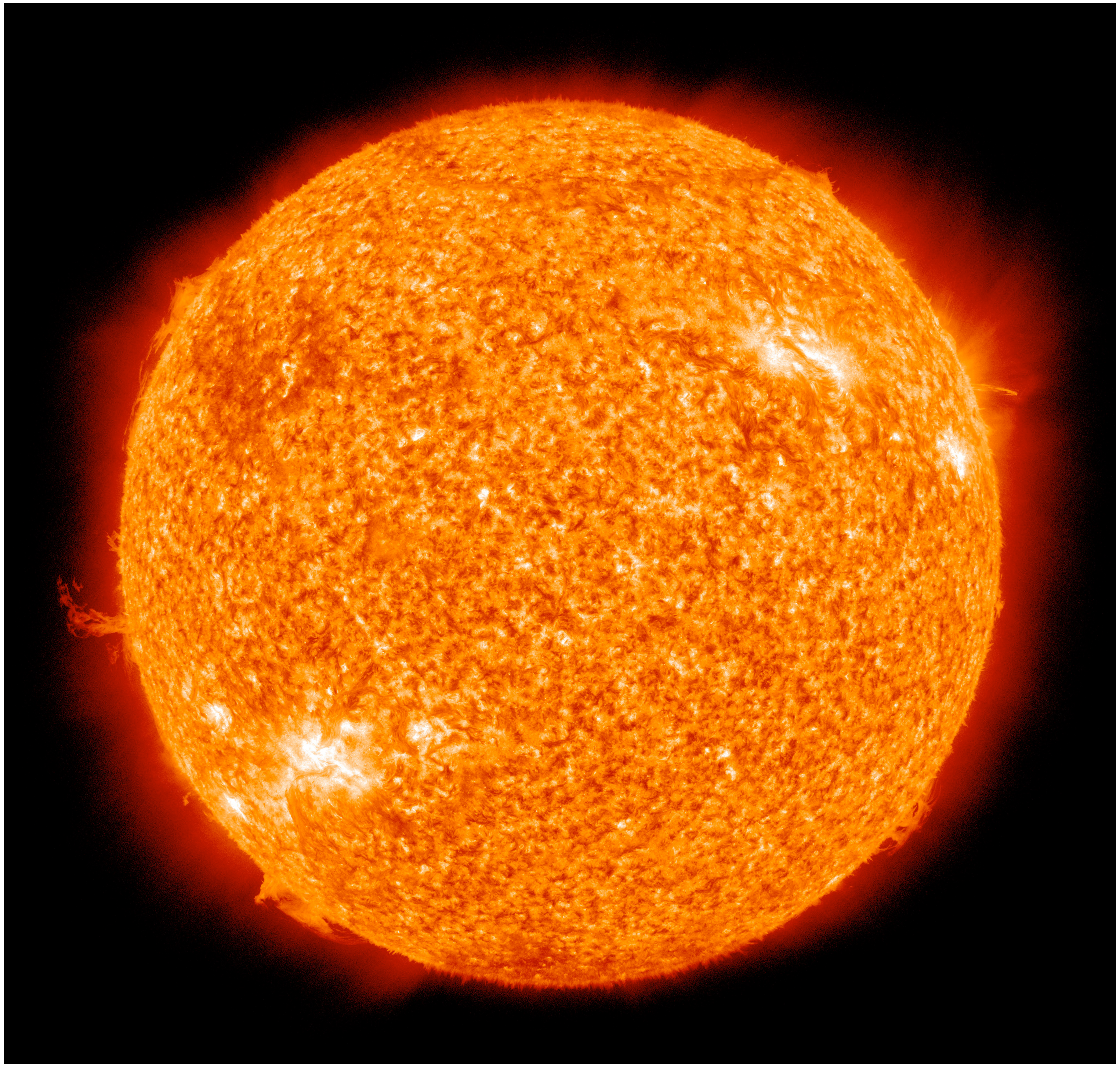
Some big questions about small numbers

- Hierarchy problem: why is $m_W^2/M_{\mathrm{Pl}}^2 \approx 10^{-33}$?
- Strong CP problem: why is $|\bar{\theta}| \lesssim 10^{-10}$?
- Flavor: why the wide range of Yukawa couplings and of mixings, e.g. $y_e \approx 3 \times 10^{-6}$ but $y_t \approx 0.95$?
- Neutrino masses: why so small? $m_\nu \sim \dfrac{v^2}{10^{15}\ \mathrm{GeV}}$ or $m_\nu \sim 10^{-13} v$?
- Cosmological constant problem: why $\rho_\Lambda \sim 10^{-120} M_{\mathrm{Pl}}^4$?
- Matter/antimatter asymmetry: why $(n_B - n_{\bar{B}})/n_\gamma \sim 10^{-9}$?
- Dark matter abundance: why $n_{\mathrm{DM}}/n_\gamma \sim 10^{-12} m_{\mathrm{DM}}/\mathrm{TeV}$?
- Primordial density perturbations: why $\delta\rho/\rho \sim 10^{-5}$?

Common theme: when we see small numbers, we're not satisfied until we can *explain* them in terms of some underlying mechanism.

The electroweak hierarchy and our world

The mysterious number that best motivates new colliders is the electroweak hierarchy. We should not lose sight of how important it is to understand.



The electroweak hierarchy is not just an obscure fact about high-energy physics. It is crucial for the existence of large objects like stars and planets.

$$\begin{aligned} M_\odot &\approx 2 \times 10^{30}\ \mathrm{kg} \\ &\approx 1.1 \times 10^{57}\ \mathrm{GeV} \\ &\approx 0.6 \left(\frac{M_{\mathrm{Pl,unred}}}{m_{\mathrm{proton}}}\right)^3 m_{\mathrm{proton}} \end{aligned}$$

The electroweak hierarchy and our world

It's possible to do a more detailed estimate of both the minimum and maximum size of an ordinary star. A star should be hot enough for nuclear fusion to happen in its core.

$$P_{\text{fuse}}(E) \sim \exp\left(-E/T - \mathcal{O}(\alpha)\sqrt{m_p/E}\right)$$

Boltzmann

Gamow (WKB)

The rate peaks at $E_g \sim \alpha^{2/3} m_p^{1/3} T^{2/3}$. In order to not have too much suppression, we need

$$T \gtrsim E_g \quad \Rightarrow \quad T \gtrsim \alpha^2 m_p$$

We need thermal pressure to balance gravitational attraction, and for the star *not* to be so compact that electron degeneracy pressure is important. Putting the pieces together gives a bound on stellar mass.

[Details: V. Weisskopf, *Science* **187**(4177):605–612 (1975); Burrows and Ostriker, *PNAS* 111 (7):2409-2416 (2014).]

The electroweak hierarchy and our world

The detailed estimate, assuming a ball of hydrogen gas that is hot enough for nuclear fusion to work despite Coulomb repulsion, leads to a scaling like:

$$\frac{M_{\text{star}}}{m_{\text{proton}}} \gtrsim \left(\frac{M_{\text{Pl}}}{m_{\text{proton}}}\right)^{3} \left(\frac{m_{\text{proton}}}{m_{\text{electron}}}\right)^{3/4} \alpha^{3/2}$$

In fact, a star also cannot be too much heavier than this without collapsing.

Similar reasoning reveals that the maximum mass of a rocky planet scales like

$$\frac{M_{\text{rocky planet}}}{m_{\text{proton}}} \lesssim \left(\frac{M_{\text{Pl}}}{m_{\text{proton}}}\right)^{3} \alpha^{3/2}$$

If the Higgs VEV were near the Planck scale, the Universe would be a very different place!

[Details: V. Weisskopf, *Science* **187**(4177):605–612 (1975); Burrows and Ostriker, *PNAS* 111 (7):2409-2416 (2014).]

What is the hierarchy problem?

A good solution to the hierarchy problem should leave us feeling like we *understand* the origin of a scale in terms of some more fundamental physics.

A good example comes from QCD: we can ***compute*** the QCD scale from the gauge coupling measured at some higher energy, and it comes out exponentially small in a robust manner:

$$\Lambda_{\text{QCD}} \sim M\,\mathrm{e}^{-8\pi^2/(bg(M)^2)}$$

Or BCS superconductivity: Cooper pairing from similar running of marginal interaction. (Shankar, Polchinski)

We want something similar for the EW hierarchy. Not literally the same, but same qualitative character of allowing us to compute the scale from something more microscopic.

What is the hierarchy problem?

A further remark about the QCD scale:

$$\Lambda_{\mathrm{QCD}} \sim M\, \mathrm{e}^{-8\pi^2/(bg(M)^2)}$$

By some simple fine-tuning measures, this is “fine-tuned”; e.g. Barbieri-Giudice,

$$\frac{\partial \log \Lambda_{\mathrm{QCD}}}{\partial \log g} = 2 \log \frac{\Lambda_{\mathrm{QCD}}}{M} \sim 100$$

This doesn’t bother me. Shouldn’t be too quick to dismiss a theory because of moderate sensitivity to an underlying parameter.

What is the hierarchy problem?

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where did the weak scale come from?

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Various refinements of this question, or related questions, are:

- Can we *explain* or *compute* the weak scale in terms of a more fundamental theory beyond the Standard Model?
- Are there microscopic *dynamics* that tell us why electroweak symmetry breaking happened, or that make it more likely?
- What is the shape of the Higgs potential? (Strong motivation for measuring the Higgs self-coupling.)
- Is the Higgs boson a fundamental particle, or is it composite?
- What would happen if we heated up the universe above the weak scale?

What is the hierarchy problem NOT?

The question is ***NOT***

how do I regulate a loop diagram?

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how do I regulate a loop diagram?

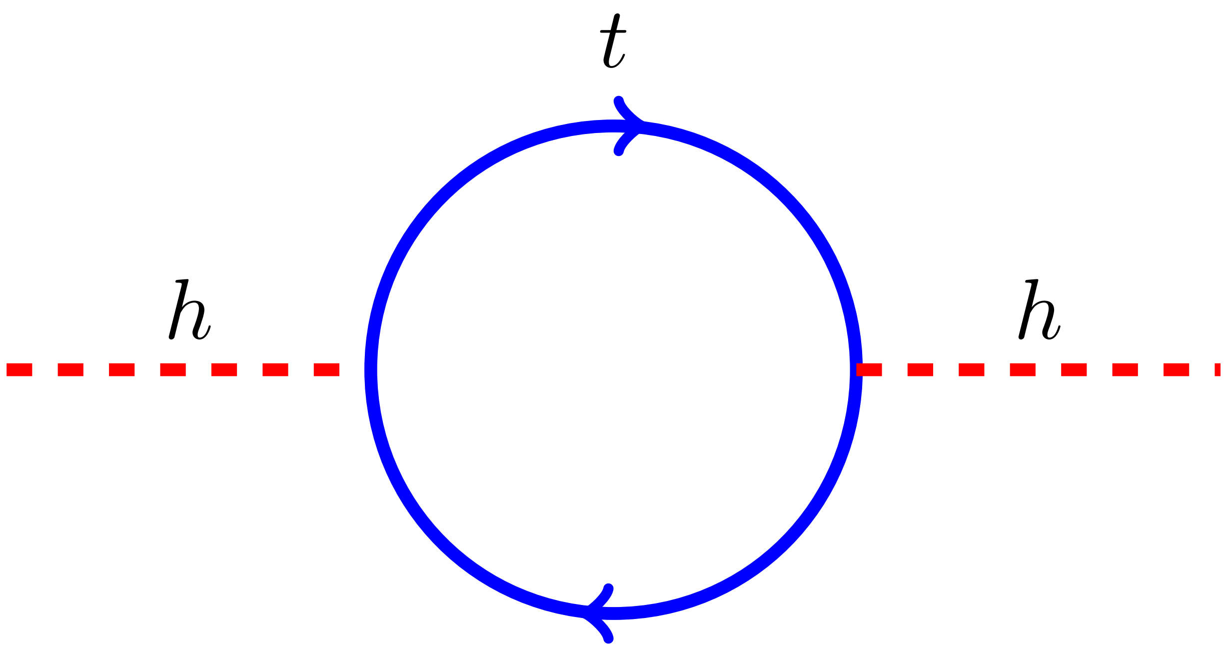
- The problem will *not* go away just because you like to use dimensional regularization, which has no power divergences.

- The problem will *not* go away simply because you like a different choice of “fine-tuning measure.”

- The fact that you can measure Standard Model parameters and do calculations to high precision that match data at the weak scale does *not* mean there is nothing to explain.

What not to say

Like many other people, I have given talks where, due to lack of time or wanting to focus on other points, I have just said things like:



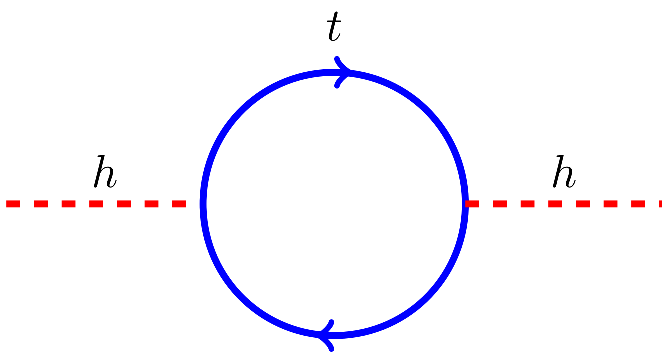
$$\delta m_H^2 \sim \frac{y_t^2}{16\pi^2}\Lambda_{\rm UV}^2$$

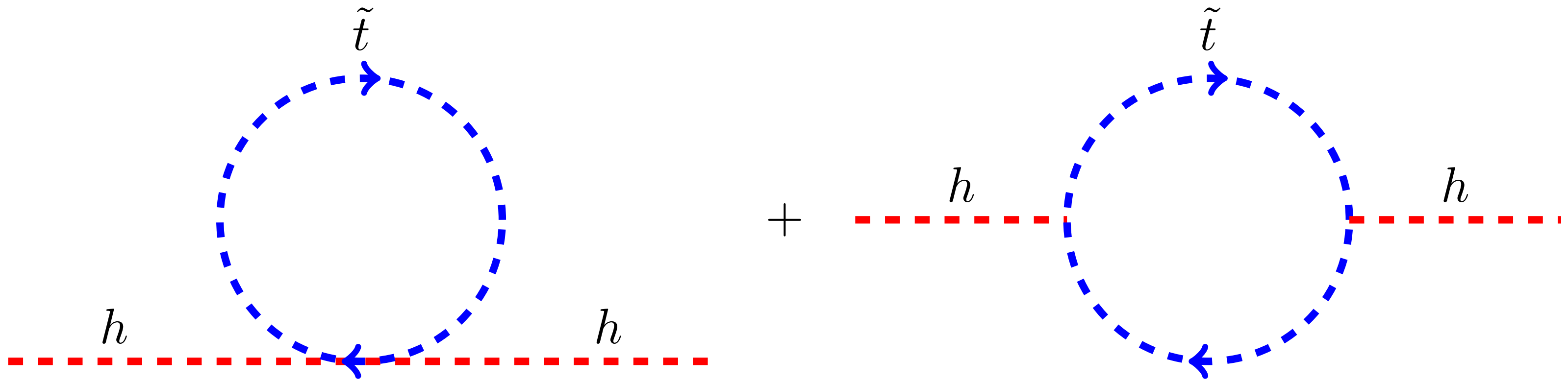
This diagram is quadratically divergent, so the weak scale is quadratically sensitive to UV scales. We need a low cutoff or a cancelation of this divergence.

Because then…

Some people respond “power divergences are unphysical” or “when you use the renormalized mass in a calculation, there is no problem” or any number of other things you’ve probably heard before.

Or maybe we are a little more careful and we say something like:





$$\delta m^2_{H_u} = -\frac{3}{8\pi^2} y_t^2 \left(m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} + |A_t|^2\right) \log \frac{\Lambda}{\mathrm{TeV}}.$$

What we have is *quadratic sensitivity* to *physical scales*.

Can we do better?

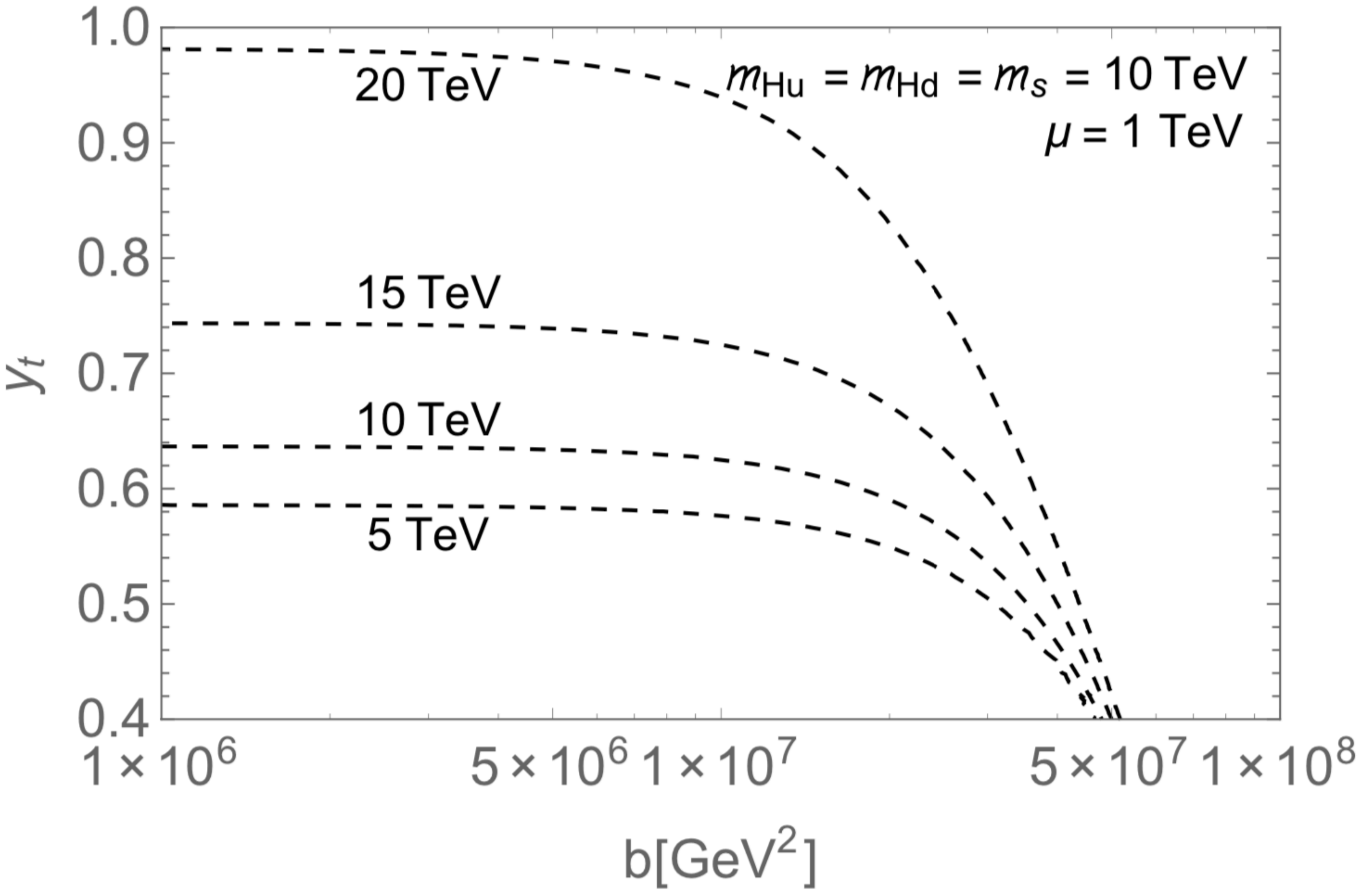
The better way to frame the problem, and the role of fine-tuning, is that we are seeking a theory that explains the *origin* of the EW scale.

If, within that theory, the EW scale is extremely sensitive to input parameters, it’s not a very good explanation. The theory does not generically describe a universe like the one we live in.

If moving around in parameter space just produces modest changes in the low-energy physics, that’s a compelling theory that predicts a world like ours.

SUSY and the hierarchy problem

When we say that weak-scale SUSY solves the hierarchy problem, we mean something simple:



The weak scale can be ***computed*** from input parameters, and is *typically* of order* the SUSY breaking parameters.

* leaves room for small accidents

Technical naturalness

A theory in which the hierarchy becomes “technically natural”—that is, in which you can compute radiative corrections and don’t find dramatic changes—*might or might not* solve the hierarchy problem.

If the theory introduces a tiny number by hand, from my viewpoint it hasn’t solved the problem, even if that number is stable. But it has, perhaps, *made the problem more tractable*.

Putting too much emphasis on radiative stability would discard other problems, like the Strong CP problem, which in my mind are every bit as important as the hierarchy problem.

Indeed, tiny technically natural couplings seem to be problematic in UV-complete gravitational theories (this is a whole other talk, about the Weak Gravity Conjecture).

Recasting the hierarchy problem

Many known solutions to the hierarchy problem really *recast* the problem into a different problem: what is the origin of the…

- scale of supersymmetry breaking?
- compositeness scale?
- volume of extra dimensions?
- extreme flatness of the relaxion potential?

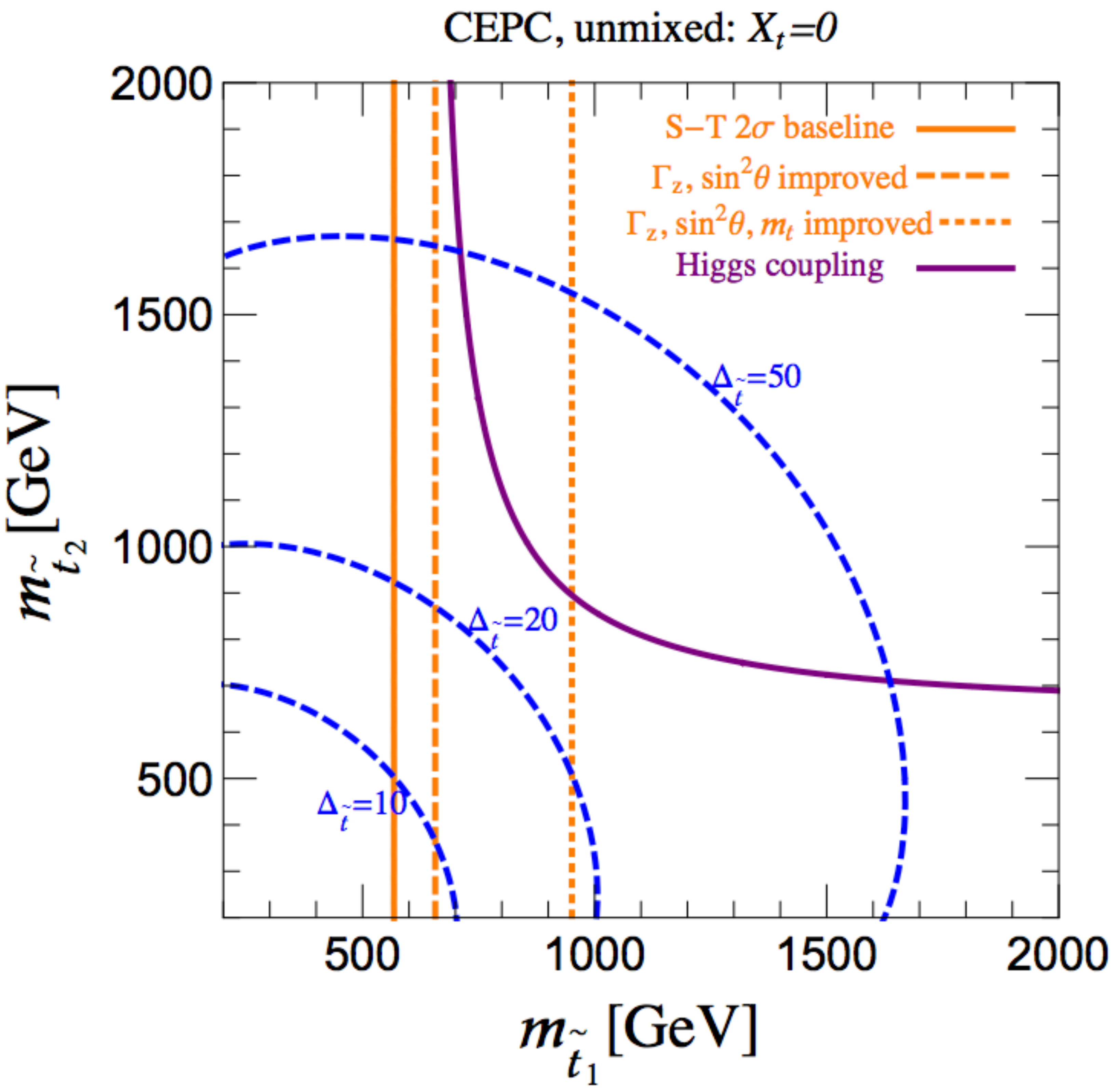
They allow the electroweak scale to be **computed** from other inputs, but explaining the origin of those inputs is a new problem.

By changing the character of the problem, they allow for new kinds of solutions—often dimensional transmutation.

Higgs and EWPT Constraints for Hierarchy Problem Models: Quick Reminder

(work from 2014, not new results)

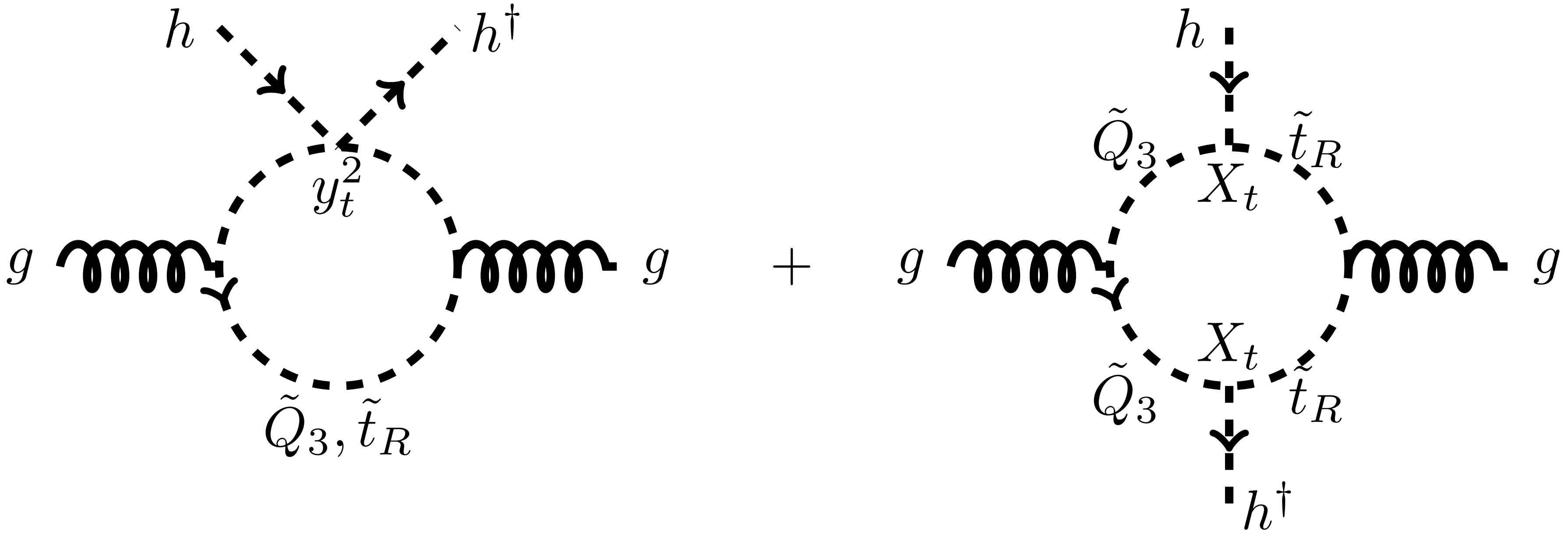
Precision EW Constraints on SUSY Stops



Higgs couplings (gluons and photons) probe left- and right-handed stops roughly equally well.

The *T* parameter probes left-handed stops.

Higgs couplings



$$r_G^{\tilde{t}} \equiv \frac{c_{hgg}^{\tilde{t}}}{c_{hgg}^{\mathrm{SM}}} \approx \frac{1}{4}\left(\frac{m_t^2}{m_{\tilde{t}_1}^2} + \frac{m_t^2}{m_{\tilde{t}_2}^2} - \frac{m_t^2 X_t^2}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}\right)$$

Familiar low-energy theorem: beta function coefficients times $\sum \frac{\partial \log M}{\partial \log v}$

Similar result for photons (except SM contribution dominated by *W* loop)

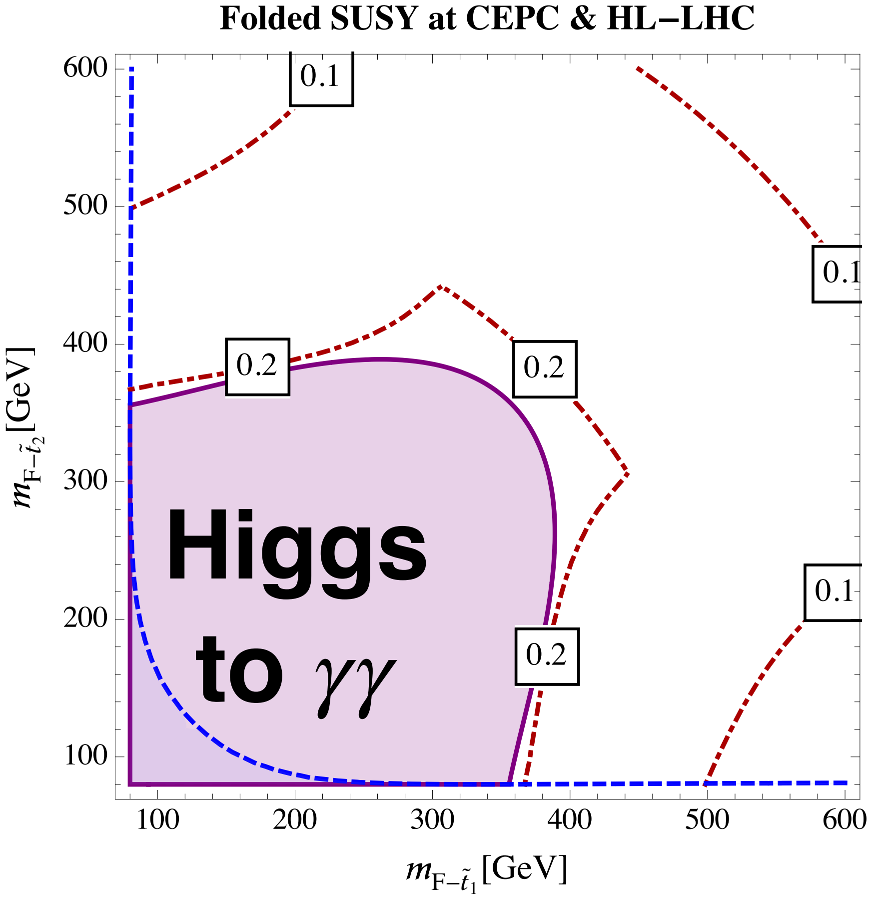
Folded SUSY: Uncolored Naturalness

In folded SUSY, stops have **no QCD color** (makes life difficult at LHC). But still have electroweak interactions.

Measuring Higgs decays to photons and the *T* parameter can help constrain folded SUSY stops.

The *T*-parameter bounds previously shown for stops are *exactly* the same for folded stops!

Another way the CEPC has exciting potential for uncolored naturalness!


Folded SUSY at CEPC & HL−LHC

Strengths of Higgs Factories

The Folded SUSY example illustrates how CEPC could probe a scenario where LHC constraints, even after the HL-LHC run, can be fairly weak.

New electroweak physics and “neutral naturalness” is one arena where Higgs factories have an advantage over hadron colliders, compared to more “standard” SUSY or composite Higgs explanations of the hierarchy.

Another big theoretical motivation of Higgs factories is the *Higgs portal*—the possibility of light “Hidden Valley” physics that is only accessed through the Higgs.

In some models, such physics can be related to the hierarchy problem.

Can Kilic’s talk on Twin Higgs in this session will cover some of this in more detail.

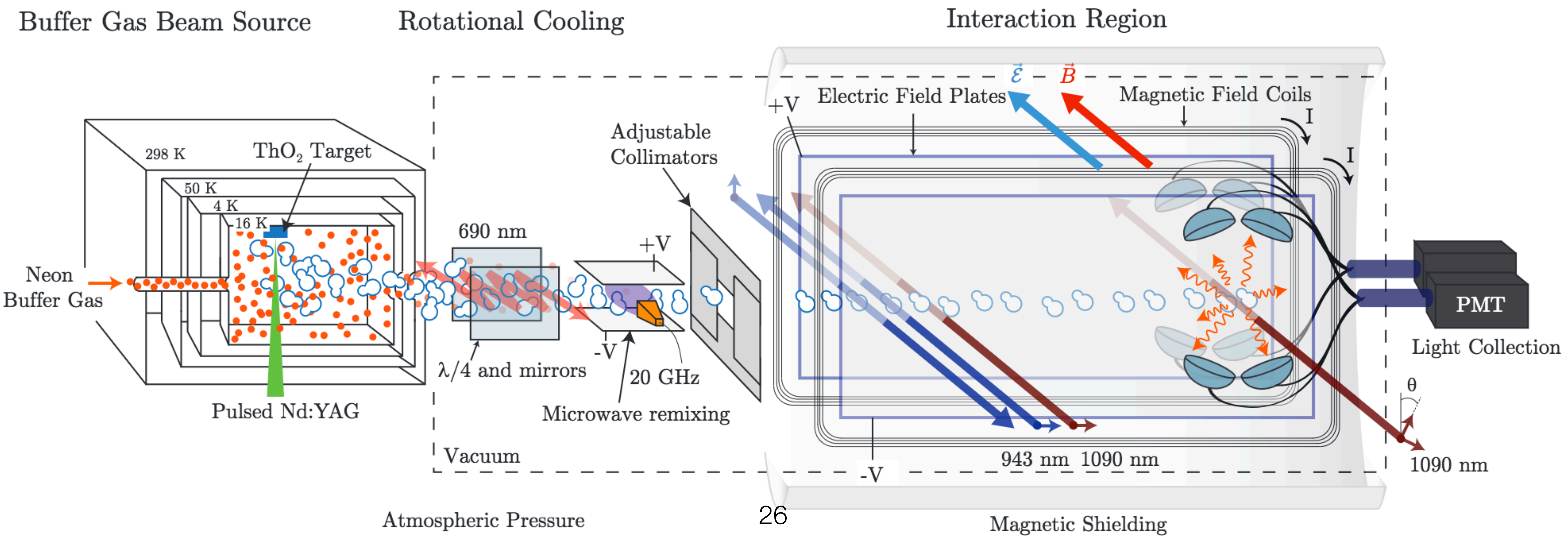
Remarks on EDMs

Atomic Physics Testing the Standard Model

Colliders may not be our first sign of new physics!

Recent dramatic progress in AMO physics: searches for the electron EDM.

ACME 2 (source: electronedm.org) DeMille, Doyle, Gabrielse and collaborators. New result last year.



Electron EDM

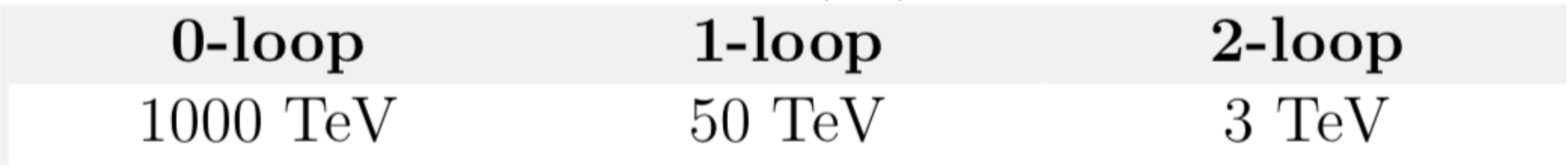
The 2018 bound from ACME is: $|d_e| \lesssim 1.1 \times 10^{-29}\, e\, \mathrm{cm}$

This improves on the previous, 2013, ACME bound by about an order of magnitude.

EDMs violate chirality, so putting in the electron mass a spurion, we expect an effect of order:

$$d_e \sim \delta_{\mathrm{CPV}} \left(\frac{\lambda}{16\pi^2} \right)^k \frac{m_e}{M^2}$$

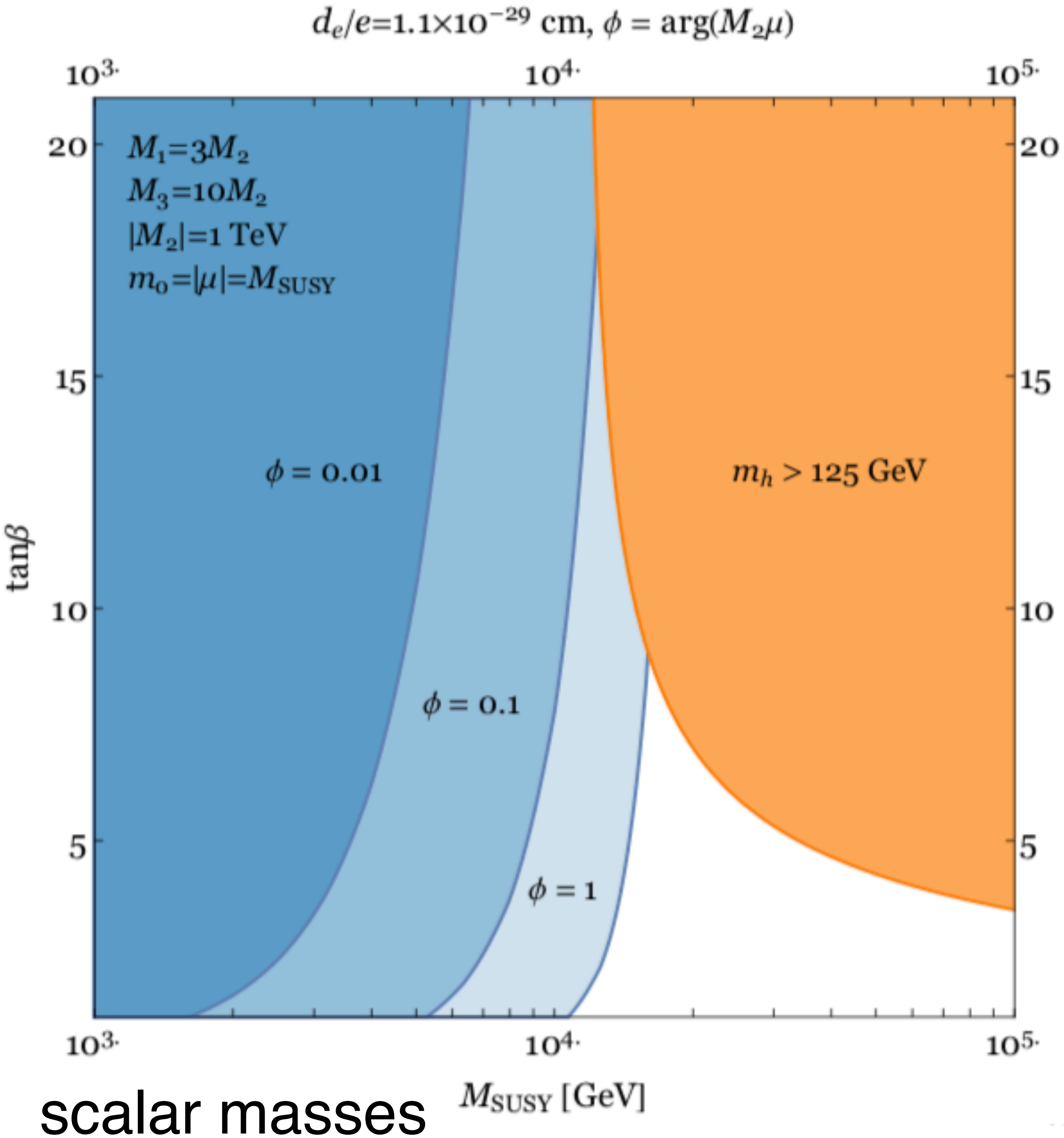
Then dimensional analysis tells us that the experiment probes masses



0-loop	1-loop	2-loop
1000 TeV	50 TeV	3 TeV

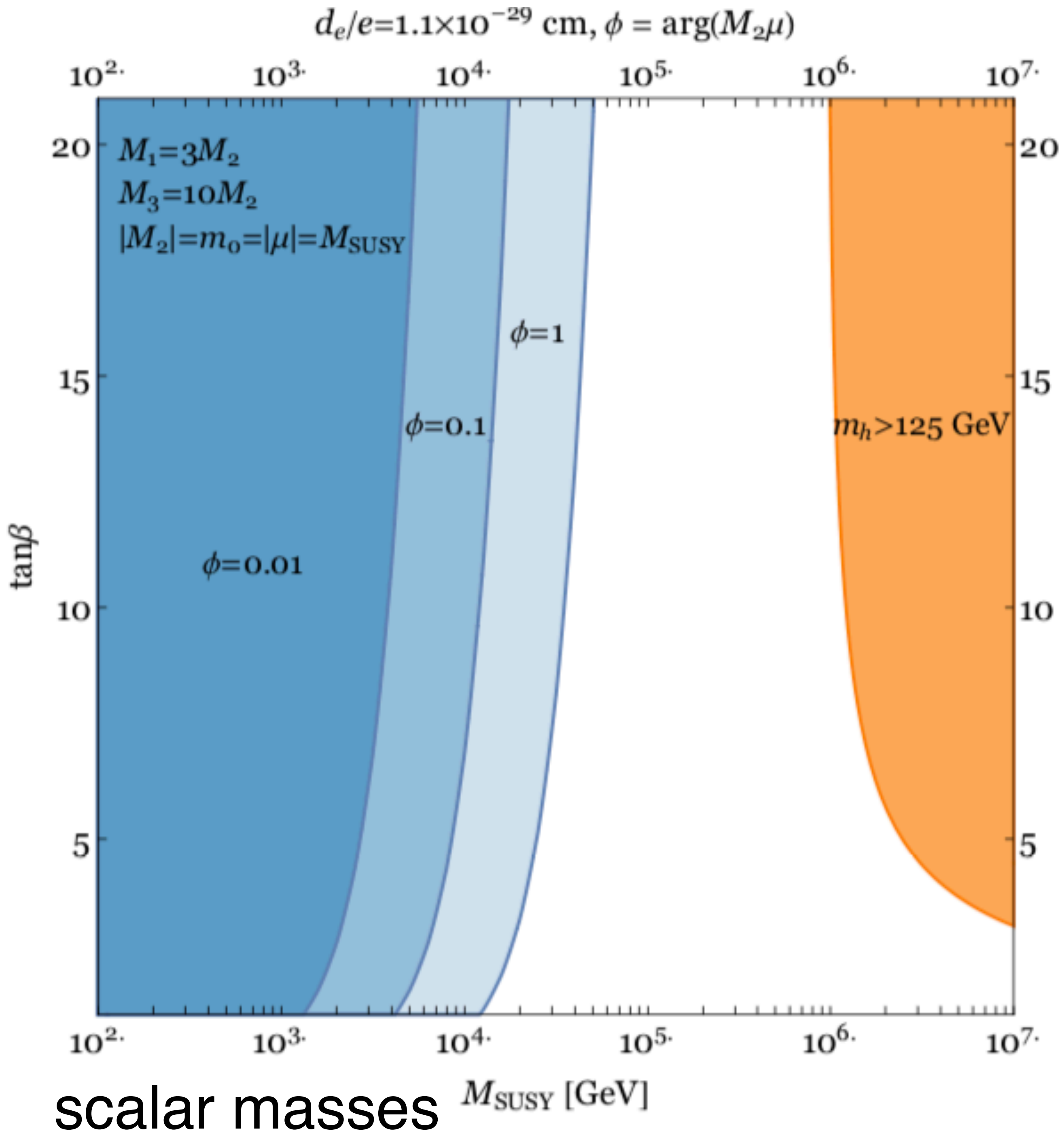
for order-one CPV phases this often exceeds LHC reach!

Electron EDM vs. MSSM



scalar masses

Split SUSY



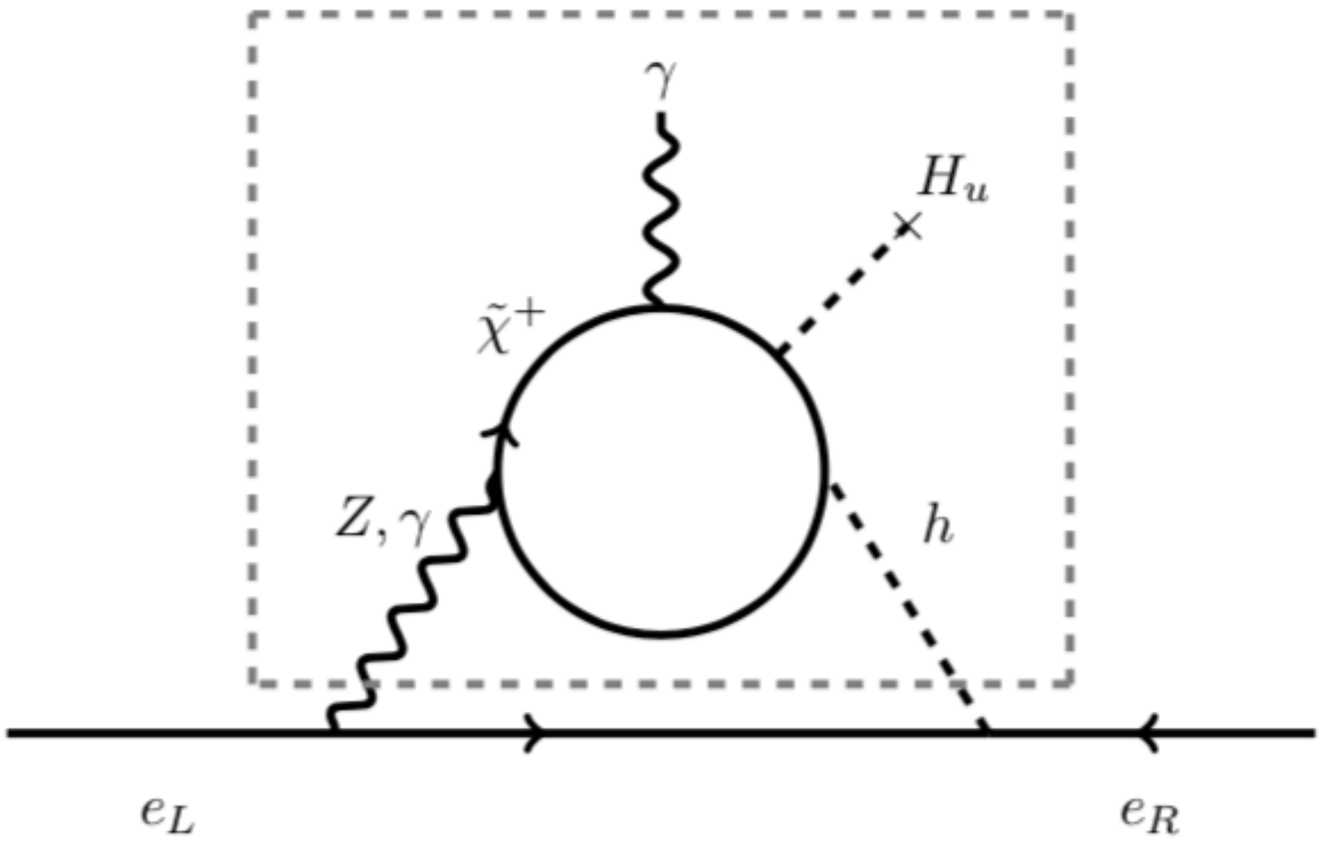
scalar masses

High-Scale SUSY

One-loop effects: Cari Cesarotti, Qianshu Lu, Yuichiro Nakai, Aditya Parikh, MR, ’18

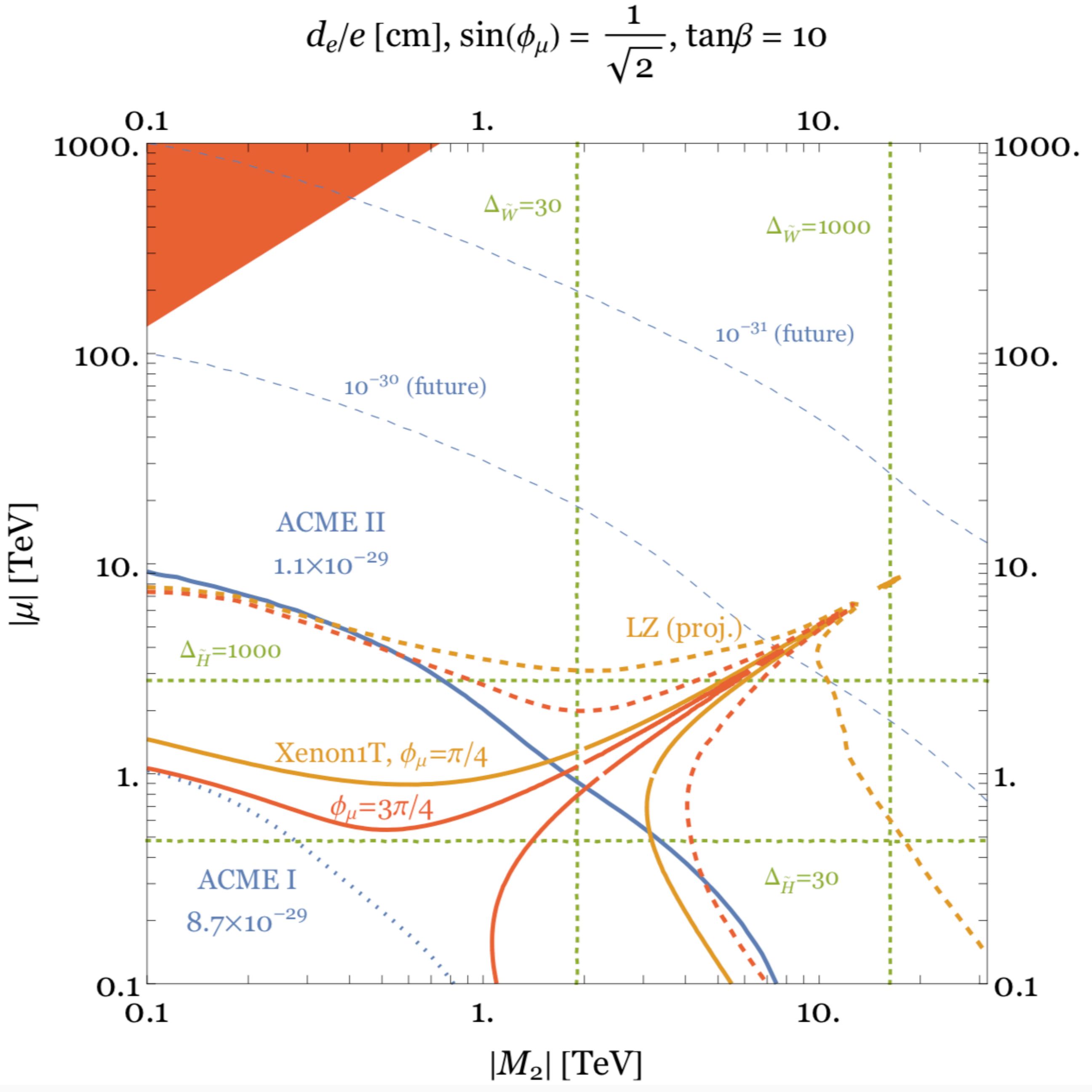
Electron EDM vs. Electroweak Physics

Quite generally, electroweak new physics coupling to the Higgs boson gives rise to an electron EDM (Barr-Zee).



[Cesarotti, Lu, Nakai, Parikh, MR, ’18]

Powerful split SUSY electroweakino constraints from ACME 2!



How Convincing Can Null Results Be?

New physics ***discoveries*** or strong hints would strengthen the motivation for future energy-frontier colliders.

Converse: can a strong ***null result*** ever convince you that we *should not* build a collider? Usually not—still many possibilities remain.

Generic new physics with EW interactions allows for new CP phases and hence an electron EDM. So orders-of-magnitude stronger null results in EDM experiments would be mysterious if there is TeV-scale new physics.

Possibility: **CP** is a ***spontaneously broken*** symmetry. If all the breaking is *correlated* with flavor breaking, this could explain why the CP phase in the CKM is large but the CP phase in the EDM is small. (Nir & Rattazzi, 1996) Could be timely to revisit this idea—how small can phases naturally be?

Or, recall Stefania Gori's comment yesterday: *what if Higgs doesn't couple to electrons?*

Outlook, 1

The LHC has discovered what appears to be an elementary, spin-0 boson.

So far, it acts just like a Standard Model Higgs boson.

This is a *big deal*! We have learned something important about nature. But ***we do not understand the weak scale***, and we still have every reason to think *the answer is most likely to lie at nearby energies*.

We, as a field, are failing to convey this to younger physicists and possibly to funding agencies. A larger fraction of the field is spending time on *unmotivated* models that do not confront fundamental questions.

I don't know the answer, but I think that we need to talk *more* but also *more clearly* about why the big questions matter.

Outlook, 2

There are many smaller scale, non-collider experiments that are happening, and many more proposed for the future.

These include dark matter, flavor, EDMs, ….

Our first discovery of physics beyond the Standard Model could come from these experiments, but only a collider will allow us to directly characterize the properties of the new particles and interactions.

These experiments have the potential to strengthen our arguments for future colliders. However, in most cases null results at these experiments will not *weaken* the case, because a wide range of collider-accessible, motivated new physics can evade them.

Particle physics needs new energy frontier colliders!