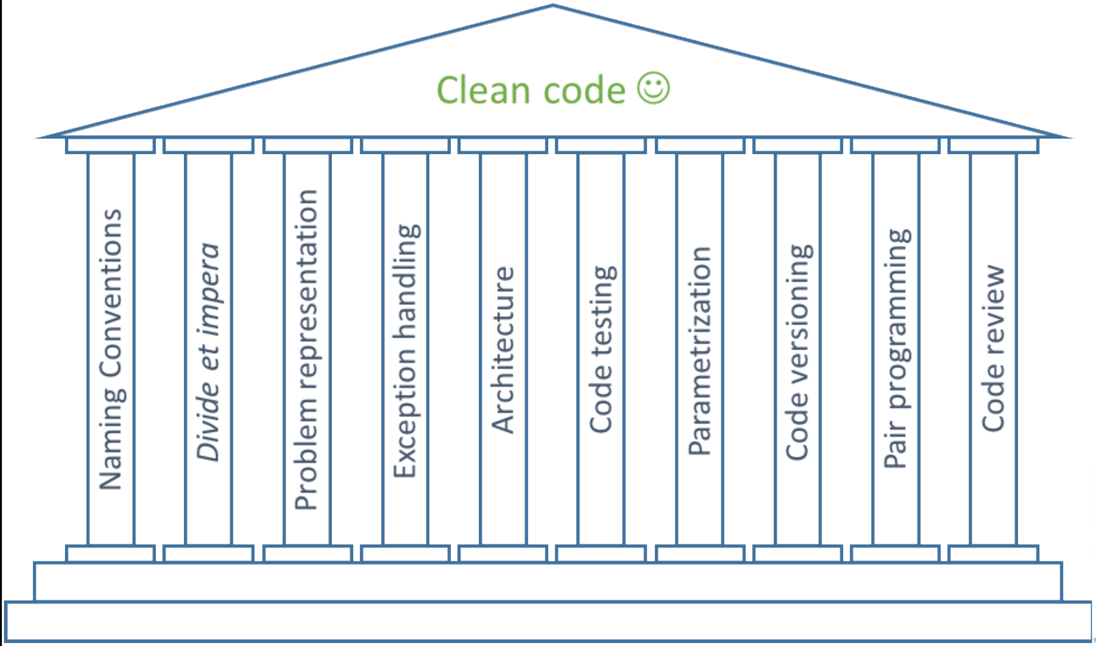
C. Wiesner

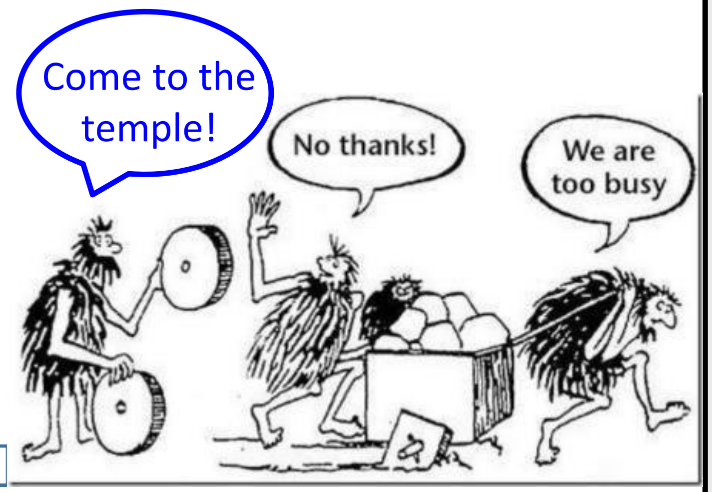
PE Mini Lectures „How to describe a particle beam?“

29.05.2019

Recap last session: Good Coding Style

The Temple of Clean Code





https://www.quora.com/I-work-really-very-hard-for-CSIR-life-science-but-still-my-paper-was-not-so-good-What-should-I-have-to-do-so-I-can-crack-it

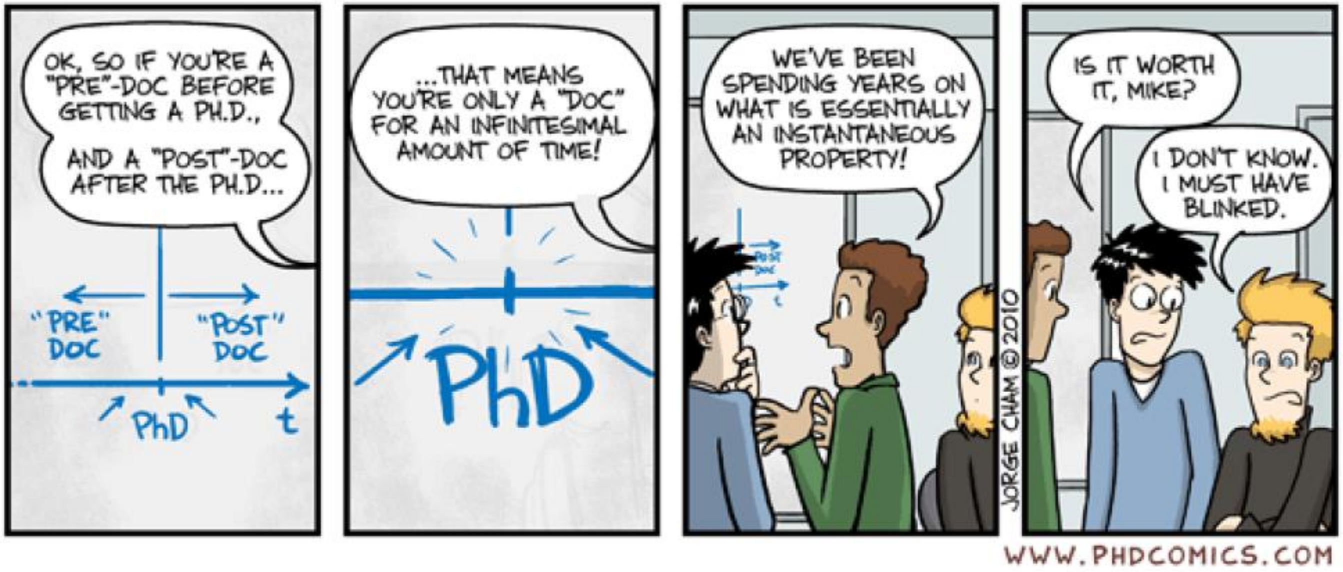
https://indico.cern.ch/event/821596/

Recap last session: Good Coding Style

1. **Naming conventions**: think twice if the name describes its purpose… and is short
2. **Use of functions**: break down the problem into small parts and solve one after another
3. **Representation**: failure at planning is planning a failure
4. **Exception handling**: a problem one expects and handles is not a problem anymore
5. **Architecture**: think about layers. For example Acquire → Analyse → Present
6. **Code testing**: no more fear while modifying the code
7. **Parametrization**: stop commenting bits of code to execute different parts
8. **Code versioning**: you no longer need *script_v1.m*, *script_v2.m*, *script_v3a.m*
9. **Pair programming**: two heads are better than one
10. **Code reviewing**: everyone knows what's happening around – expertise continuity

https://indico.cern.ch/event/821596/

Recap last session: Celebrating…



Beam and Accelerator Physics

1) Basics: How to describe a particle beam?
- Concept of phase space
- Conservation of phase-space volume (Liouville theorem)
- Emittance
- Beta function and Twiss parameters

Today

2) How to produce a particle beam?
- Ion sources
- Space charge

3) Beam transport
- How to deflect a beam?
- How to focus a beam?
- Magnet types and their beam-dynamics functions

Beam and Phase Space

What is a beam?

- A particle **beam** is a particle ensemble with a much higher longitudinal than transverse velocity.

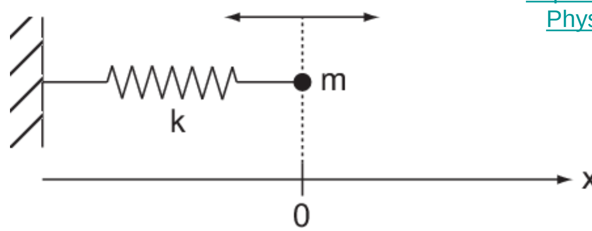
How to describe a beam?

- To describe the behaviour of a dynamic system, classical (Hamiltonian) mechanics uses the concept of **phase space**: an abstract space formed by the (generalized) coordinates and the (generalized) momenta, e.g. by the positions x, y, z and the momenta p_x, p_y, p_z.
- One point in phase space represents state of the system at a certain time, e.g. for one particle: (x, y, z, p_x, p_y, p_z).
- Therefore, the trajectory of a point in the phase space represents how the state of the dynamic system changes.

Example: Linear harmonic oscillator

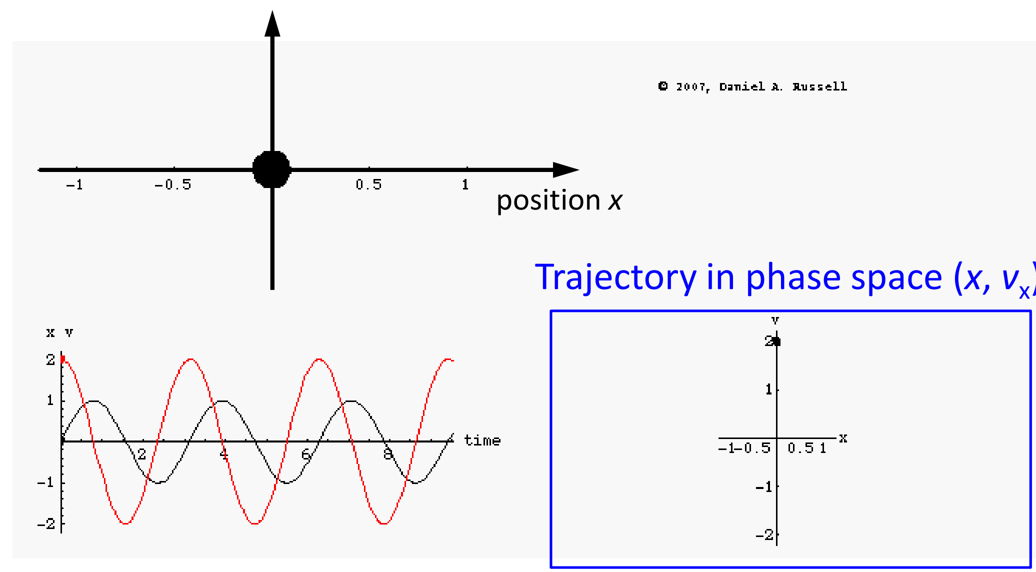
- Simple example: Point particle exposed to a linear restoring force (spring).
- 1d problem represented in 2d phase space: (x, p_x) or (x, v_x).
- Phase space trajectory shows dynamic change of the system state.
- Periodic motion → phase space trajectory is a closed curve.
- Linear force → point moves on an ellipse in phase space.

https://qudev.phys.ethz.ch/content/science/BuchPhysikIV/PhysikIVch10.html rev. 2019-05-27



$$F = -k\,x$$

Linear harmonic oscillator

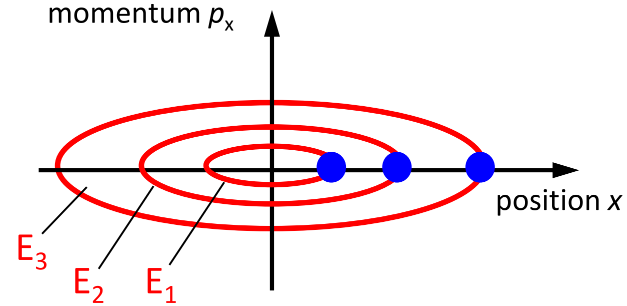


Plots adapted from https://www.acs.psu.edu/drussell/Demos/phase-diagram/phase-diagram.html rev. 2019-05-27

Phase Space for Accelerator Physics

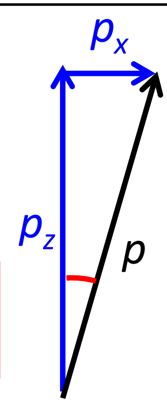
- Accelerator physics: A beam with *N* particles is represented in the same 6d phase space (strictly speaking this assumes that the particles do not interact with each other…*).

These points represent 3 different initial conditions of the same system… or 3 *non-interacting* particles in the same phase space.



- In accelerator physics, we typically use the transverse angles (instead of the momenta) to describe the particles, e.g. we use (x, x'), (y, y') instead of (x, p_x), (y, p_y).**

$$x' = \frac{dx}{dz} = \frac{v_x}{v_z} = \frac{p_x}{p_z} = \tan\theta_x$$



For $v_z \gg v_x$

→ small angles: $x' \approx \theta_x$

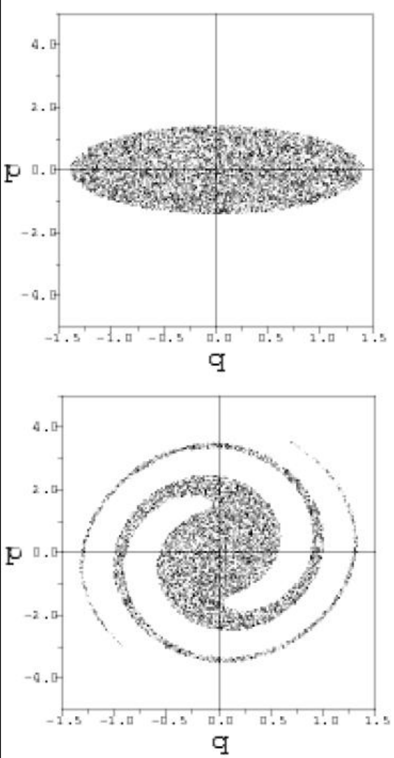
* This implies neglecting the Coulomb interaction or approximating it by a smooth 'external' space-charge field.

**The longitudinal plane will be discussed later. Note that the (position, angle) space is often called trace space.

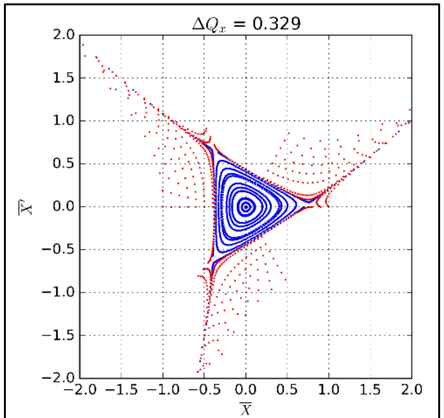
The Art (and Science) of Phase Space

The **phase space** is “one of the most powerful inventions of modern science”

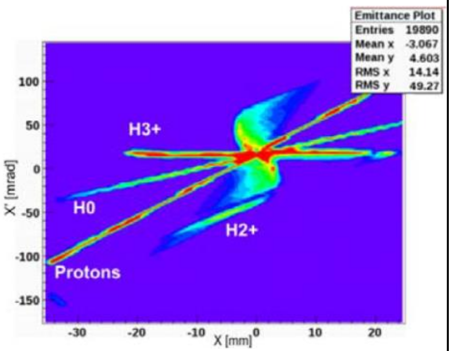
(J. Gleick, Chaos: Making a New Science, Viking, New York (1987), cited by D. Nolte, Physics Today, April 2010, p. 34)



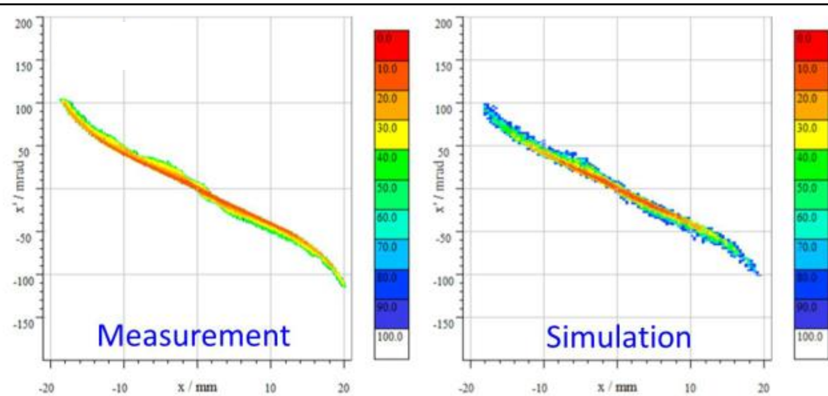
STRUCKMEIER, J. (2005). Ion Beam Optics. Lecture, Goethe-Universität Frankfurt am Main, http://www-linux.gsi.de/~struck/> rev. 2008-05-14



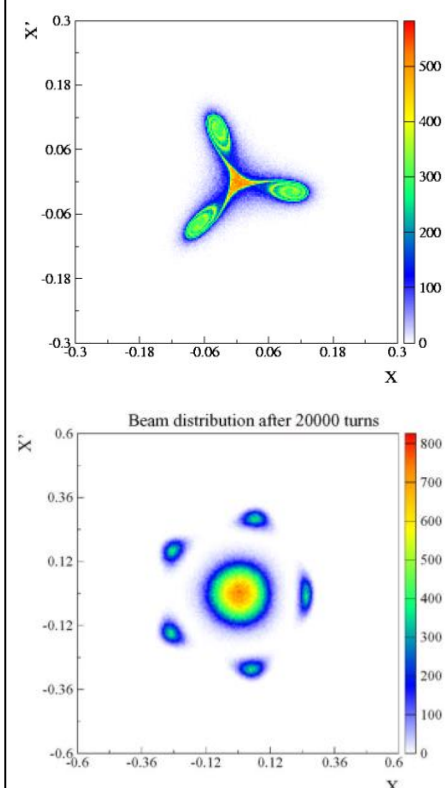
C. Wiesner, sim. 3rd order resonance



Measurement LINAC4, CERN-BE-2011-005_BI



C. Wiesner, Ph.D. thesis



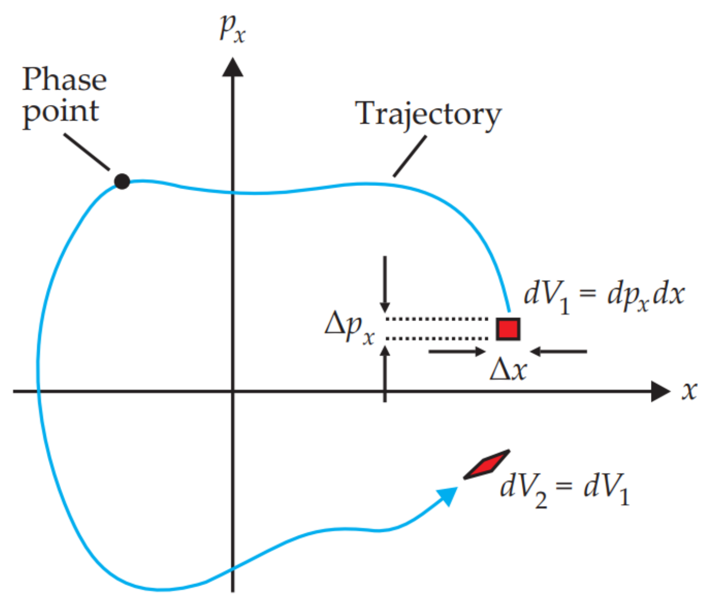
M. Giovannozzi: MTE Design Report, CERN-2006-011, 2006

Phase Space Conservation and Liouville

- Liouville's theorem (see appendix for general formulation): The local point density of an ensemble of N particles in the 6d phase space is conserved – *given that there are no interactions between the particles, no binary collisions, no dissipative forces and no particle losses or charge exchanges*.

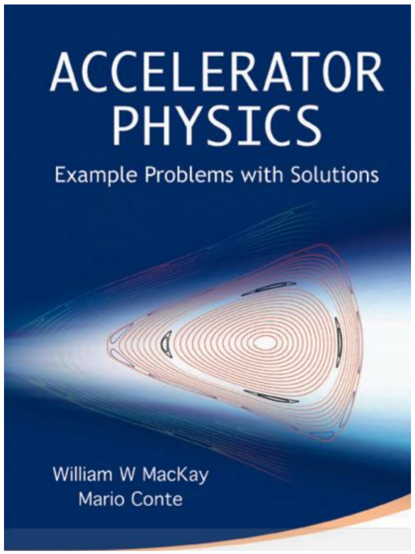
- If (and only if) the motions in the three orthogonal directions are not coupled, also the local point densities in the 2d subspaces (i.e. x-p_x, y-p_y, z-p_z) are conserved.

Imagine it like the phase-space points behaving as an incompressible fluid → the shape of the phase-space volume can change, but the absolute value remains constant (under certain conditions).



[D. Nolte, Physics Today, April 2010, p. 34]

Beating Liouville?



Chapter 12

Problems of Chapter 12: How to Baffle Liouville

12.1 Problem 12–1: New cooling ideas

Invent a new method of beating Liouville and win the Nobel prize.

This is really a suggested problem for further research (in the vein of Knuth's[38] rating "HM50"), so we will not be giving a solution here. As for winning the Nobel prize: Simon van der Meer[67] did it, so why not you?

W. MacKay/M. Conte, *Accelerator Physics. Example Problems with Solutions*, Singapore 2012, p.233

Nobel prize (1984) for Simon van der Meer for stochastic cooling.

Phase Space and Emittance

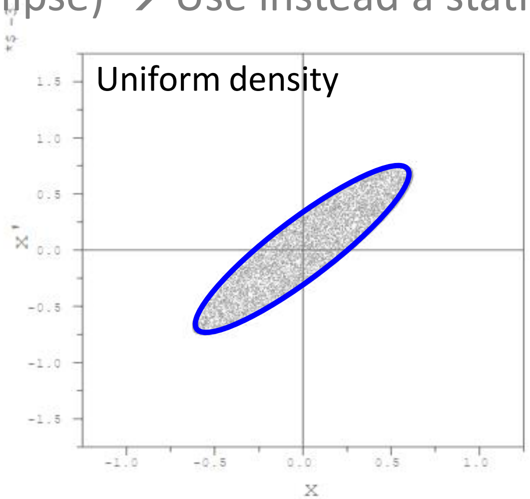
Area in the 2d phase-space projections:

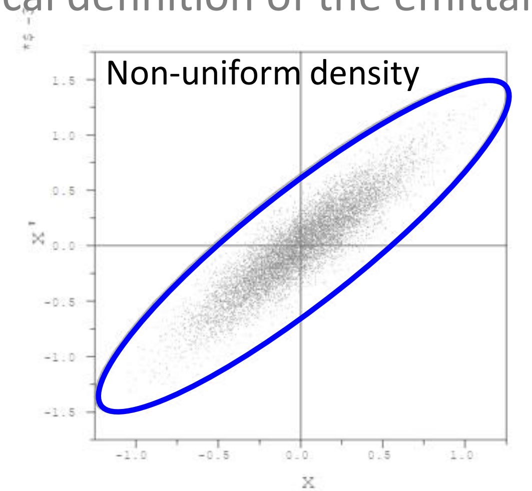
$$A_x = \int\int dx\, dx' = \epsilon_x \pi$$

$$A_y = \int\int dy\, dy' = \epsilon_y \pi$$

The quantity $\epsilon = \dfrac{A}{\pi}$ is called **emittance**.

- The emittance quantifies the occupied area in phase space.
- It is an important figure of merit for the beam quality.
- Challenge: not all phase-space distributions can be (easily) described by a limiting contour (ellipse) → Use instead a statistical definition of the emittance (see appendix).





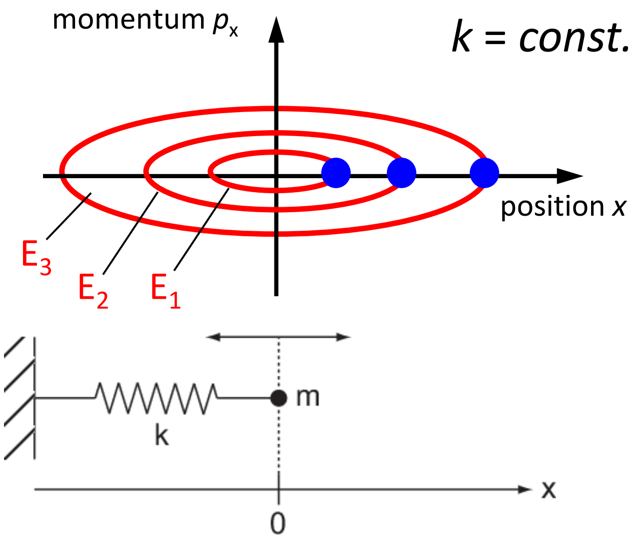
Plots from STRUCKMEIER, J. (2005). Ion Beam Optics. Lecture, Goethe-Universität Frankfurt am Main, http://www-linux.gsi.de/~struck/> rev. 2008-05-14

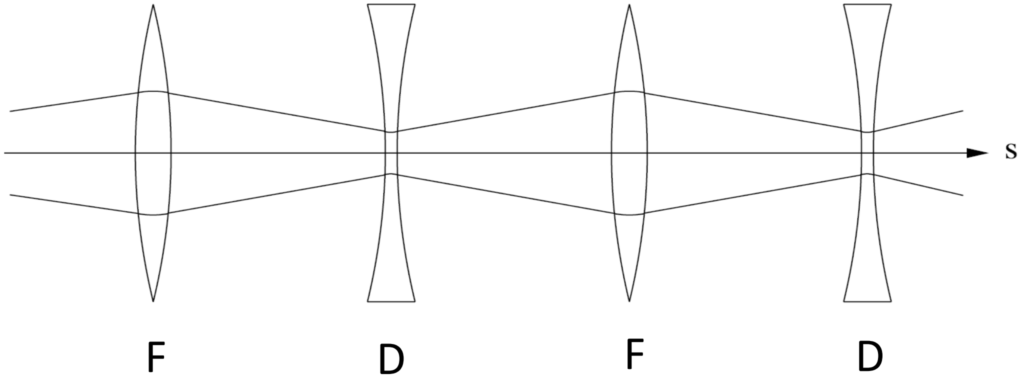
Periodic Focusing Channel

- In the presence of only linear forces, all particle trajectories in the 2d subspaces lie on ellipses → particle ensembles can be represented by their surrounding ellipse.

- Imagine a particle beam propagating through a periodic channel of focusing and defocusing lenses.
- → The restoring forces $k(s)$ depend on the longitudinal position s, i.e. k differs between a focusing lens and a defocusing lens or a drift…

Linear harmonic oscillator





Reference particle trajectory s (or time t)

Hill’s equation

Hill’s equation: differential equation for motions with periodic focusing properties (e.g. lunar motion)

$$x''(s) - k(s)x(s) = 0$$

General solution of Hill’s equation:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cdot\cos(\psi(s)+\phi)$$

→ “All right, let's buckle down and work.”

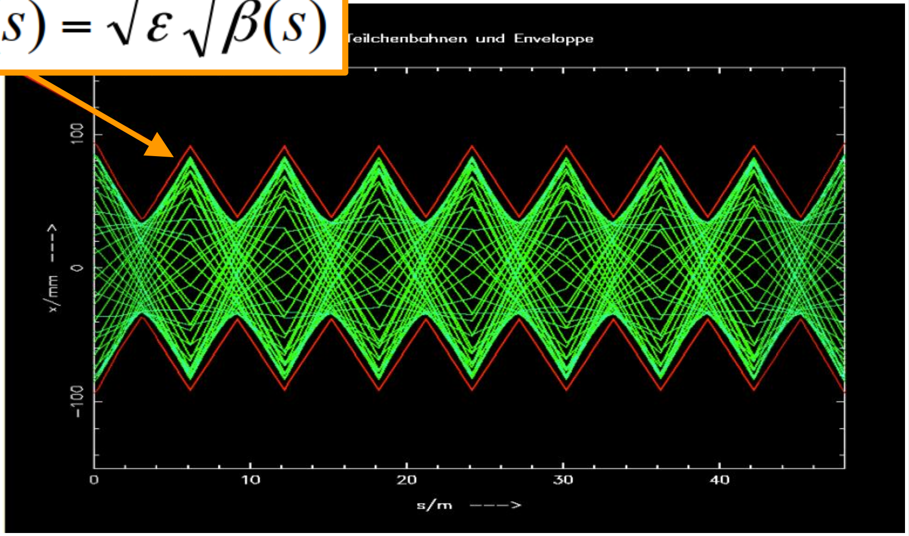
Emittance ε is constant of motion (independent of *s*). It is an intrinsic beam parameter that cannot be changed by the focusing properties.

β(s) is the “beta function” (or “envelope function”). It is determined by the focusing properties of the lattice (optics).

Together, ε and β(*s*) determine the beam envelope or beam size at a position *s*.

$$\beta(s+L) = \beta(s)$$

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



B. Holzer, Introduction to Transverse Beam Dynamics, CAS 2016

Twiss Parameters

- Based on the solution of Hill's equation, one can derive the parametric representation of an ellipse in the (x, x') space:

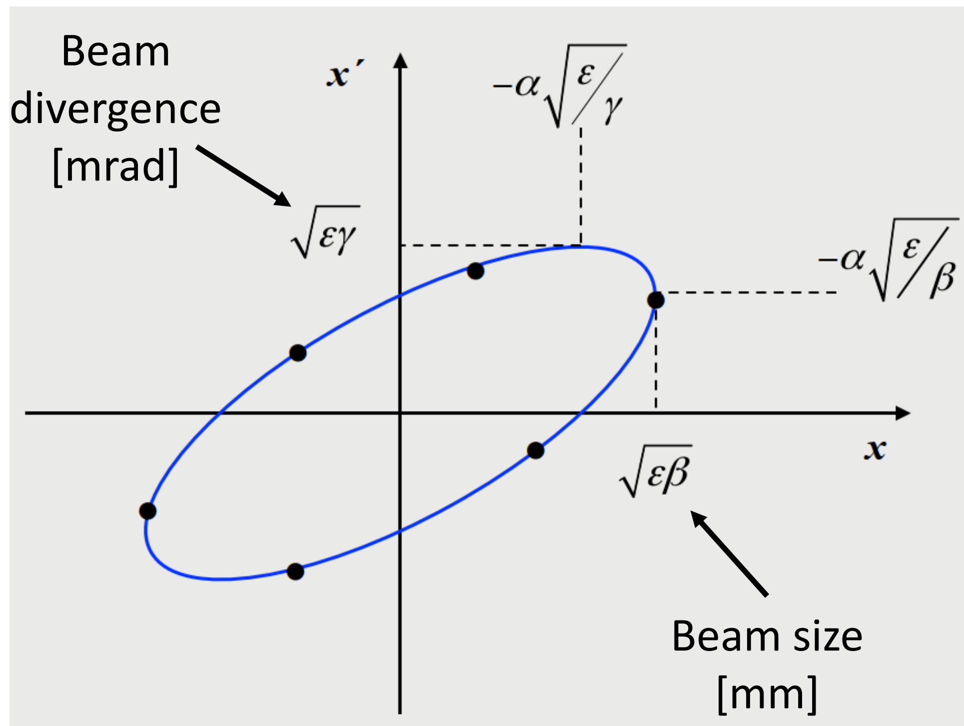
$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

- Shape and orientation of the phase ellipse are given by the Courant-Snyder or Twiss parameters α, β, γ.

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

- Given the Twiss parameters at any point in a known optical lattice, we can transform them and calculate their values at any other point in the ring.

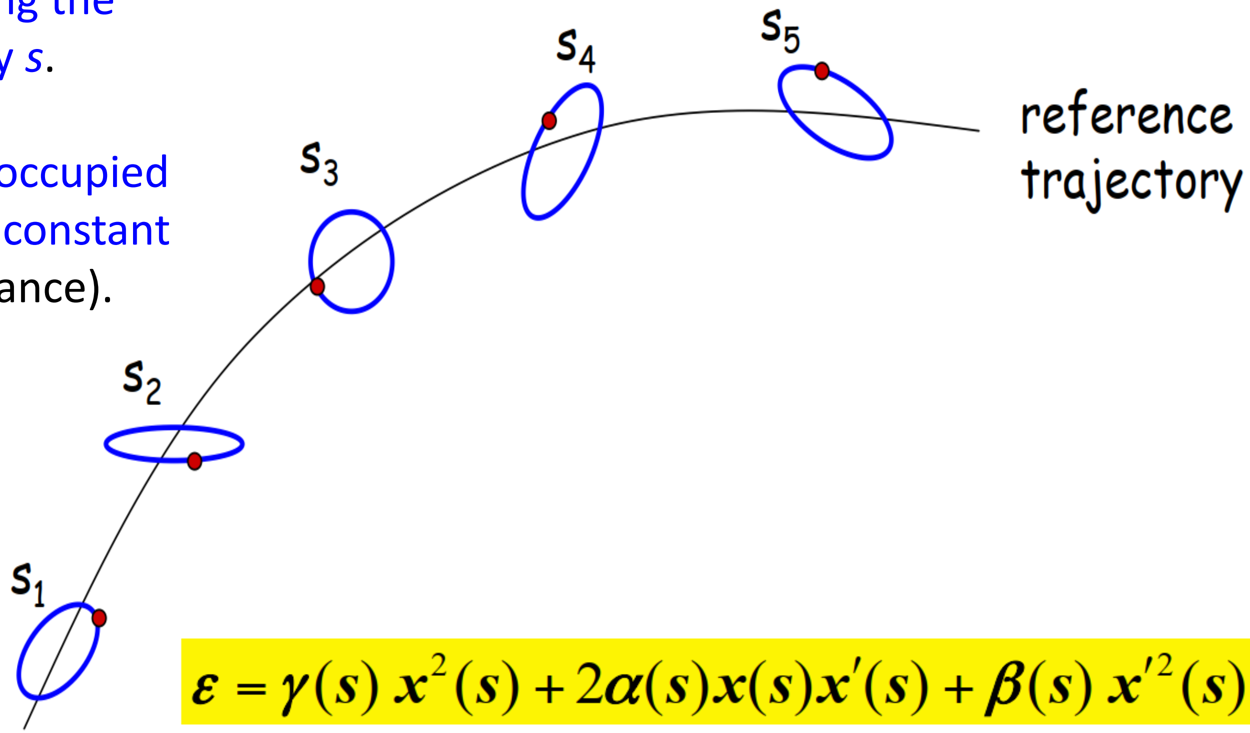


B. Holzer, Introduction to Transverse Beam Dynamics, CAS 2016

Twiss Parameters II

- Particles move on ellipses (given linear forces), but the orientation and shape of the ellipse changes along the reference trajectory *s*.

- However, the area occupied by the ellipse stays constant (given by the emittance).

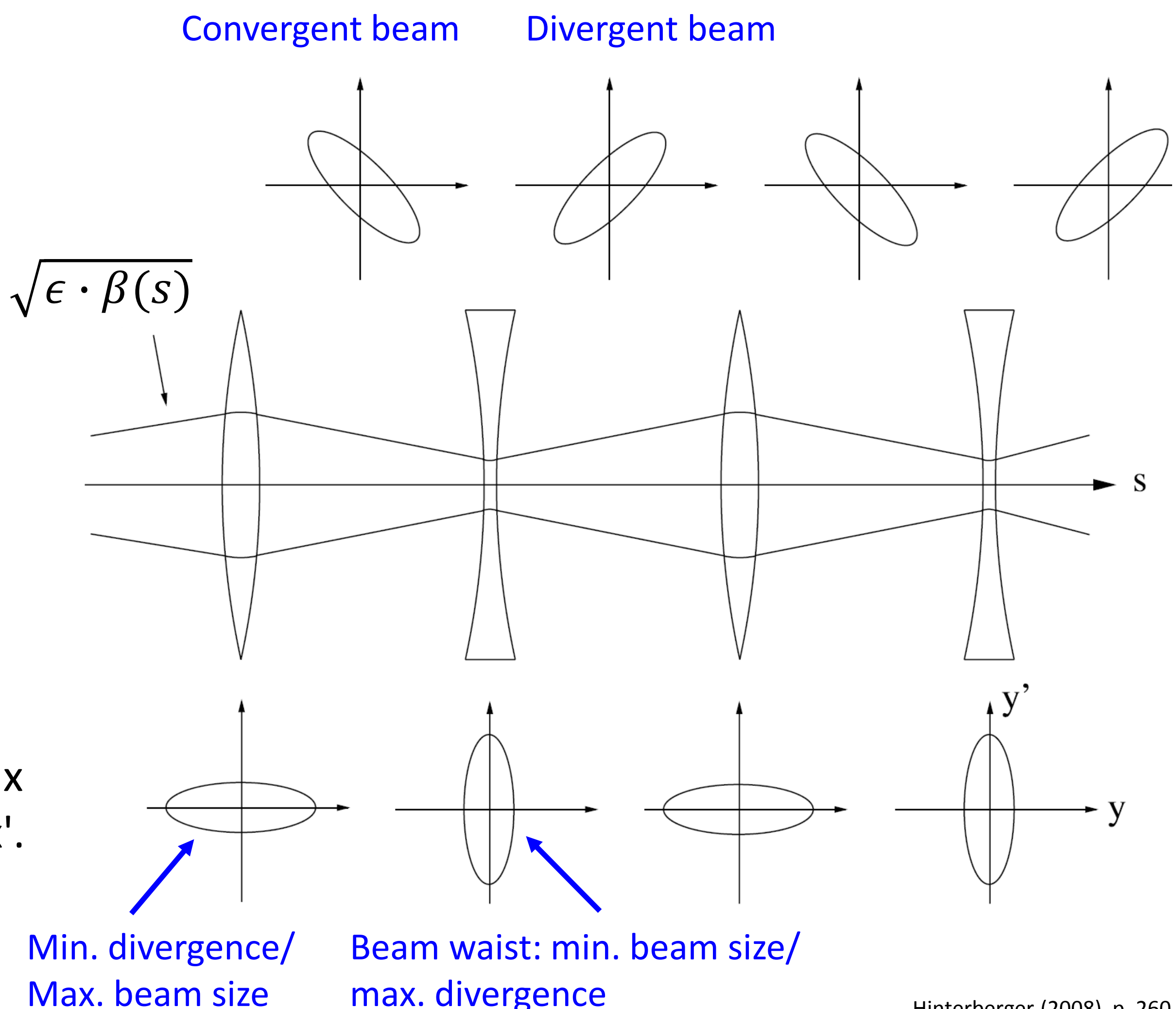


$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$

Phase Ellipse and Beam Properties

The form and orientation of the phase ellipse gives important information about the beam properties.

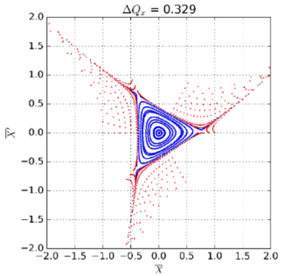
Small beam size x
→ large angles x'.

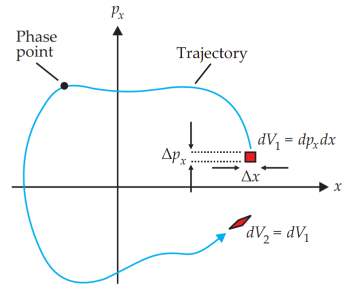


Hinterberger (2008), p. 260.

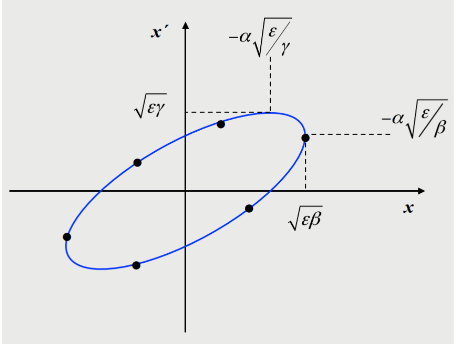
Conclusions

- A particle **beam** is a particle ensemble with a much higher longitudinal than transverse velocity.
- Dynamic behaviour is described using concept of phase space.
  - 6d phase space: (x, y, z, p_x, p_y, p_z)
  - Transverse (4d) phase space: (x, x', y, y')
- Linear forces → point moves on an ellipse in phase space.
- **Liouville's theorem**: The (6d) phase space volume is conserved (under certain conditions). For uncoupled motion in the perpendicular planes, also the 2d phase space area is conserved.
- Occupied area in 2d phase space is quantified by the **emittance**.
- In an accelerator, the restoring force $k(s)$ is s-dependent → Solve Hill's equation.
- Shape and orientation of the phase ellipse are given by the Courant-Snyder or Twiss parameters α, β, γ. They are used to describe the beam dynamics along the lattice.
- The beta function $\beta(s)$ is determined by the focusing properties of the lattice (optics).
- Beam size is given by $\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$





$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$



Questions?
Comments?

"Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a much higher level." (Enrico Fermi)

Next Mini Lecture: Wednesday, 19.06.2019, 10.30h, Room 30-6-19

Liouville’s Theorem

Given a Hamiltonian system $H(q^j, p_j, t)$ of n degrees of freedom, the volume form $dV = dq^1 \dots dq^n dp_1 \dots dp_n$ is invariant with respect to canonical transformations.

Canonical transformations are transformations that preserve the form of the canonical equations

$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \tag{2.1}$$

and

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q^i}, \qquad \text{for } i = 1 \dots n. \tag{2.2}$$

[REISER 1994, 62-65; STRUCKMEIER 2006, 4]

RMS Emittance

Based on a statistical approach, the RMS (root mean square) emittance is defined as:

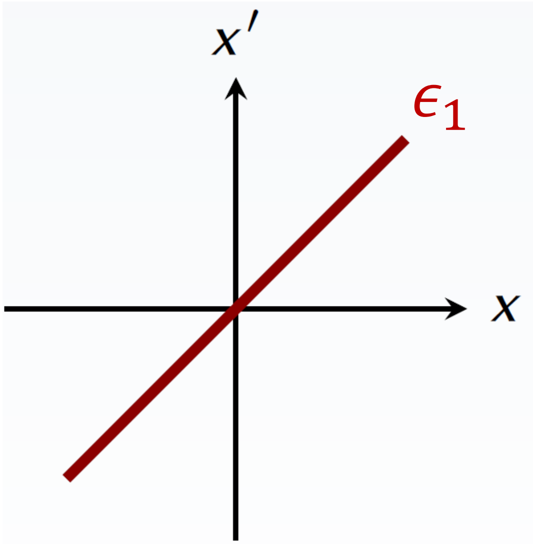
$$\overline{xx'} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})(x'_i - \overline{x'})$$

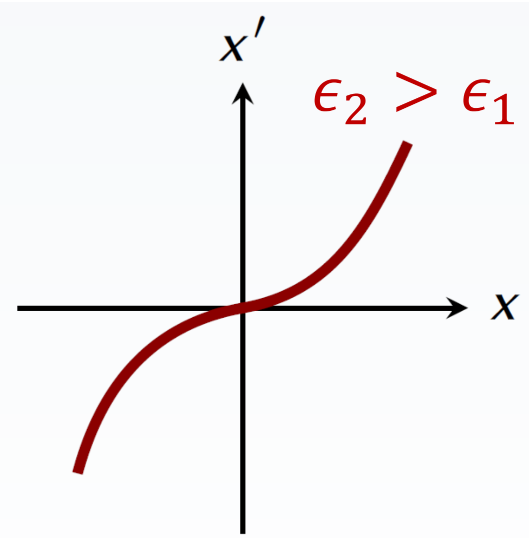
$$\epsilon_{\mathrm{rms}} = \sqrt{\overline{x^2}\;\overline{x'^2} - \overline{xx'}^2}$$

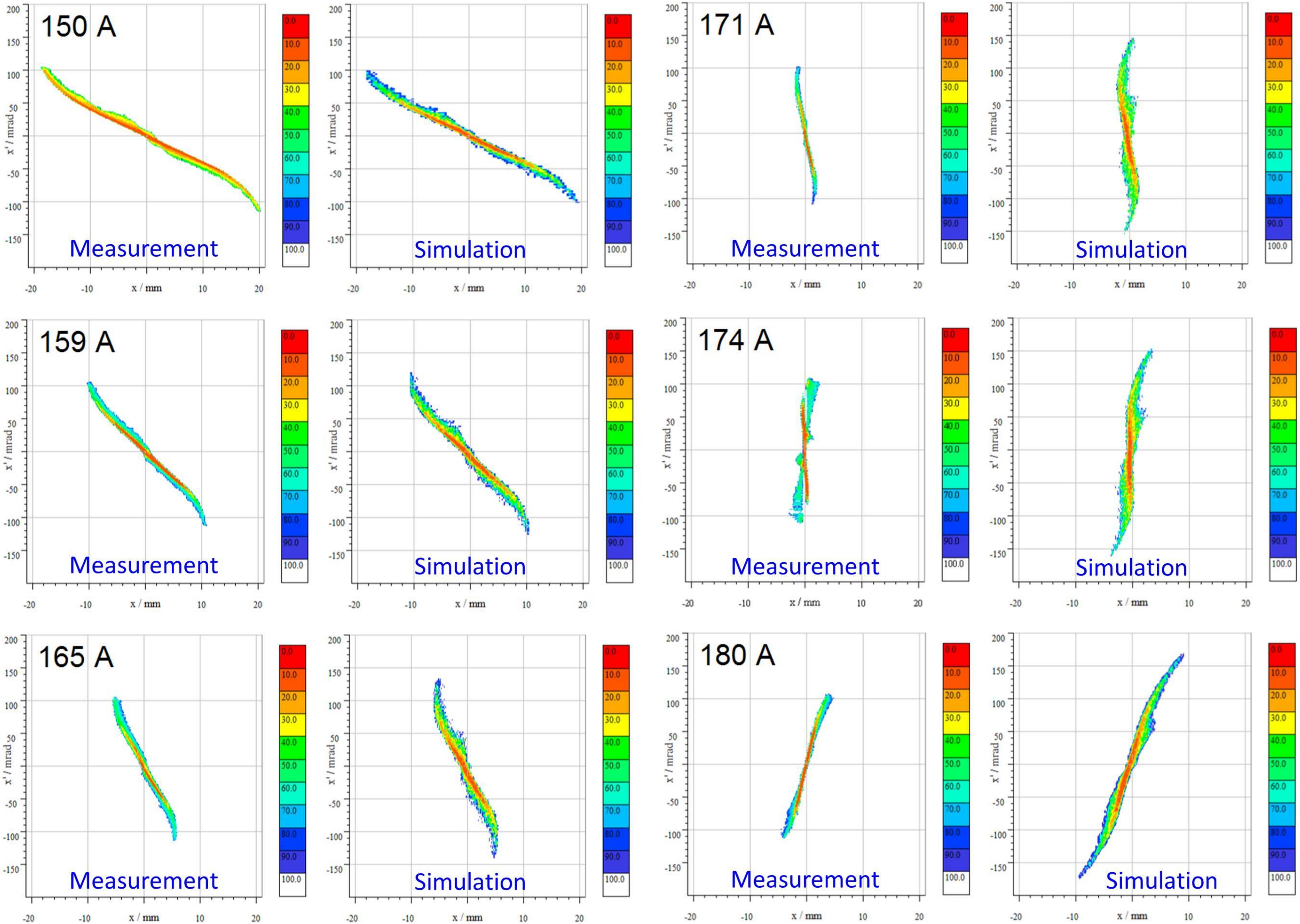
$$\overline{x^2} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2 \qquad \overline{x'^2} = \frac{1}{N}\sum_{i=1}^{N}(x'_i - \overline{x'})^2$$

The RMS emittance depends not only on the occupied phase space area, but also on the shape of the particle density distribution.

Example:







150 A
Measurement
Simulation
159 A
Measurement
Simulation
165 A
Measurement
Simulation
171 A
Measurement
Simulation
174 A
Measurement
Simulation
180 A
Measurement
Simulation
x' / mrad
x / mm
200
150
100
50
-50
-100
-150
-20
-10
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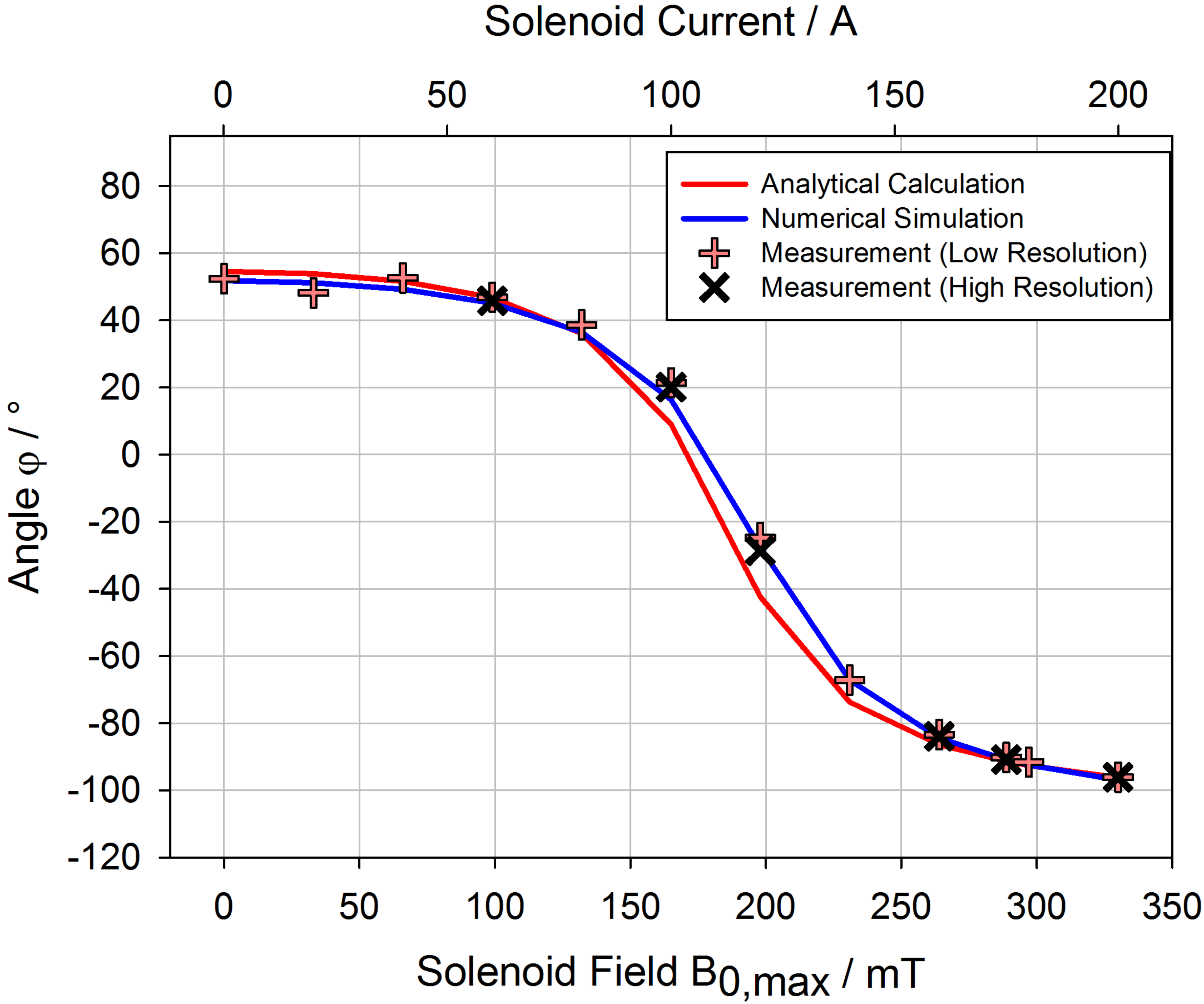
Solenoid Current / A
0
50
100
150
200
80
60
40
20
0
-20
-40
-60
-80
-100
-120
Angle φ / °
Analytical Calculation
Numerical Simulation
Measurement (Low Resolution)
Measurement (High Resolution)
0
50
100
150
200
250
300
350
Solenoid Field $B_{0,max}$ / mT

Mini Lectures: Proposed Topics

Beam & accelerator physics

How to describe a particle beam? 1)
- Phase-space, Liouville theorem, emittance, optical functions (α, β, γ), σ

How do accelerators work? 2)
- Beams production: ion sources
- Beam transport, FODO lattice
- Beam acceleration: linacs and acc. cavities 5)
- Beam collision: synchrotron, collider, luminosity, β^*
- Acc. hardware: beam dump, cavities, …

What types of magnets do we need? And how do we get them? 3)
- Dipoles, quadrupoles, and more: beam-dynamics and hardware realization
- Kicker and septa

Magnets

How do superconducting acc. magnets work? 4)
- Basics of superconductivity
- Basics of superconducting magnet and cable design
- Why use superfluid helium?

Why and how to protect a s.c. magnet?
- How to quench a s.c. magnet?
- How to protect a s.c. magnet?
- Quench/damage limits

What can go wrong? Beam-related failures
- Failure classification (risk, slow/fast/ultrafast failures)
- Failure examples: magnet powering, injection/extraction failures, UFOs, QH firing
- Failure criticality for different machines

How does the CERN accelerator complex work?
- Injectors: LINACs, PSB, PS, SPS
- LHC operation and cycle
- LHC availability and faults

Reliability and availability 6)
- Basic definitions (for CERN and other accelerators)
- Introduction to risk assessment
- Lifetime distributions and bathtub curve

Reliability & Availability

What happens if the beam is lost?
- Beam-matter interaction
- Hydrodynamic tunnelling

MP Systems
- Main MP systems at LHC (BIS, PIC, WIC, QPS, LBDS, COLL)
- Electronics for MP

Machine Protection

Special Topics… Visits…

Computational Methods

- Coding conventions and good practice / Object-oriented programming 0)
- Basics of co-simulation
- Introduction to machine learning
- How to simulate a particle beam? How to simulate a magnetic field?