Cosmological predictions from infinite classical action

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- Integration over "geometries"

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Finite Action Principle- field configurations with infinite classical action do not contribute to the path integral [Barrow and Tipler, 1988].

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- Unitary gravity realization [JC and Kwapisz, 2021]

Ghost appear in the quadratic gravity making it non-unitary. The propagator is proportional to:

$$R \to \frac{1}{k^2},$$
$$R^{\mu\nu}R_{\mu\nu} \to \frac{1}{k^2} - \frac{1}{k^2 - G_N^{-1}}.$$

The pole corresponds to a massive ghost with negative kinetic energy. This is a direct consequence of Ostrogradsky's theorem, since $R^{\mu\nu}R_{\mu\nu}$ is quartic in time derivatives.

Avoid Ostrogradsky's ghost by including only second-order time derivatives in the gravitational Lagrangian.

Regulate UV behavior of the theory with higher-order *spacial* curvature scalars.

Restore Lorentz Invariance in the IR limit.

Direct breaking of the Lorentz Invariance must be performed with a great care. Anisotropic scaling:

$$t \to b^{-z} t$$
$$x^i \to b^{-1} x^i.$$

The mass-dimension of the space and time is different:

$$[t] = -z, \quad [x^i] = -1.$$

Lorentz Invariance is restored if $z = 1$. For $z \geq d$ the theory becomes power-counting renormalizable.

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The spacetime is foliated, and the full diffeomorphism invariance is replaced by foliation-preserving diffeomorphisms:

$$t \to \xi^0(t)$$
$$x^i \to \xi^i(x^j, t)$$

Building blocks of the Lagrangian are constructed from the ADM-decomposed metric tensor:

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N_i N^i & N_i \\ N_i & g_{ij} \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N_i}{N^2} \\ \frac{N_i}{N^2} & g^{ij} - \frac{N^i N_i}{N^2} \end{pmatrix}.$$

N-lapse function, N_i- shift vector.

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The theory is composed of scalars constructed with:

$$R_{ij}, \quad K_{ij}, \quad a_i, \quad \nabla_i,$$

where $a_i = \frac{N_{,i}}{N}$, while R_{ij}, ∇_i are respectively, Ricci curvature and covariant derivative of g_{ij}. The extrinsic curvature tensor K_{ij} is defined as:

$$K_{ij} := \frac{1}{2N}\Big(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i \Big)$$

Curvature scalars become singular at $r = 0$ for black-hole spacetimes (Kerr, Schwarzschild). Finite Action Principle dynamically removes these singularities.

This is also the case for H-L gravity already in second-order of curvature scalars. There are additional complications, absent in GR.

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- There are no known solutions of regular black holes in H-L gravity
- Stable, traversable wormholes solve H-L field equations with finite action and similar asymptotic behavior to black holes.

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Conclusions

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- Only finite action field configurations contribute the path integral
- This leads to a homogeneous, isotropic beginning of the universe and resolves the black hole singularity problem in the R^2 gravity.
- Ostrogradsky's ghosts may be avoided by considering Lorentz Invariance violating theory- Horava-Lifshitz gravity.
- Big Bang is possible in H-L theories with maximally second-order in curvature scalars.
- Finite Action Principle indicates that H-L black holes are replaced by wormhole spacetimes.

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