Electroweak Phase Transition with an SU(2) Dark Sector

In collaboration with Tathagata Ghosh, Huai-ke Guo, Tao Han

[arXiv:2010.12109]

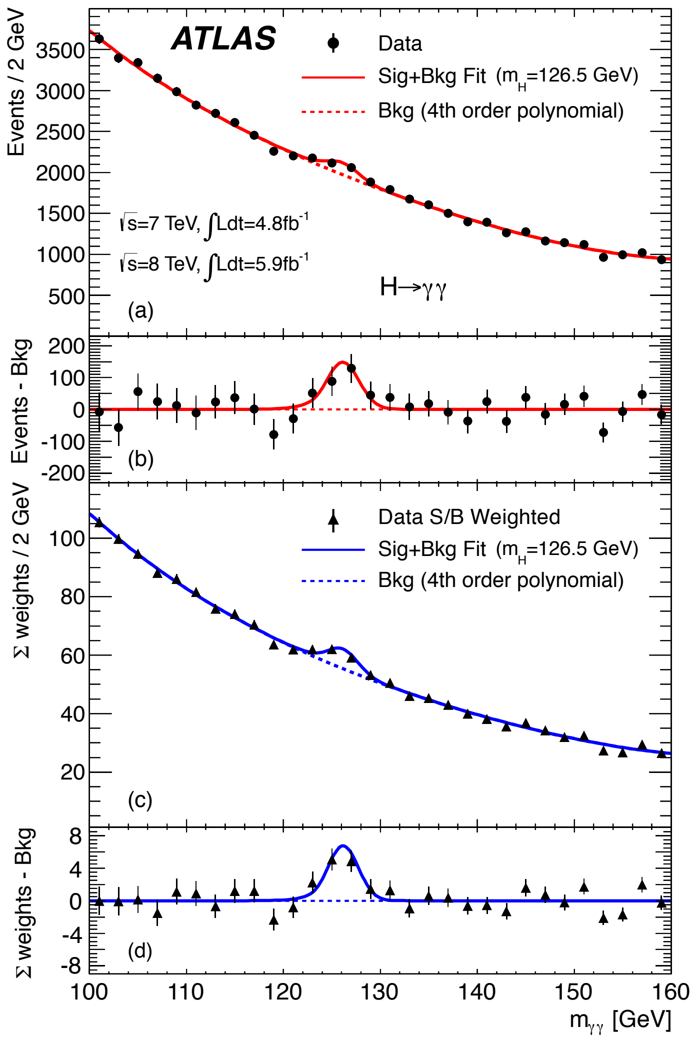
Hongkai Liu

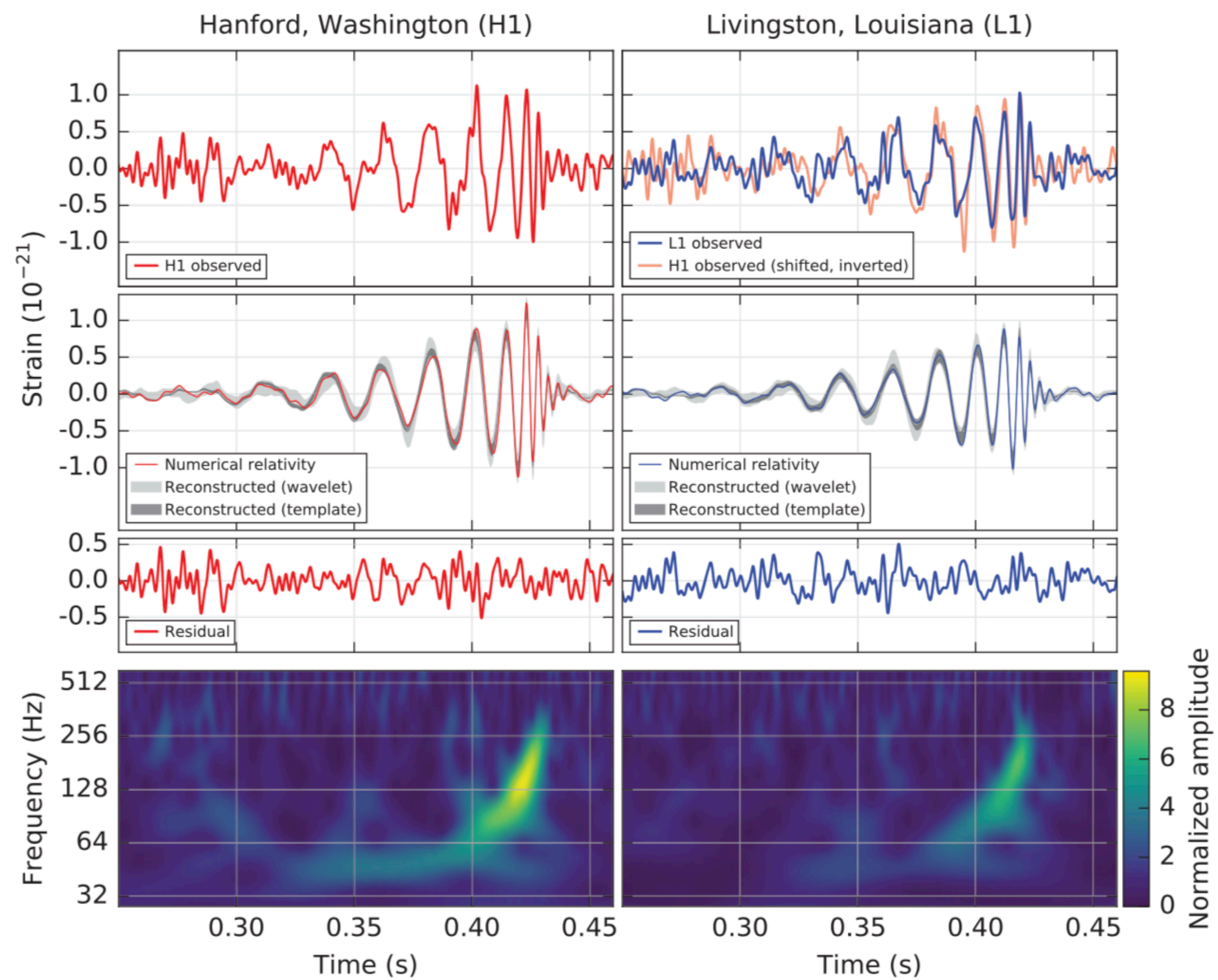
University of Pittsburgh

PCC 2021
May 20. 2021

Two big events in the past decade

- The discover of Higgs boson in 2012 completed the SM
- Gravitational waves predicted by GR was observed in 2015

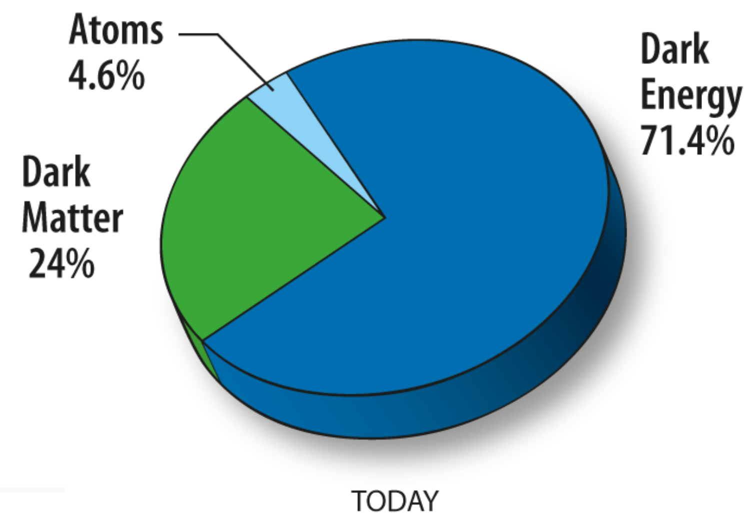




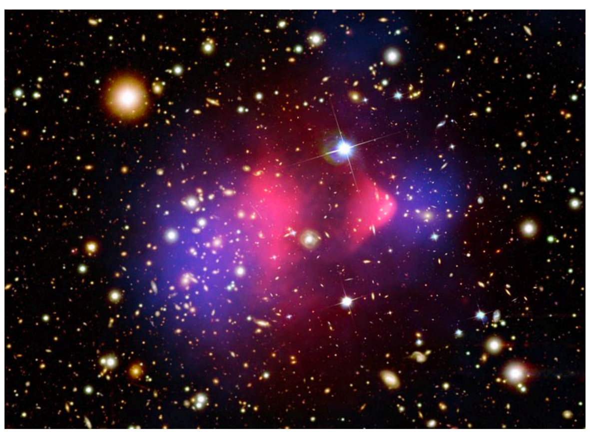
[arXiv: 1207.7214] **[arXiv: 1602.03837]**

Unsolved questions

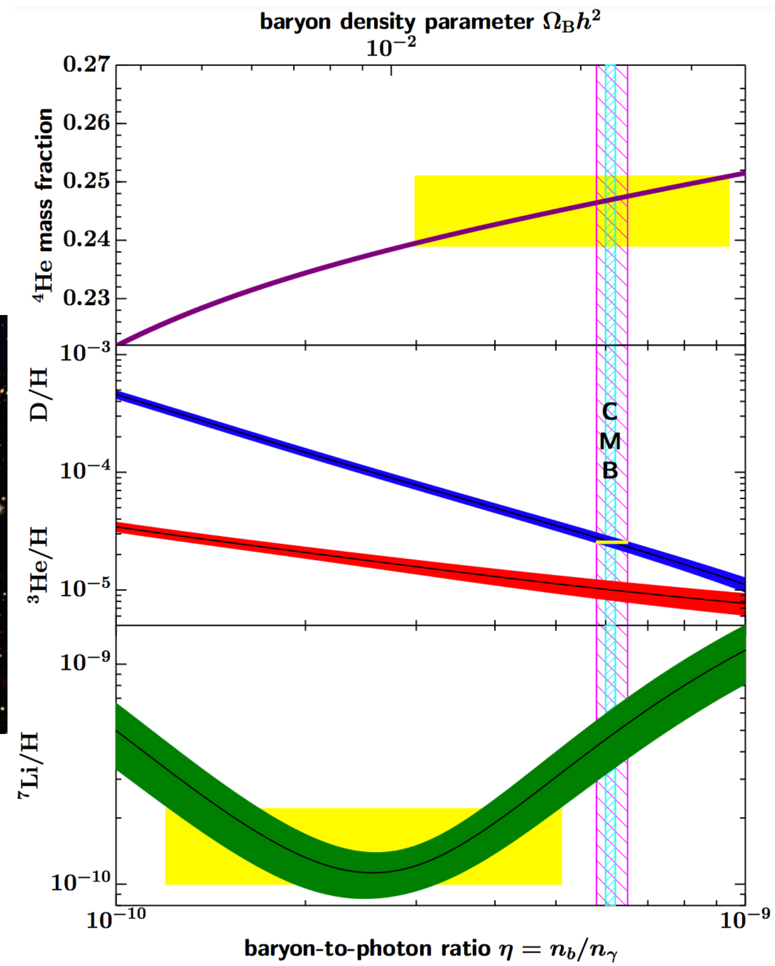
- Dark matter
- Baryon asymmetry
- Neutrino masses
- …



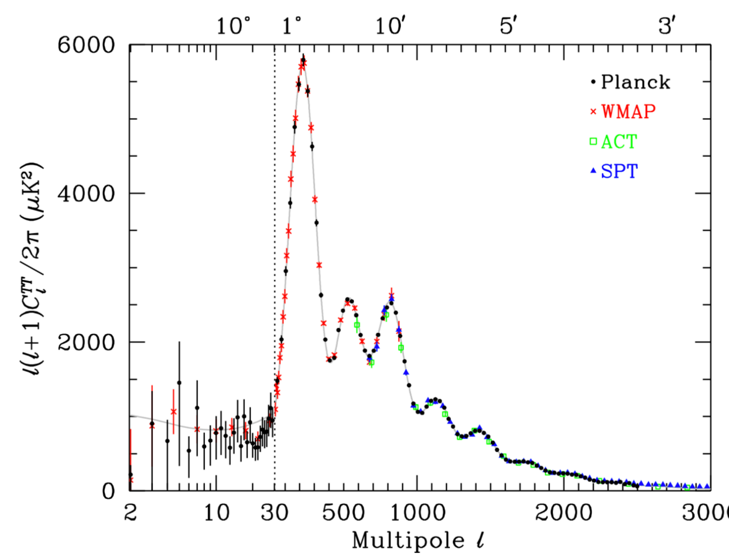
[NASA]



[NASA]

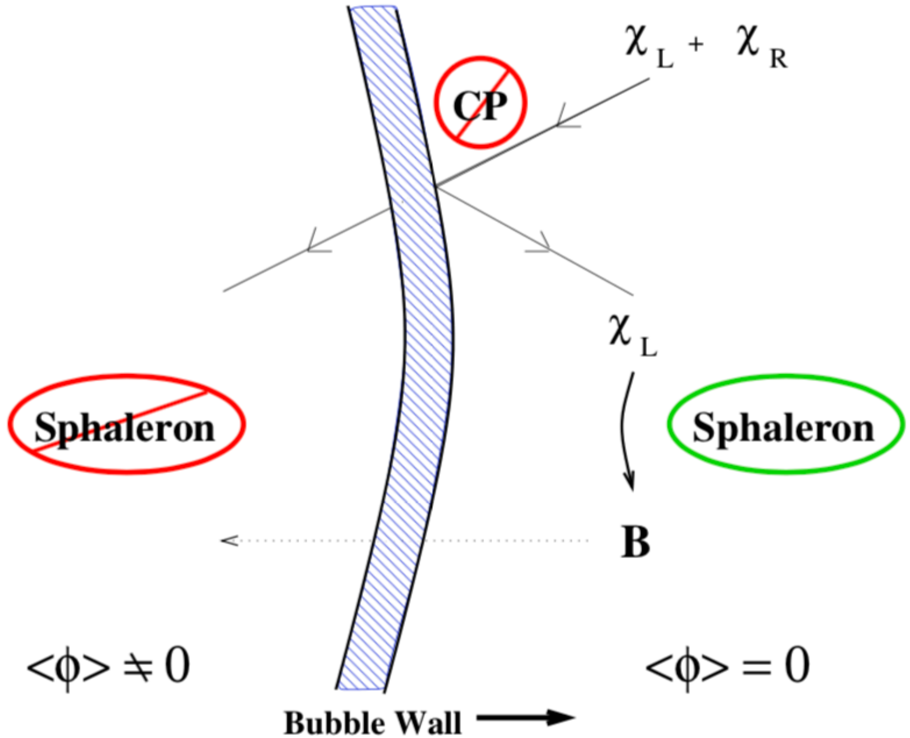


[PDG]



[PDG]

Electroweak baryogenesis



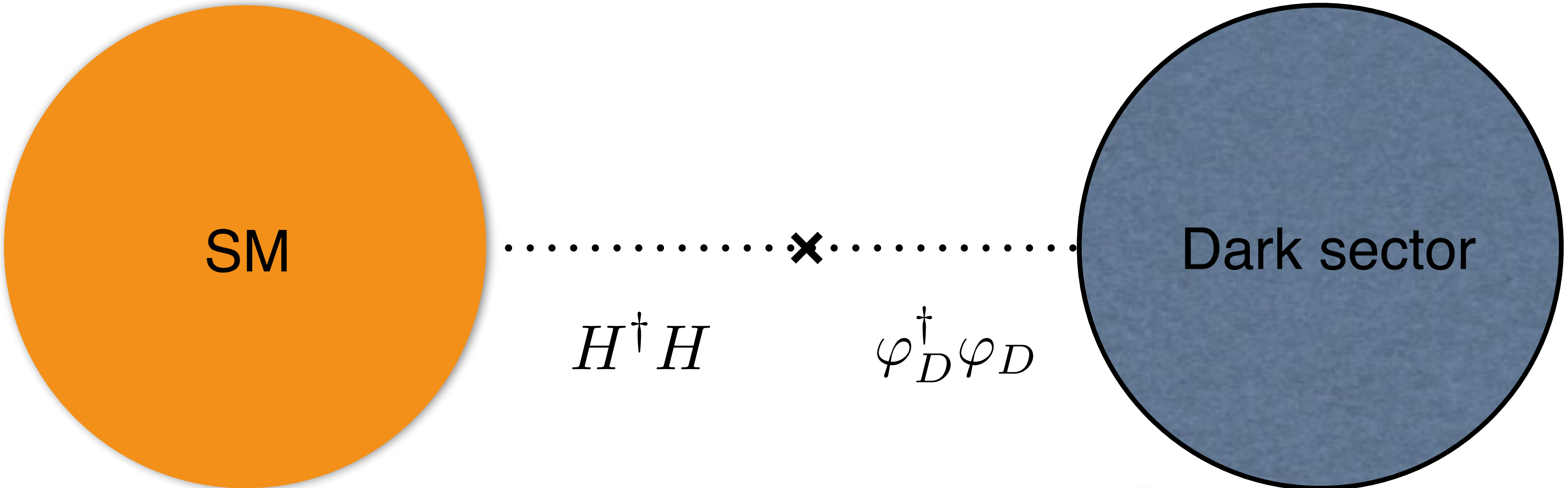
[arXiv:1206.2942]

Problems in the SM:

- First-order electroweak phase transition is precluded in the SM [hep-lat/9510020] [hep-ph/9809291]
- Not enough CP violation [hep-ph/9302210]

To alter the EW phase transition pattern, new physics must couple to the Higgs field

SU(2) dark sector and Higgs portal



$$H^{\mathrm{T}} = (G^+, (v_h + h_0 + iG_0)/\sqrt{2})$$

Dark sector = dark SU(2) gauge sector + dark scalar sector

Dark scalar sector consists of two scalar multiplets: $\Phi_1,\ \Phi_2$

$$\Phi_1 = \begin{cases} \mathrm{ST} & \frac{1}{\sqrt{2}}(v_1 + \omega) \\ \mathrm{TT} & \frac{1}{\sqrt{2}}(\omega_1,\ \omega_2,\ v_1 + \omega_3)^{\mathrm{T}} \end{cases}, \quad \Phi_2 = \frac{1}{\sqrt{2}}(\varphi_1,\ v_2 + \varphi_2,\ \varphi_3)^{\mathrm{T}}$$

Lagrangian

Total Lagrangian can be written as

$$\begin{aligned}
\mathscr{L} &= \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm portal} + \mathscr{L}_{\rm DS}, \\
-\mathscr{L}_{\rm SM} &\supset V_{\rm SM} = m_H^2|H|^2 + \frac{\lambda_H}{2}|H|^4, \\
-\mathscr{L}_{\rm portal} &\supset V_{\rm portal} = \lambda_{H11}|H|^2|\Phi_1|^2 + \lambda_{H22}|H|^2|\Phi_2|^2, \\
\mathscr{L}_{\rm DS} &= -\frac{1}{4}\tilde{W}^a_{\mu\nu}\tilde{W}^{a\mu\nu} + |D_\mu\Phi_1|^2 + |D_\mu\Phi_2|^2 - V_{\rm DS},
\end{aligned}$$

Z2 symmetries are imposed: $\Phi_1 \to -\Phi_1,\ \Phi_2 \to -\Phi_2$

$$V_{\rm DS} = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2$$

We set $v_1 = 0$ to partially break dark SU(2) into dark U(1)

$$\Phi_1 = \frac{1}{\sqrt{2}}(\omega_1,\, \omega_2,\, v_1 + \omega_3), \quad \Phi_2 = \frac{1}{\sqrt{2}}(\varphi_1,\, v_2 + \varphi_2,\, \varphi_3)$$

- φ_1 and φ_3 are eaten by dark gauge bosons to form two massive vector DM
- $\omega_1,\ \omega_2,$ and ω_3 are three massive scalar DM
- φ_2 mixes with SM Higgs serving as a portal to the dark sector

Higgs signal rate

Higgs-SM interactions

$$\mathscr{L} \supset \frac{h_1 \cos\theta - h_2 \sin\theta}{v_h}\left(2m_W^2 W_\mu^+ W^{\mu-} + m_Z^2 Z_\mu Z^\mu - \sum_f m_f \bar{f} f\right)$$

- Compared with the SM case, the Higgs couplings are universally suppressed by $\cos\theta$

Higgs signal strength

$$\mu_{h_1} \equiv \frac{\sigma_{h_1}\ \mathrm{BR}(h_1 \to \mathrm{SM})}{\sigma_{h_1}^{\mathrm{SM}}\ \mathrm{BR}^{\mathrm{SM}}(h_1 \to \mathrm{SM})}$$

$$\sigma_{h_1} = \cos^2\theta\,\sigma_{h_1}^{\mathrm{SM}} \qquad \mathrm{BR}(h_1 \to \mathrm{SM}) = \frac{\Gamma_{h_1}^{\mathrm{SM}} \cos^2\theta}{\Gamma_{h_1}^{\mathrm{SM}} \cos^2\theta + \Gamma_{h_1}^{\mathrm{DS}}} \qquad \mathrm{BR}^{\mathrm{SM}}(h_1 \to \mathrm{SM}) \equiv 1$$

$$\Rightarrow \quad \mu_{h_1} = \frac{\Gamma_{h_1}^{\mathrm{SM}} \cos^4\theta}{\Gamma_{h_1}^{\mathrm{SM}} \cos^2\theta + \cancel{\Gamma_{h_1}^{\mathrm{DS}}}} \approx \cos^2\theta$$

Current bound from Higgs couplings measurements

$$|\sin\theta| < 0.35$$

[arXiv:1509.00672]

Higgs phenomenology

- High mass region

  W mass corrections

  [arXiv:1406.1043]

- Medium mass region

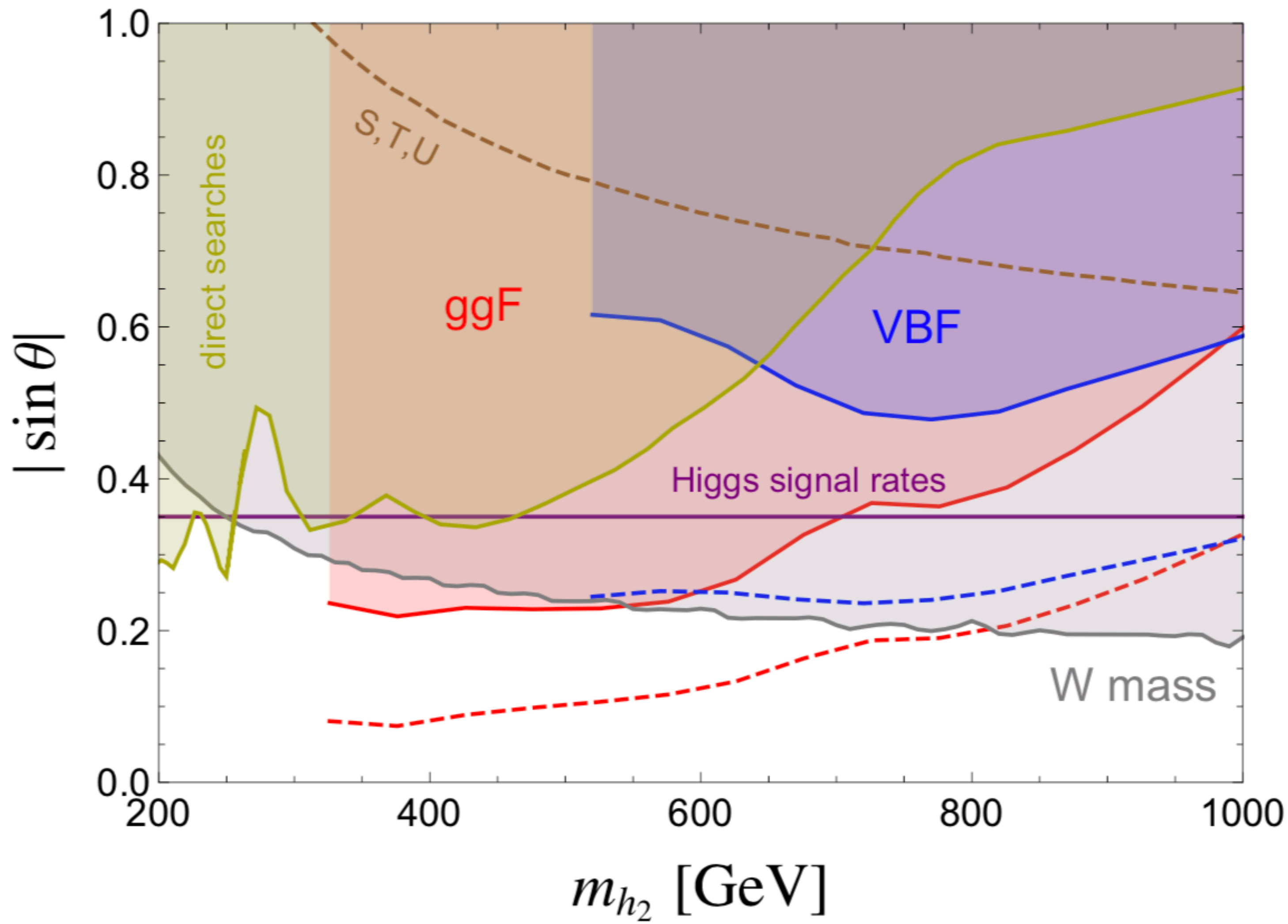
  Di-boson searches (ggF and VBF)

  [arXiv:1808.02380]

- Low mass region

  LEP and LHC ($\sqrt{s} = 7\ \mathrm{TeV}$)

  [arXiv:1502.01361]



Benchmark points

Criteria:

(1) Vacuum stability, partial wave unitarity, and electroweak precision measurements.

(2) Higgs, DM and DR phenomenology bounds.

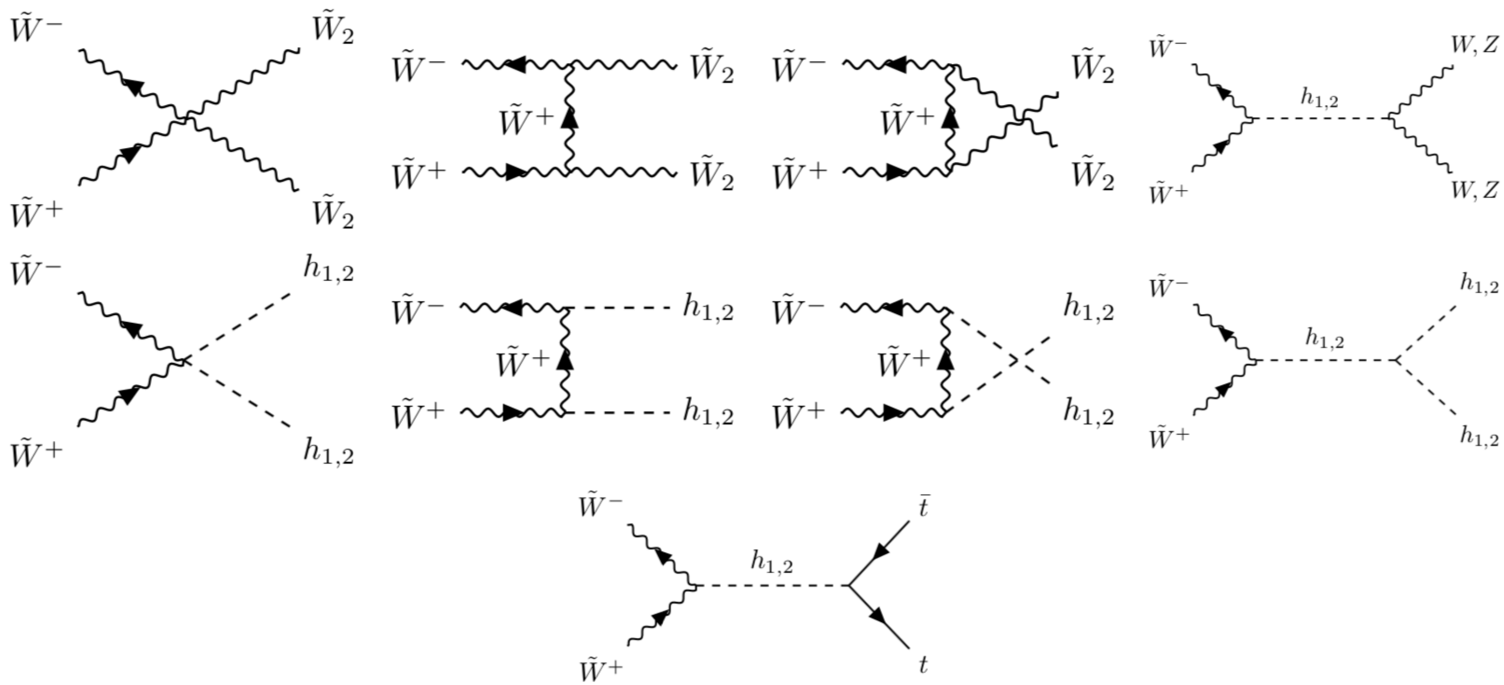
(3) Strong first-order phase transition and produce strong GW signal.

DM Masses ordering

$$m_{\tilde{W}^+} \ll m_{\omega^+} \lesssim m_{\omega_2}$$

Parameters	BM1	BM2
$\sin\theta$	-0.25	-0.12
$\tilde{g}$	0.094	0.133
$m_{\tilde{W}^\pm}$	94 GeV	133 GeV
m_{h_2}	200 GeV	290 GeV
$m_{\omega^\pm}$	1.2 TeV	1.3 TeV
m_{ω_2}	2.0 TeV	1.9 TeV
λ_1	3.5	3.5
λ_{H11}	2.0	2.0
λ_3	3.0	3.5
λ_H	0.28	0.27
λ_2	3.8×10^{-2}	8.3×10^{-2}
λ_{H22}	2.4×10^{-2}	3.2×10^{-2}
λ_4	5.0	4.0
v_2	1 TeV	1 TeV

Dark matter annihilation

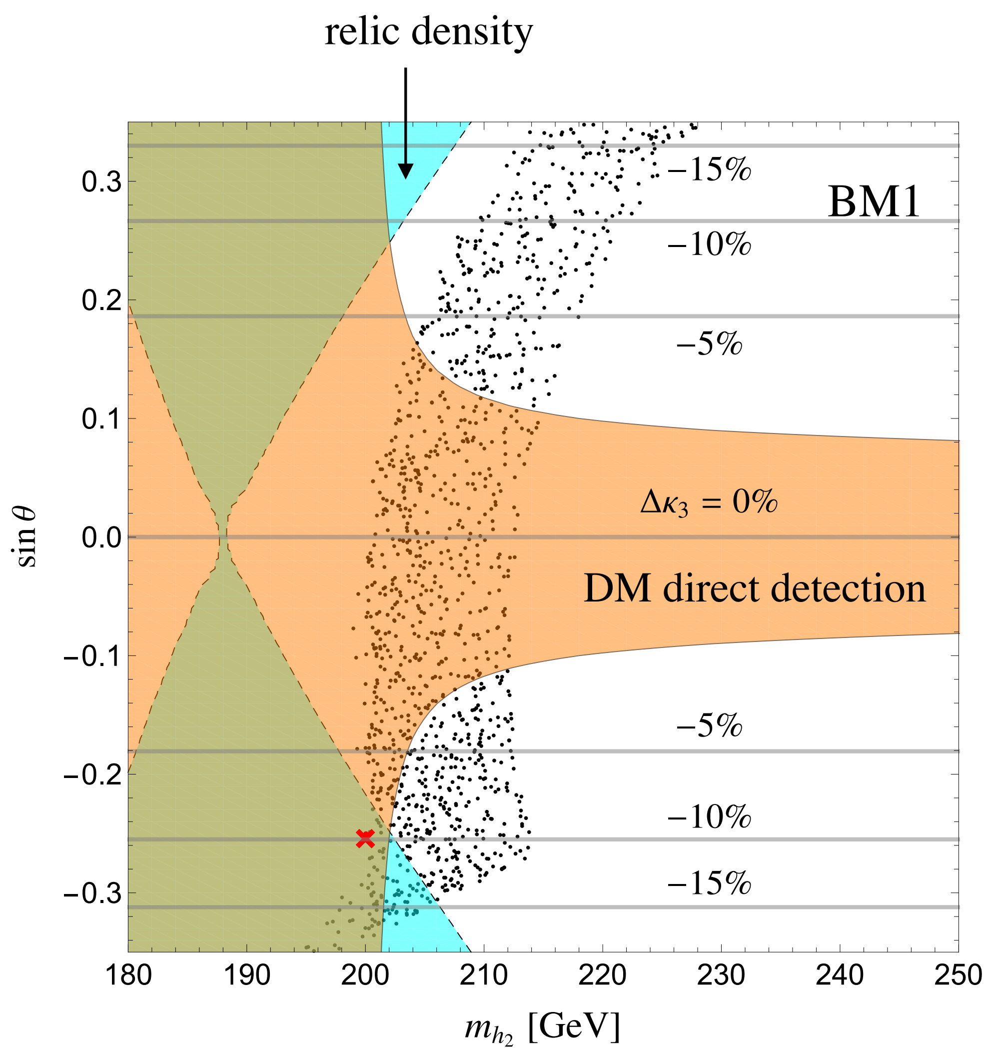


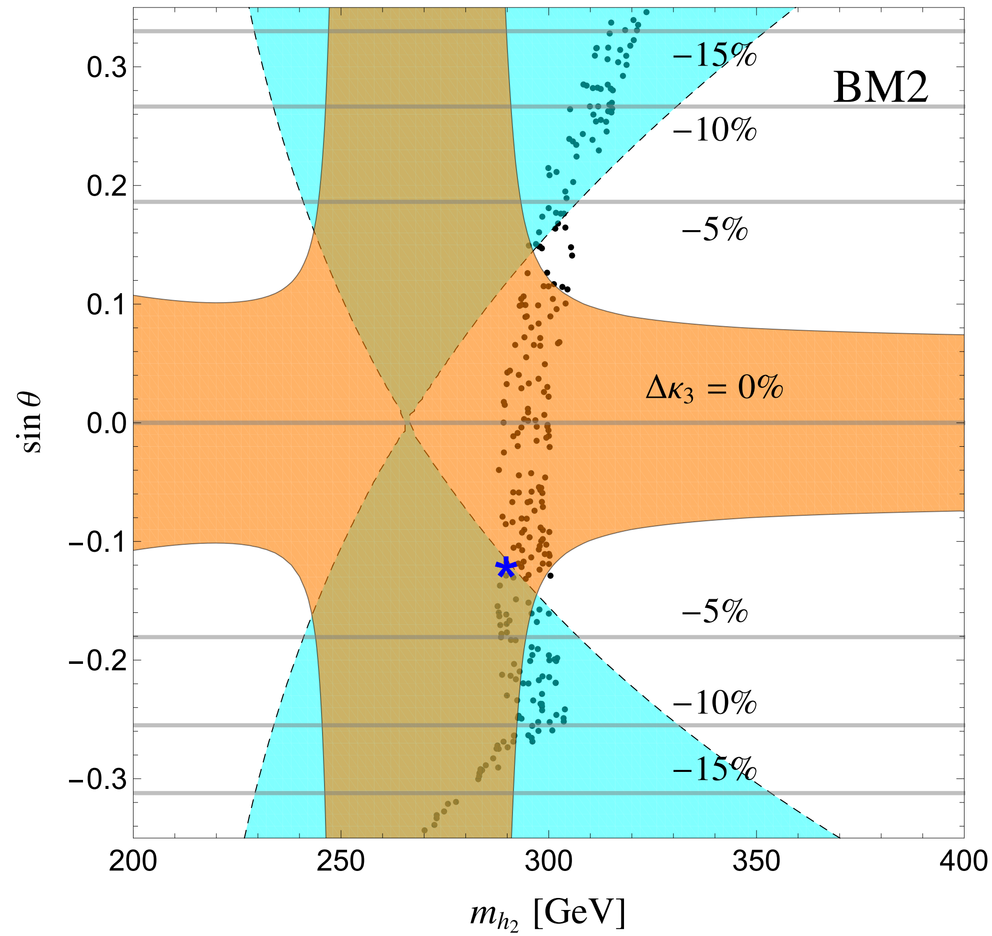
$$\tilde{W}^+\tilde{W}^- \to W^+W^-,\ ZZ,\ \bar{t}t,\ h_1h_1,\ h_1h_2,\ h_2h_2,\ \tilde{W}_2\tilde{W}_2,$$

$$\omega^+\omega^- \to W^+W^-,\ ZZ,\ \bar{t}t,\ h_1h_1,\ h_1h_2,\ h_2h_2,\ \tilde{W}^+\tilde{W}^-, \tilde{W}_2\tilde{W}_2,$$

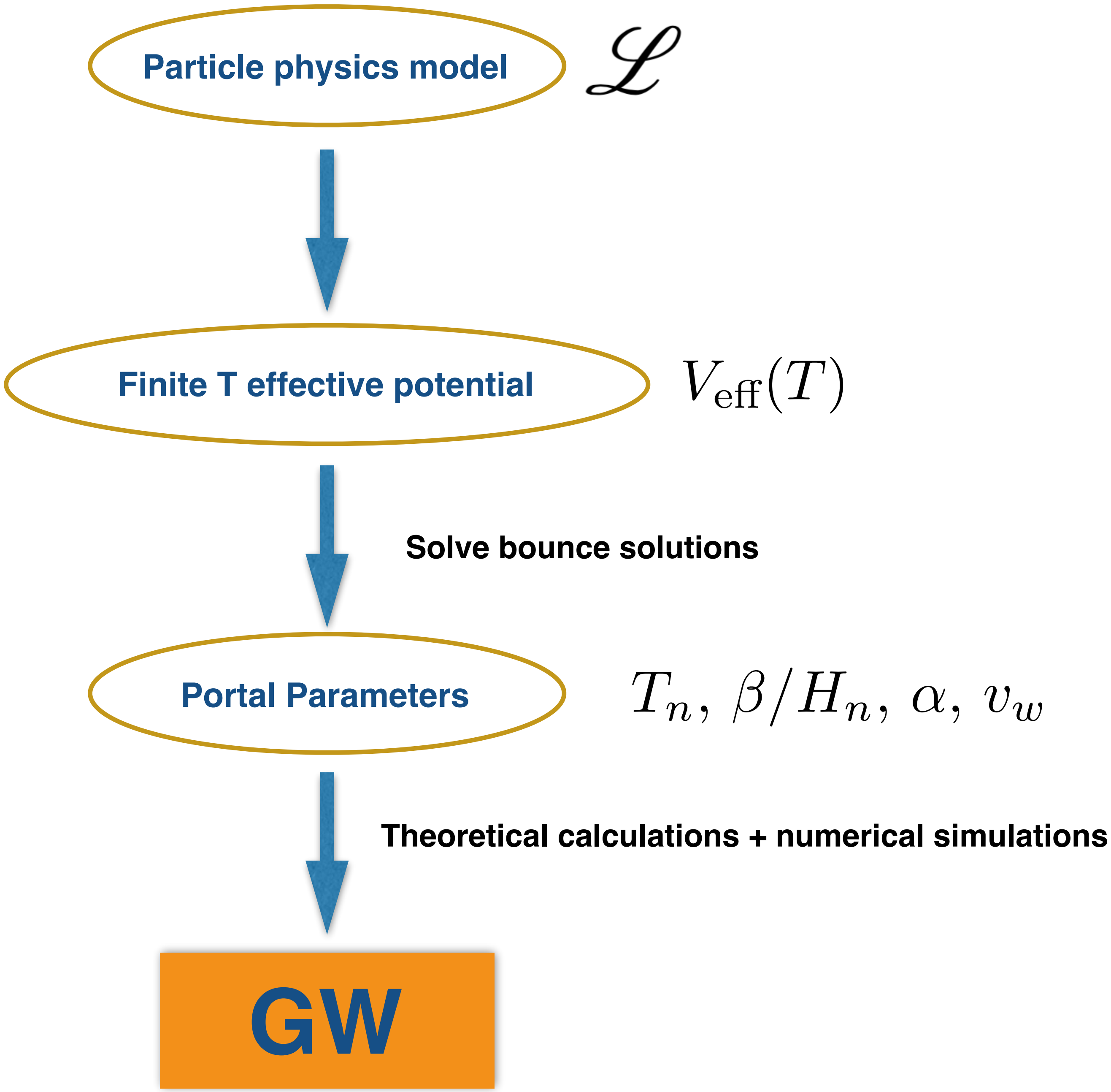
$$\omega_2\omega_2 \to W^+W^-,\ ZZ,\ \bar{t}t,\ h_1h_1,\ h_1h_2,\ h_2h_2,\ \tilde{W}^+\tilde{W}^-,\ \omega^+\omega^-.$$

Phenomenology constraints





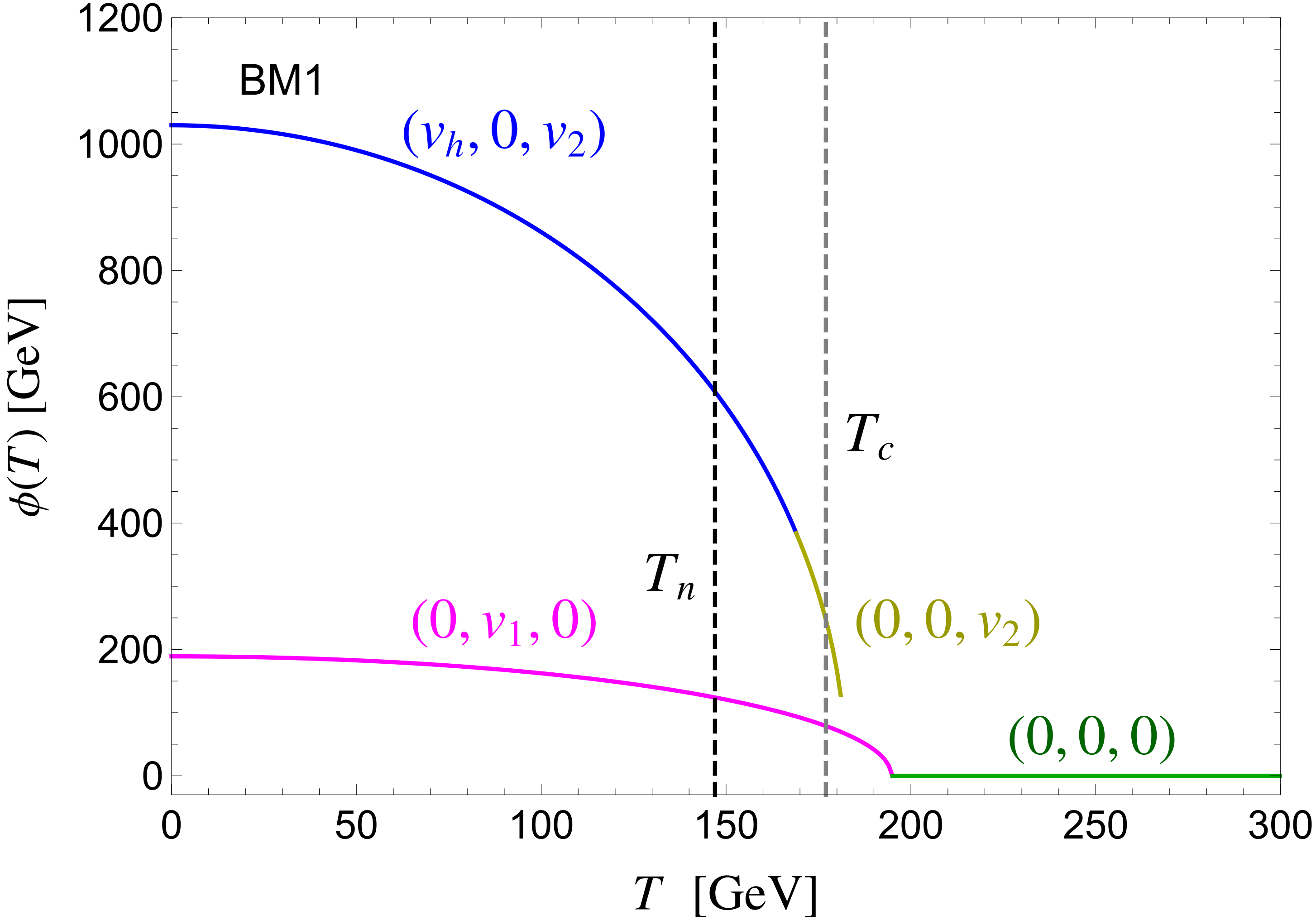
Procedure of gravitational wave calculations



Phase transition pattern

Two-step

$$\phi \equiv \sqrt{v_h^2 + v_1^2 + v_2^2}$$



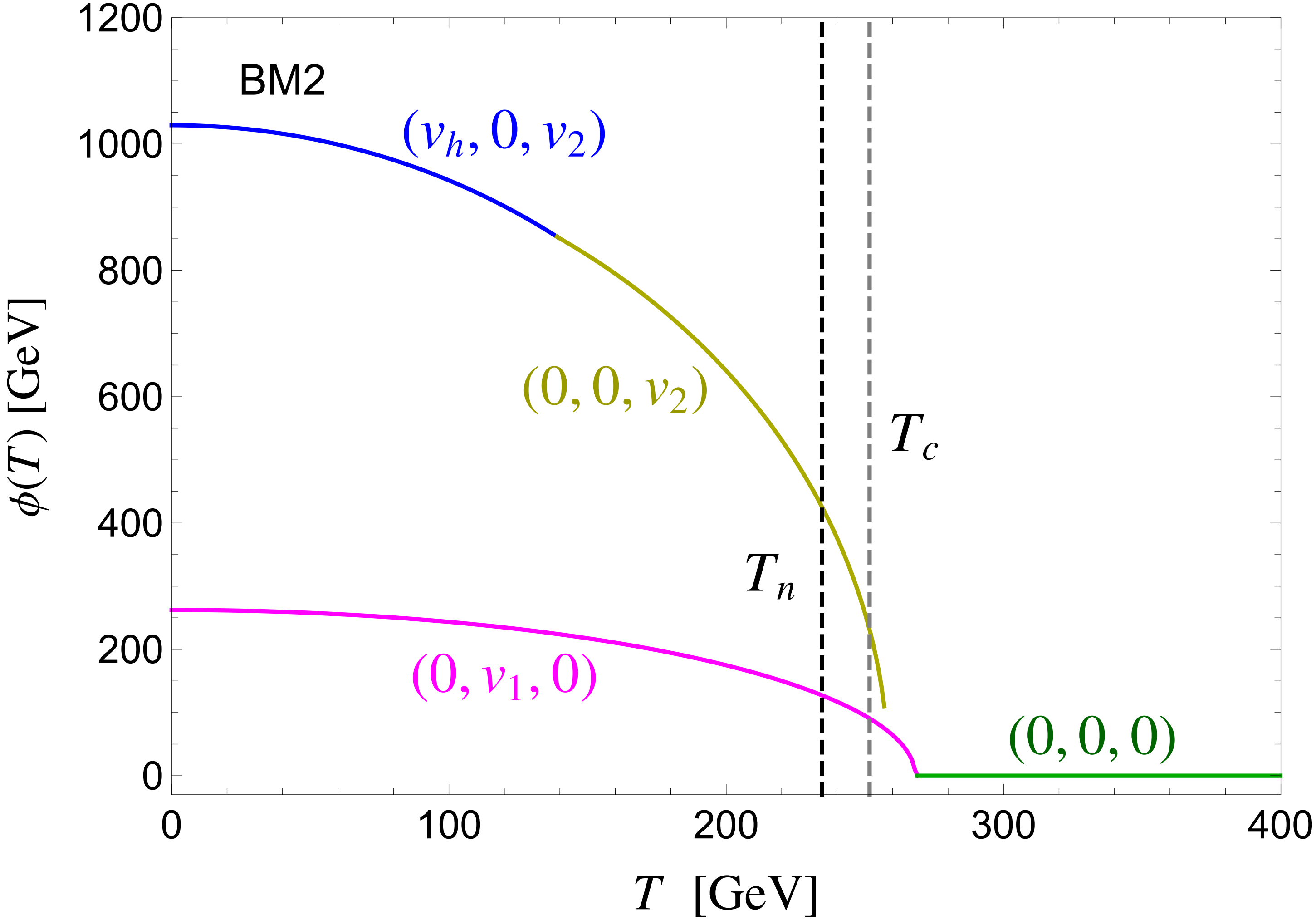
two-step: $(v_h, v_1, v_2) : (0,0,0) \to (0, v_1, 0) \Rightarrow (v_h, 0, v_2),$

FOPT is at both dark and electroweak sector

Phase transition pattern

Three-step

$$\phi \equiv \sqrt{v_h^2 + v_1^2 + v_2^2}$$



$$\text{three-step: } (v_h, v_1, v_2) : (0,0,0) \to (0, v_1, 0) \Rightarrow (0, 0, v_2) \to (v_h, 0, v_2)$$

FOPT is at dark sector

Benchmark points

We use **CosmoTransitions** to solve the bounce solutions

[arXiv: 1109.4189]

A smaller β/H_*

and larger α

will result in stronger GWs

Parameters	BM1	BM2
$\sin\theta$	-0.25	-0.12
$\tilde{g}$	0.094	0.133
$m_{\tilde{W}^\pm}$	94 GeV	133 GeV
m_{h_2}	200 GeV	290 GeV
$m_{\omega^\pm}$	1.2 TeV	1.3 TeV
m_{ω_2}	2.0 TeV	1.9 TeV
λ_1	3.5	3.5
λ_{H11}	2.0	2.0
λ_3	3.0	3.5
λ_H	0.28	0.27
λ_2	3.8×10^{-2}	8.3×10^{-2}
λ_{H22}	2.4×10^{-2}	3.2×10^{-2}
λ_4	5.0	4.0
v_2	1 TeV	1 TeV
$\Omega_{\tilde{W}^\pm}h^2$	0.096	0.12
$\sigma_{\rm SI}$ (cm^2)	7.8×10^{-47}	8.0×10^{-47}
T_c (GeV)	177	252
T_n (GeV)	147	234
β/H_n	297	760
α	0.32	5.1×10^{-2}
phase transition pattern	2-step	3-step

GW spectrum

GW sources: $\Omega_{\rm GW}h^2 \simeq \Omega_{\rm sw}h^2 + \Omega_{\rm turb}h^2$

Sound wave

$$\Omega_{\rm sw}h^2 = 2.65\times 10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_s}\right)^{\frac{1}{3}} v_w \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{\frac{7}{2}}\times\Upsilon(\tau_{\rm sw})$$

$$f_{\rm sw} = 1.9\times 10^{-2}\ \mathrm{mHz}\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\ \mathrm{GeV}}\right)\left(\frac{g_s}{100}\right)^{\frac{1}{6}} \qquad \Upsilon = 1-\frac{1}{\sqrt{1+2\tau_{\rm sw}H_*}}$$

[arXiv: 2007.08537]

[arXiv: 1512.06239]

Magnetohydrodynamic turbulence

$$\Omega_{\rm turb}h^2 = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_s}\right)^{\frac{1}{3}} v_w \frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}(1+8\pi f/H_0)}$$

$$f_{\rm turb} = 2.7\times 10^{-2}\ \mathrm{mHz}\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\ \mathrm{GeV}}\right)\left(\frac{g_s}{100}\right)^{\frac{1}{6}}$$

[arXiv: 1512.06239]

GW spectrum

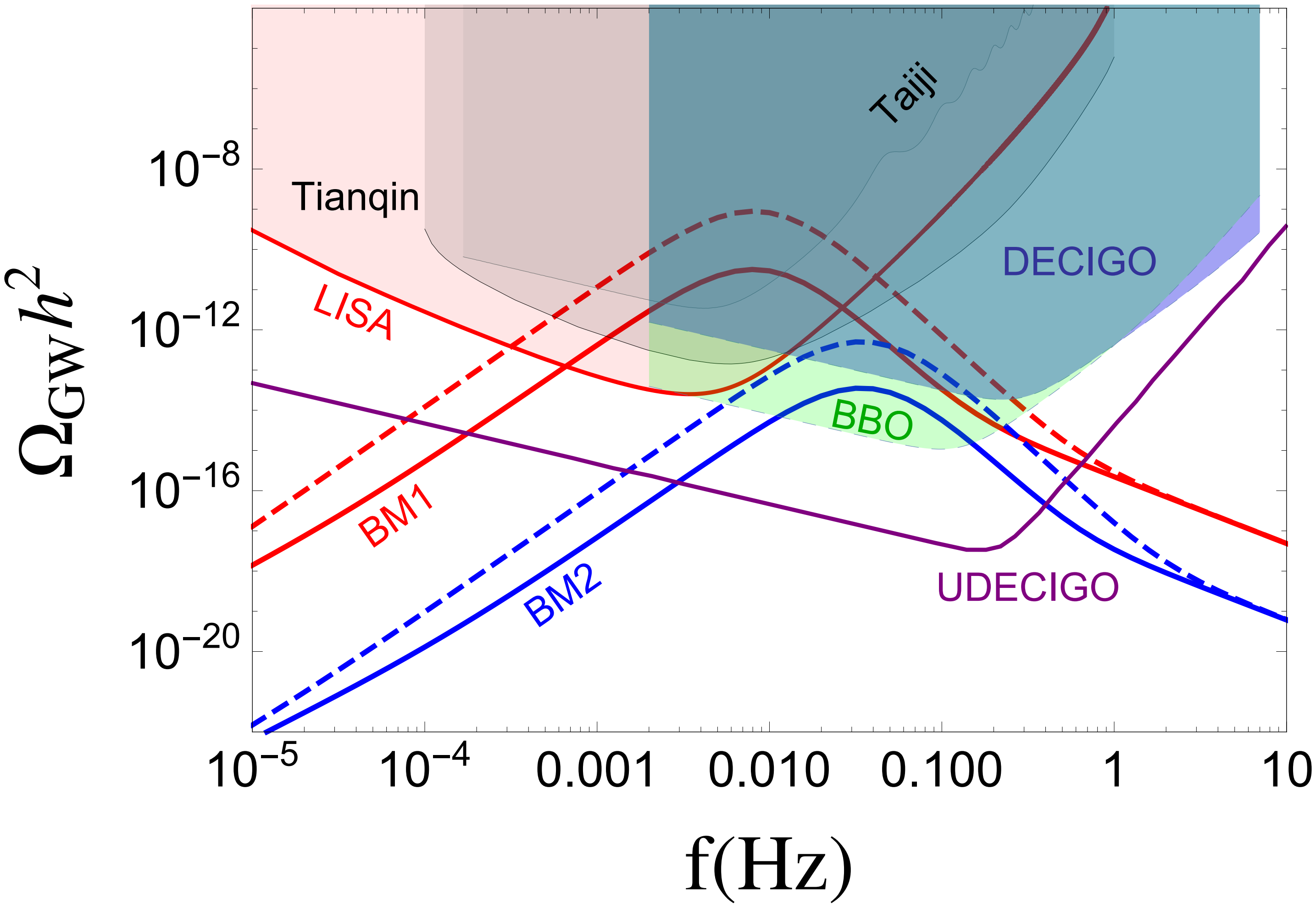
Signal-to-background ratio

$$\mathrm{SNR} = \sqrt{\delta \times \mathcal{T} \int_{f_{min}}^{f_{max}} df \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\exp}(f)}\right]^2}$$

Independent channel

$\delta = 1$ for LISA, $\delta = 2$ for BBO

Duration $\mathcal{T} = 5$ (yr)



$$\mathrm{BM1:}\quad \mathrm{SNR} = 1.08 \times 10^2\ (\mathrm{LISA}),\ \mathrm{SNR} = 8.56 \times 10^2\ (\mathrm{BBO})$$

$$\mathrm{BM2:}\quad \mathrm{SNR} = 9.95 \times 10^{-3}\ (\mathrm{LISA}),\ \mathrm{SNR} = 8.25\ (\mathrm{BBO})$$

Conclusions

- The two stable massive gauge bosons associated with the broken dark gauge group and the pseudo-Goldstone boson can serve as cold DM candidates.

- We have found both the two-step and three-step phase transitions with the cooling of the universe. Due to the rich vacuum pattern, the scalar sectors can introduce a strong FOPT, for the benchmark points BM1 with a successful EW FOPT, and BM2 with a FOPT in the dark sector.

- We found that the two-step EWPT in our BM1 can produce strong GW signals and can be detectable using the future space-based interferometers LISA and BBO, while the GW signal for BM2 may be difficult to observe at LISA due to the rather low signal-to-noise ratio.

Thanks!

Back-up slides

Phenomenological constraints

Vacuum stability:
$$\lambda_H > 0, \quad \lambda_1 > 0, \quad \lambda_2 > 0,$$
$$\lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} > 0,$$
$$\lambda_{H11} + \sqrt{\lambda_H \lambda_1} > 0, \quad \lambda_{H22} + \sqrt{\lambda_H \lambda_2} > 0.$$

Partial wave unitarity:

$$|\lambda_H| < 8\pi, \quad |\lambda_{H11}| < 8\pi, \quad |\lambda_{H22}| < 8\pi,$$
$$|\lambda_3 - \frac{1}{2}\lambda_4| < 8\pi, \quad |\lambda_3 + \frac{1}{2}\lambda_4| < 8\pi, \quad |\lambda_3 + 2\lambda_4| < 8\pi,$$
$$|\lambda_1 + \lambda_2 - \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_4^2}| < 16\pi, \quad |\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_4^2}| < 16\pi,$$
$$|\mathrm{Eigenvalues}[\mathcal{P}]| < 8\pi,$$

where

$$\mathcal{P} = \frac{1}{2}\begin{pmatrix} 5\lambda_1 & 3\lambda_3 + \lambda_4 & 2\sqrt{3}\lambda_{H11} \\ 3\lambda_3 + \lambda_4 & 5\lambda_2 & 2\sqrt{3}\lambda_{H22} \\ 2\sqrt{3}\lambda_{H11} & 2\sqrt{3}\lambda_{H22} & 6\lambda_H \end{pmatrix}.$$

Dark matter relic density

DM relic density can be estimated by

$$\Omega_{\rm DM}h^2 = 1.07 \times 10^9 \frac{x_f \ {\rm GeV}^{-1}}{(g_{*S}/\sqrt{g_*})M_{pl}\langle\sigma v_{\rm rel}\rangle} \qquad x_f \equiv m_\chi/T_f$$

[arXiv: 0810.5126]

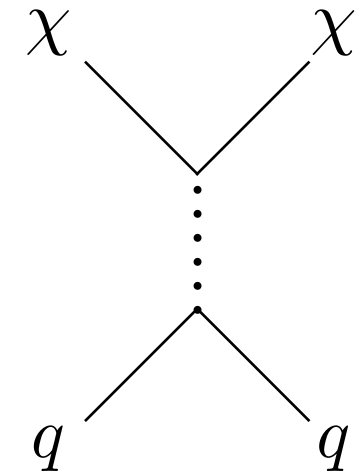
$$x_f = \ln\left[0.038\frac{g}{\sqrt{g_*}}M_{pl}m_\chi\langle\sigma v_{\rm rel}\rangle\right] - \frac{1}{2}\ln\ln\left[0.038\frac{g}{\sqrt{g_*}}M_{pl}m_\chi\langle\sigma v_{\rm rel}\rangle\right]$$

The s-wave annihilation cross section at leading order

$$\langle\sigma v_{\rm rel}\rangle = \frac{1}{32\pi}\frac{\sqrt{1-4M_W^2/s}}{m_\chi^2}|M_{\rm annihilation}(s)|^2$$

[Nucl. Phys. B 310 (1988) 693]

Dark matter direct detection



The effective interactions of DM with light quarks and gluons

$$\mathscr{L}_{q,g}^{\mathrm{eff}} = \sum_{q=u,d,s} f_q^{\chi} m_q \chi\chi\bar{q}q + f_G^{\chi}\chi\chi\frac{\alpha_s}{\pi}G^{a\mu\nu}G^a_{\mu\nu}$$

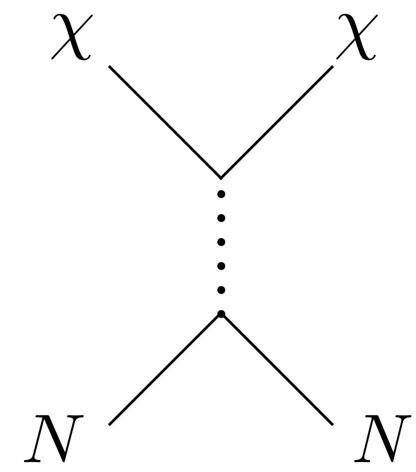
f_q^{χ} is the effective couplings between DM and light quarks

$$\begin{aligned}
f_q^{\tilde{W}^{\pm}} &= \tilde{g}^2\frac{v_2}{v_h}\sin\theta\cos\theta\left(\frac{1}{m_{h_2}^2} - \frac{1}{m_{h_1}^2}\right), \\
f_q^{\omega^{\pm}} &= \frac{1}{v_h}\left(\frac{c_2\cos\theta}{m_{h_2}^2} - \frac{c_1\sin\theta}{m_{h_1}^2}\right), \\
f_q^{\omega_2} &= \frac{1}{v_h}\left(\frac{d_2\cos\theta}{m_{h_2}^2} - \frac{d_1\sin\theta}{m_{h_1}^2}\right).
\end{aligned}$$

The coupling between DM and gluon comes from the effective coupling after integrating-out of heavy quarks

$$f_G^{\chi} = -\frac{1}{12}\sum_{Q=c,b,t} f_Q^{\chi} = -\frac{1}{4}f_q^{\chi}$$

Dark matter direct detection



To obtain the DM-nucleon scattering cross section, we have to evaluate the DM-quark operator in the nucleon matrix elements.

$$\langle N|m_q\bar{q}q|N\rangle \equiv f^N_{Tq}m_N, \; \langle N|\frac{\alpha_s}{\pi}GG|N\rangle = -\frac{8}{9}m_N f^N_{TG}$$

The effective interactions of DM and nucleon

$$\mathscr{L}^{\mathrm{eff}}_N = f^\chi_N \chi\chi\bar{N}N \qquad f^\chi_N = m_N\Big(\sum_{q=u,d,s} f^N_{Tq}f^\chi_q - \frac{8}{9}f^N_{TG}f^\chi_G\Big)$$

The mass-fraction parameters

[arXiv: 1305.0237]

$$f^p_{Td} = 0.0191, \; f^p_{Tu} = 0.0153, \; f^p_{Ts} = 0.0447, \;\; f^p_{TG} \equiv 1 - \textstyle\sum_{q=u,d,s} f^p_{Tq} = 0.925$$

The spin-independent cross section of DM with nucleon can be calculated with

$$\hat{\sigma}^\chi_{\mathrm{SI}} = \frac{1}{\pi}\left(\frac{m_N}{m_\chi + m_N}\right)^2 (f^\chi_N)^2 \qquad \sigma_{\mathrm{SI}} = \left(\frac{\Omega_\chi h^2}{\Omega_{\mathrm{obs}}h^2}\right)\hat{\sigma}^\chi_{\mathrm{SI}}$$

Effective potential at finite temperature

The dynamics of the phase transition is determined by the effective potential at the finite temperature. At high temperature approximation: $y \equiv m/T \ll 1$

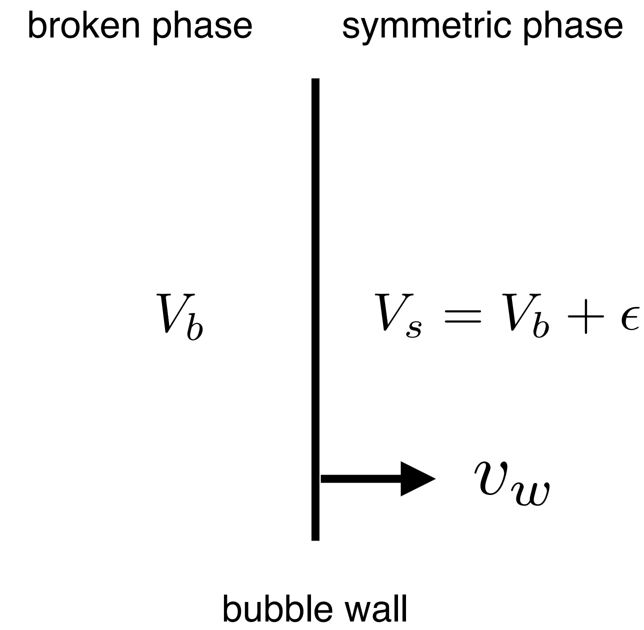
$$V^{(1)}(T) = V_{\text{tree}} + \Delta V^{(1)}(T)$$

$$\Delta V^{(1)}(T) = \frac{T^4}{2\pi^2}\left\{\sum_b n_b J_B[\frac{m_b^2(\phi_i)}{T^2}] - \sum_f n_f J_F[\frac{m_f^2(\phi_i)}{T^2}]\right\}$$

$$V_S(T) = \frac{m_H^2(T)}{2}h_0^2 + \frac{\lambda_H}{8}h_0^4 + \frac{m_{11}^2(T)}{2}\omega_3^2 + \frac{\lambda_1}{8}\omega_3^4 + \frac{m_{22}^2(T)}{2}\varphi_2^2 + \frac{\lambda_2}{8}\varphi_2^4 + \frac{\lambda_{H11}}{4}h_0^2\omega_3^2 + \frac{\lambda_{H22}}{4}h_0^2\varphi_2^2 + \frac{\lambda_3}{4}\omega_3^2\varphi_2^2.$$

$$\begin{aligned}
m_H^2(T) &= m_H^2 + \frac{T^2}{16}(g_1^2 + 3g_2^2 + 2(2\lambda_H + \lambda_{H11} + \lambda_{H22} + 2y_t^2)), \\
m_{11}^2(T) &= m_{11}^2 + \frac{T^2}{24}(12\tilde{g}^2 + 5\lambda_1 + 3\lambda_3 + \lambda_4 + 4\lambda_{H11}), \\
m_{22}^2(T) &= m_{22}^2 + \frac{T^2}{24}(12\tilde{g}^2 + 5\lambda_1 + 3\lambda_3 + \lambda_4 + 4\lambda_{H22}).
\end{aligned}$$

The bag model



In the bag model, pressure (p), energy density (e) and enthalpy (ω) in the symmetric and broken phase are

$$p_s = \frac{1}{3}a_s T_s^4 - \epsilon,\ e_s = (T\frac{\partial p}{\partial T} - p)_s = a_s T_s^4 + \epsilon,\ \omega_s = (T\frac{\partial p}{\partial T})_s = p_s + e_s = \frac{4}{3}a_s T_s^4$$

$$p_b = \frac{1}{3}a_b T_b^4,\ e_b = a_b T_b^4,\ \omega_b = \frac{4}{3}a_b T_b^4$$

ϵ is the latent heat, which is the difference of the potential in two phases. The strength of the phase transition can be parametrized by the ratio of the latent heat to the total radiation density

$$\alpha = \frac{e_s(T_s) - e_b(T_s)}{a_s T_s^4} = \frac{1}{a_s T_s^4}[T\frac{\partial \Delta V}{\partial T} - \Delta V] = \frac{\epsilon}{a_s T_s^4};\ \Delta V = V_b - V_s = -\epsilon$$

The efficiency factor κ indicates how much of the latent heat can go to the fluid kinetic energy

$$\kappa = \frac{\rho_{fl}}{\epsilon}$$

The kinetic energy fraction

$$K = \frac{\rho_{fl}}{e_s} = \frac{\rho_{fl}}{a_s T_s^4 + \epsilon} = \frac{\alpha\kappa}{\alpha + 1}$$

Source lifetime

$$\Upsilon = 1 - \frac{1}{\sqrt{1 + 2\tau_{\rm sw} H_*}}$$

[arXiv: 2007.08537]

$$\tau_{\rm sw} \sim \frac{R_*}{\bar{U}_f}$$

$$R_* = (8\pi)^{1/3} v_w / \beta$$ mean bubble separation

$$\bar{U}_f = (\frac{\rho_{fl}}{\omega_s})^{1/2} = (\frac{K}{\omega_s / e_s})^{1/2}$$ root mean squared velocity

For an ultra-relativistic fluid $\omega_s / e_s \to 4/3$

At bag model

$$\bar{U}_f = \sqrt{(3\kappa_v \alpha / 4)}$$