

Enlarging the Space of Viable Inflation Models: A Slingshot Mechanism



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Based on work done in collaboration with:

- **Keith Dienes and Jeff Kost [arXiv:1907.10074].**

PPC 2021, May 19th, 2021.

Initial Conditions for Inflation

- In order to give rise to a prolonged period of accelerated expansion, the inflation potential $V(\phi)$ must satisfy the slow-roll criteria:

$$\epsilon = \frac{M_P^2}{2} \left[\frac{V'(\phi)}{V(\phi)} \right]^2 \ll 1$$

Slow Roll

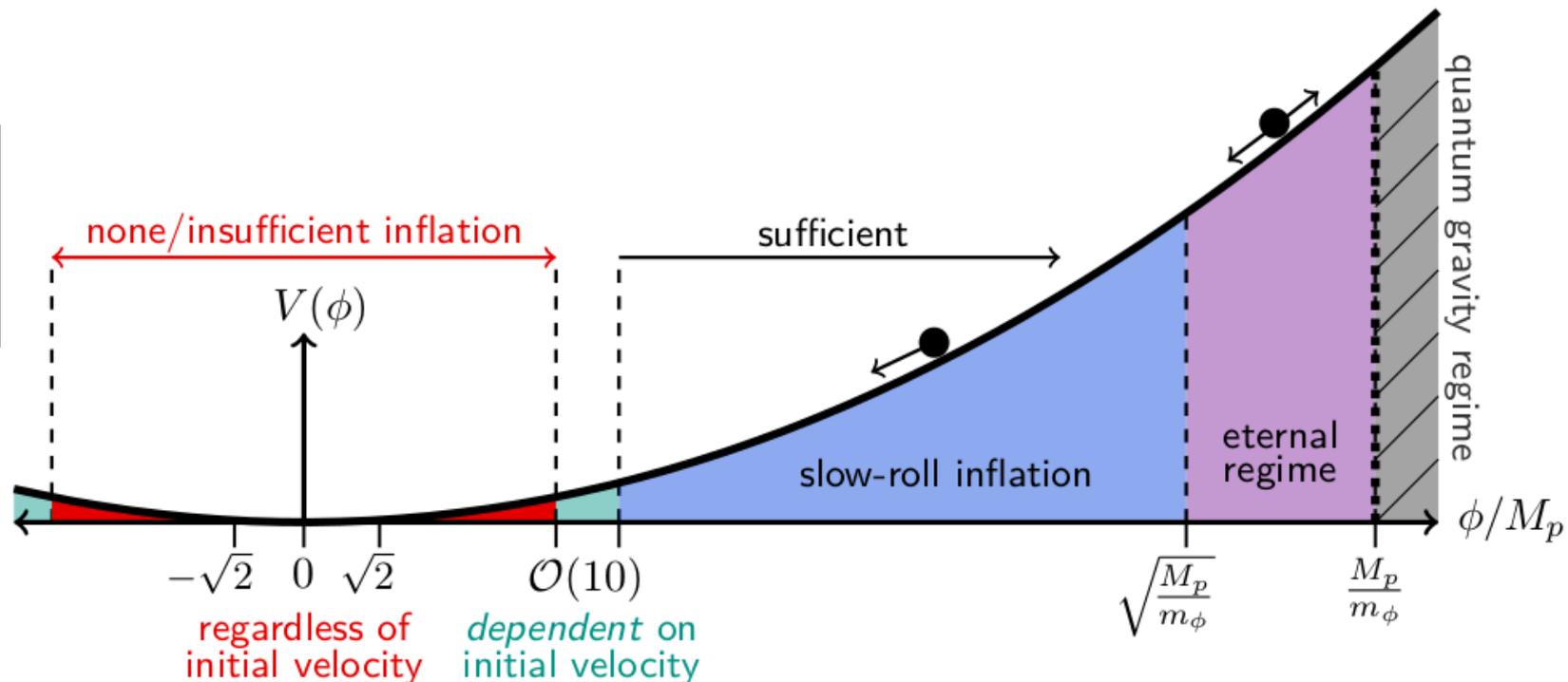
$$\eta = M_P^2 \left| \frac{V''(\phi)}{V(\phi)} \right| \ll 1$$

Prolonged Duration

- Successful inflation is not contingent on the shape of $V(\phi)$ alone, however. Appropriate ***initial conditions*** for ϕ and $\dot{\phi}$ must also be established.

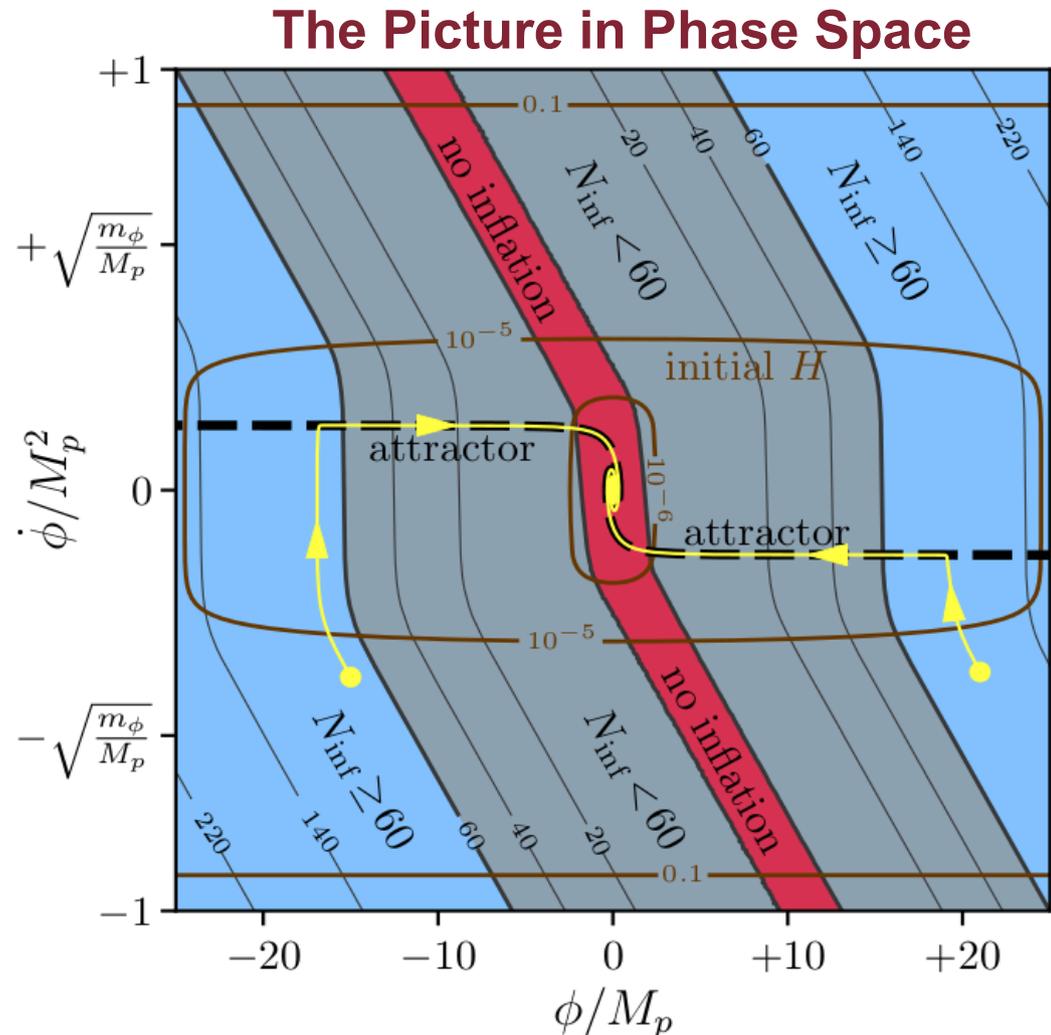
Example:

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$$



Initial Conditions for Inflation

- Appropriate initial conditions for inflation that include large or even trans-Planckian field VEVs arise naturally many scenarios, e.g.,
 - Chaotic inflation [Linde '83]
 - Natural inflation [Freese, Frieman, Olinto, '90; Freese, Kinney '04]
- ***The question***: might we be able to “salvage” regions of phase space that might not give rise to successful inflation?
- In other words, might there be a ***dynamical mechanism*** which could drive a scalar field into a region of phase space from which inflation of sufficient duration can occur?

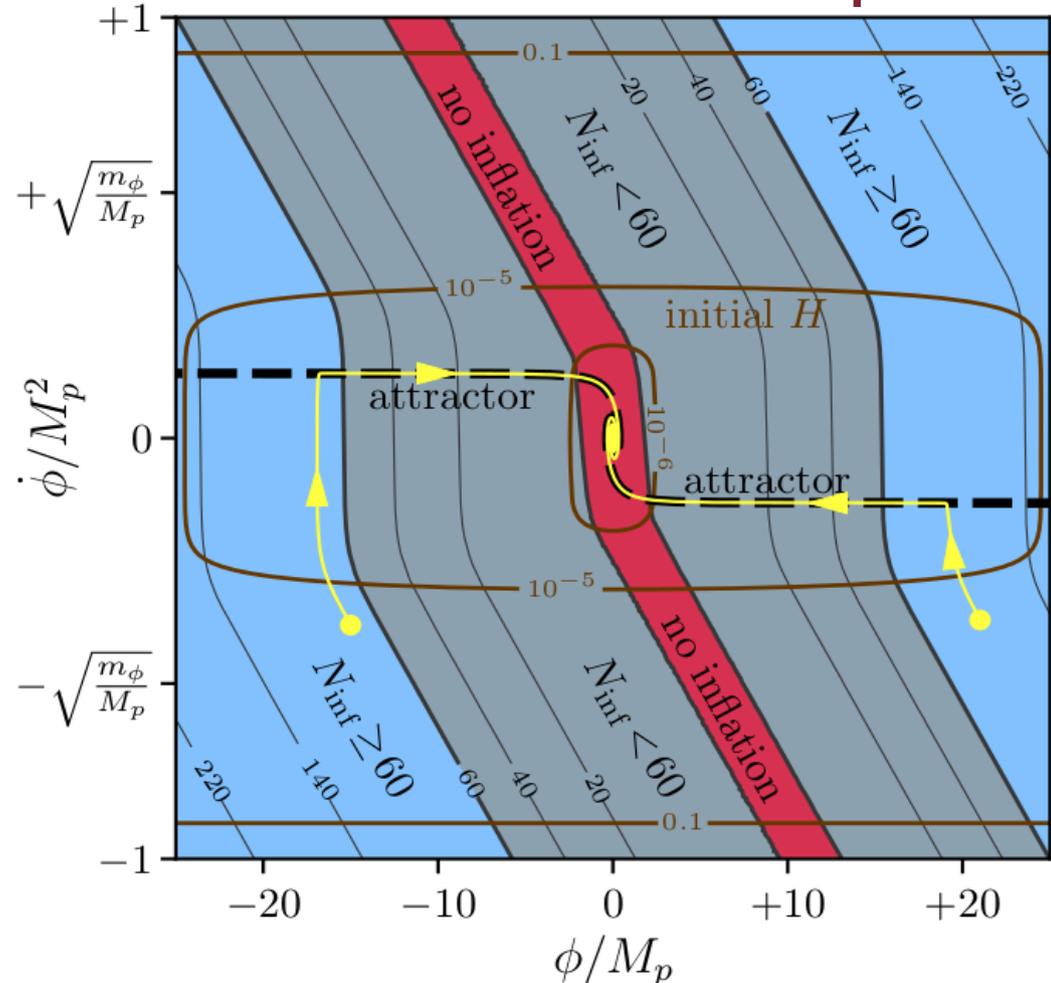


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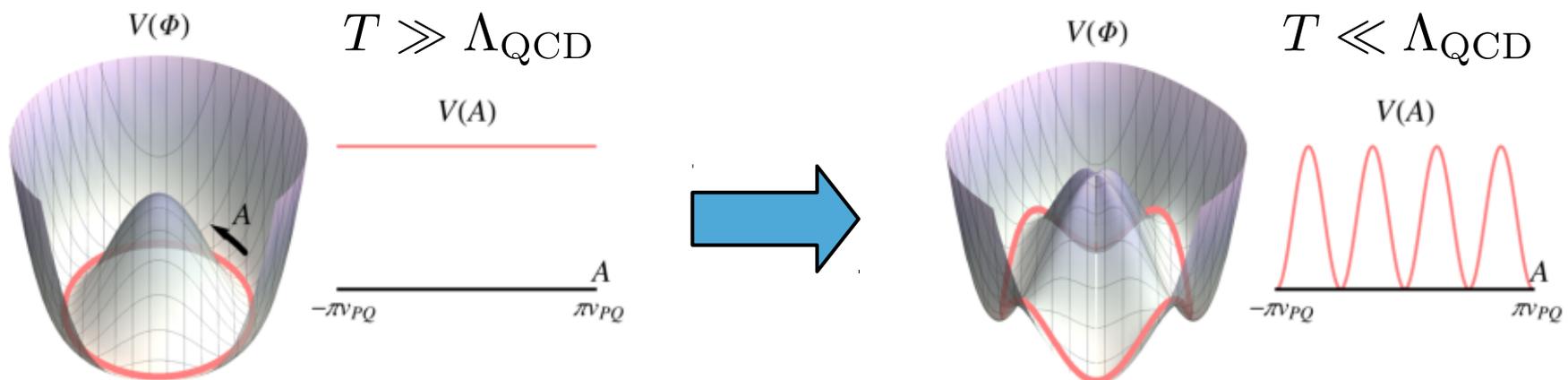
The answer: yes, there is, and the ingredients for this mechanism are standard tools in the cosmologist's toolbox.

The Picture in Phase Space



Scalars and Phase Transitions

- Additional ***light scalars*** are a common feature of many scenarios for physics beyond the Standard Model.
- Such fields are often light because they are protected from acquiring masses by classical symmetries which are broken at the quantum level by non-perturbative effects.
- Such non-perturbative effects can arise dynamically, as a consequence of a ***cosmological phase transition***.
- A canonical example is the QCD axion, which is protected from acquiring a mass at high temperatures $T \gg \Lambda_{\text{QCD}}$ by a shift symmetry, but acquires a small mass at temperatures $T \lesssim \Lambda_{\text{QCD}}$ due to instanton effects associated with the QCD phase transition.



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Important:

Scalar masses generated in this way are *time-dependent* and evolve dynamically over the course of the phase transition. As we shall see, this time-dependence will play a crucial role in establishing the initial conditions for the inflaton field at the beginning of inflation.

Dynamics of the Phase Transition

[Dienes, Kost, BT '15]

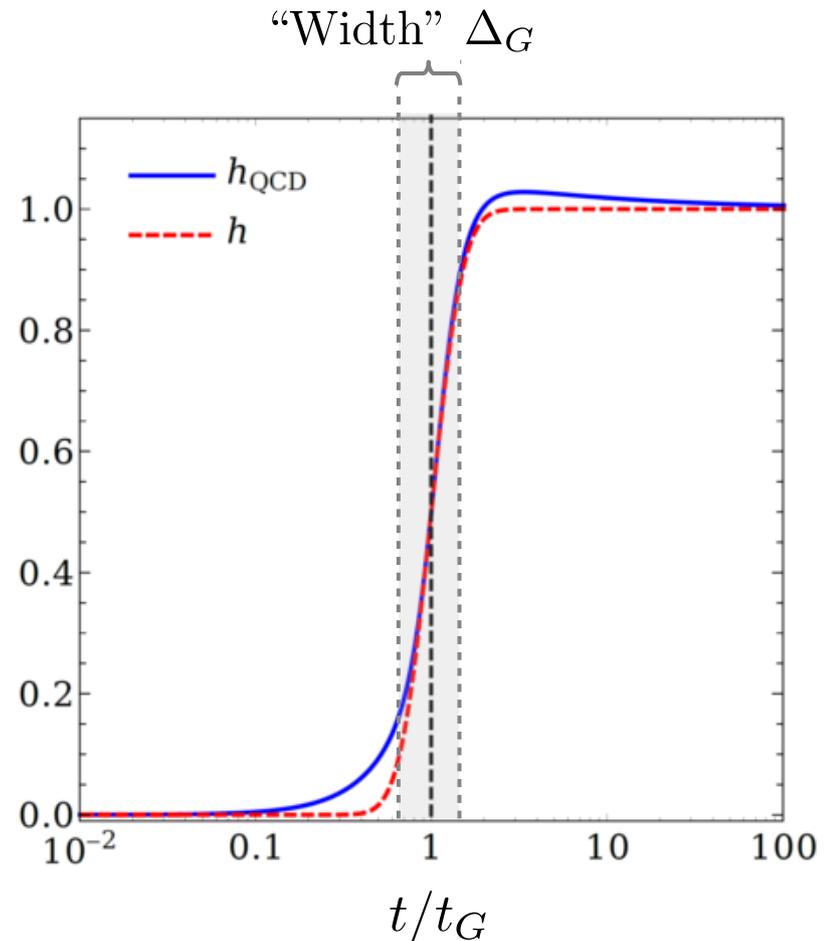
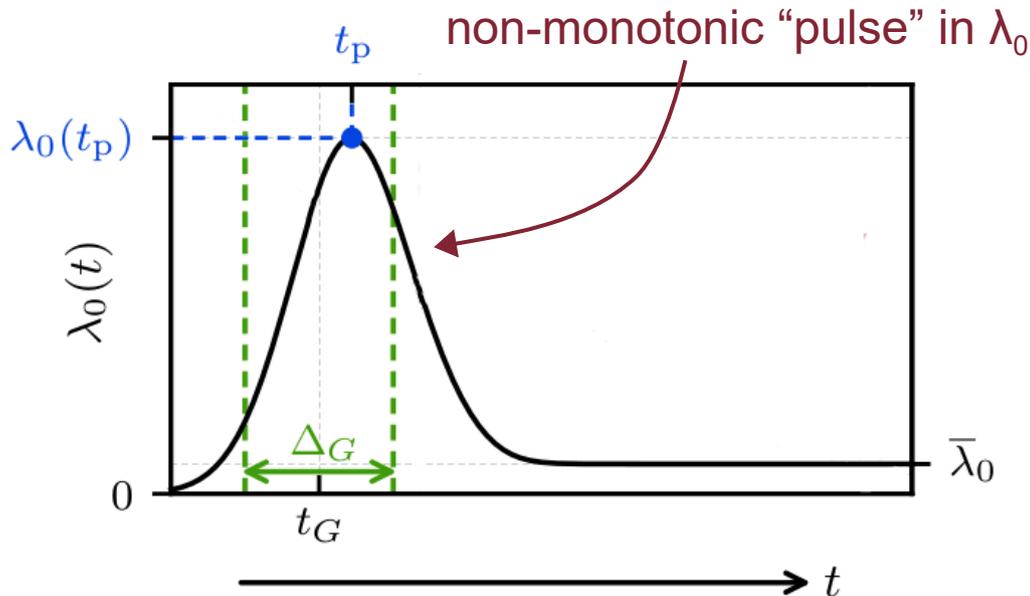
- We consider a mass-squared matrix of the form

$$\mathcal{M}^2 = \begin{array}{c} \text{static} \\ \left[\begin{array}{cc} 0 & 0 \\ 0 & M^2 \end{array} \right] \end{array} + \begin{array}{c} \text{dynamical} \\ \left[\begin{array}{cc} m_{00}^2(t) & m_{01}^2(t) \\ m_{01}^2(t) & m_{11}^2(t) \end{array} \right] \end{array}$$

$$m_{ij}^2 = \overline{m}_{ij}^2 h(t)$$

constant uniform time dependence

- The mass eigenvalues λ_0 and λ_1 evolve as well, and can even be ***non-monotonic*** functions of t . [Dienes, Kost, BT '15]



Consequences of Non-Monotonicity

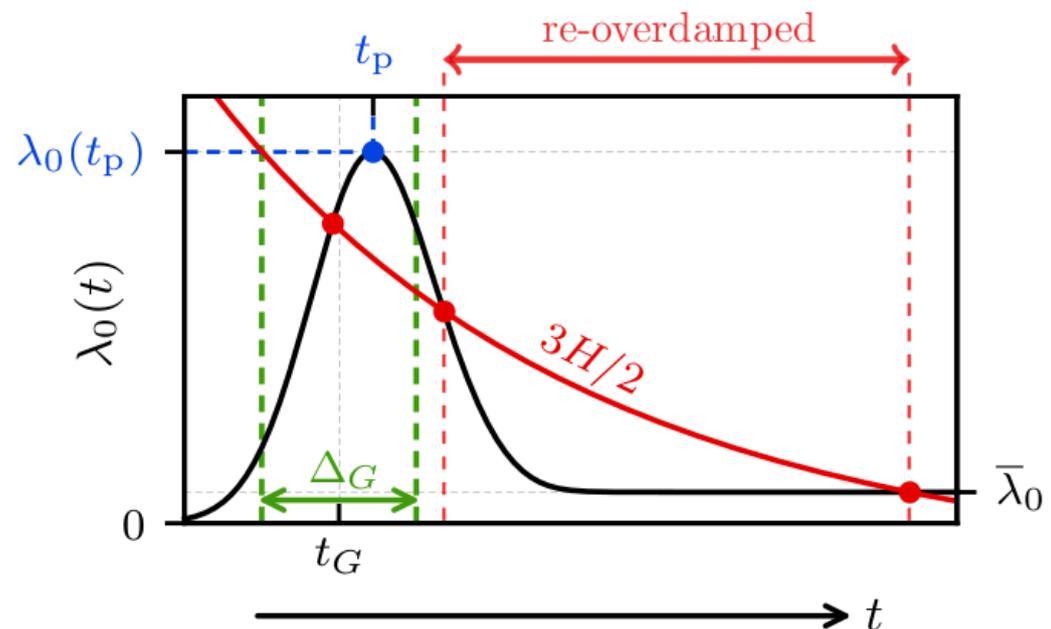
1. Re-Overdamping

- The way in which the energy density ρ_{λ_0} associated with the lighter mass eigenstate ϕ_{λ_0} evolves depends on its mass.

$3H \gtrsim 2\lambda_0 \Rightarrow$ **Overdamped**: behaves like vacuum energy ($\rho_{\lambda_0} \sim [\text{const.}]$)

$3H \lesssim 2\lambda_0 \Rightarrow$ **Underdamped**: behaves like massive matter ($\rho_{\lambda_0} \propto a^{-3}$)

- Thus, a typical field will behave like vacuum energy at early times and like matter at late times.
- However, when $\lambda_0(t)$ is non-monotonic, a field can transition from the overdamped to the underdamped regime and then **back** to the overdamped regime.



Consequences of Non-Monotonicity

2. Parametric Resonance

- The “pulse” behavior in which λ_0 rises and falls during the phase transition occurs over a particular timescale.
- Indeed, one can view this pulse as having an effective “frequency”:

$$\lambda_0^2(t) \sim \frac{1}{2} \lambda_0^2(t_p) \{1 + \cos[\omega_{\text{eff}}(t - t_p)]\} \Rightarrow$$

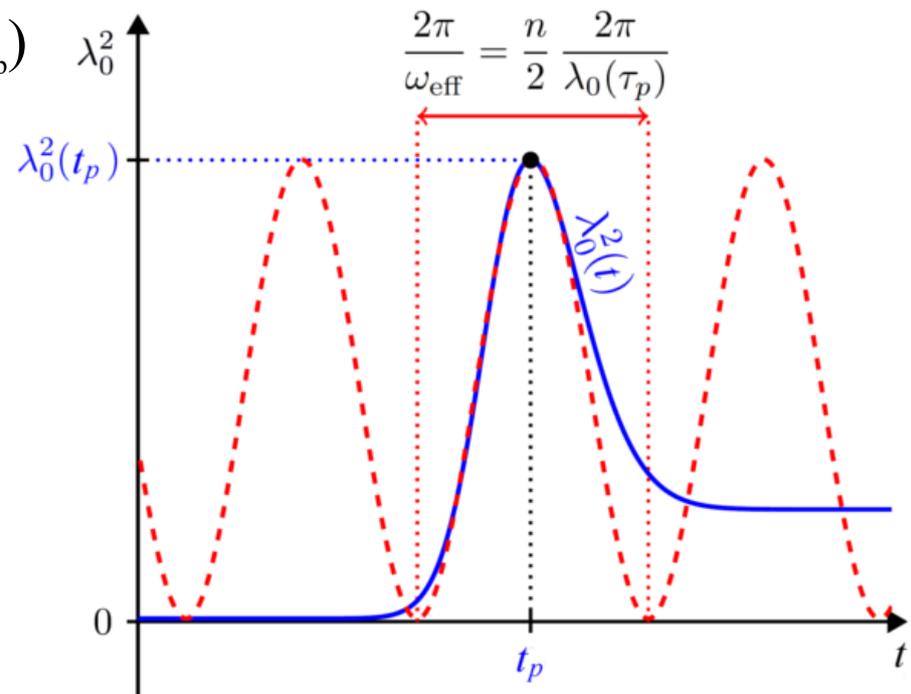
$$\omega_{\text{eff}}^2 = -4 \left(\frac{\ddot{\lambda}_0}{\lambda_0} \right) \Big|_{t=t_p}$$

- It therefore follows that when a certain relationship exists between ω_{eff} and $\lambda_0(t_p)$ itself, **parametric resonances** arise.

Resonance Condition

$$\left(\frac{\ddot{\lambda}_0}{\lambda_0^3} \right) \Big|_{t=t_p} = \frac{1}{n^2} \quad n \in \mathbb{Z}^+$$

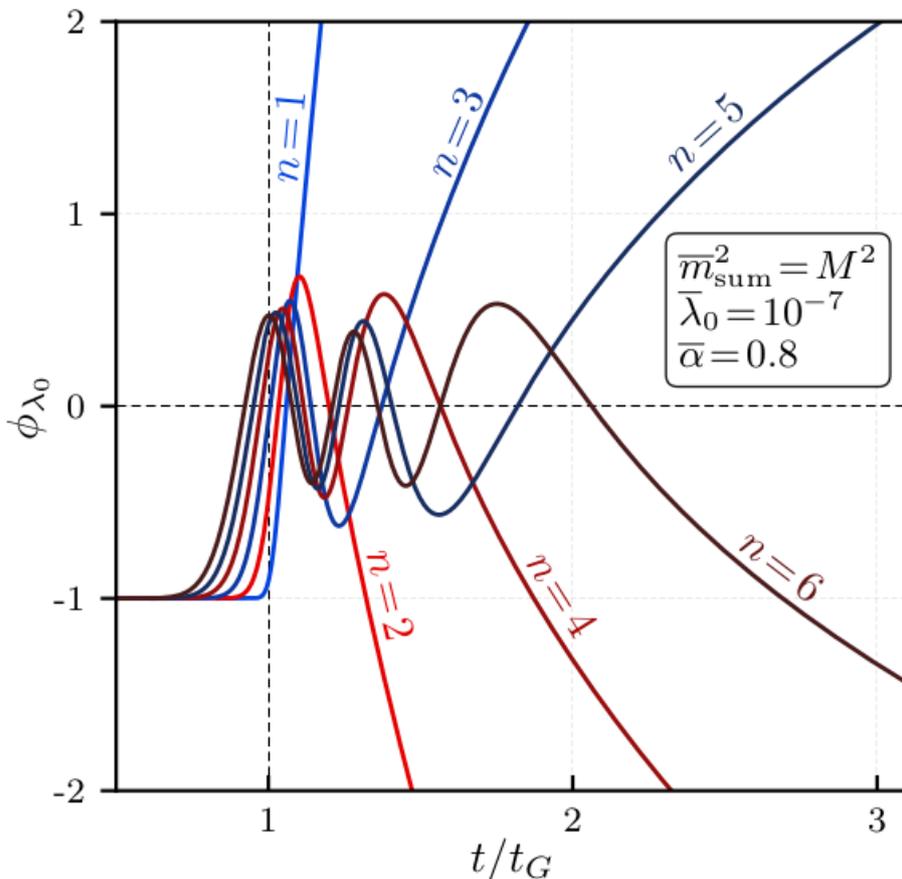
- Even though this resonance exists for only half an “oscillation,” it can have a dramatic effect on ϕ_{λ_0} and ρ_{λ_0} .



“Propulsion” of the Field

- The interplay between these two effects can serve to propel the field to a large value of $|\phi_{\lambda_0}|$.
- The phase transition carries the field through $n/2$ oscillations and then releases it at maximum velocity $\dot{\phi}_{\lambda_0}$ into the re-overdamped phase.

Field trajectories (where Δ_G has been chosen appropriately for each value of n)

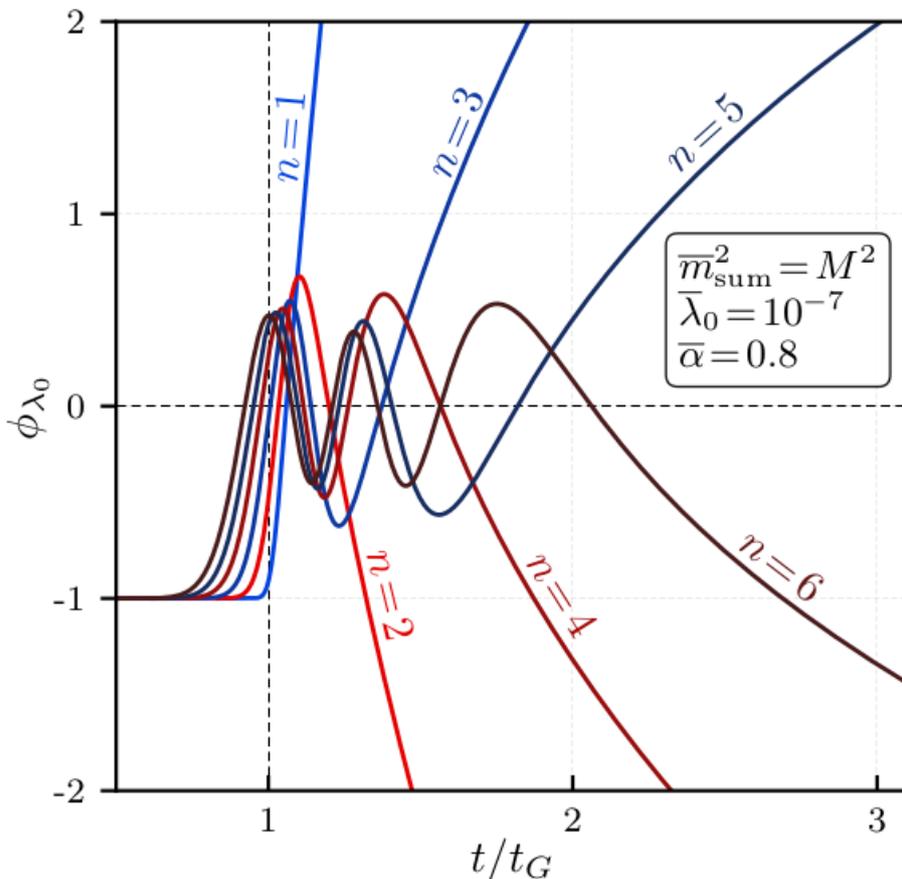


- During the subsequent ***kination-dominated*** epoch, $|\phi_{\lambda_0}|$ grows as the kinetic energy of the field is damped away.
- When it reaches its maximum $|\phi_{\lambda_0}^{(I)}|$, inflation begins.

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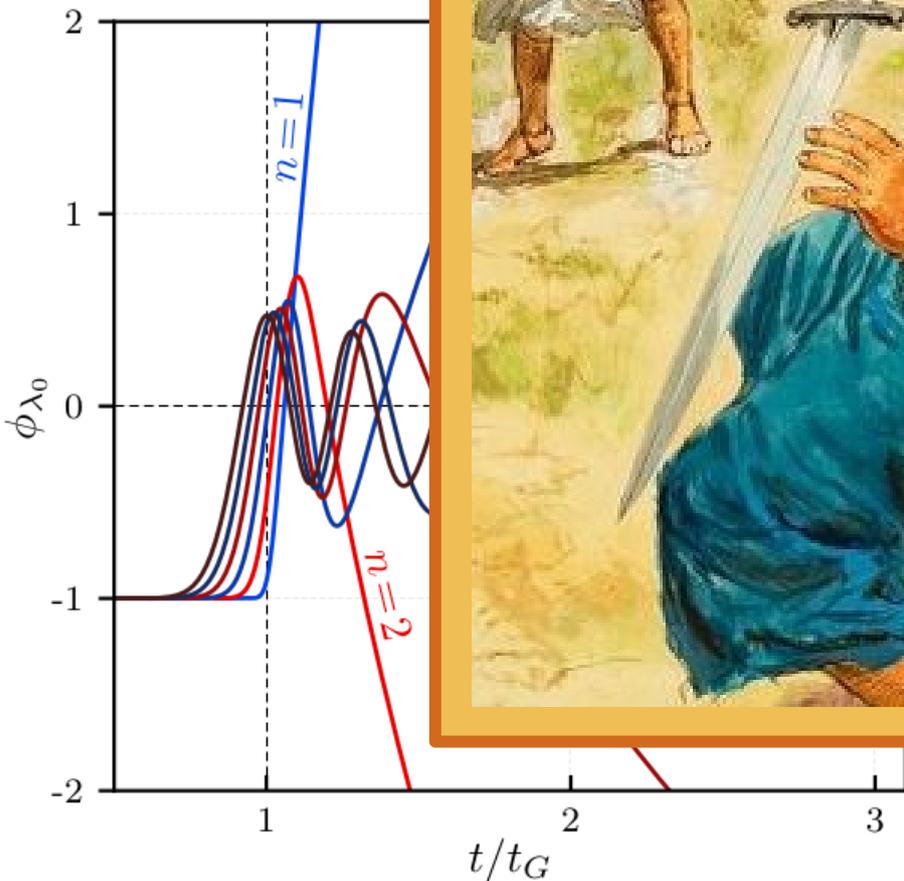


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- **So...**
in other words...

“Propulsion” of the Field

- The interplay of the field to a large value of n
- The phase transition releases it and then phase.

Field trajectories chosen appropriately



It's a slingshot!

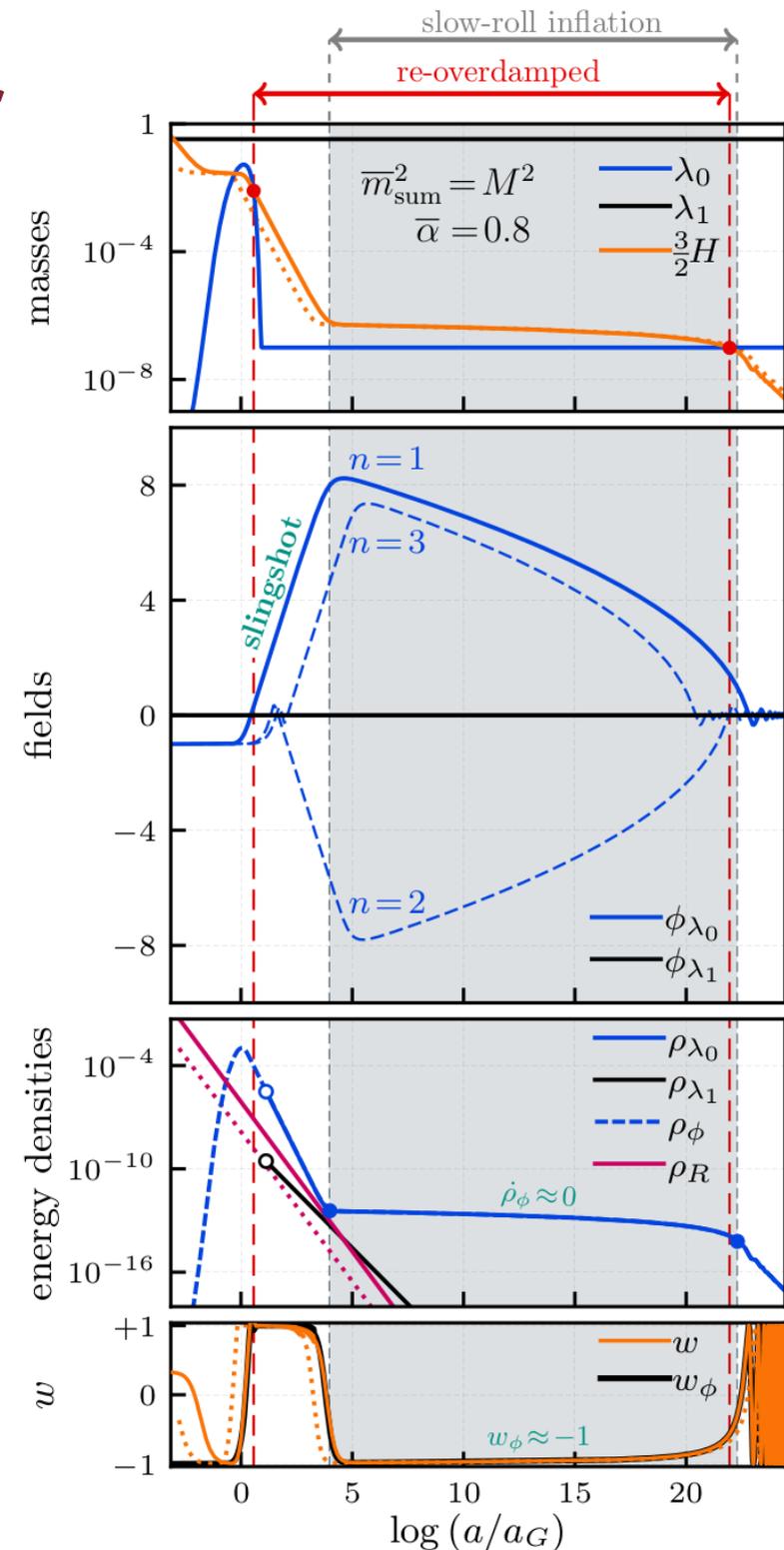
el the field to a
s and then
phase.

Excitation-
grows as the
d is damped

(Yeah, I know... it's really technically a sling rather than a slingshot, but “slingshot” has a nicer ring to it.)

Evolution of the Scalar Sector

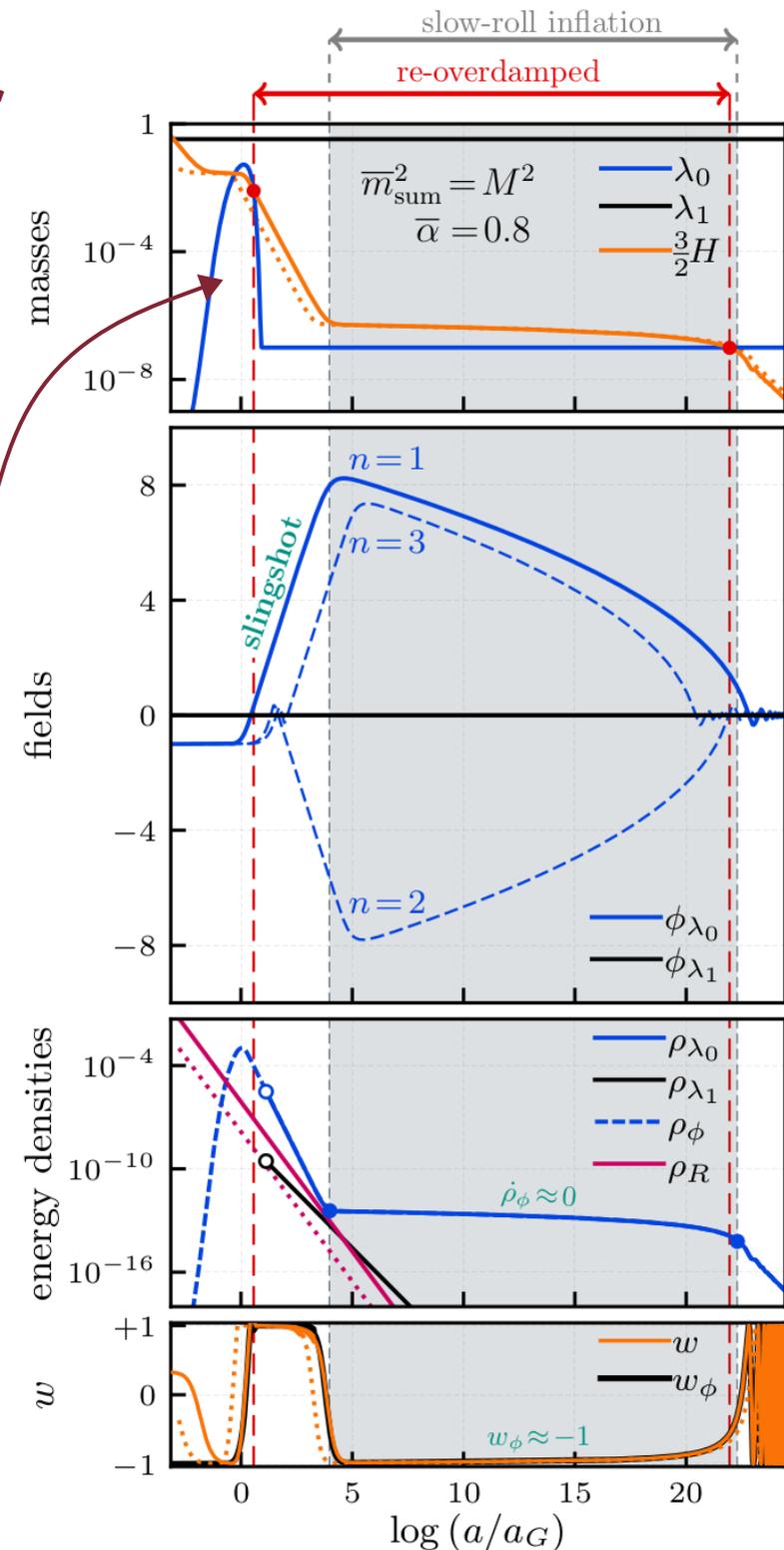
- Assume that the universe is initially radiation-dominated, with no energy density stored in the scalar sector.
- For concreteness, start from an initial field value $\mathcal{A}_\phi \equiv \phi_{\lambda_0}(t \ll t_G) \sim -M_P$.



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The evolution of λ_0 during the phase transition drives the resonance.

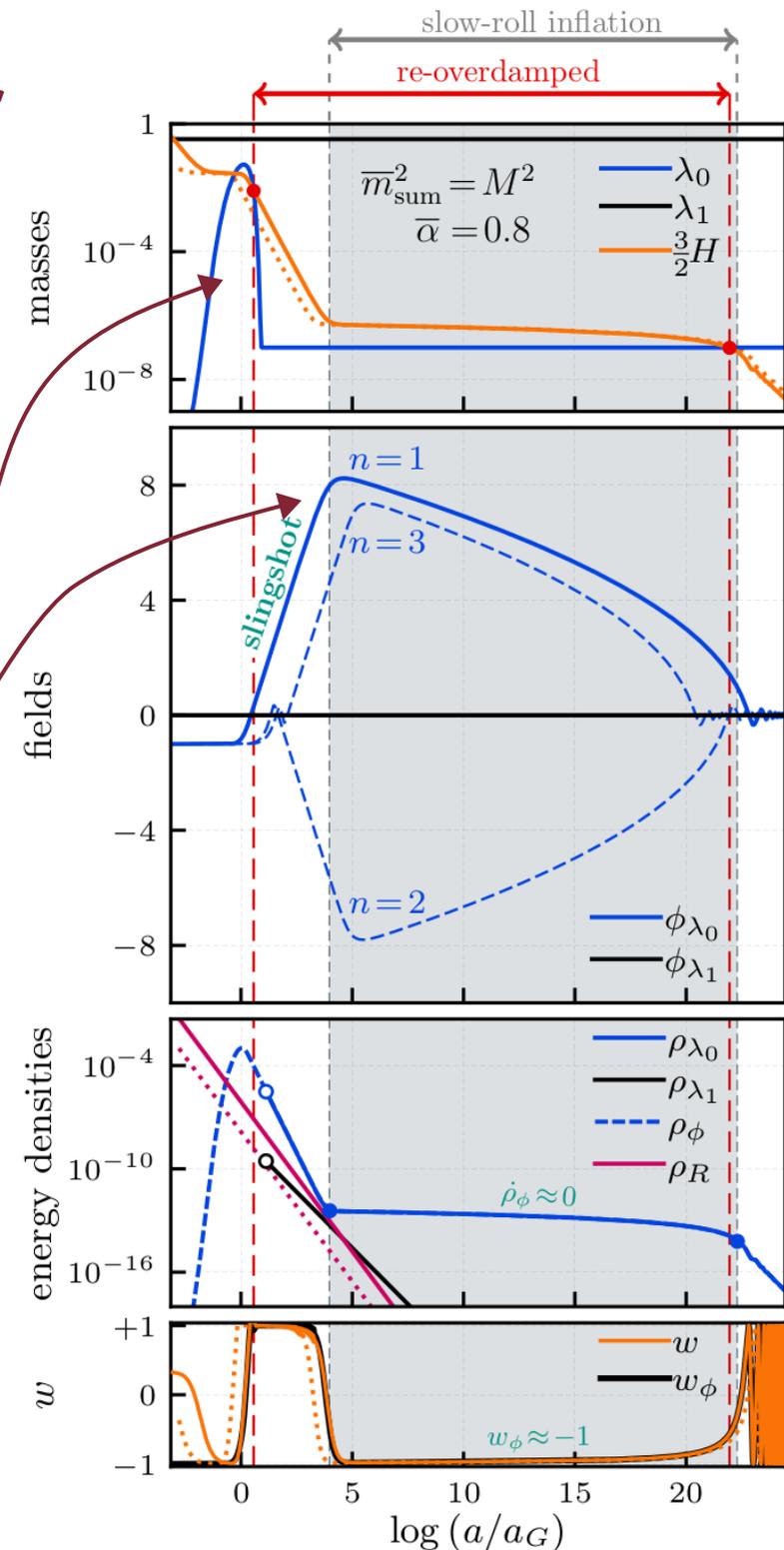


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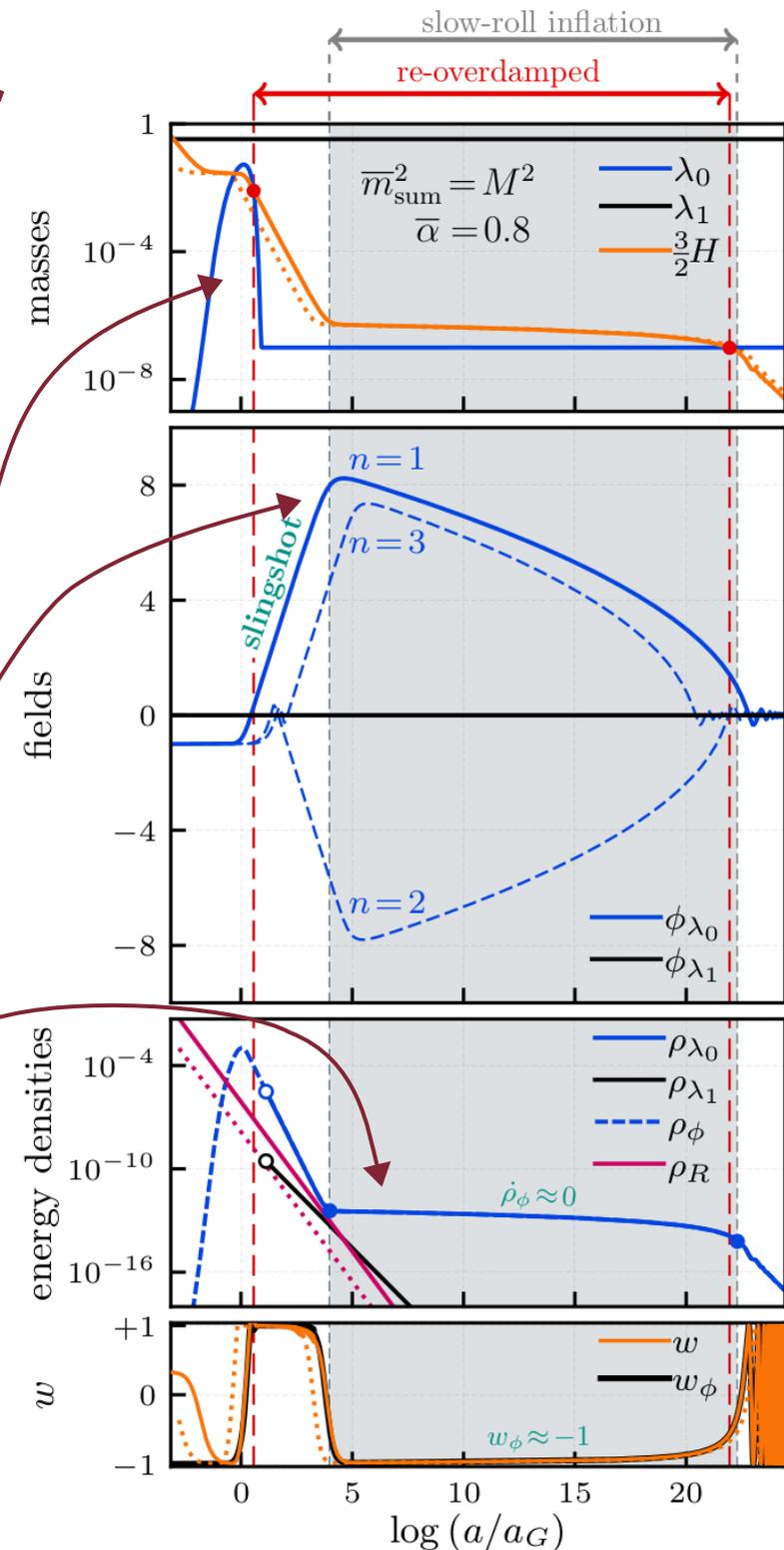
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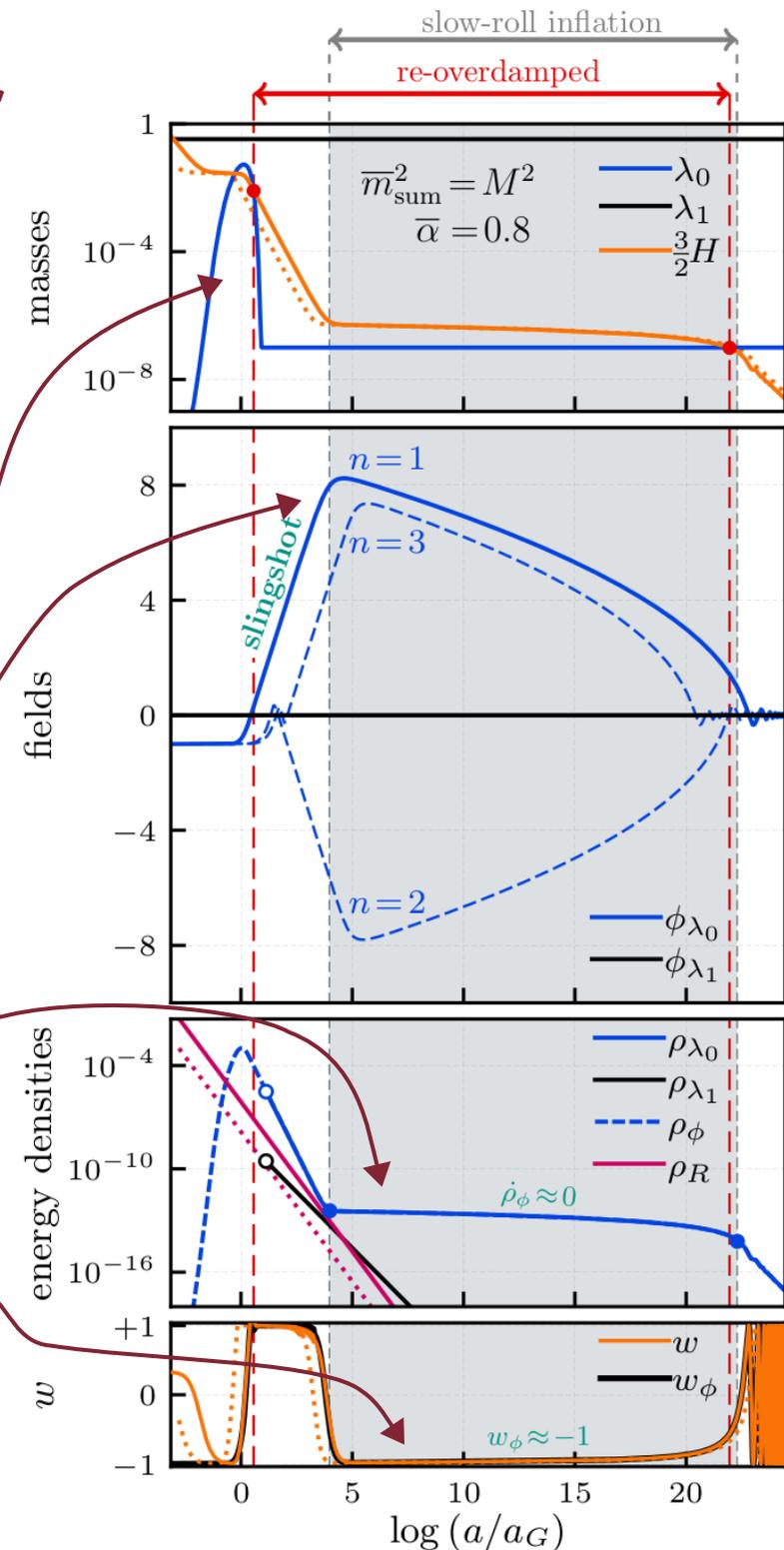
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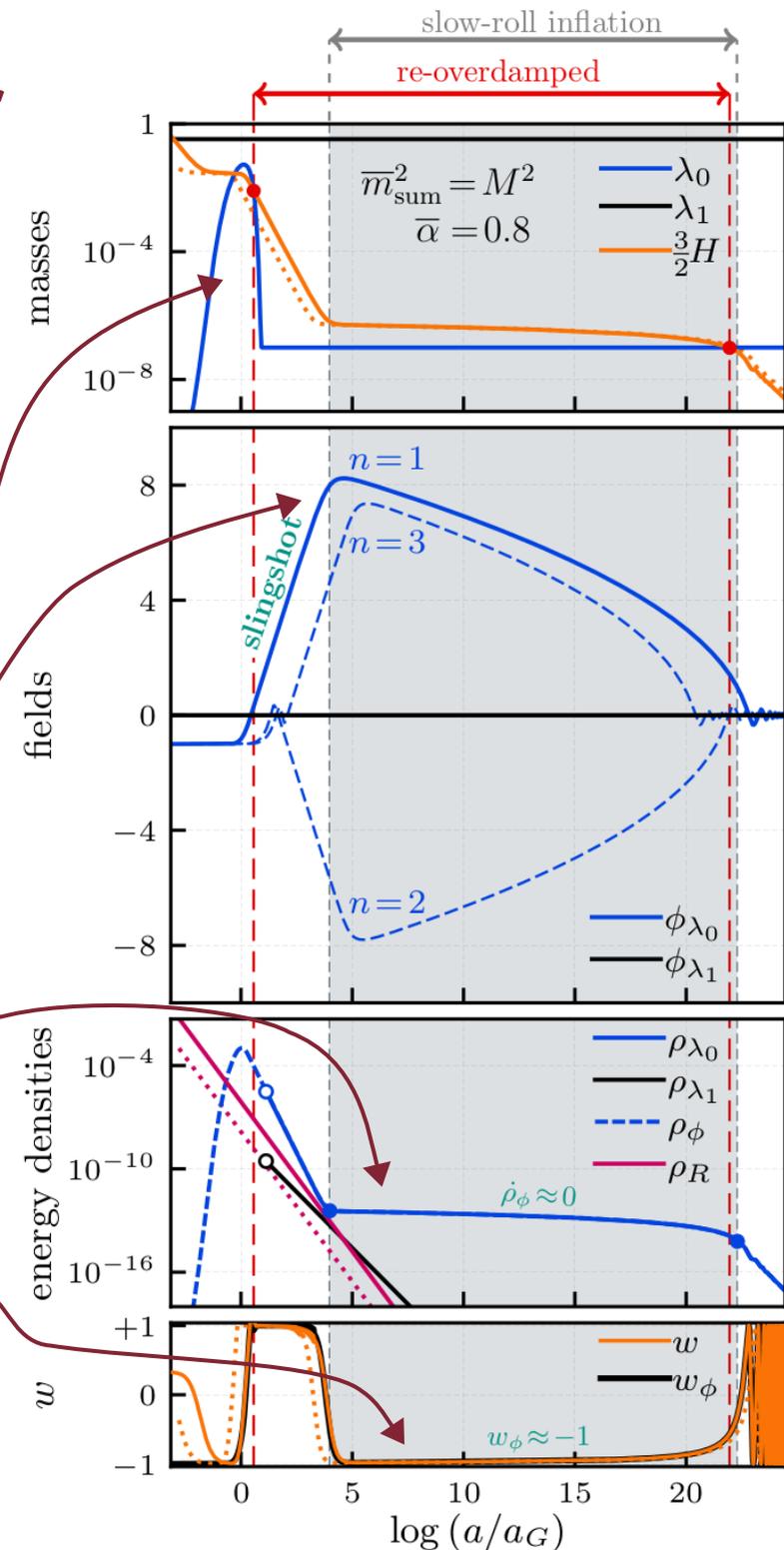
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In this way, an inflationary epoch emerges!



Mapping to Inflationary Quantities

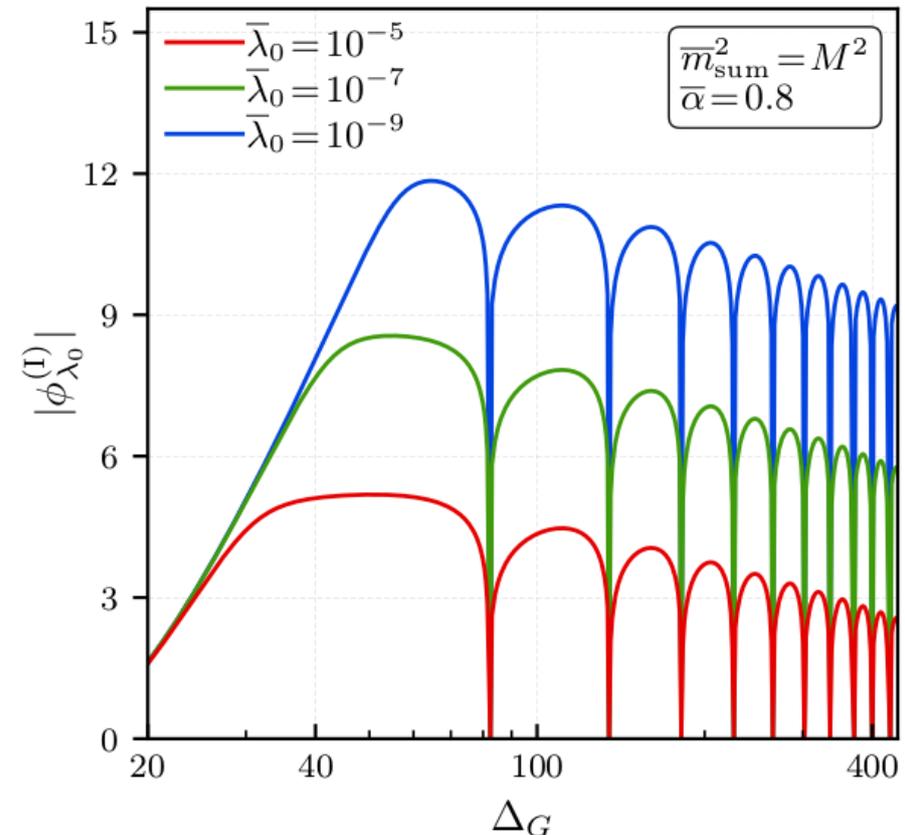
- In general, we can construct a map between the parameters of our two-field model and initial conditions for the inflaton field.

$$\{M^2, \bar{m}_{ij}^2, t_G, \Delta_G\} \longrightarrow \phi_{\lambda_0}^{(I)}$$

- Certain regions of our parameter space give rise to suitable initial field VEVs, while others do not. The most auspicious region of parameter space is that within which...

- ...there is **substantial mixing**.
- ...the late-time inflaton mass is **small**.
- ...the width of the phase transition is at least **approximately on resonance**, though fine-tuning is not required.

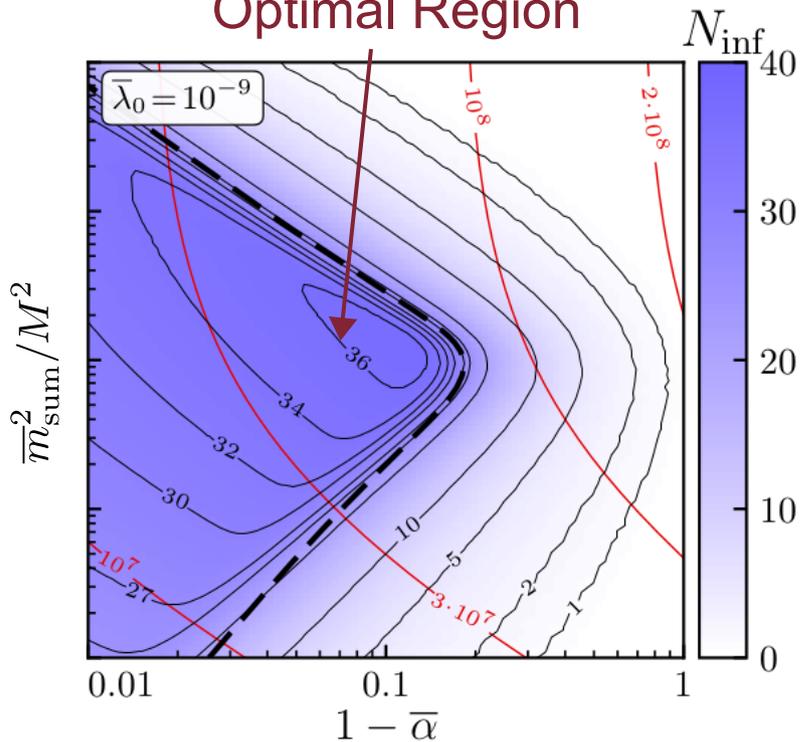
$$\Delta_G \approx \Delta_G^{(n)}$$



Number of e -folds

- The number of e -folds of inflation N_{inf} can be computed numerically across the parameter space of this two-field model.

Optimal Region



(\bar{m}_{sum} and $\bar{\alpha}$ parametrize \mathcal{M}^2)

--- Contour where $\rho_{\lambda_0} = \rho_{\lambda_1}$ at the start of inflation.

— Contours of the ratio Q , where...

$$Q \equiv \frac{\lambda_0(t_p)}{\bar{\lambda}_0}$$

Mass at the peak of the pulse

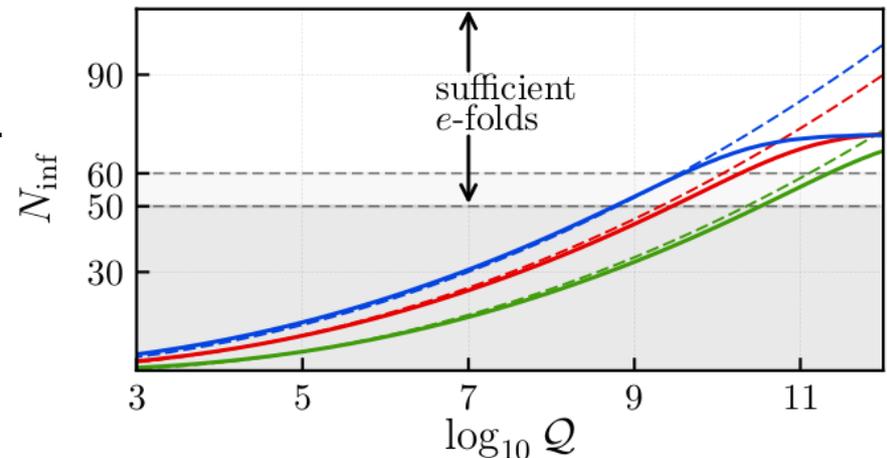
Late-time asymptotic mass

- Within our optimal region of parameter space, N_{inf} turns out to be well approximated (for $n = 1$ and $|\mathcal{A}_\phi| = M_P$) by

$$N_{\text{inf}} \approx \frac{1}{6} \log^2 \left(\frac{2Q}{1 + \sqrt{3} \log Q} \right)$$

- A full numerical calculation reveals that this mechanism **can** generate $N_{\text{inf}} \gtrsim (50 - 60)$ e -folds of inflation!

Solid lines: full numerical results
Dashed lines: approximate values



Summary

- In this talk, we have presented a novel mechanism for realizing the initial conditions for cosmic inflation.
- This mechanism can be realized in theories involving multiple scalar fields in which a cosmological phase transition dynamically gives rise to additional contributions to the scalar mass matrix..
- The time-dependent mixing which arises in the mass matrix during the phase transition give rise to two crucial effects – a **parametric resonance** and a period of **re-overdamping** – which work in concert to propel or “slingshot” the inflaton VEV to larger values and precipitate a subsequent epoch of accelerated expansion.
- We have shown that this scenario is capable of yielding an appropriate number of e -folds $N_{\text{inf}} \gtrsim (50 - 60)$ for successful cosmic inflation.



Future Directions

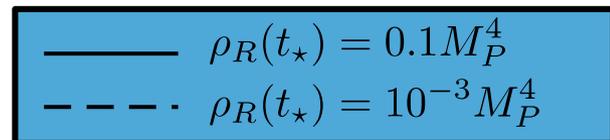
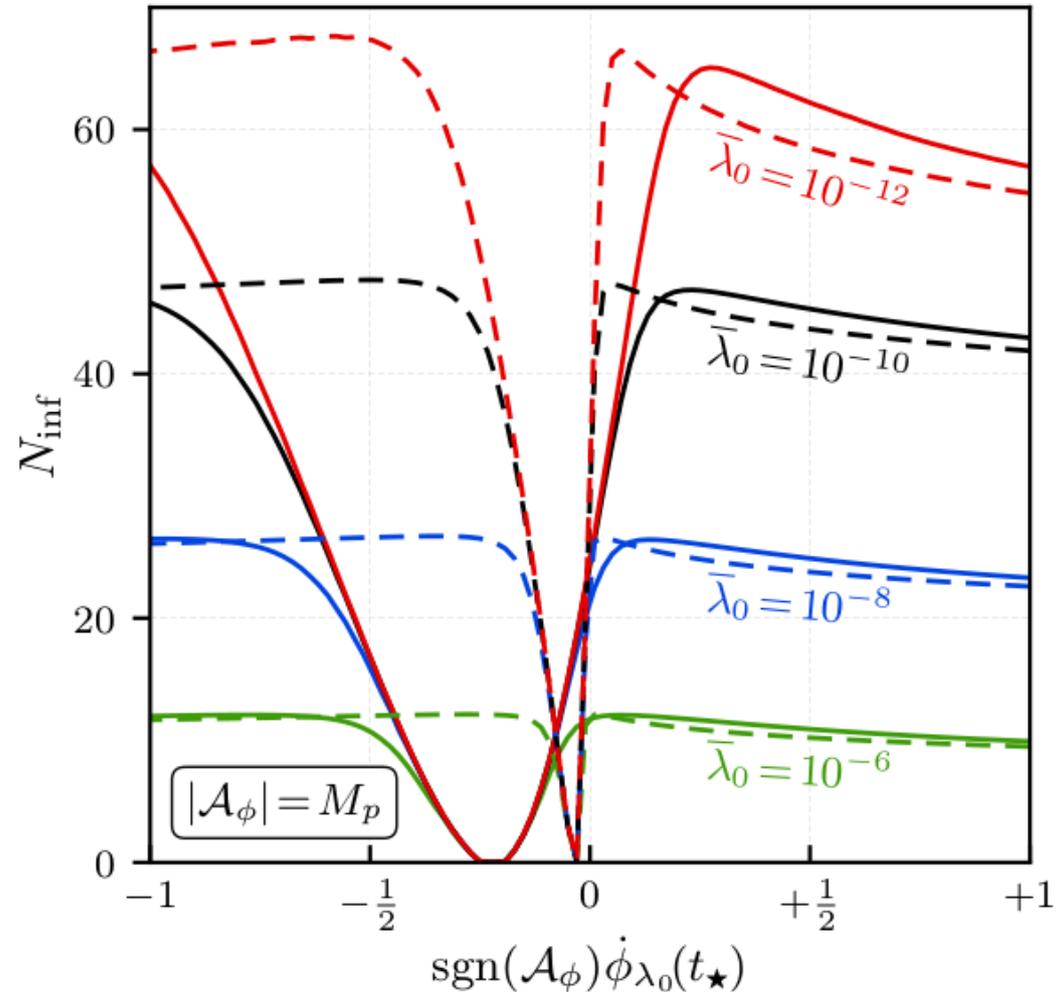
- While challenging to arrange in practice, it may be possible to driving the inflaton VEV to higher values via multiple successive “slingshots.”
- It would be interesting to generalize our results for a quadratic inflaton potential to more general functional forms for $V(\phi)$.
- In scenarios in which $N_{\text{inf}} \sim 60$, the initial conditions which determine the spectrum of large-scale perturbations prior to inflation differ from those associated with the Bunch-Davies vacuum. This has potentially observable consequences. [Ramirez, Schwarz '11; Ramirez '12]
- The presence of additional heavy fields during inflation can affect the pattern of non-Gaussianities in the CMB. This could potentially give rise to “cosmological-collider” signals. [Arkani-Hamed, Maldacena '15]



Extra Slides

Initial Field Velocities

- It is also interesting to consider how results are modified in the case in which the fields in our scalar sector have nonzero initial velocities – and in particular, when $\dot{\phi}_{\lambda_0}(t_\star) \neq 0$, where $t_\star \ll t_G$.
- In cases in which \mathcal{A}_ϕ and $\dot{\phi}_{\lambda_0}(t_\star)$ have opposite sign and the field is propelled to smaller field values by its initial velocity, N_{inf} is significantly suppressed for certain values of $\phi_{\lambda_0}(t_\star)$.
- However, other than this local suppression, N_{inf} remains large, even in the presence of an initial field velocity.
- Thus, our slingshot mechanism is robust against modifications of the initial field velocities.



Inflationary Observables

Important:

Once the slingshot has done its job and inflation begins, we just have regular old quadratic inflation.

- The power spectra for scalar and tensor perturbations are typically parameterized as

$$\mathcal{P}_s = A_s \left(\frac{k}{k_*} \right)^{n_s - 1} \quad \mathcal{P}_T = A_T \left(\frac{k}{k_*} \right)^{n_T}$$

← pivot scale →

- Quantities like $r = A_T/A_s$, the spectral index n_s , etc., are essentially the same as in other models of quadratic inflation.
- By contrast, A_s also depends of the scale of inflation – or, equivalently in our scenario, on the late-time inflaton mass $\bar{\lambda}_0$:

$$A_s = \frac{1}{12\pi^2 M_P^6} \left[\frac{V^{3/2}(\phi_{\lambda_0})}{V'(\phi_{\lambda_0})} \right]^2 \Big|_{\phi_{\lambda_0} = \phi_{\lambda_0}^*} = \frac{1}{6} \left[\frac{(\phi_{\lambda_0}^*)^2 \bar{\lambda}_0}{M_P^3} \right]^2$$

- Planck data with $k_* = 0.05 \text{ Mpc}^{-1}$ yield $A_s = 2 \times 10^{-10}$. [Akrami et al. '18] This implies that $\bar{\lambda}_0 \sim \mathcal{O}(10^{-6} M_P)$.
- While some tension exists between this bound and the bound on N_{inf} , it can be reconciled in many ways – a curvaton, modifications to $V(\phi)$, etc.

Particle Production

- The masses of our scalars change quite abruptly during the phase transition. This non-adiabatic evolution can lead to the production of *inflaton field quanta*.
- The total energy density associated with such quanta can be determined by evaluating the *Bogoliubov coefficients* between the asymptotic states at $t \ll t_G$ and the asymptotic states at $t \gg t_G$.
- These turn out to be mathematically equivalent to the transmission coefficients for scattering off a potential $V(x) \propto -\text{sech}^2(x)$ in non-relativistic quantum mechanics.

Total energy density of background fields \rightarrow

$$\frac{\rho_{\lambda_0}^{(p)}}{\rho_\lambda} \approx \frac{12\zeta(3)}{\pi^6 \lambda_0^2(t_p) \mathcal{A}_\phi^2} \left(\frac{1}{\Delta_G^4} \right)$$

Total energy density of inflaton quanta

- This implies a constraint on Q of the form

$$Q \ll Q_{\max} \approx \frac{\pi^9 \lambda_0^3(t_p) \Delta_G^6 |\phi_{\lambda_0}^{(I)}|^2 \mathcal{A}_\phi^2}{24\sqrt{3}\zeta^{3/2}(3)}$$

Plugging in some numbers \rightarrow

$$Q_{\max} \approx 3.2 \times 10^{11}$$

$\mathcal{A}_\phi = M_P, \bar{m}_{\text{sum}} = M$
 $\bar{\alpha} = 0.9, \Delta_G = \Delta_G^{(1)}$

...which is a non-trivial constraint, but one we can easily satisfy.