Extracting Dark-Matter Velocities from Halo Masses: A Reconstruction Conjecture

Fei Huang

ITP CAS and UC Irvine

arXiv: 2001.02193, 2101.10337

in collaboration with

Keith Dienes, Jeff Kost, Kevin Manogue, Shufang Su, Brooks Thomas

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Basic idea…

arXiv: 2001.02193

early-universe dynamics

dark-matter phase-space distribution $g(p)$

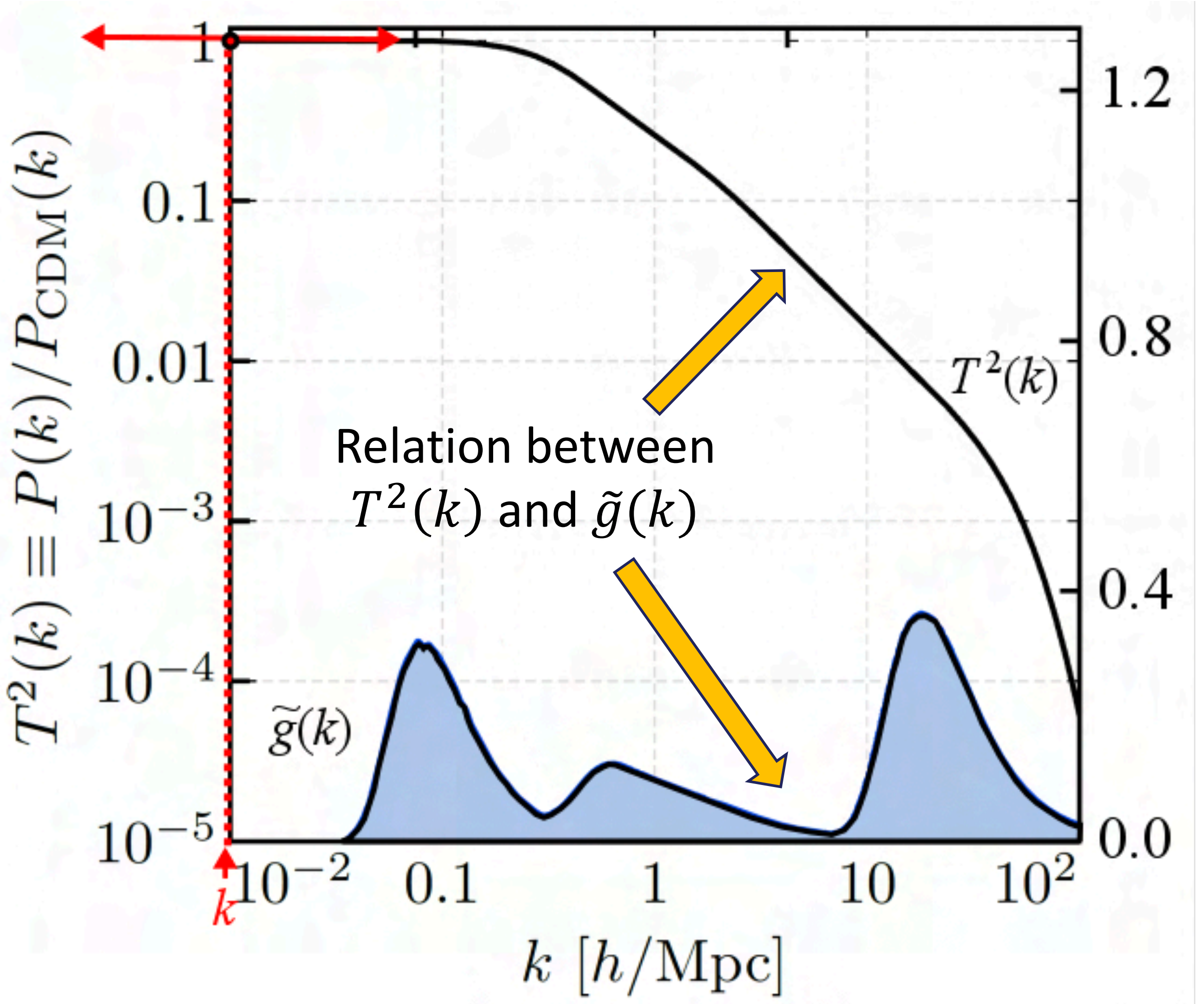
linear matter power spectrum $P(k)$

reconstructed $g(p)$

- **What can we learn from the structure formation?**
- **To what extent is an inversion possible?**

DM velocity v → Wavenumber $k \to k_{hor} \sim 1/d_{hor}(v)$

Distribution in k-space $g(v) \to \tilde{g}(k)$

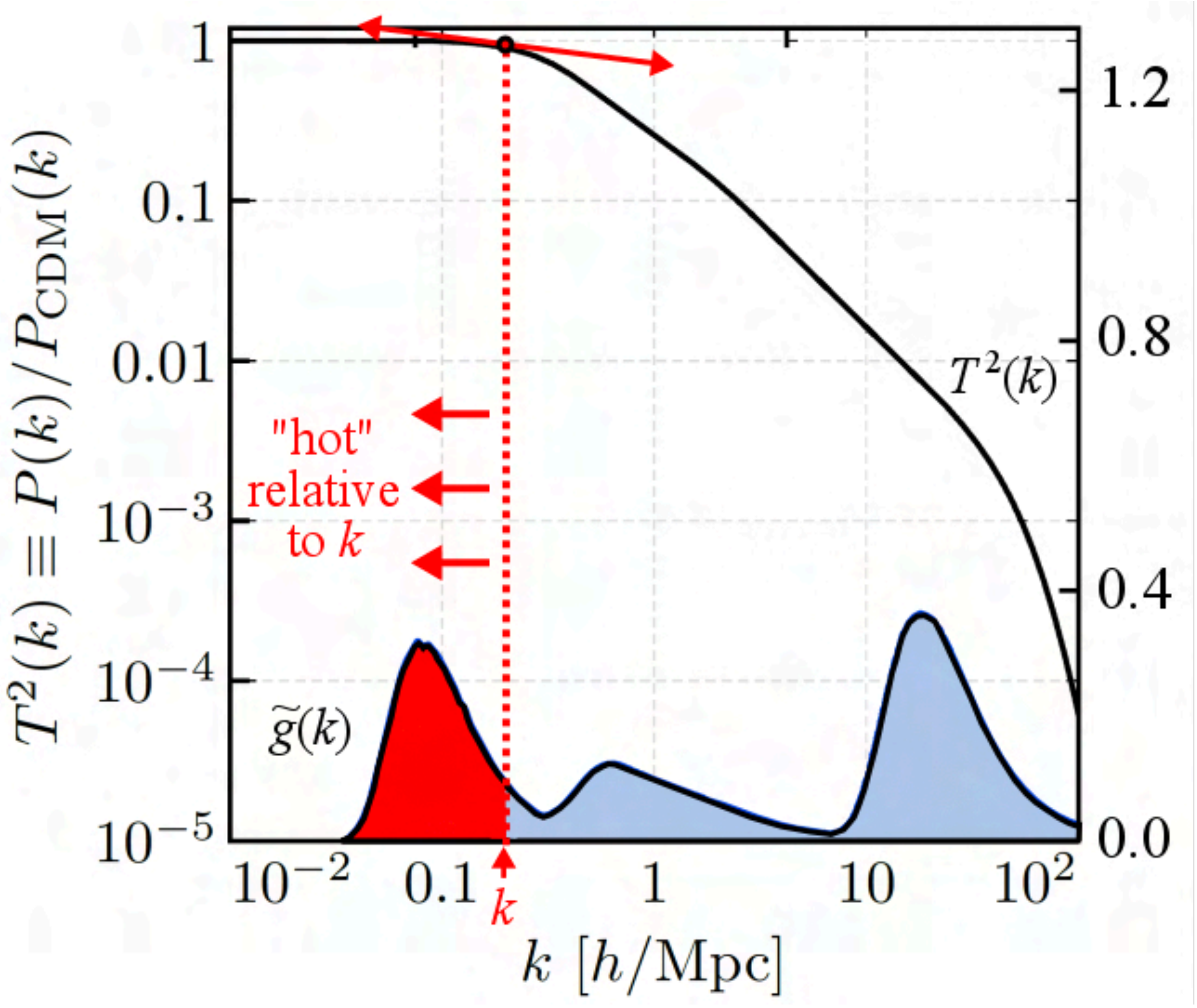


DM velocity v → Wavenumber $k \to k_{hor} \sim 1/d_{hor}(v)$

Distribution in k-space
$g(v) \to \tilde{g}(k)$

The slope of
$T^2(k) \equiv P(k)/P_{CDM}(k)$
is related to the ***"hot-fraction function"***

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k')\, d\log k'}{\int_{-\infty}^{\infty} \tilde{g}(k')\, d\log k'}$$



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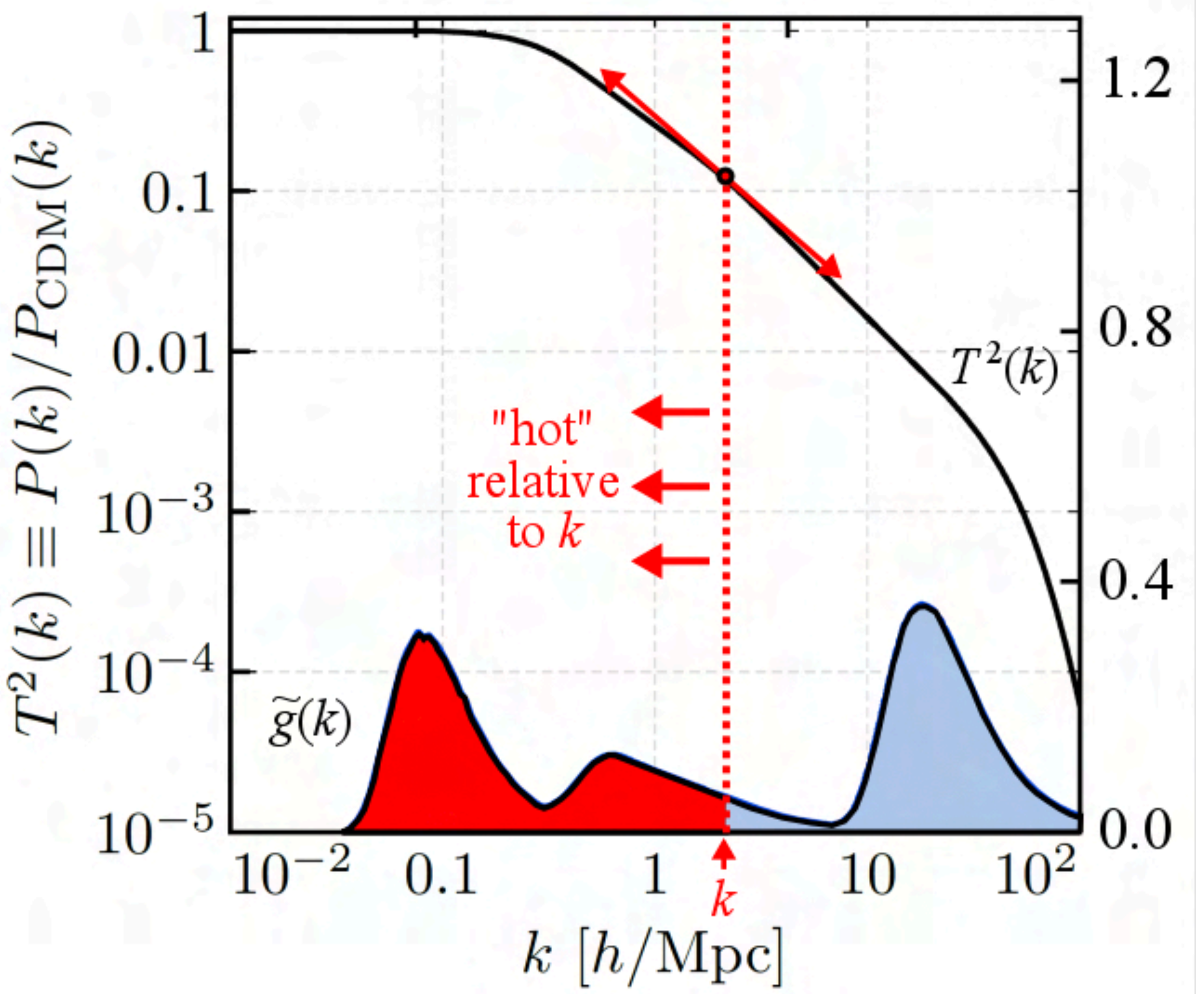

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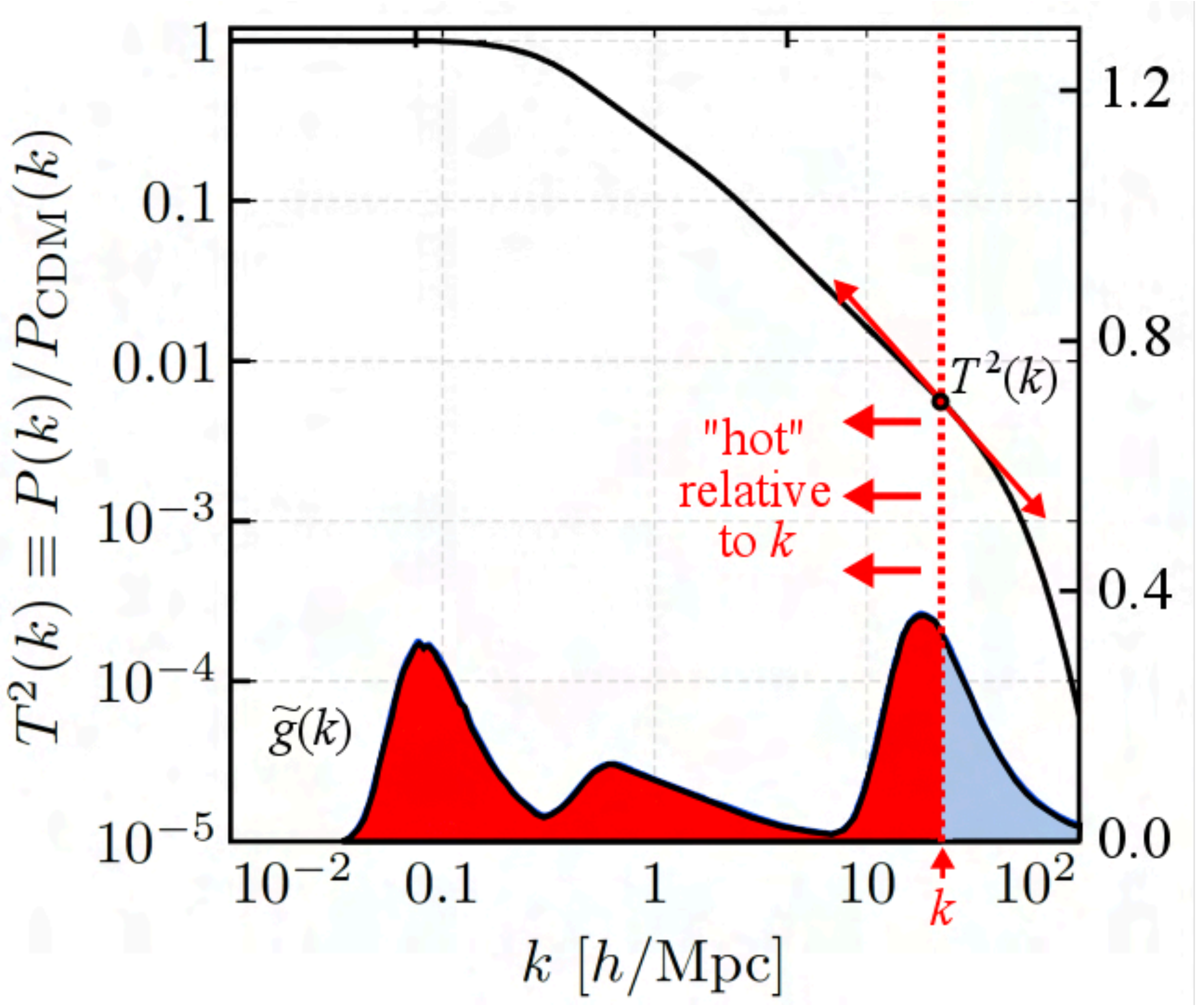

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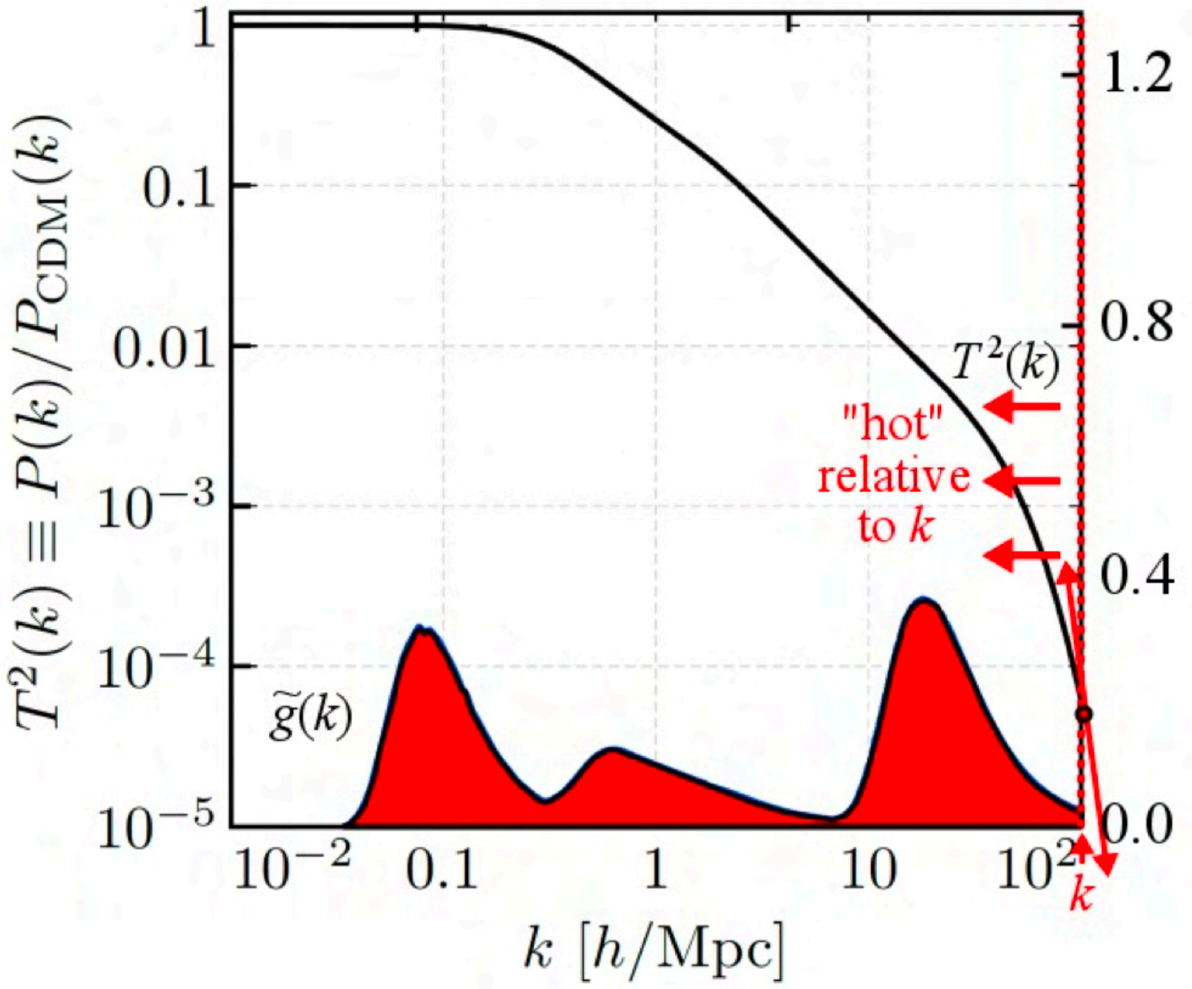

Linear regime: Reconstruction Conjecture

Reconstruct the phase-space distribution $\tilde{g}(k)$
directly from the transfer function $T^2(k)$

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16}+\left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2}\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

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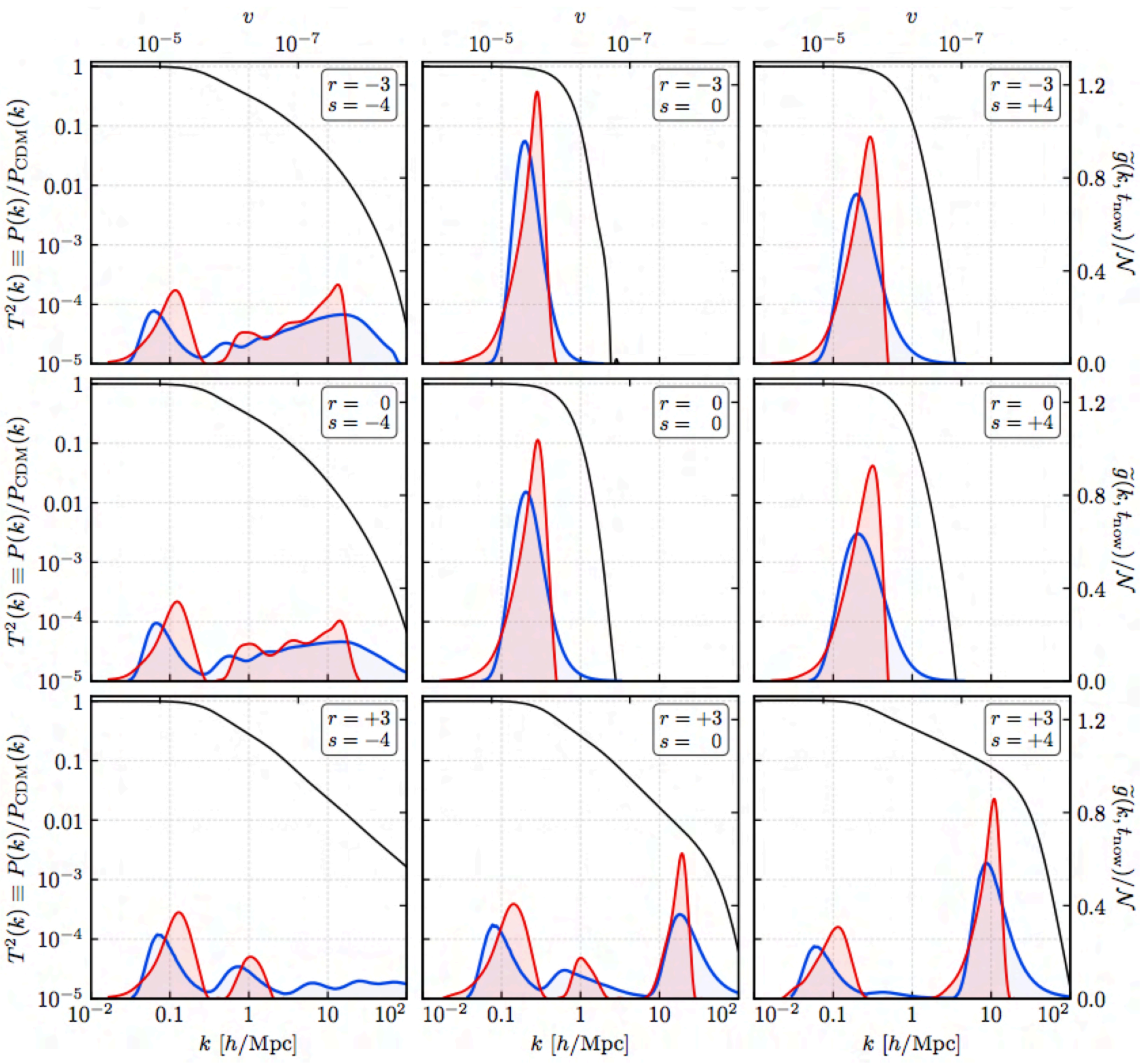
To what extent can we "***resurrect***" the DM phase-space distribution from the transfer function?

Blue: original DM distribution in k-space
Red: reconstruction directly from $T^2(k)$

Archaeological reconstruction is surprisingly accurate for a **variety** of possible DM distributions.
Able to resurrect the **salient features** of the original distribution!



One step further: How about the *nonlinear* regime?

arXiv: 2101.10337

early-universe dynamics

dark-matter phase-space distribution $g(p)$

linear matter power spectrum $P(k)$

Density perturbations eventually go nonlinear

reconstructed $g(p)$

?

nonlinear regime

Is it possible to reconstruct DM velocities in the non-linear regime?

What happens in the nonlinear regime?

In the nonlinear regime, density perturbations $\delta > 1$.

Overdense regions *collapse and* form ***dark-matter halos.***

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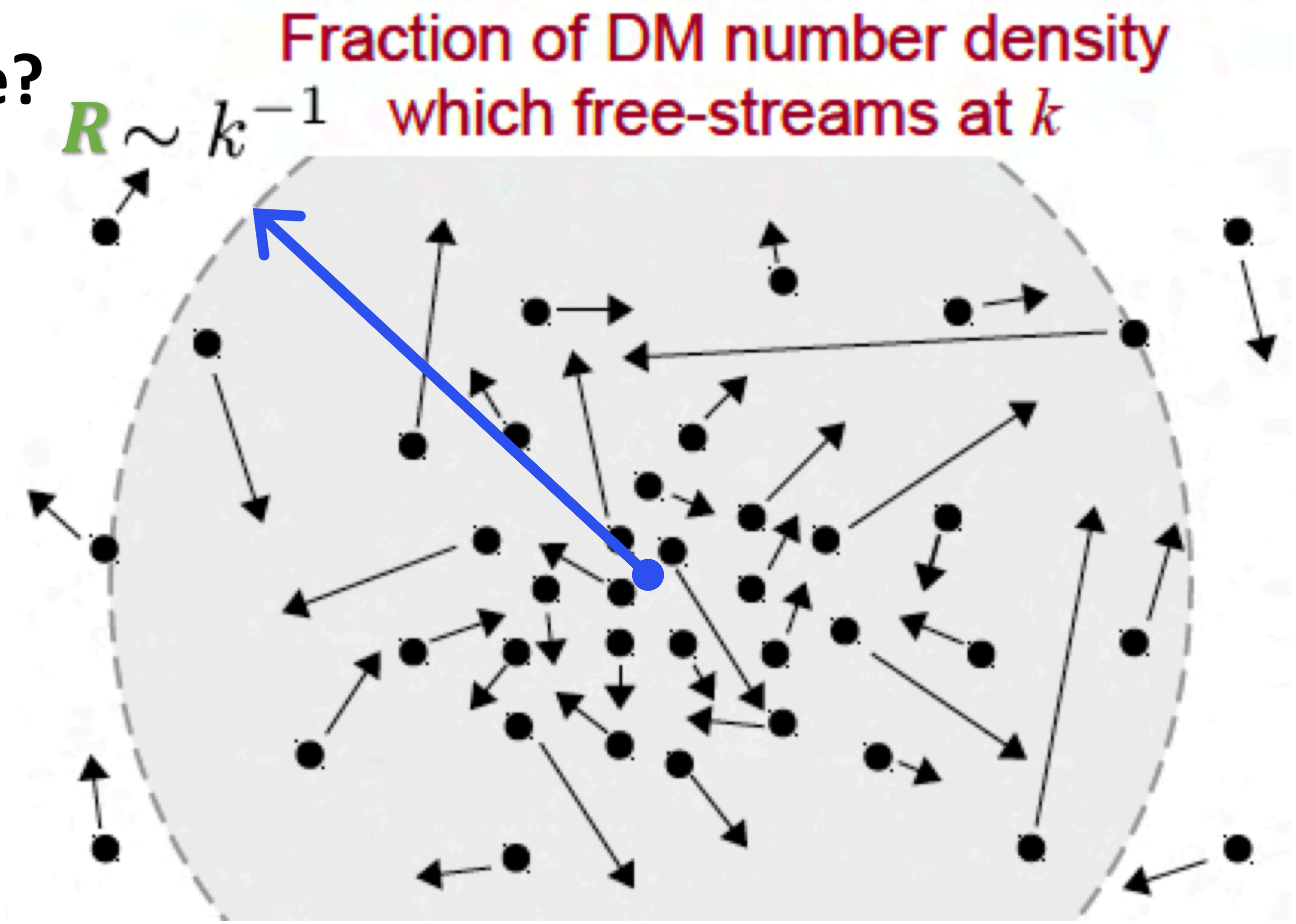
If DM particles have non-negligible velocities, DM halos forming from regions of size R would be ***suppressed*** by DM particles with $k_{hor}^{-1}(v) \gtrsim R$



Halos of mass $M \sim \bar{\rho} \times R^3$

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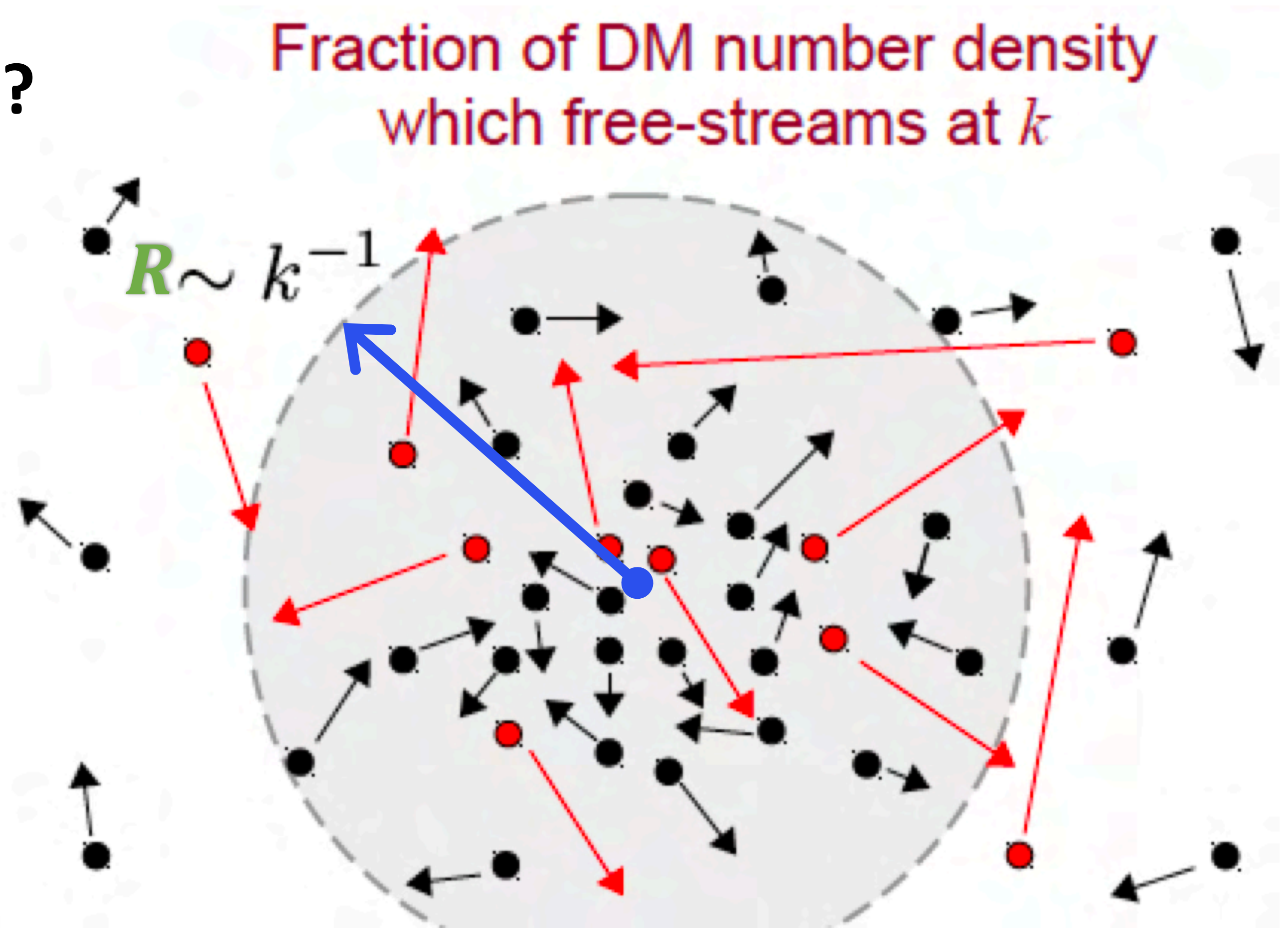

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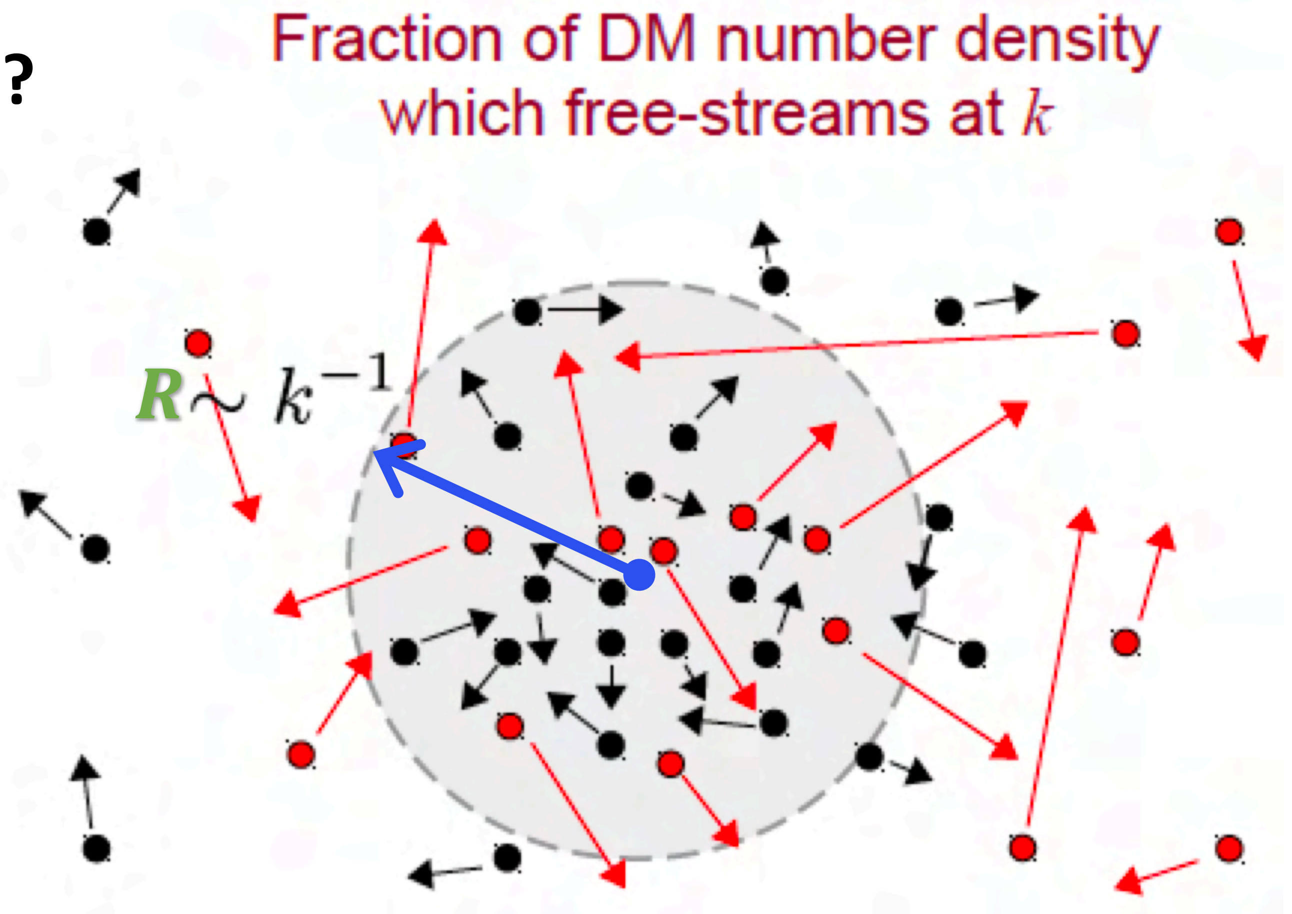

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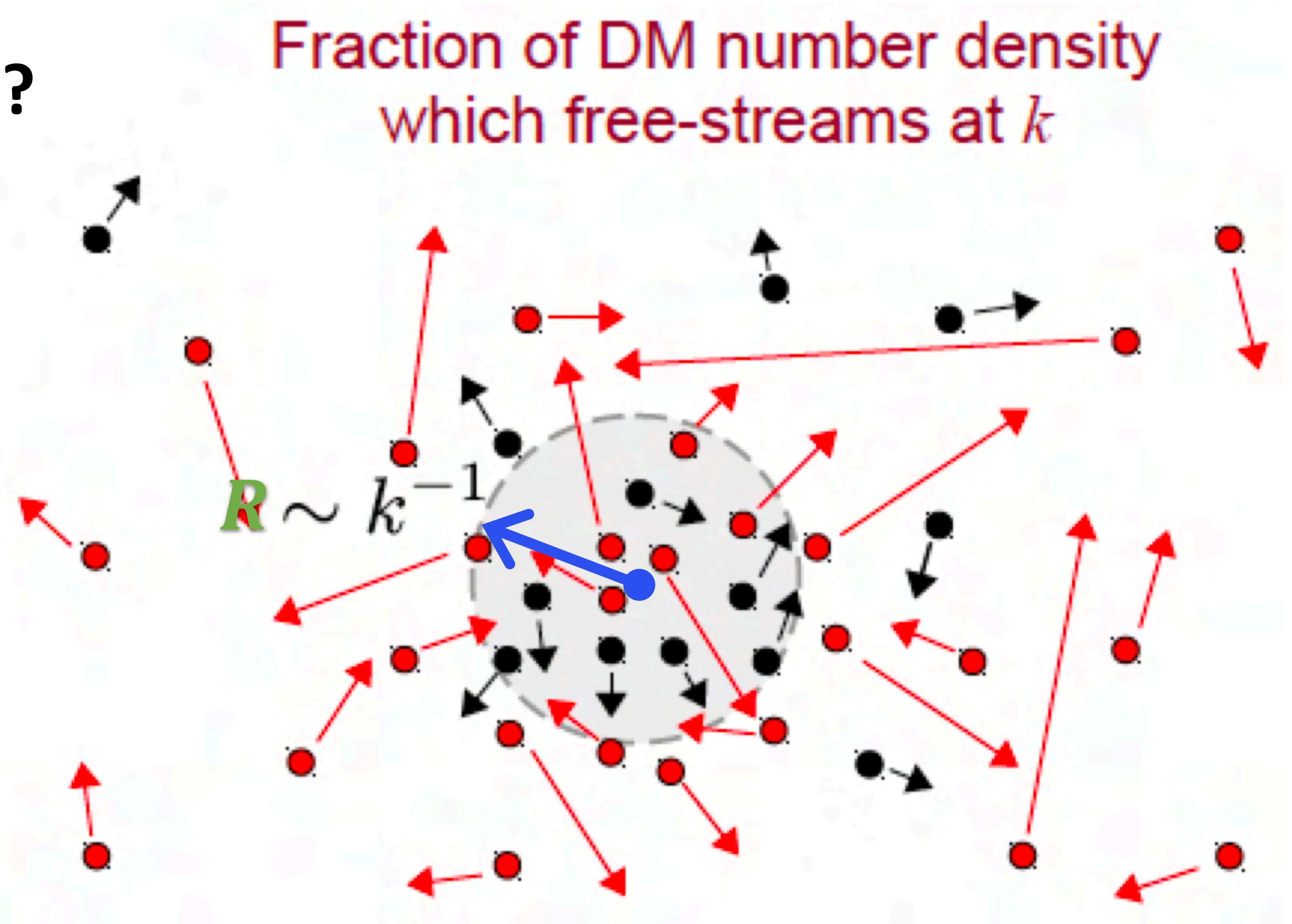

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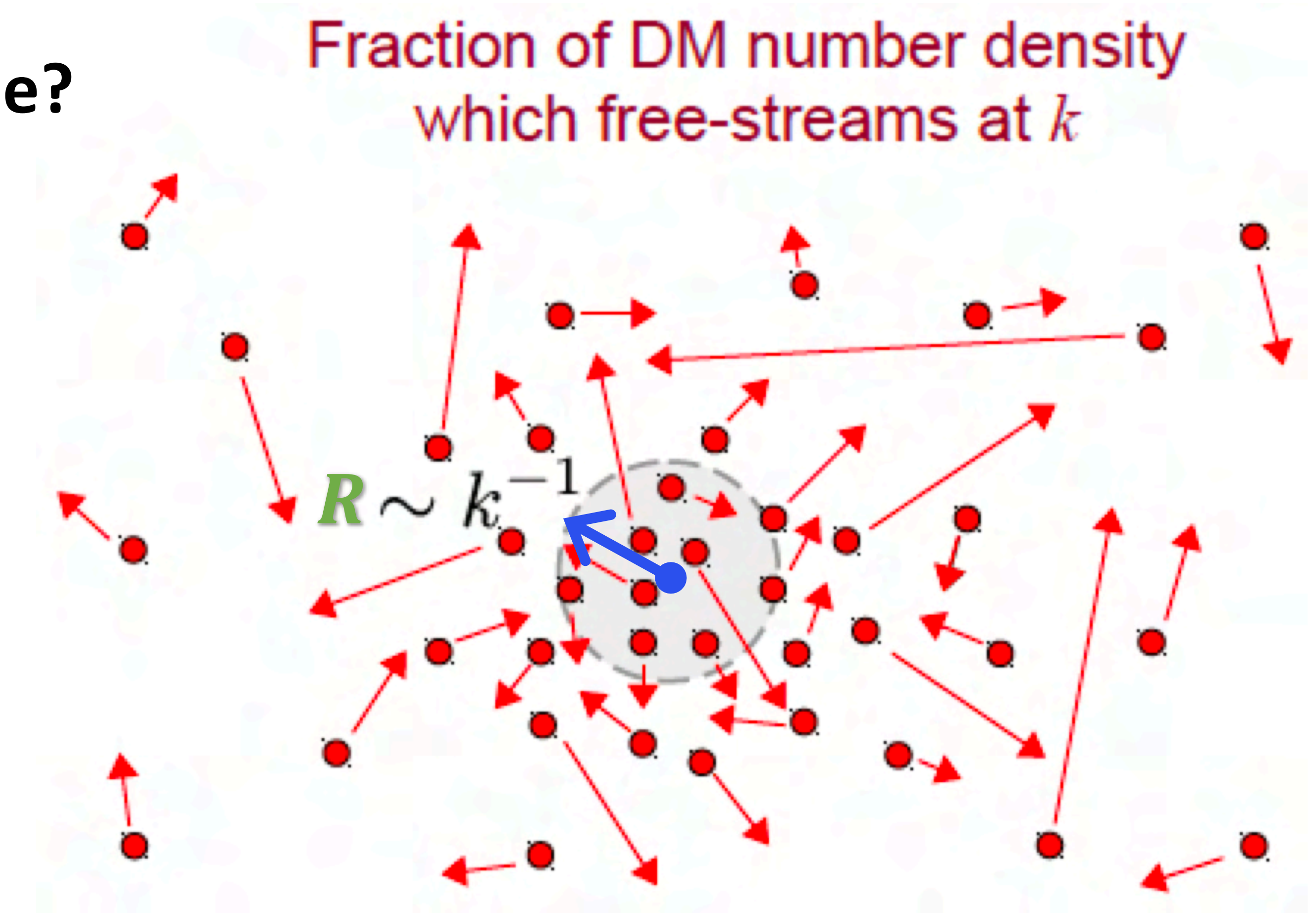

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Overdense regions *collapse and* form ***dark-matter halos.***

If DM particles have non-negligible velocities, DM halos forming from regions of size R would be ***suppressed*** by DM particles with $k_{hor}^{-1}(v) \gtrsim R$ → Halos of mass $M \sim \bar{\rho} \times R^3$

Naturally led to think about the ***halo mass function, $dn/d\log M$***, *i.e.*, comoving number density of halos in a logarithmic mass interval

Press-Schechter formalism

- Spherical collapse model: *linearly* extrapolate $\delta(\vec{x},t)$, regions have collapsed and formed DM haloes by time t if

$$\delta(\vec{x},t) > \delta_c \approx 1.686$$

- Probability that $\boldsymbol{\delta_M > \delta_c}$ proportional to fraction of halos with mass $> M$:

$$\mathcal{P}(m > M, t) = 2\mathcal{P}(\delta_M > \delta_c, t, M)$$

- Gaussian:

$$\mathcal{P}(\delta_M > \delta_c, t, M) = \int_{\delta_c}^{\infty} d\delta_M \frac{1}{\sqrt{2\pi\sigma^2(t,M)}} \exp\left[-\frac{\delta_M^2}{2\sigma^2(t,M)}\right]$$

$$\sigma^2 \equiv \int \frac{d^3k}{(2\pi)^3} P(k) W^2(k,R)$$

With these simple ingredients

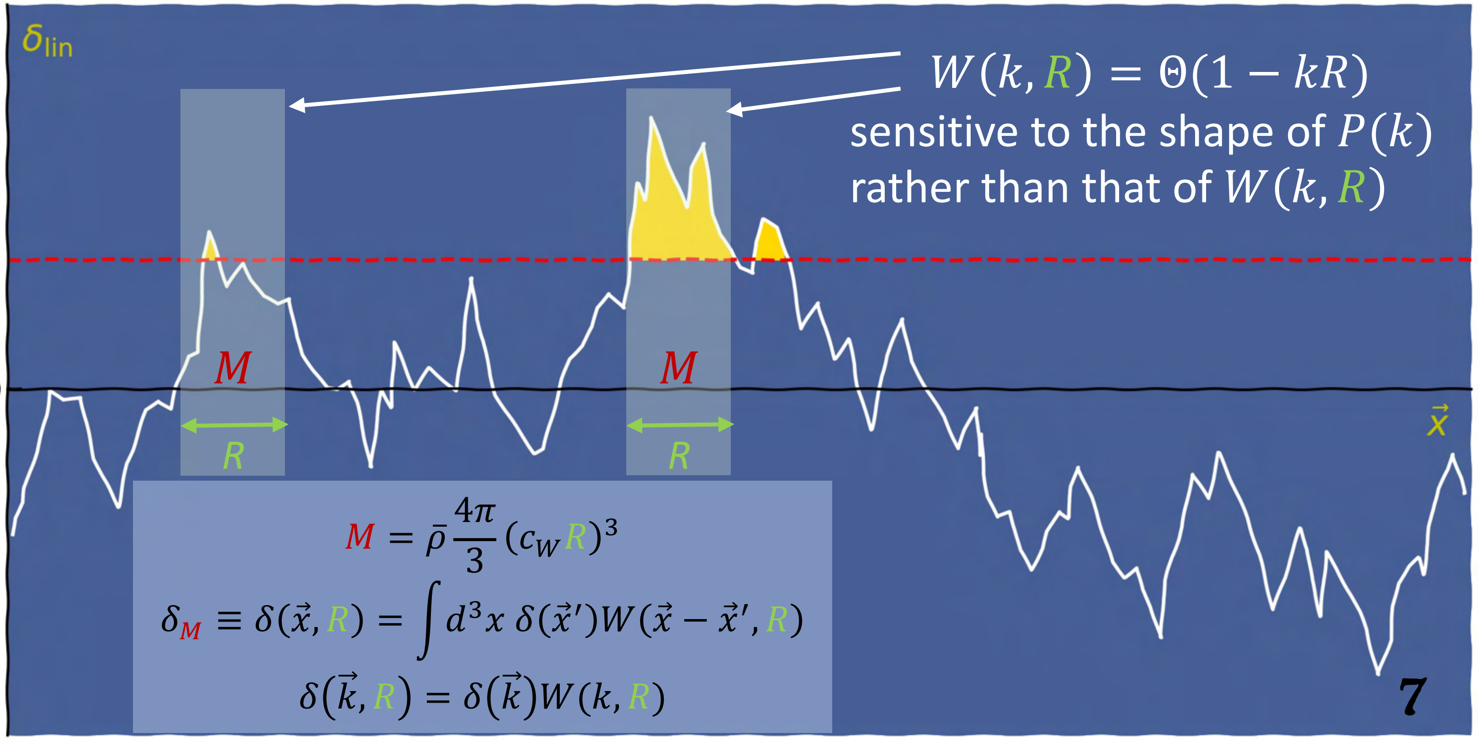
$$\frac{dn}{dM} = \frac{\rho_0(t)}{M}\frac{\partial\mathcal{P}(m > M,t)}{\partial M}$$

which can be related to $P(k)$

$$\frac{dn}{d\log M} = \frac{\bar{\rho}}{12\pi^2 M}\nu(M)\eta(M)\frac{P(1/R(M))}{\delta_c^2 R^3(M)}$$

$g(v)$ → **CLASS** → $P(k)$ → PS → $\dfrac{dn}{d\log M}$

Inversion possible?



Relating DM velocity v and the halo mass M

Recall the functional map $v \to k$

$$k \to k_{\rm hor}(v) \sim 1/d_{hor}(v)$$

Minimum scale k that DM particles with v can suppress.

In the Press-Schechter formalism

$$M = \bar{\rho} \times \frac{4\pi}{3}(c_W R)^3$$

$$W(k, R) = \Theta(1 - kR)$$

tells us the *maximum* halo mass that density perturbations k could have effect on is $M \sim \bar{\rho} \times k^{-3}$.

Let us instead take this threshold relation as defining a functional map $k \to M$

$$M(k) \equiv \bar{\rho} \times \frac{4\pi}{3}\left(\frac{c_W}{k}\right)^3$$

⇓

Together, we define a functional map $v \to M$

Maximum M that DM particles with v can suppress

$$g(v) \to g_M(M)$$

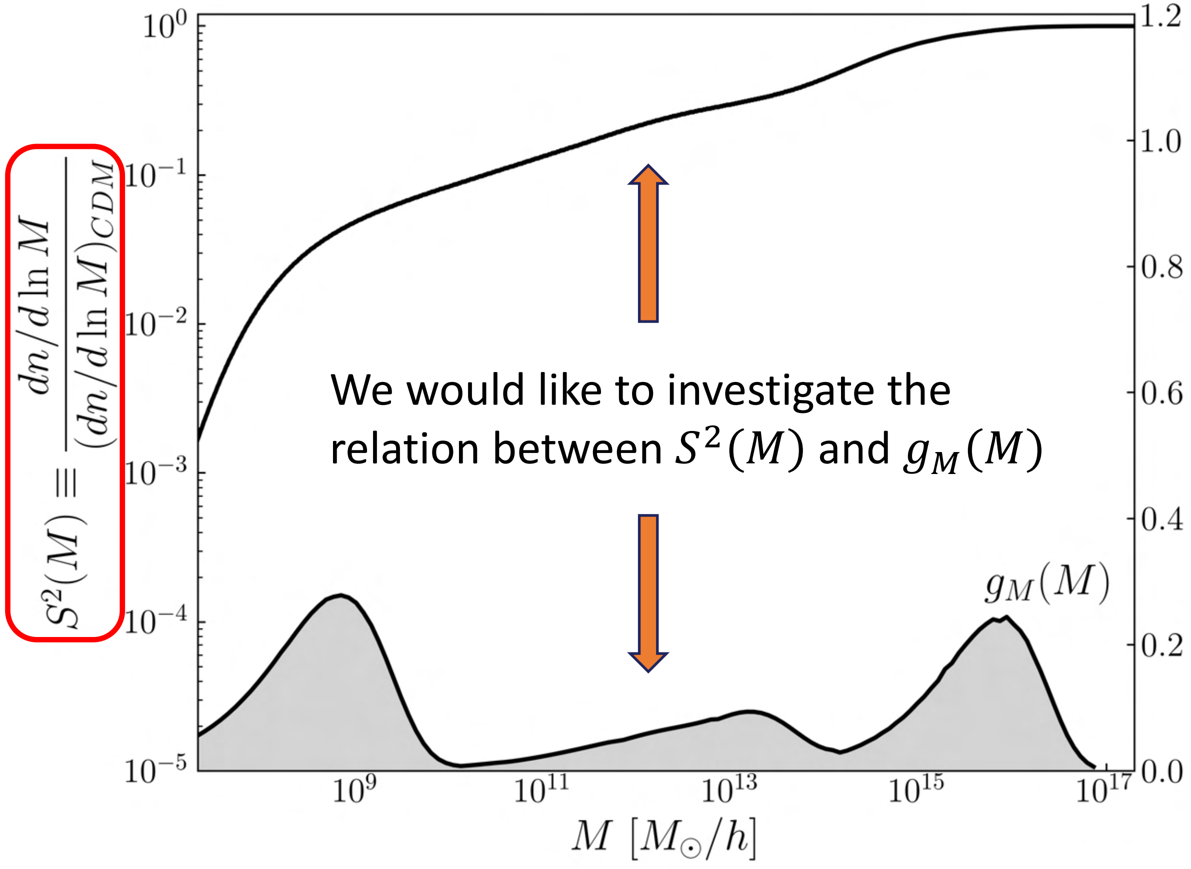
Distribution in the M-space!

Can be plotted together with the halo mass function to investigate their relations!

Halo mass function and distribution in *M-space*

Analogous to the transfer function, we define the ***structure-suppression function***

$$S(M) \equiv \sqrt{\frac{dn/d\log M}{(dn/d\log M)_{CDM}}}$$



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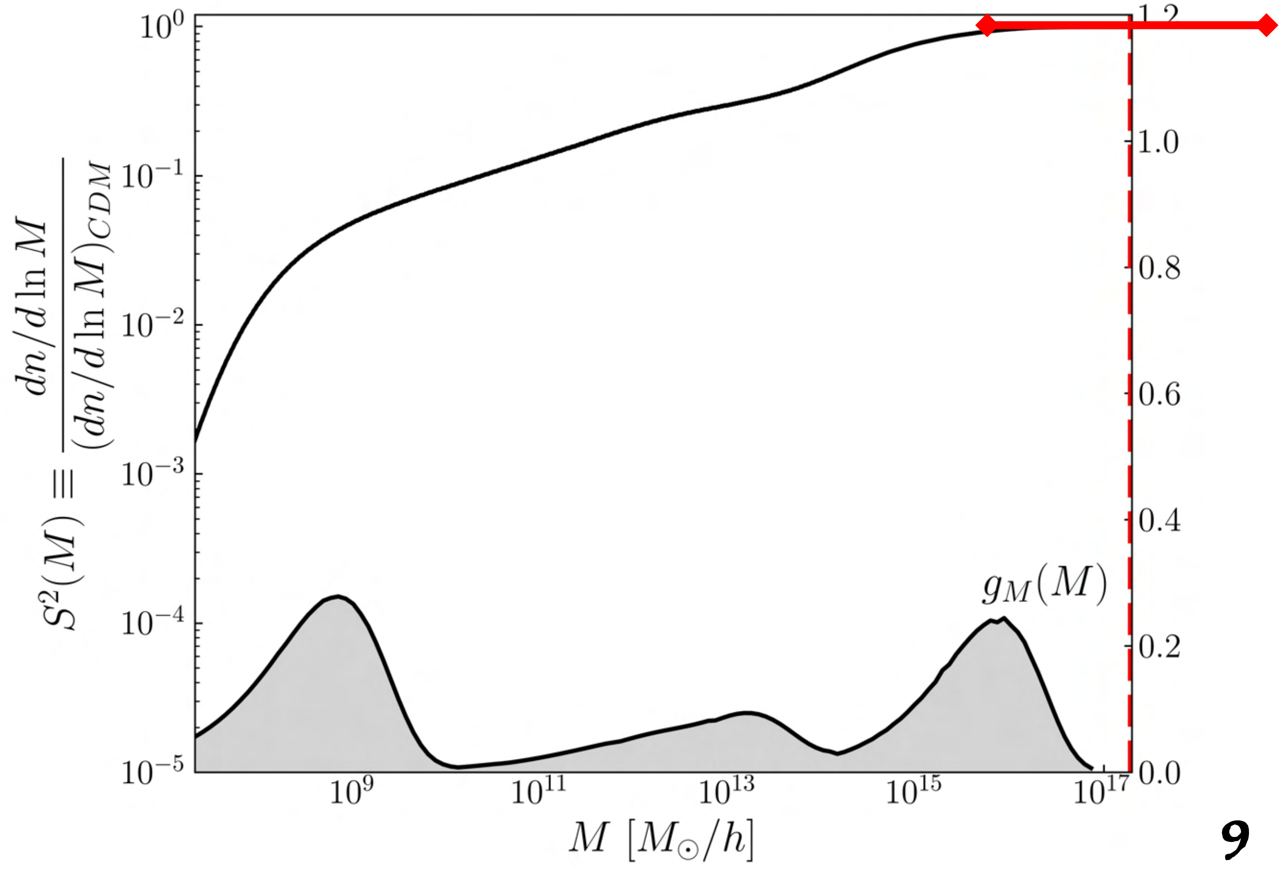
Hot-fraction function in the M-space:

$$F(M) \equiv \frac{\int_{\log M}^{\infty} d\log M'\, g_M(M')}{\int_{-\infty}^{\infty} d\log M'\, g_M(M')}$$

fraction of DM with velocities large enough to escape a region that would collapse into a halo of mass M



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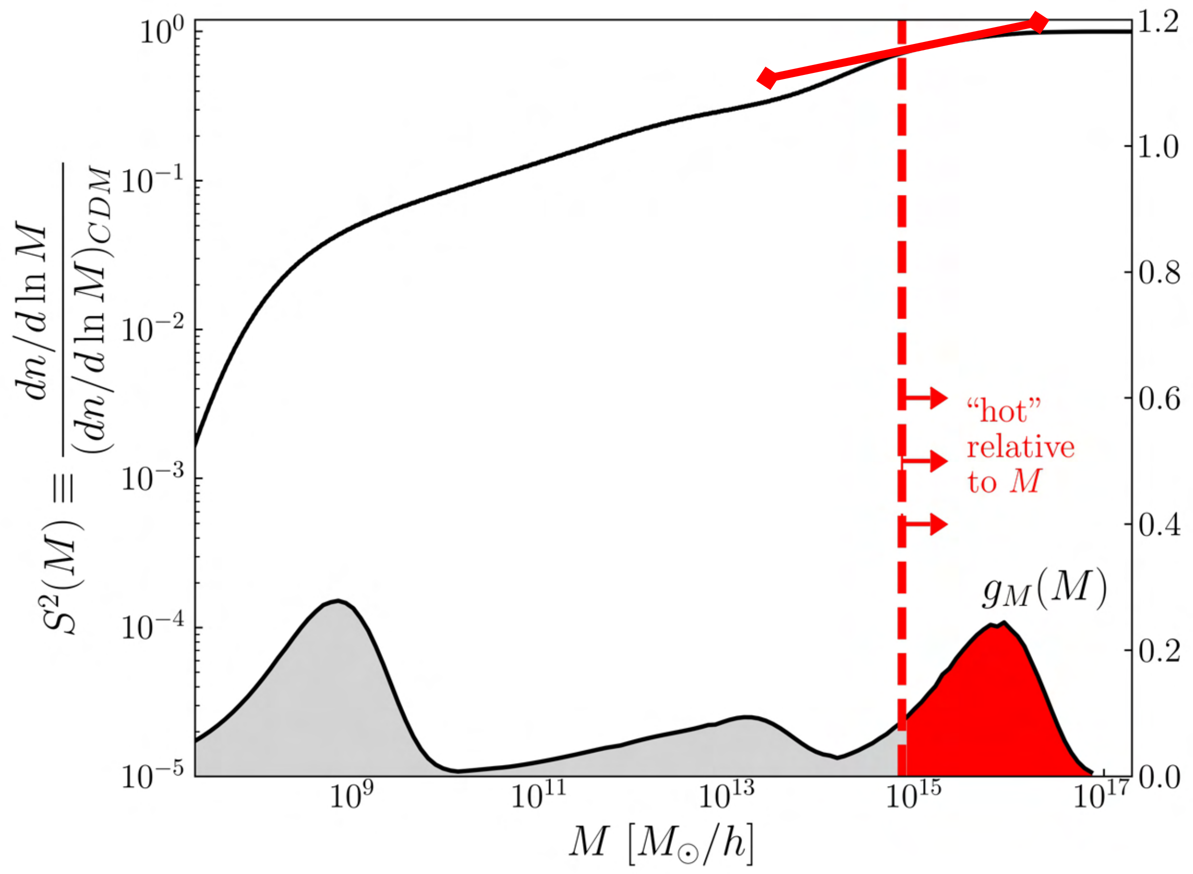

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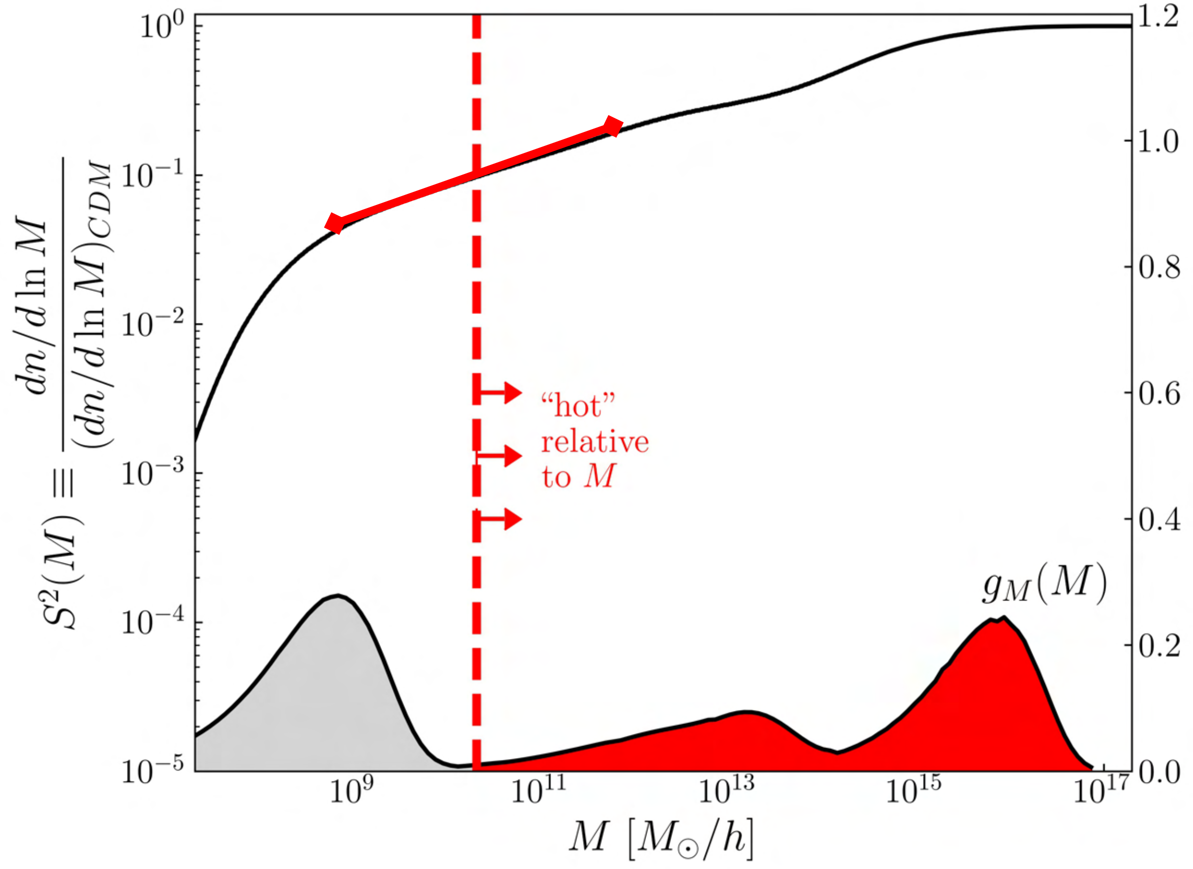

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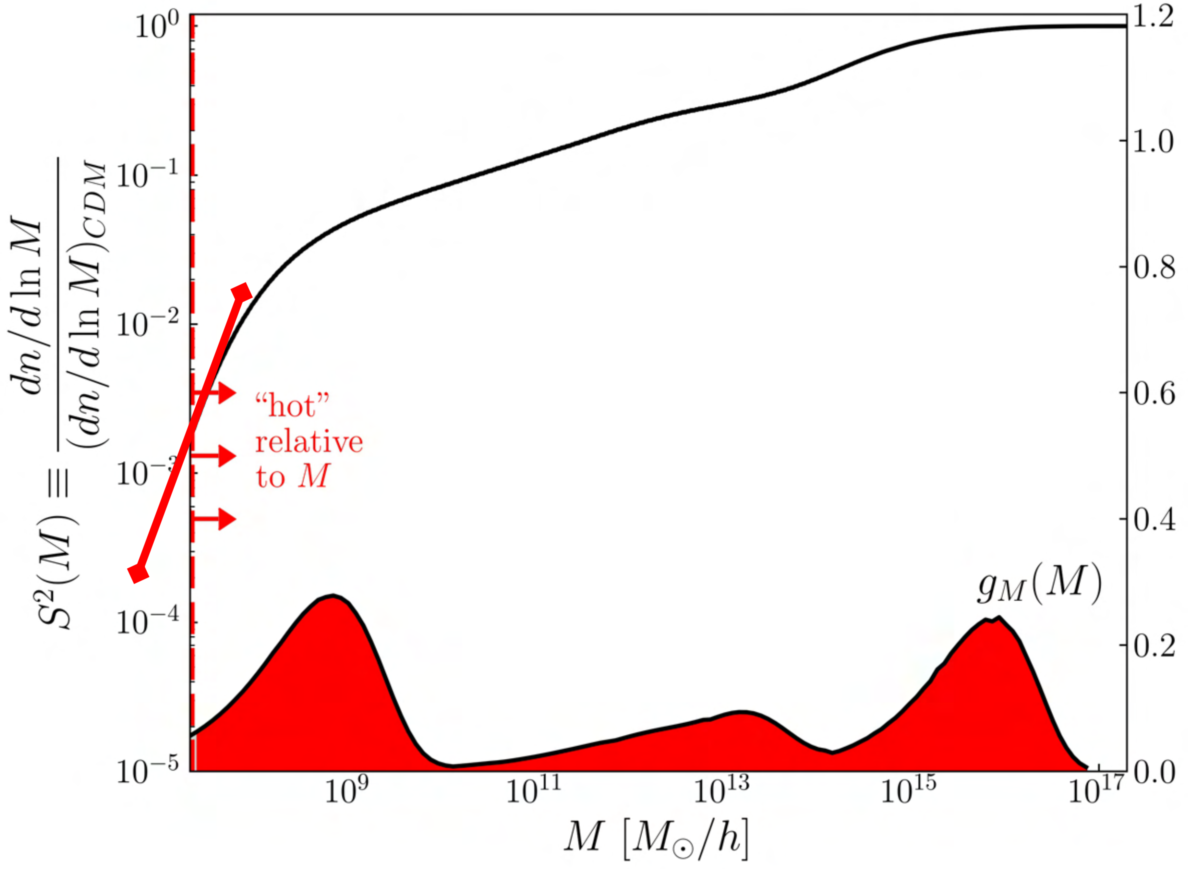

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Once again, we find the **logarithmic slope** of $S^2(M)$ is ***directly*** related to $F(M)$!

$$\frac{d\log S^2(M)}{d\log M} \approx \frac{7}{10} F^2(M)$$

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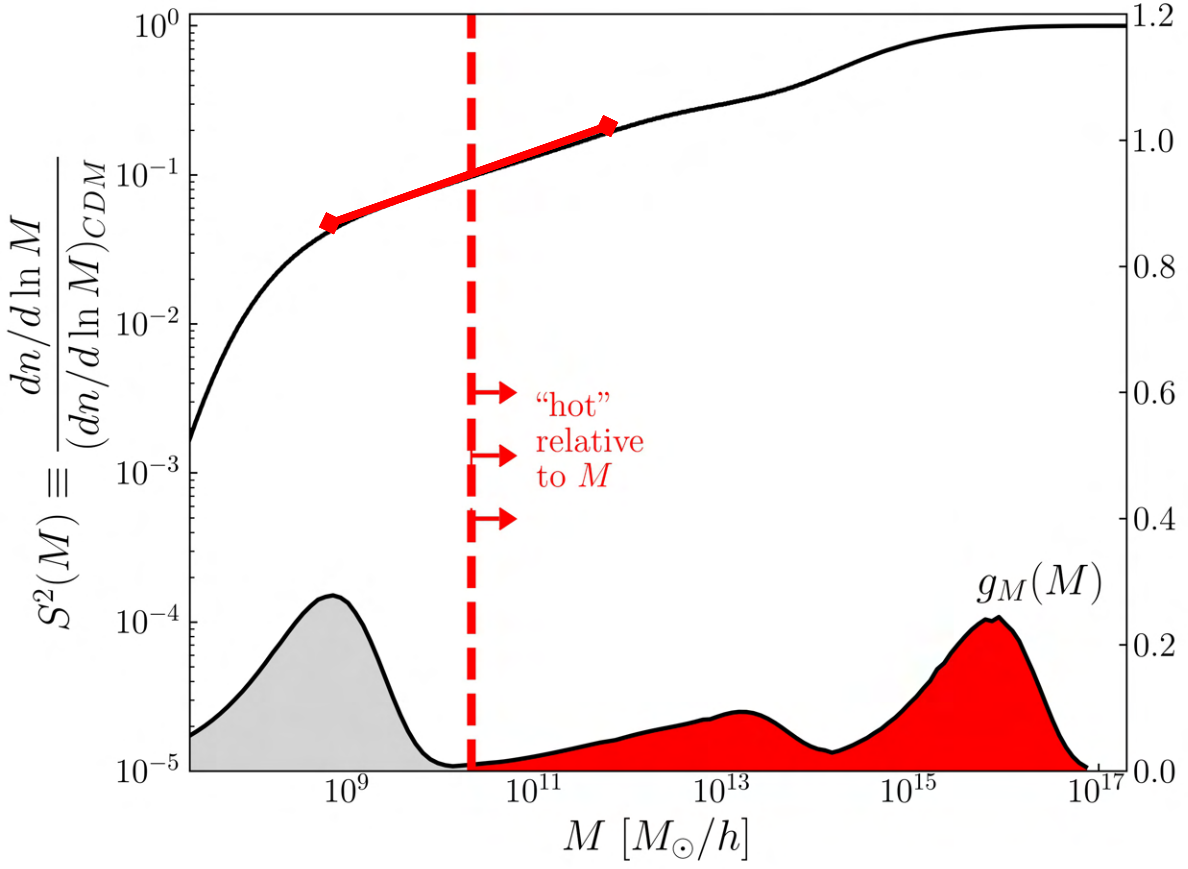

Non-linear regime: Reconstruction Conjecture

With $$\frac{d\log S^2(M)}{d\log M} \approx \frac{7}{10} F^2(M)$$

and $$\frac{dF}{d\log M} = \frac{g_M(M)}{\mathcal{N}}$$

take derivatives on both sides and find

$$\frac{g_M(M)}{\mathcal{N}} \approx \sqrt{\frac{5}{14}} \left(\frac{d\log S^2(M)}{d\log M}\right)^{-\frac{1}{2}} \left|\frac{d^2\log S^2(M)}{(d\log M)^2}\right|$$

This allows us to ***reconstruct*** the DM phase-space distribution ***directly*** from $S^2(M)$!

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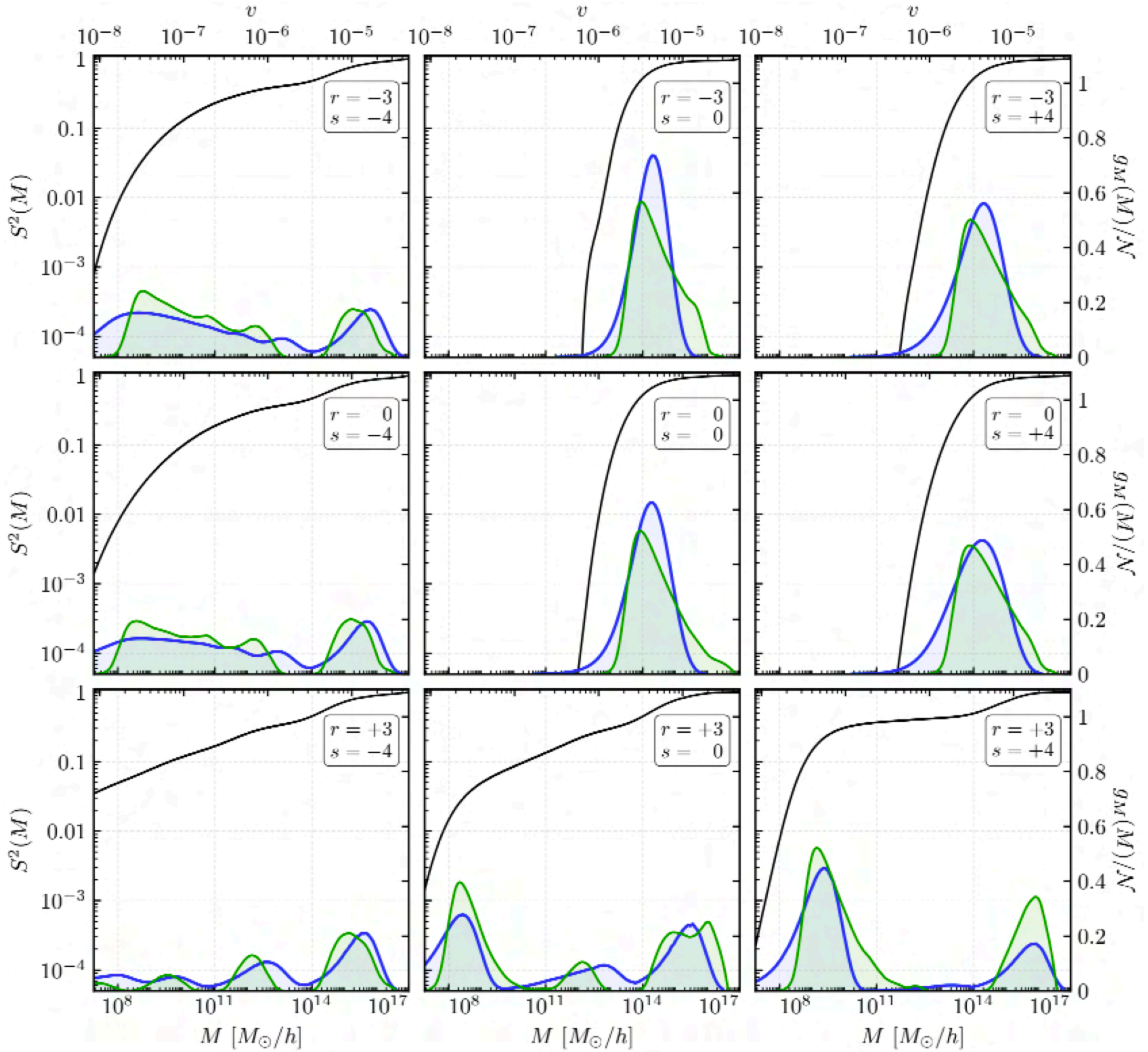
Blue: original DM distribution in M-space
Green: reconstruction directly from $S^2(M)$

Once again, able to resurrect the **salient features** of the original distribution!



Conclusions

- ***Identifiable patterns*** in the phase-space distribution $g(v)$ of dark matter are ***imprinted*** on the cosmic structure even in the non-linear regime.
- The DM phase-space distribution $g(v)$ is ***correlated*** with the halo mass function $dn/d\log M$ through the ***hot-fraction function*** $F(M)$.
- We proposed a ***reconstruction conjecture*** in the nonlinear regime which enables us to reproduce $g(v)$. The reconstruction conjecture is simple and allows us to ***resurrect the salient features*** of the phase-space distribution directly from $dn/d\log M$.
- Such approaches allow us to learn about dark-sector dynamics even ***if the dark sector has only gravitational couplings to the SM***.
- There are recent efforts aimed at probing $dn/d\log M$ observationally. Our work provides motivation to probe $dn/d\log M$ with increased precision.