Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors

Jeff Kost
[University of Sussex]

[arXiv:2001.02193]
[arXiv:2101.10337]

collaborators on this work:
Keith R. Dienes [U. Arizona]
Fei Huang [U.C. Irvine/ITP-CAS]
Kevin Manogue [Lafayette College]
Shufang Su [U. Arizona]
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PPC 2021

Wednesday, May 19th, 2021

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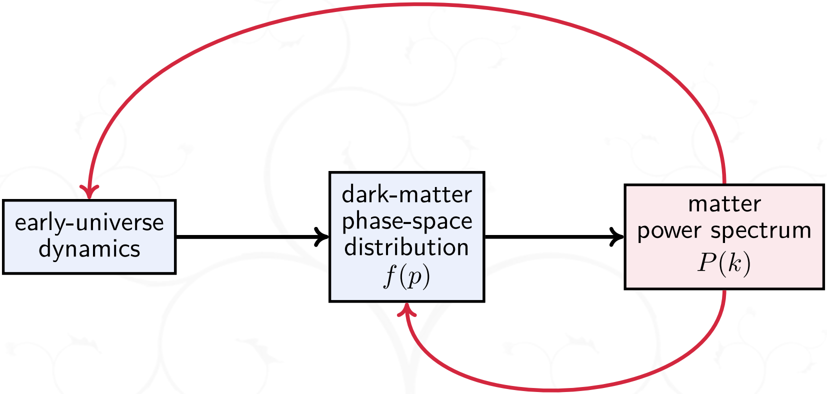

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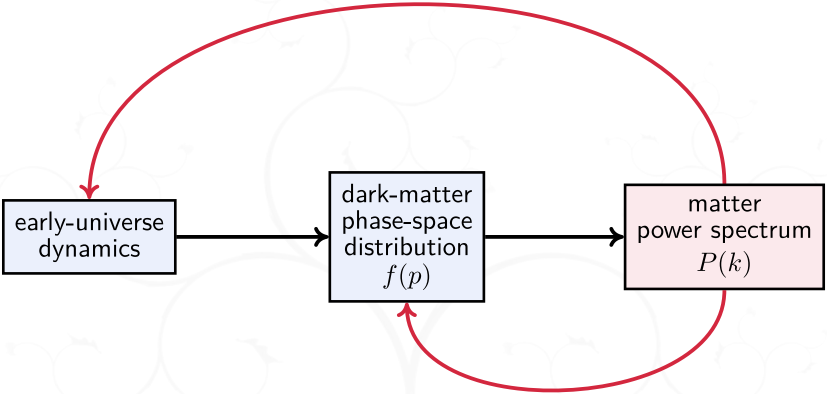

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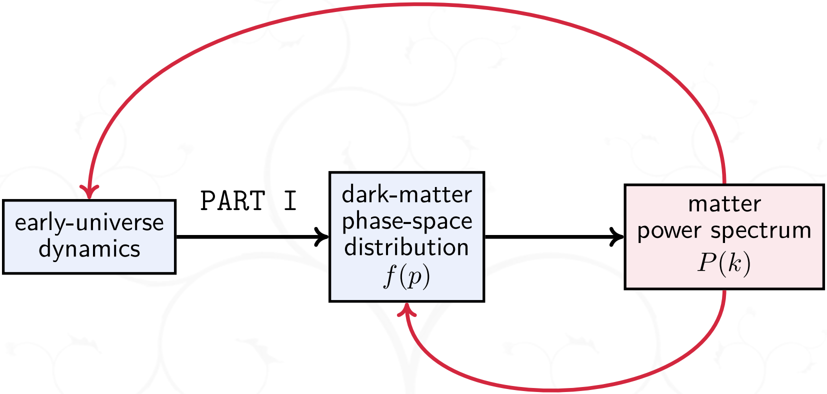

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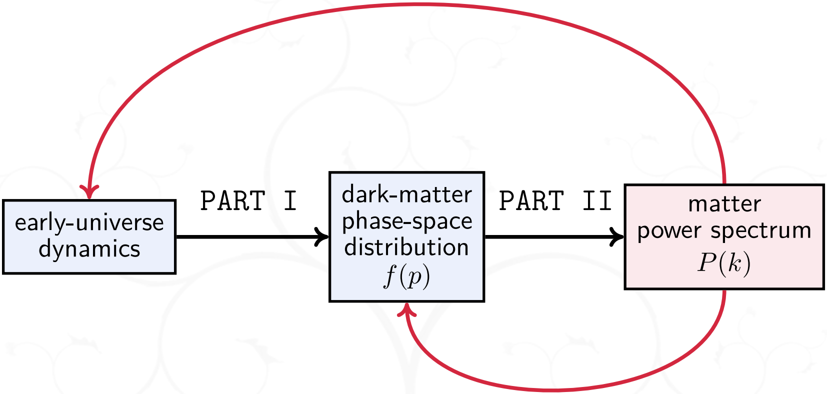

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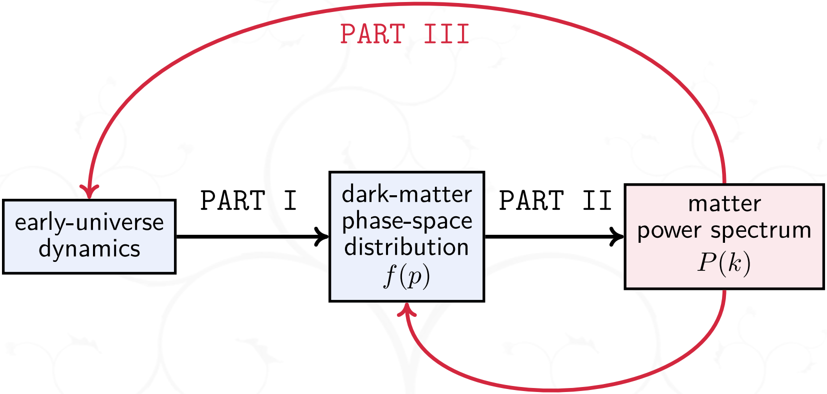

To **what extent** can we find signatures or patterns in $P(k)$ tell us about early universe dynamics that produced the dark matter?

PART I

early-universe dynamics $\longrightarrow$ DM phase-space distribution

• Once DM is produced, many of its properties are described by its primordial phase-space distribution $f(\vec{x},\vec{p},t) \underbrace{\approx f(p,t)}_{\text{homogeneity/isotropy}}$:

comoving number density:

$$N(t) = g_{\text{int}} \int \frac{d^3p}{(2\pi)^3} a^3 f(p,t)$$

energy density:

$$\rho(t) = g_{\text{int}} \int \frac{d^3p}{(2\pi)^3} E f(p,t)$$

pressure:

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$\Rightarrow$ the distribution is the **central quantity** in understanding cosmological properties of the dark sector

The distribution $f(p,t)$ is often assumed thermal and/or unimodal, but this *need not be the case*.

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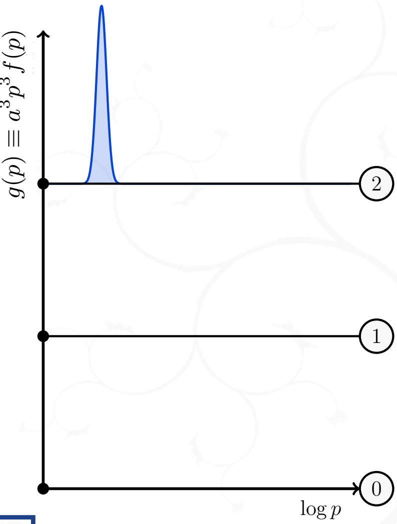
For example:

If the dark sector consists of an **ensemble** of states with different masses, then DM phase-space distributions of a much different form can arise from **decays *within* dark sector**

First, let's consider this qualitatively...

For instance, take a three-state system with $m_2 > m_1 > m_0$ and consider two-body decays. Assume heaviest state initially populated (for simplicity).



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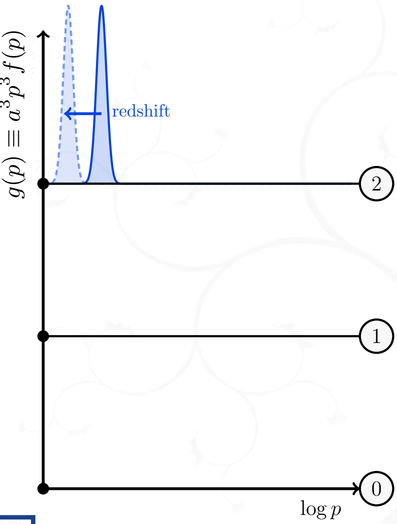

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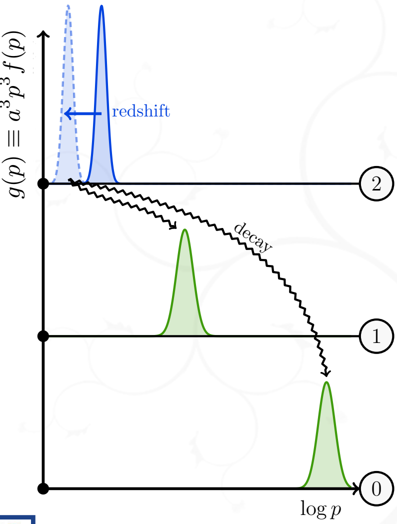

BASIC OBSERVATIONS:

2 $\longrightarrow$ 1 + 0: daughter packets get extra kinetic energy and width (Δp) compared to parent packet

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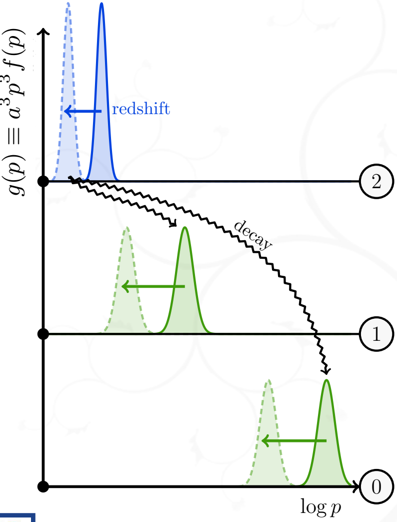

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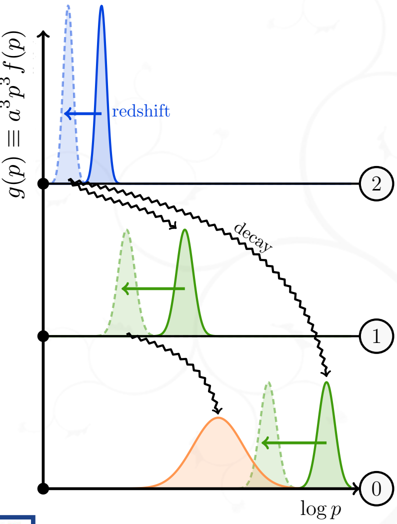

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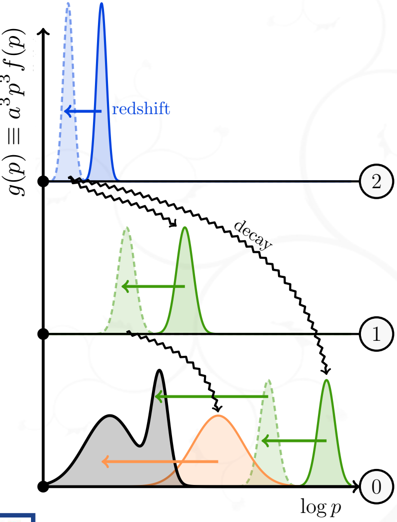

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resulting distribution $g(p)$ is **superposition** of deposits from *two seperate decay chains*—carries imprints of the early decay dynamics

We can verify that these features by (numerically) solving the **Boltzmann system**:

$$\frac{\partial f_\ell(p_\ell, t)}{\partial t} = \underbrace{H(t) p_\ell \frac{\partial f_\ell}{\partial p_\ell}}_{\text{redshifting}} + \underbrace{\frac{C[f]}{\sqrt{p_\ell^2 + m_\ell^2}}}_{\text{collision terms}}$$

for the three-state system.

Survey different combinations of decay rates Γ_{ij}^ℓ for two-body decays $\ell \longrightarrow i + j$.

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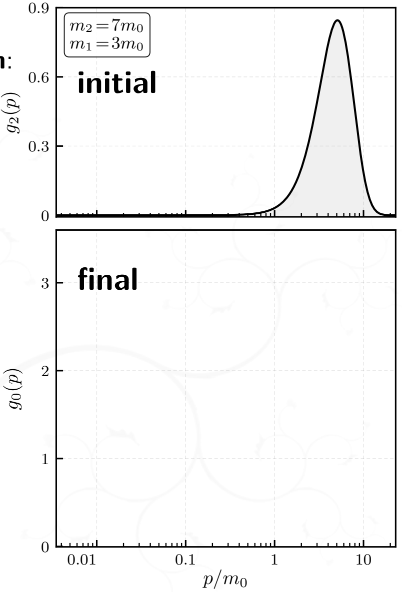

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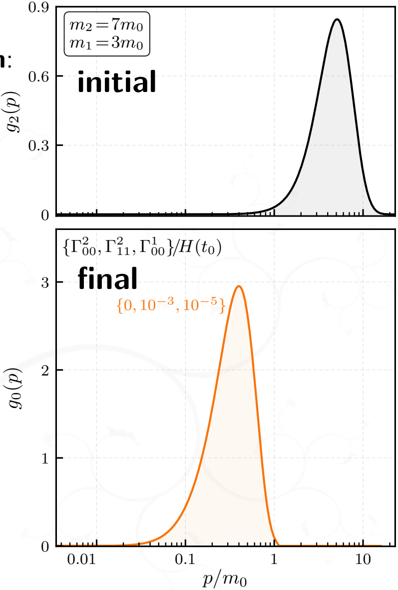

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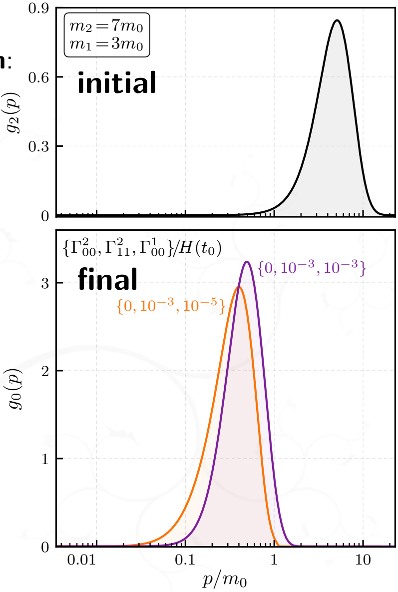

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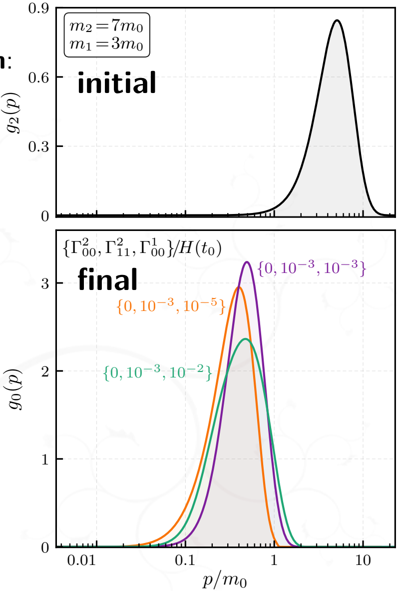

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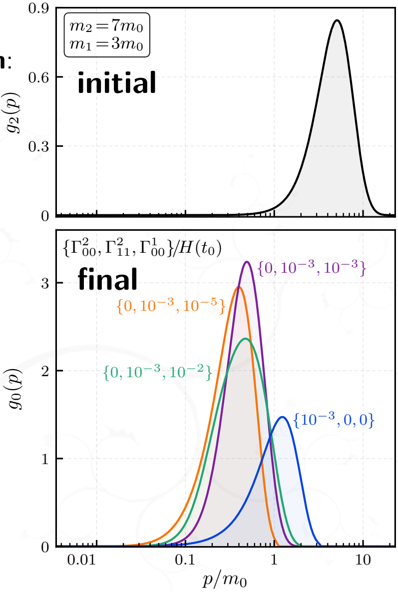

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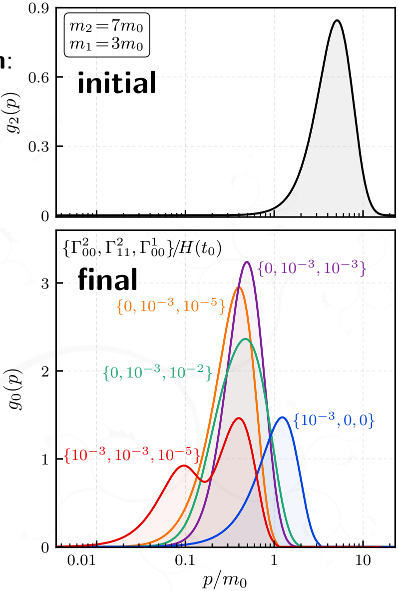

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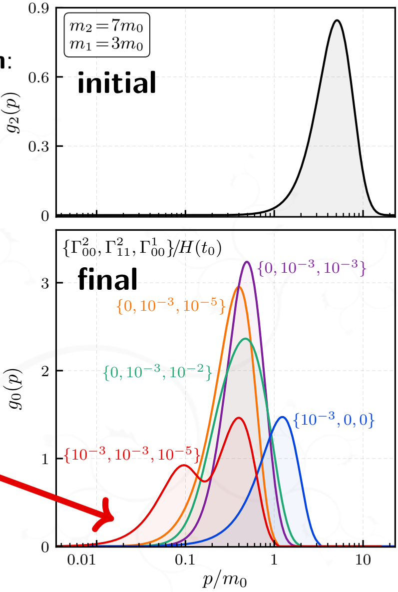

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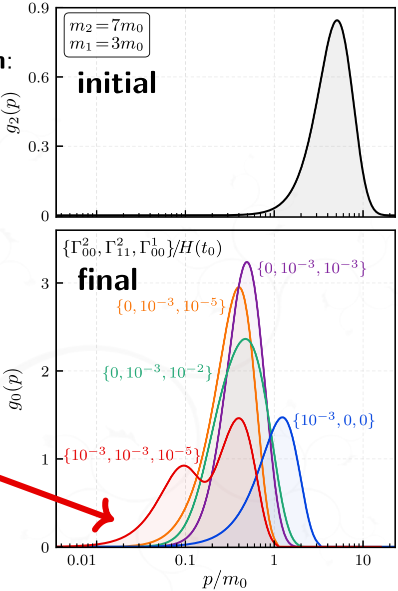

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much more can be inferred about decay process...

(see paper for details `[arXiv:2001.02193]`)

PART II

DM phase-space distribution $\longrightarrow$ matter power spectrum

- (Cold) **dark matter** drives the growth of **structure**

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$$\ddot{\delta} + 2H\dot{\delta} + \underbrace{\frac{k^2}{a^2}\overbrace{c_s^2}^{\text{sound speed}}\delta}_{\text{pressure}} - \underbrace{4\pi G\rho}_{\text{gravity}}\delta = 0$$

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$$\delta \equiv \frac{\delta\rho_{\mathrm{DM}}}{\rho_{\mathrm{DM}}}$$

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- A standard approach is to define a free-streaming horizon

$$k_{\mathrm{FSH}}^{-1} \equiv \int_{t_{\mathrm{prod}}}^{t_{\mathrm{now}}} dt \frac{\langle v(t) \rangle}{a(t)}$$

as a benchmark for the scale below which structure is suppressed.

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We'll consider a different approach...

Our Approach:

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- We begin by considering *momentum slices* through the distribution:

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• In other words, we identify $k_{\rm hor}(p)$ with k and consider $g(p)$ as having a corresponding profile in k-space:

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which retains $\mathcal{N} = \int d\log p\, g(p) = \int d\log k\, \widetilde{g}(k)$.

we are finally equipped to ask:

Can we conjecture the ***relationship***

$$\widetilde{g}(k) \longleftrightarrow P(k)$$

between distributions/power spectra?

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let's do a bit of exploring...

[using `CLASS` to compute $P(k)$]

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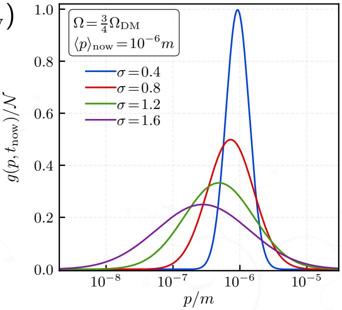

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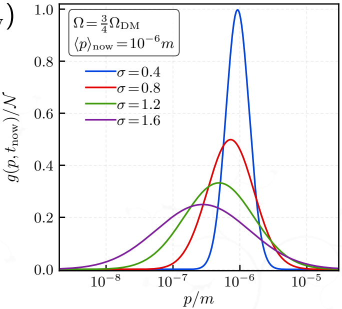

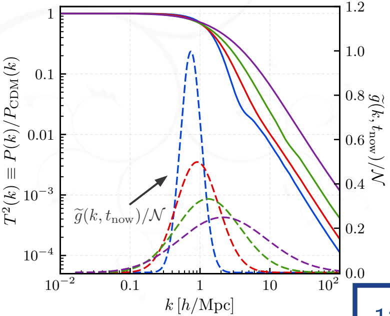

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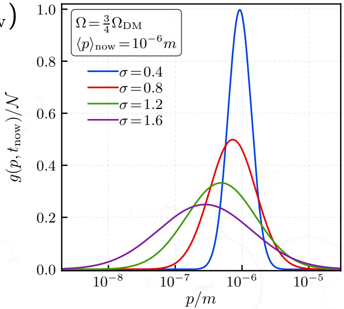

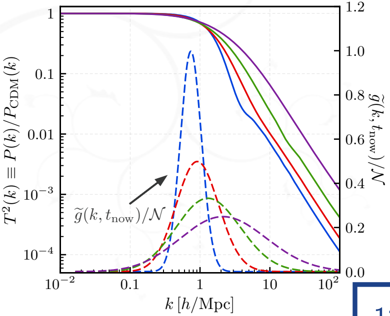

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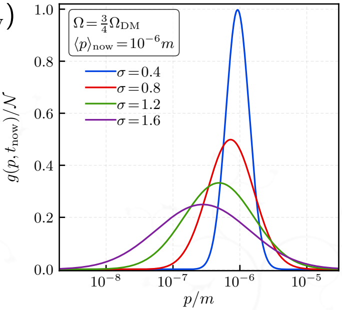

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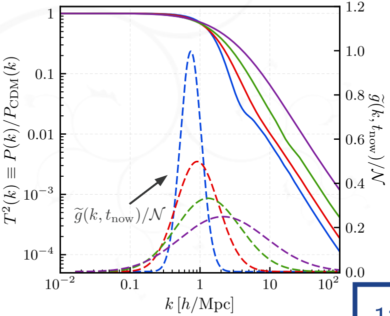
• as we widen the distribution:
- ○ slope of $T^2(k)$ changes more slowly
- ○ the power suppression becomes smaller
- ○ HOWEVER, the slope of $T^2(k)$ itself **remains the *same*** at large k



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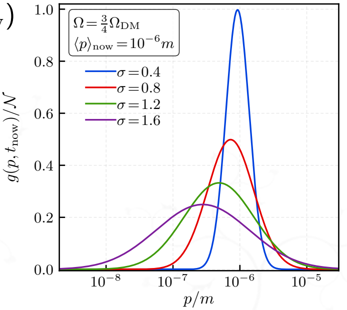

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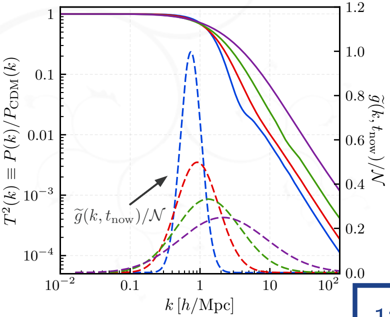

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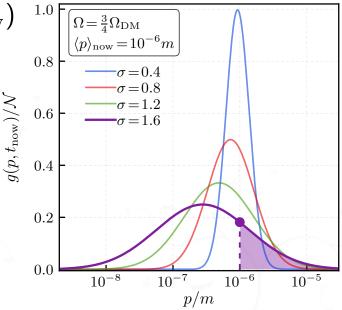

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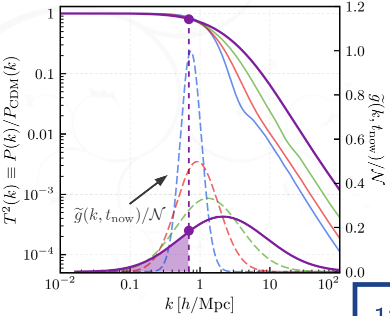

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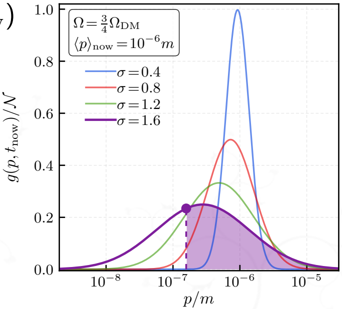

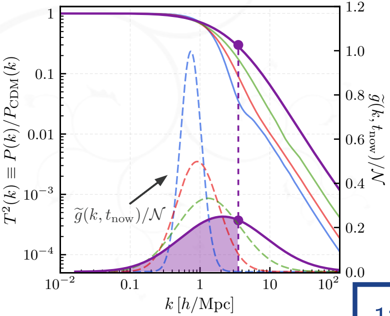

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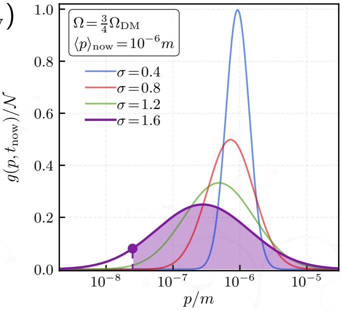

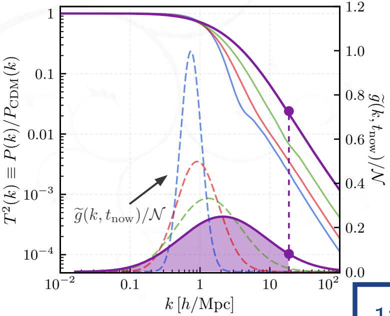

PART III

The "Archaeological" Inverse Problem

- At any particular k this accumulated abundance is:

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \widetilde{g}(k')d\log k'}{\int_{-\infty}^{+\infty} \widetilde{g}(k')d\log k'} ~,$$

or equivalently, the fraction of our DM which is effectively "hot" (*i.e.*, free-streaming).

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$$F(k) \approx \eta\left(\left|\frac{d\log T^2}{d\log k}\right|\right)$$

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so that the relationship has the form

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How well does the inverse map work?
let's ***test*** the conjecture...

An Illustrative Model of Multi-Component Decay Chains

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- Consider a model with $N+1$ real scalars $\{\phi_0, \phi_1, \ldots \phi_N\}$

with a mass spectrum

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and Lagrangian

$$\mathcal{L} \;=\; \sum_{\ell=0}^{N} \left(\frac{1}{2} \partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell} \sum_{j=0}^{i} c_{\ell ij} \phi_\ell \phi_i \phi_j \right) + \cdots$$

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fix parameters:

$N = 9$
$\delta = 1$
$\Delta m = 2m_0$
$\mu = 0.1 m_0$

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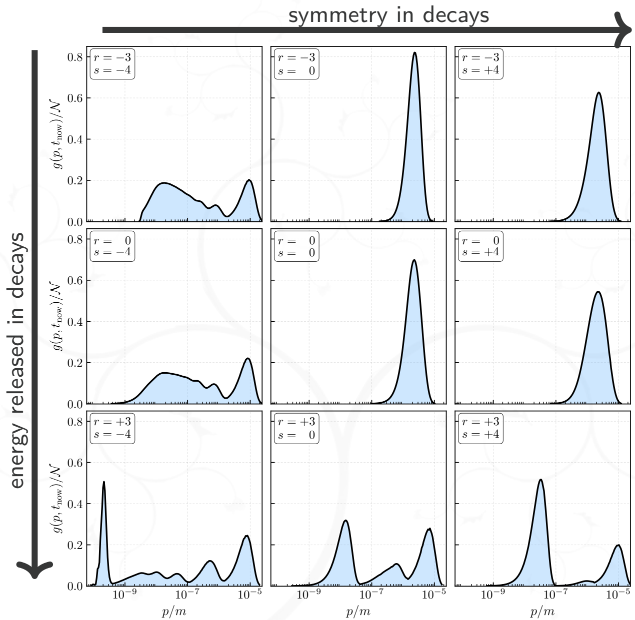

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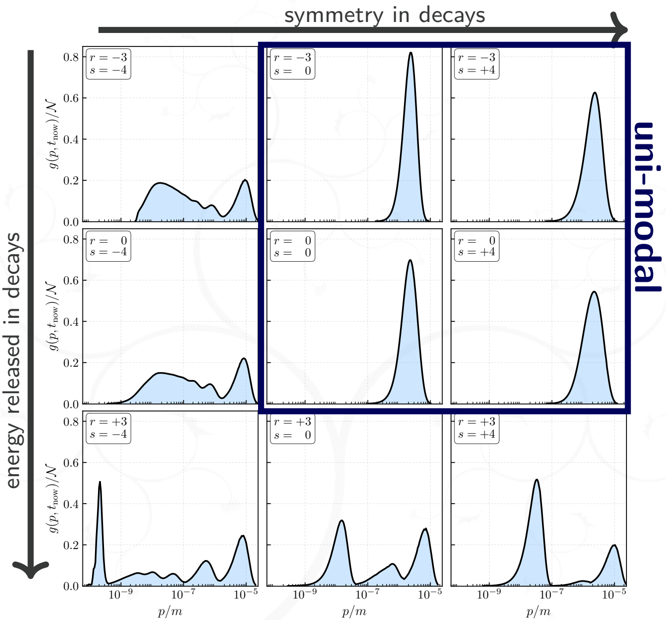

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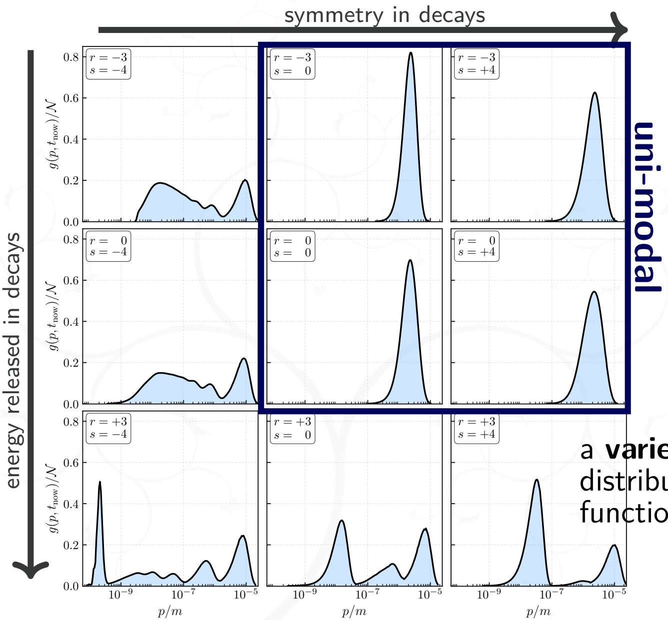

a **variety** of distribution functions emerge!

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Recall our conjecture:

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What features can we "resurrect" from this relation?

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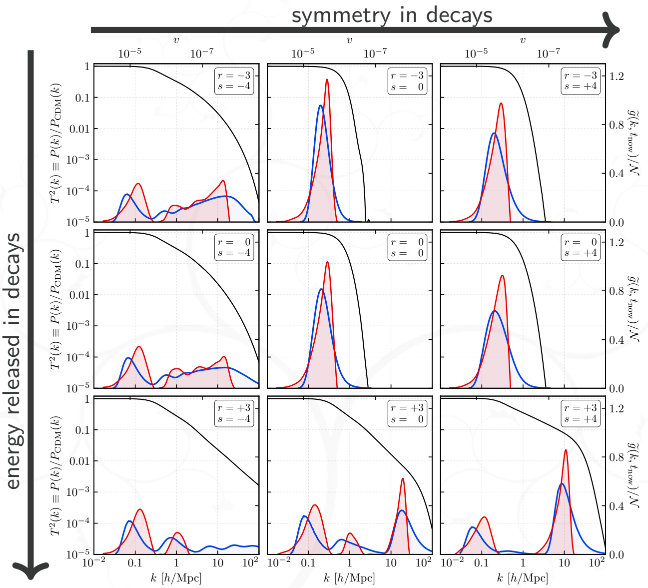

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- If the dark sector is non-minimal, early-universe processes such as decays *within* the dark sector can leave identifiable imprints in $f(p)$ and the matter power spectrum $P(k)$—certain features may allow us to study the inverse problem and **reconstruct** the dark-matter momentum distribution.
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• We have focused on decays, but we should also examine models in which distributions arise from other non-thermal particle production processes.

TAKE-HOME MESSAGE:

• If the dark sector is non-minimal, early-universe processes such as decays *within* the dark sector can leave identifiable imprints in $f(p)$ and the matter power spectrum $P(k)$—certain features may allow us to study the inverse problem and **reconstruct** the dark-matter momentum distribution.

• Such approaches may be *only* probes for dark sectors decoupled from SM.

FUTURE WORK/DIRECTIONS:

• Addressing the non-linear regime (*e.g.*, halo mass function). Can our conjecture be extended to the nonlinear regime? **(NEXT TALK!)**

• Implications of observational bounds/constraints (Lyman-α, *etc.*). In particular, do existing methods of recasting Lyman-α constraints behave consistently with such non-minimal dark sectors? (in preparation)

• If the dark sector includes light states, non-trivial distributions of dark radiation will be produced: what are the implications? (in preparation)

• We have focused on decays, but we should also examine models in which distributions arise from other non-thermal particle production processes.

THANK YOU FOR YOUR ATTENTION!