Supersymmetry, naturalness and the landscape

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Overview

The Standard Model

Although, the Standard Model is the most celebrated theory till date, it has certain drawbacks as follows :

- Existence of Dark Matter [LSP from RPC SUSY + QCD Axion]
- The Higgs mass instability problem in the EW sector [SUSY]
- The strong CP problem and the gravity spoliation problem [$\mathbb{Z}_{24}^{R}$ symmetry $\rightarrow$ PQ symmetry]
- Gravity, Dark energy, Cosmological Constant [Landscape]

SUSY as a BSM Theory

- Supersymmetry or SUSY is a highly motivated extension of SM which obeys a new quantum symmetry which relates fermions to bosons.
- In SUSY, the SM fields are elevated to superfields containing both fermionic and bosonic components. Supersymmetrizing the SM leads to the MSSM.
- Quadratic Divergences in Higgs Mass due to each SM particle is cancelled by its *Superpartner*. This idea solves the Big Hierarchy problem which is one of the main motivations of SUSY.
- But **no** sparticles have been seen in LHC yet.

Where are the sparticles ?

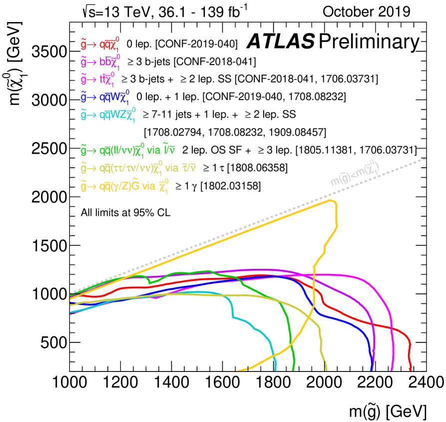


Figure: Results of ATLAS searches for gluino pair production in SUSY for various simplified models with up to 139 fb^{-1} of data at $\sqrt{s} = 13$ TeV.

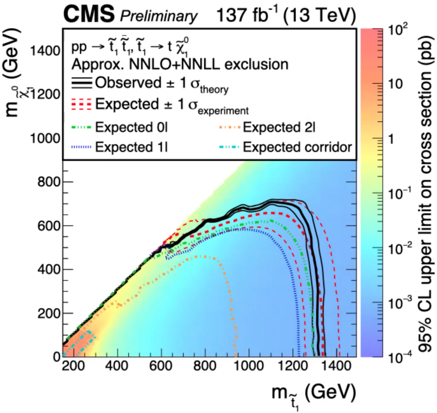


Figure: Results of CMS searches for top squark pair production in SUSY for various simplified models with up to 137 fb^{-1} of data at $\sqrt{s} = 13$ TeV.

Naturalness

$$m_{sparticles} >> m_{SMparticles}$$

LHC Limits: $m_{\tilde{g}} > 2.2$ TeV, $m_{\tilde{t}_1} > 1.3$ TeV $\Longrightarrow$ **Is SUSY Unnatural?**

Various notions of Naturalness found in literature include : Δ_{BG}, Δ_{HS} and Δ_{EW}.

Δ_{HS} and Δ_{BG} measure put a stringent upper bound on the masses of the sparticles. Hence, these notions of naturalness, along with the above-mentioned experimental limits, render weak scale SUSY unnatural/highly fine-tuned.

However, a critical assessment of these older measures of Naturalness reveal that they must be updated to the model-independent electroweak measure of Naturalness (Δ_{EW}) so as to follow the notion of *Practical Naturalness* which states that

> *An Observable $\mathcal{O}$ is natural if all independent contributions to $\mathcal{O}$ are comparable to or less than $\mathcal{O}$.*

Δ_{BG}

Traditionally proposed by Ellis *et. al.* and later investigated more thoroughly by Barbieri and Giudice, the Δ_{BG} measure of Naturalness is calculated as :

$$\Delta_{BG} \equiv max_i\left[c_i\right] \quad \text{where} \quad c_i = \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right| = \left|\frac{p_i}{m_Z^2}\frac{\partial m_Z^2}{\partial p_i}\right| \tag{1}$$

where p_i = fundamental parameters of the model at high scale.

There is ambiguity in 'free parameters' since almost all parameters are correlated i.e. not independent, in string theory.

$\Delta_{BG} \implies \Delta_{EW}$ when all soft terms correlated, as is expected in string theory.

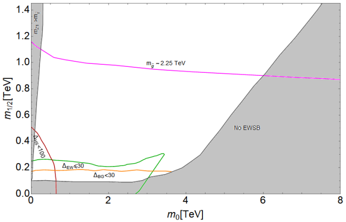


Figure: (a) mSUGRA/CMSSM

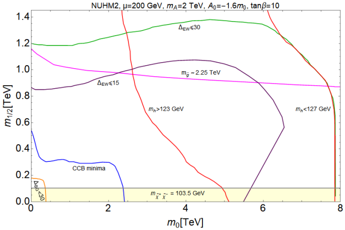


Figure: (b) NUHM2

Figure: Contours of various finetuning measures in m_0 vs. $m_{1/2}$ plane.

arXiv : 2002.03013 by Baer, Barger, Salam, **DS** and Sinha.

Δ_{HS}

The Large Log Measure Δ_{HS} starts with relating the mass of higgs boson in terms of weak scale SUSY parameters as follows :

$$m_h^2 \approx \mu^2(weak) + m_{H_u}^2(weak) + mixing + rad.corr. \tag{2}$$

In terms of some high-energy cut-off scale Λ,

$$m_{H_u}^2(weak) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 \tag{3}$$

Taking $\Lambda \sim m_{GUT}$, a simplified formula to calculate Δ_{HS} is :

$$\Delta_{HS} = \delta m_{H_u}^2 / m_{H_u}^2 \tag{4}$$

$\Delta_{HS} \leq 1 \implies m_{\tilde{t}_1} < 500$ GeV $\implies$ excluded by LHC searches.

The simplification ignores the fact that $\delta m_{H_u}^2$ is highly dependent on $m_{H_u}^2(\Lambda)$, which is set to zero in the simplification. Since $\delta m_{H_u}^2$ and $m_{H_u}^2(\Lambda)$ are not independent, hence Δ_{HS} violates the notion of *Practical Naturalness*.

Δ_{EW}

A more conservative measure of Naturalness is the Electroweak fine-tuning parameter (Δ_{EW}) which is defined as

$$\Delta_{EW} = max_i|C_i|/(M_Z^2/2) \tag{5}$$

Where, C_i is any one of the parameters on the RHS of the following equation :

$$\frac{M_Z^2}{2} \approx -m_{H_u}^2 - \mu^2 - \Sigma_u^u(\tilde{t}_{1,2}) \tag{6}$$

Since all the terms on RHS of Eqn. 6 must be comparable to $M_Z^2/2$, it implies

- $\mu \leq 300$ GeV $\Longrightarrow$ Light higgsinos.
- top squarks must be highly mixed

Understanding Δ_{EW}

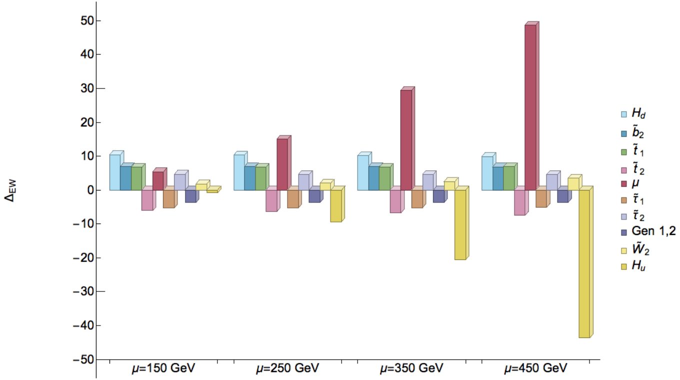


Figure: Top ten contributions to $\Delta_{EW} = max_i|C_i|/(M_Z^2/2)$ from NUHM2 model benchmark points with $\mu = 150$, 250, 350 and 450 GeV.

arXiv: 1702.06588 by Baer, Barger, Gainer, Huang, Savoy, Serce and Tata.

Radiatively-Driven Natural SUSY

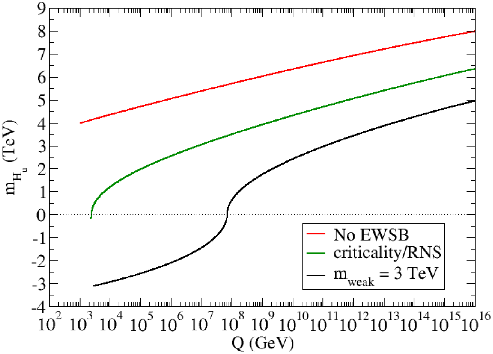


Figure: Evolution of the term $sign(m^2_{H_u})\sqrt{m^2_{H_u}}$ for the case of *No EWSB*, criticality as in *RNS* and $m_{weak} = 3$ TeV.

arXiv: 1602.07697 by Baer, Barger, Savoy and Serce.

Models with Radiatively-Driven Natural SUSY

- nNUHM2 Model *(Nucl.Phys. B435 (1995) 115-128; JHEP 0507 (2005) 065.)*
  m_0, $m_{1/2}$, A_0, $\tan\beta$, μ, m_A
- nNUHM3 Model *(Nucl.Phys. B435 (1995) 115-128; JHEP 0507 (2005) 065.)*
  $m_0(1,2)$, $m_0(3)$, $m_{1/2}$, A_0, $\tan\beta$, μ, m_A
- nGMM Model *(Phys. Rev. D 94 (2016) no.11, 115017.)*
  α, $m_{3/2}$, c_m, c_{m3}, a_3, $\tan\beta$, μ, m_A
- nAMSB Model *(Nucl. Phys. B 557 (1999) 79; Phys. Rev. D 98 (2018) no.1, 015039.)*
  m_0, $m_{3/2}$, A_0, $\tan\beta$, μ, m_A

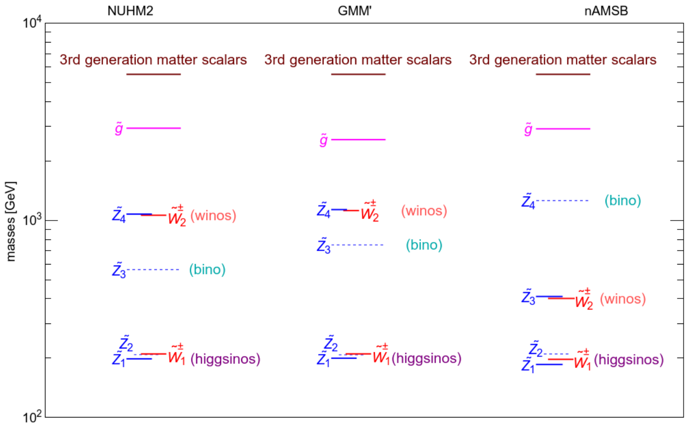


Figure: Typical mass spectra from natural SUSY in the case of NUHM2 (with gaugino mass unification), nGMM with mirage unification and compressed gauginos and natural AMSB where the wino is the lightest gaugino. In all cases, the higgsinos lie at the bottom of the spectra.

Comparison

mass	BG/DG	Δ_{EW}	Δ_{HS}
μ	< 350 GeV	< 350 GeV	-
$m_{\tilde{g}}$	$< 400 - 600$ GeV	< 6 TeV	$< 900 - 1500$ GeV
$m_{\tilde{t}_1}$	< 450 GeV	< 3 TeV	< 500 GeV
$m_{\tilde{q},\tilde{\ell}}$	$< 550 - 700$ GeV	$< 10 - 30$ TeV	-

Table: Upper bounds on sparticle masses from 3% naturalness using Δ_{BG}, Δ_{HS} and Δ_{EW} within multi-parameter SUSY effective theories.

Nucl.Phys.B 306 (1988) 63-76 by Barbieri and Giudice

Phys. Lett. B 357 (1995) 573 by Dimopoulos and Giudice

arXiv: 1509.02929 by Baer, Barger and Savoy

arXiv: 1808.04844 by Baer, Barger, Gainer, DS, Serce and Tata

arXiv: 1110.6926 by Papucci, Ruderman and Weiler

arXiv: 1110.6670 by Brust, Katz, Lawrence and Sundrum

Dark Matter in SUSY

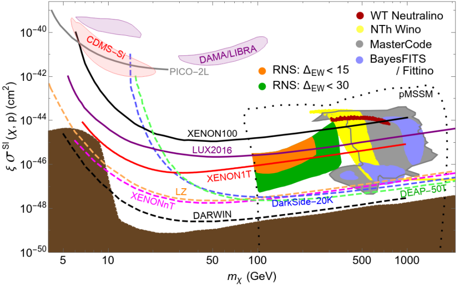


Figure: Plot of rescaled spin-independent WIMP detection rate $\xi\sigma^{SI}(\chi, p)$ versus m_χ. For RNS and pMSSM, $\xi = \frac{\Omega_{\tilde{Z}_1} h^2}{0.12} < 1$.

In Natural SUSY, the higgsino-like neutralino (WIMPs) are still allowed experimentally, provided they form only 10-20 % of the total Dark Matter. The rest of the DM can be formed by the Axion, which is anyway necessary to solve the Strong CP problem.

SUSY μ problem

- The MSSM superpotential contains term $\mu H_u H_d$ which leads to $\mu \approx m_P$.
- $\mu \approx m_{weak}$ phenomenologically (otherwise no proper Electroweak Symmetry Breaking)

This is the famous SUSY μ problem

- A promising approach to solve the SUSY μ problem is to first forbid μ, perhaps via some symmetry, and then re-generate it of order the scale of soft SUSY breaking terms.
- However, present LHC limits suggest the soft breaking scale m_{soft} lies in the multi-TeV regime whilst naturalness requires $\mu \sim m_{W,Z,h} \sim$ 100 GeV so that a Little Hierarchy (LH) appears with $\mu \ll m_{soft}$.

Strong CP Problem and its solution

'Strong CP Problem' $\rightarrow$ Due to a complicated structure of the QCD vacuum, an additional term arise in the Lagrangian : $\bar{\theta}\frac{g^2}{32\pi^2}F_a^{\mu\nu}\tilde{F}_{a\mu\nu}$ And the experimental observation (neutron electric dipole moment $< 2.9 \times 10^{-26}$ ecm), gives the bound $\bar{\theta} < 10^{-9}$ - 10^{-10}.Now the question arise why $\bar{\theta}$ is so small ? And this is the strong CP problem

'Peccei-Quinn Solution' $\rightarrow$ Introduction of a global $U(1)_{PQ}$ symmetry dynamically drives $\bar{\theta} \rightarrow 0$ by replacing the static CP violating phase $\bar{\theta}$ by a dynamical CP conserving field : the axion

Though PQ symmetry solves the strong CP problem, it is a global symmetry and global symmetries are not compatible with inclusion of quantum gravity and hence the theory suffers from **gravity-spoliation problem**.

Simultaneous solution to the SUSY μ problem, Strong CP problem and the gravity-spoliation problem

When $\mathbb{Z}_{24}^{R}$ symmetry is imposed as the fundamental symmetry in a supersymmetric model, it yields the following benefits :

$\mu_{eff} \sim m_{weak}$ is generated

~~R-Parity Violating operators~~

~~Dim-5 proton decay operators~~

Solves the gravity-spoliation problem because no terms with suppression less than $1/m_P^8$ are allowed in the scalar potential

arXiv : hep-th/9202003 by Kamionkowski and March-Russell.

Stringy Naturalness and the Landscape from the Multiverse

- Why is the experimentally measured value of cosmological constant (CC) Λ so tiny ($\Lambda \simeq 10^{-120} m_P^2$) when there is no known symmetry to suppress its magnitude?
- Assuming an eternaly inflating multiverse with a huge assortment of vacua($\sim 10^{500}$) states with cosmological constant uniformly distributed, then those pocket universes with Λ somewhat larger than our measured value would lead to such rapid expansion that galaxies wouldn't condense, and presumably observors wouldn't arise. Weinberg used such reasoning (anthropic principle) to predict the value of Λ to within a factor of several well before it was experimentally measured.

Stringy Naturalness and the Landscape from the Multiverse

- Given the success of the landscape in predicting Λ, can multiverse arguments also be used to predict the scale of SUSY breaking in a fertile patch of string landscape which has MSSM as the low energy EFT ?
- A statistical approach to understand the SUSY breaking scale has been advocated by Douglas. In this approach, naturalness is replaced by stringy naturalness wherein

  > *observable $\mathcal{O}_2$ is more natural than observable $\mathcal{O}_1$ if more phenomenologically viable vacua lead to $\mathcal{O}_2$ than to $\mathcal{O}_1$.*

- *phenomenologically viable vacua* $\Longrightarrow$ such vacua that lead to pocket universes that can admit life as we understand it.

Stringy Naturalness and the Landscape from the Multiverse

Specifically, the distribution of vacua might be written as

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWSB} \cdot f_{CC} \cdot dm^2_{hidden} \quad (7)$$

where, For the prior distribution f_{SUSY}, Douglas proposed on rather general grounds a power law ansatz

$$f_{SUSY}(m^2_{hidden}) \sim (m^2_{hidden})^{2n_F+n_D-1} \quad (8)$$

where n_F is the number of hidden sector F-breaking fields and n_D is the number of contributing D-breaking fields.

$f_{CC} \sim \Lambda/m^4_{string} \implies$ Influence of Cosmological Constant on selecting *phenomenologically viable* vacua. This does NOT influence SSB scale.

Stringy Naturalness and the Landscape from the Multiverse

Agrawal *et al.* showed if $m_{weak}^{PU} \geq (2-5)m_{weak}^{OU}$ then atoms, as we know them, will not form in such a universe. For the case of Natural SUSY i.e., $\mu \sim m_{weak}$, the condition $m_{weak}^{PU} < 4 \times m_{weak}^{OU}$ corresponds to vetoing pocket universes with $\Delta_{EW} > 30$. Thus,

$$f_{EWSB} = \Theta(30 - \Delta_{EW}). \tag{9}$$

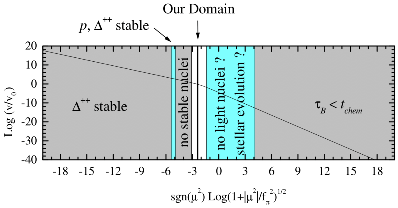


Figure: Allowed values of m_{weak}^{PU}.

arXiv : hep-ph/9801253 by Agrawal, Barr, Donoghue and Seckel.

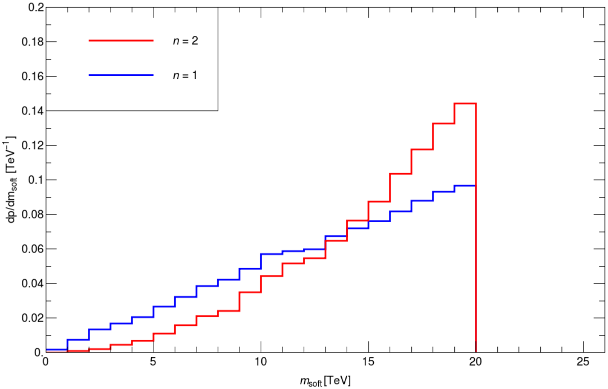


Figure: Multiverse prefers large soft terms provided they fulfill the anthropic requirements.

Mirage Mediation from the Landscape

arXiv : 1912.01672 by Baer, Barger and **DS**

- KKLT flux compactification gives rise to mirage-mediation spectra.
- It is a mixed gravity/moduli plus anomaly-mediated soft SUSY breaking (SSB) mechanism where we can choose how much each of gravity/moduli-mediated and anomaly-mediated SUSY breaking contribute.
- The model considered here is the natural General Mirage Mediation (nGMM) model (discussed earlier under topic RNS models).
- Here, we shall see the effect of Landscape on nGMM (mirage mediation) model.
- We scan the input SSB parameters of the nGMM model with non-uniform pull and try to simulate multiverse selection of our universe.

Higgs mass prediction

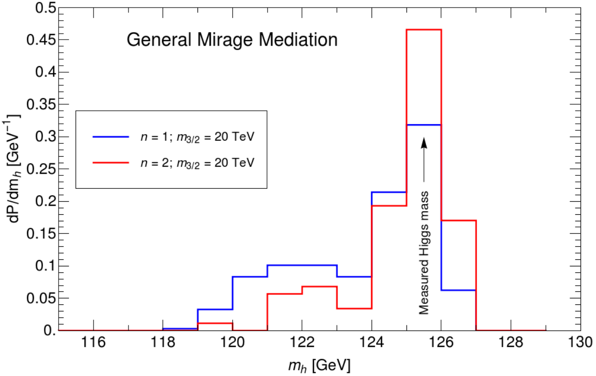


Figure: Probability distribution for mass of light Higgs boson (m_h) from n$=1$ (blue) and n$=2$ (red) statistical scans with $m_{3/2}$= 20 TeV.

The String Landscape predicts $m_h \sim 125$ GeV statistically.

Sparticle mass prediction

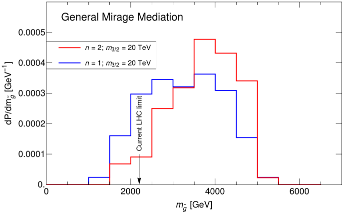


Figure: (a)

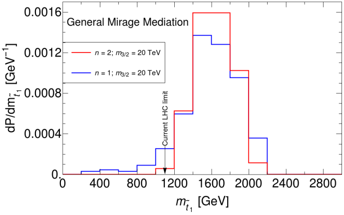


Figure: (b)

Figure: Probability distribution for mass of (a) gluino ($m_{\tilde{g}}$) and (b) stop quark ($m_{\tilde{t}_1}$) from n= 1 (blue) and n= 2 (red) statistical scans with $m_{3/2}$= 20 TeV.

The String Landscape predicts that the gluino and top squark are well-above the LHC mass limits.

A Landscape solution to SUSY flavor and CP problems

arXiv : 1910.00090 by Baer, Barger and **DS**

By scanning over SUSY models with soft terms generated according to m_{soft}^n for $n = 1$ (blue) and 2 (red), along with the anthropic vetos from $m_{weak}^{PU} < 4 \times m_{weak}^{OU}$:

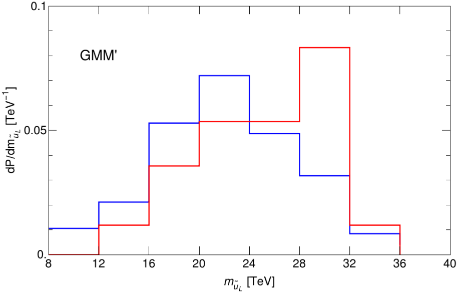
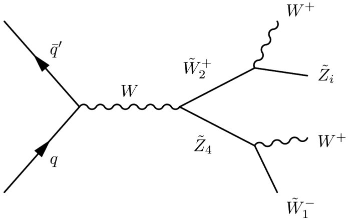


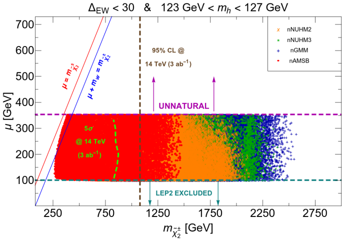
Figure: As seen in the above distribution of $m_{\tilde{u}_L}$, first and second generation matter scalars (squarks and sleptons) are pulled up to $m(\tilde{q}, \tilde{\ell}) \sim 30 \pm 10$ TeV.

A Landscape solution to SUSY flavor and CP problems

- As seen in previous figure, the landscape pull with $n = 1$ ($n = 2$) results in $m_{\tilde{f}} = 22$ TeV (30 TeV) to be the most probable value with some non-zero probability for $m_{\tilde{f}} = 36$ TeV. This results in decoupling. Because of this decoupling, limits on off-diagonal terms can float as high as tens of TeV, comparable to the tens of TeV for diagonal terms.
- Since this upper bound depends only on gauge quantum numbers, so it is same for both first and second generation. With strong enough pull, this results in quasi-degeneracy, thereby, suppressing FCNC effects.
- This quasi-degeneracy, along with decoupling, helps in solving the SUSY flavor problem.
- The decoupling alone is enough to solve the SUSY CP problem.

Wino





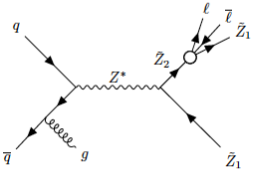
The Landscape Models with light higgsino (small μ) would give rise to a distinct same-sign diboson signal from wino pair production.

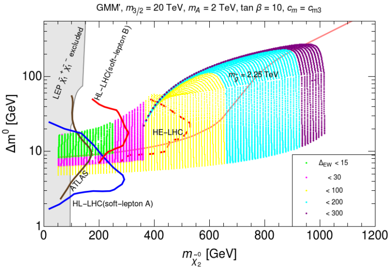
arXiv : 1710.09103 by Baer, Barger, Gainer, Savoy, **DS** and Tata.

May need HE-LHC to see winos.

arXiv : 1808.04844 by Baer, Barger, Gainer, **DS**, Serce and Tata.

Higgsino

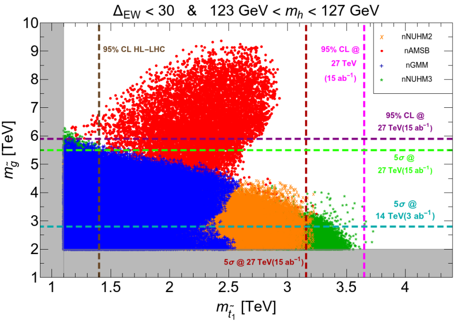




A distinctive feature of Landscape models is that $m_{NLSP} - m_{LSP} \sim 7 \pm 3$ GeV which HE-LHC is likely to see via OSDLMET signal arising from higgsino pair-production. Here, $m_{\tilde{Z}_1} \sim m_{\tilde{Z}_2} \sim \mu \sim$ 100-350 GeV.

arXiv : 2007.09252 by Baer, Barger, Salam, **DS** and Tata.

Gluino and Top squark



The Landscape Models predict gluinos and top squarks well above the LHC limits. May need HE-LHC to see (natural) gluinos and top squarks.

arXiv : 1808.04844 by Baer, Barger, Gainer, **DS**, Serce and Tata.

Summary

- Supersymmetry is one of the most motivated BSM framework.
- The older notions of Naturalness $\Longrightarrow$ more conservative electroweak naturalness measure (Δ_{EW}) $\Longrightarrow$ lot of parameter space is still left to be probed yet at the LHC.
- Dark Matter in SUSY: WIMPs (LSP) + Axion.
- Simultaneous solution to SUSY μ problem, Strong CP problem and gravity-spoliation problem: $\mathbb{Z}_{24}^{R}$ symmetry.
- Landscape from Multiverse argument along with stringy naturalness can solve SUSY flavor and CP problem.
- Landscape models predict $m_h \sim 125$ GeV with sparticles beyond LHC limits: exactly what LHC is seeing.


Thank you ...

QUESTIONS ?

Back Up Slides

Naturalness

$\mathcal{O} = \mathcal{O}$ + b - b

When evaluating fine-tuning, it is not permissible to claim fine-tuning of dependent quantities one against another.

The Electroweak Measure Δ_{EW}

$$\Delta_{EW} = max_i |C_i| / (M_Z^2/2) \tag{10}$$

Where, C_i is any one of the parameters on the RHS of the following equation :

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) tan^2\beta}{(tan^2\beta - 1)} - \mu^2 \tag{11}$$

$$\approx -m_{H_u}^2 - \mu^2 - \Sigma_u^u(\tilde{t}_{1,2}) \tag{12}$$

Sensitivity to High Scale Parameters Δ_{BG}

$$m_Z^2 \approx -2m_{H_u}^2 - 2\mu^2 \tag{13}$$

The weak scale SUSY parameters $m_{H_u}^2$ and μ^2 can be replaced in terms of GUT scale parameters as follows :

$$\begin{aligned} m_Z^2 \simeq & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 \\ & - 0.42M_2^2 + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t \\ & - 0.15M_2A_t - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2 \end{aligned}$$

Then Δ_{BG} is calculated as :

$$\Delta_{BG} \equiv max_i\,[c_i] \quad \text{where} \quad c_i = \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right| = \left|\frac{p_i}{m_Z^2}\frac{\partial m_Z^2}{\partial p_i}\right| \tag{14}$$

The Large Log Measure Δ_{HS}

$$m_h^2 \approx \mu^2(weak) + m_{H_u}^2(weak) + mixing + rad.corr. \tag{15}$$

In terms of some high-energy cut-off scale Λ,

$$m_{H_u}^2(weak) = m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 \tag{16}$$

$\delta m_{H_u}^2$ is calculated from the renormalization group equation (RGE) by setting several terms in $dm_{H_u}^2/dt$ (with $t = \log Q^2$) to zero so as to integrate in a single step:

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln\left(\Lambda^2/m_{soft}^2\right). \tag{17}$$

Taking $\Lambda \sim m_{GUT}$, a simplified formula to calculate Δ_{HS} is :

$$\Delta_{HS} = \delta m_{H_u}^2 / m_{H_u}^2 \tag{18}$$

Upper bounds from Δ_{BG} and Δ_{HS}

mass	upper limit	source
$m_{\tilde{g}}$	<400 GeV	BG(1987)
$m_{\tilde{u}_R}$	<400 GeV	BG(1987)
$m_{\tilde{e}_R}$	<350 GeV	BG(1987)
$m_{\tilde{\chi}_1^\pm}$	<100 GeV	BG(1987)
$m_{\tilde{\chi}_1^0}$	<50 GeV	BG(1987)
m_h	<115 GeV	CGR(2009)
$m_{\tilde{t}_{1,2},\tilde{b}_1}$	<500 GeV	PRW,BKLS(2011)

Table: Upper bounds on sparticle and Higgs boson masses from 10% naturalness using Δ_{BG} within multi-parameter SUSY effective theories. We also include bounds from Δ_{HS}.

Expected (rough) solution to Gravity-spoliation problem

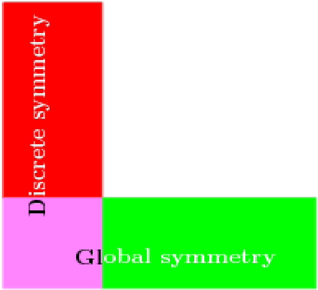


Figure: 15. **Kim diagram** where the column represents an infinite sequence of lagrangian terms obeying gravity-safe discrete symmetry while the row represents an infinite sequence of terms obeying the global symmetry. The green region terms are gravity-unsafe while red region violates the global symmetry. The lavender terms are gravity-safe and obey the global symmetry.

Twenty Solutions to the SUSY μ problem

arXiv : 1902.10748 by K.J. Bae, H. Baer, V. Barger and **D. S.**

model	Admit LH?	strong CP?	Gravity Safe?	see-saw?
GM	small λ_μ	$\times$	$--$	*SNSS*
CM	small λ_μ	$\times$	$--$	*SNSS*
R-sym	$(v_i/m_P)^{n_i}$	$\times$	$--$	*SNSS*
$\mathbb{Z}_4^R$	small λ_μ	$\times$	$--$	*SNSS*
Instanton	small $e^{-S_{cl}}$	$\times$	$--$	*SNSS*
G_2MSSM	$\langle S_i \rangle / m_P \ll 1$	$\times$	$--$	*SNSS*
NMSSM	small λ_μ	$\times$	$--$	*SNSS*
nMSSM	small λ_μ	$\times$	$--$	*SNSS*
$\mu\nu$SSM	small λ_μ	$\times$	$--$	*bRPV*

model	admit LH?	strong CP?	gravity safe?	see-saw?
$U(1)'$ (CDEEL)	small λ_μ	$\times$	$--$	$SNSS$
sMSSM	small λ_μ	$\times$	$--$	$SNSS$
$U(1)'$ (HPT)	small λ_μ	$\times$	$--$	$bRPV$
KN	$v_{PQ} < m_{hidden}$	$\surd$	?	$SNSS$
CKN	$\Lambda < \Lambda_h$	$\surd$	?	$SNSS$
BK/EWK	$\lambda_\mu \sim 10^{-10}$	$\surd$	?	$SNSS$
HFD	$v_{PQ} < m_{hidden}$	$\surd$	?	$SNSS$
MSY/CCK/SPM	$v_{PQ} < m_{hidden}$	$\surd$	$\times$	$RadSS$
CCL	small λ_μ	$\surd$	?	several
MBGW	small λ_μ	$\surd$	$\mathbf{Z}_{22}$	$SNSS$
Hybrid CCK/SPM	small λ_μ	$\surd$	$\mathbf{Z}_{24}^R$	$SNSS$

Table: Summary of twenty solutions to the SUSY μ problem and how they 1. admit a Little Hierarchy (LH), 2. solve the strong CP problem ($\surd$) or not ($\times$), 3. are expected gravity-safe and 4. Standard neutrino see-saw (SNSS) or other.

Fundamental R symmetries

- R-symmetries are characterized by the fact that superspace co-ordinates θ carry non-trivial R-charge : $+1$ being the simplest case.
- For the Lagrangian $\mathcal{L} \ni \int W d^2\theta$ to be invariant under $\mathbb{Z}_N^R$ symmetry, the superpotential W must carry R-charge $= 2 \text{ mod } |N|$

multiplet	$\mathbb{Z}_4^R$	$\mathbb{Z}_6^R$	$\mathbb{Z}_8^R$	$\mathbb{Z}_{12}^R$	$\mathbb{Z}_{24}^R$
H_u	0	4	0	4	16
H_d	0	0	4	0	12
Q	1	5	1	5	5
U^c	1	5	1	5	5
E^c	1	5	1	5	5
L	1	3	5	9	9
D^c	1	3	5	9	9
N^c	1	1	5	1	1

Table: These R-symmetries were shown to be anomaly-free and consistent with GUT by *Lee et al.* in arXiv : 1102.3595

$\mathbb{Z}_{24}^R$ discrete symmetry

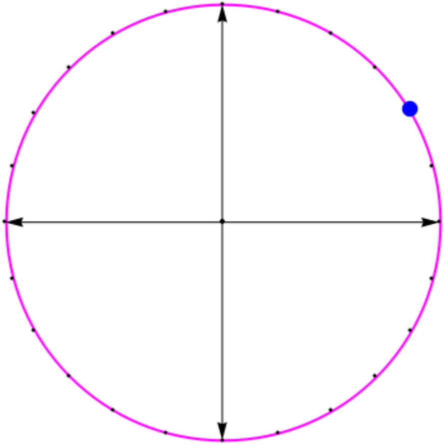


Figure: 16. All terms in superpotential (W) must have R charge :
$Q_R(W) = 2 + 24n$; (n=integer)

Radiative PQ breaking scenarios

- **MSY Model** (H. Murayama, H. Suzuki and T. Yanagida)

$$W_{PQ} \ni \frac{1}{2}h_{ij}XN_i^cN_j^c + \frac{f}{m_P}X^3Y + \frac{g_{MSY}}{m_P}XYH_uH_d \tag{19}$$

- **CCK Model** (K.Choi, E.J. Chun and J.E. Kim)

$$W_{PQ} \ni \frac{1}{2}h_{ij}XN_i^cN_j^c + \frac{f}{m_P}X^3Y + \frac{g_{CCK}}{m_P}X^2H_uH_d \tag{20}$$

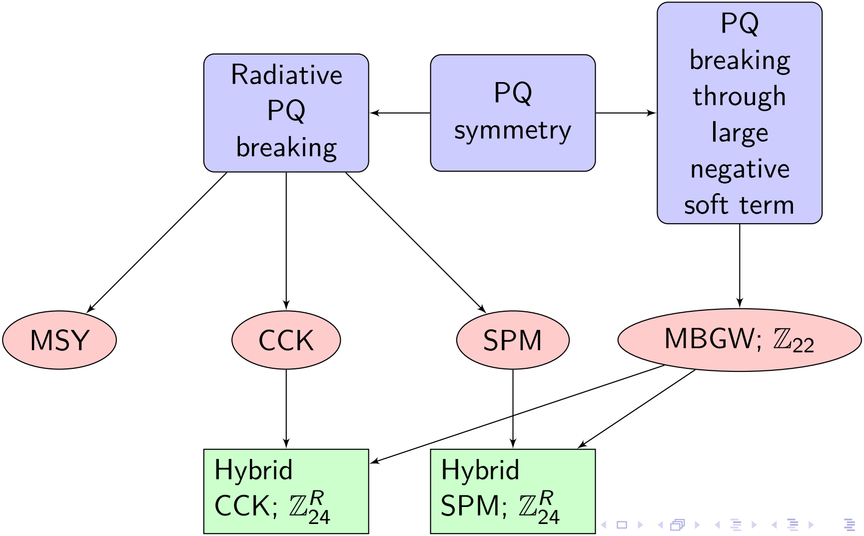
- **SPM Model** (S.P. Martin)

$$W_{PQ} \ni \frac{1}{2}h_{ij}XN_i^cN_j^c + \frac{f}{m_P}X^3Y + \frac{g_{SPM}}{m_P}Y^2H_uH_d \tag{21}$$

Unfortunately, none of these radiative PQ breaking theories are consistent with the above mentioned R symmetries and hence suffer from the gravity spoilation problem.

Hybrid Models

arXiv : 1810.03713 by H. Baer, V. Barger and **D. S.**



Hybrid CCK

$$W_{PQ} \ni \frac{f}{m_P} X^3 Y + \frac{\lambda_\mu}{m_P} X^2 H_u H_d \tag{22}$$

multiplet	Q	U^c	D^c	L	E^c	N^c	H_u	H_d	X	Y
Z_{24}^R Charges	5	5	9	9	5	1	16	12	-1	5
PQ Charges	1	0	0	1	0	0	-1	-1	1	-3

$$V = [fA_f \frac{\phi_X^3 \phi_Y}{m_P} + h.c.] + m_X^2 |\phi_X|^2 + m_Y^2 |\phi_Y|^2 + \frac{f^2}{m_P^2} [9\phi_X^4 \phi_Y^2 + \phi_X^6] \tag{23}$$

- The lowest order PQ violating terms in the superpotential are $\mathbf{X^8 Y^2 / m_P^7}$, $\mathbf{X^4 Y^6 / m_P^7}$ and $\mathbf{Y^{10} / m_P^7}$ which implies the lowest order PQ breaking term in the scalar potential is suppressed by $\mathbf{1/m_P^8}$.

- Thus the PQ symmetry arises as an accidental approximate global symmetry from the fundamental discrete $\mathbb{Z}_{24}^{R}$ symmetry.

Hence, this model is gravity-safe.

- This has been mentioned earlier by *Lee et al.* in arXiv : 1102.3595, but the PQ and $\mathbb{Z}_{24}^{R}$ breaking mechanism was conjectured to be radiative.
- Here PQ and $\mathbb{Z}_{24}^{R}$ symmetry are broken as a consequence of SUSY breaking through a large negative soft term A_f.
- Another advantage of imposing $\mathbb{Z}_{24}^{R}$ symmetry as the fundamental symmetry is that R-parity also arises accidentally from it as $\mathbb{Z}_{24}^{R}$ symmetry forbids R-parity violating terms. Hence, R-parity is no longer ad-hoc.

Hybrid CCK

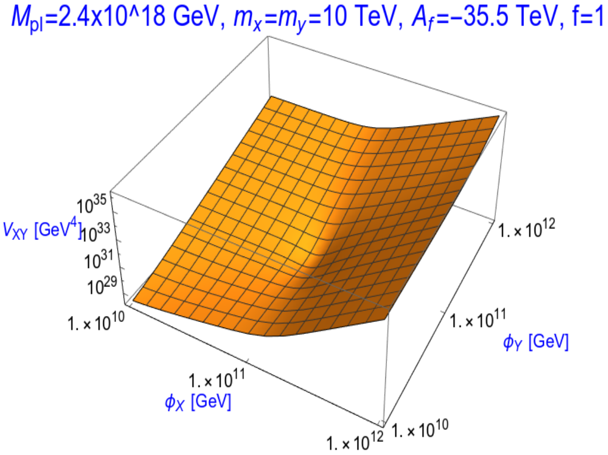


Figure: 17. Scalar potential V_{hyCCK} versus ϕ_X and ϕ_Y.

Hybrid CCK

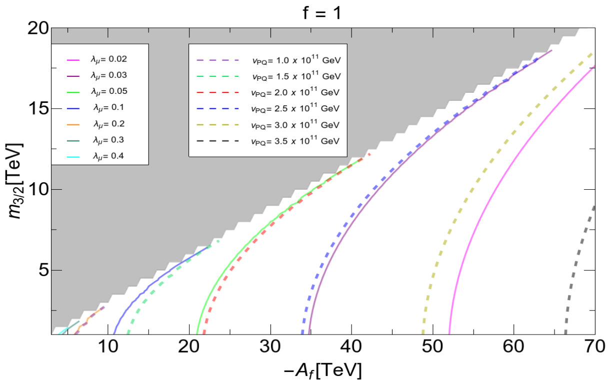


Figure: 18. Representative values of λ_μ required for $\mu = 200$ GeV in the $m_{3/2}$ vs. $-A_f$ plane of the hyCCK model for $f = 1$. We also show several contours of v_{PQ}.

Complex F_x

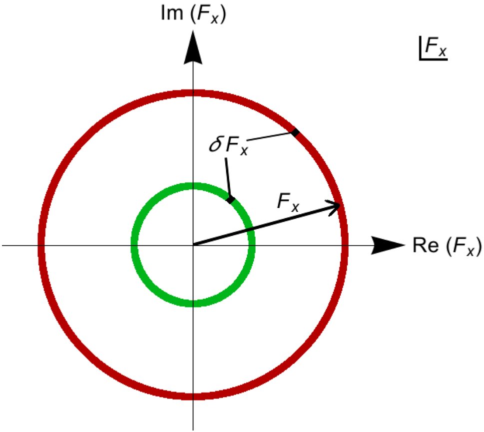


Figure: Annuli of the complex F_X plane giving rise to linearly increasing selection of soft SUSY breaking terms.

A Landscape solution to SUSY flavor and CP problems

arXiv : 1910.00090 by H. Baer, V. Barger and **D. S.**

By scanning over SUSY models with soft terms generated according to m_{soft}^n for $n = 1$ (blue) and 2 (red), along with the anthropic vetos from $m_{weak}^{PU} < 4 \times m_{weak}^{OU}$:

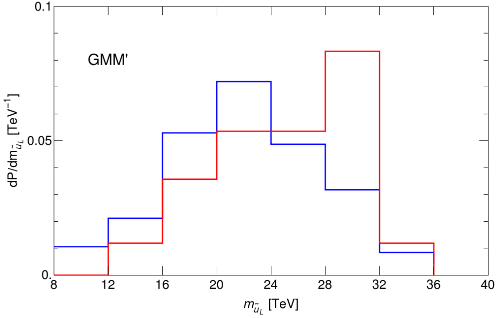


Figure: 23. As seen in the above distribution of $m_{\tilde{u}_L}$, first and second generation matter scalars (squarks and sleptons) are pulled up to $m(\tilde{q}, \tilde{\ell}) \sim 30 \pm 10$ TeV.

Living dangerously with heavy sfermions

Apparently, with first and second generation matter scalars being pulled up to the tens of TeV regime, then one is also being pulled up to a potential decoupling solution to the SUSY flavor and CP problems. The question is: how does this decoupling arise, and is it enough to actually solve these two SUSY issues?

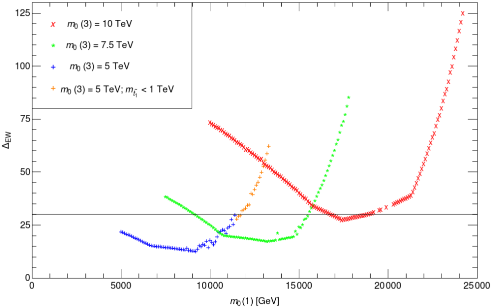


Figure: 24. $m_{1/2} = 1200$ GeV, $A_0 = -1.6 m_0(3)$ and $\tan\beta = 10$ with $\mu = 200$ GeV and $m_A = 2000$ GeV.

The SUSY flavor problem

To match experiments, flavor changing neutral current (FCNC) processes must be suppressed.
By supersymmetrizing the SM into the MSSM, then many new parameters are introduced, mainly in the soft SUSY breaking sector. These include sfermion mass matrices

$$\mathcal{L}_{soft} \ni -\tilde{f}_i^\dagger (\mathbf{m_f^2})_{ij} \tilde{f}_j \quad (24)$$

In the superCKM basis, the 6×6 sfermion mass matrices are built out of 3×3 LL, RR, LR and RL sub-matrices which have the form *e.g.*

$$(\mathbf{m}^2_{\tilde{f}})_{LL} = \begin{pmatrix} (m^2_{f1})_{LL} & (\Delta^f_{12})_{LL} & (\Delta^f_{13})_{LL} \\ (\Delta^f_{21})_{LL} & (m^2_{f2})_{LL} & (\Delta^f_{23})_{LL} \\ (\Delta^f_{31})_{LL} & (\Delta^f_{32})_{LL} & (m^2_{f3})_{LL} \end{pmatrix} \quad (25)$$

with $(m^2_{\tilde{U}})_{LL} = V^u_L \mathbf{m}^2_Q V^{u\dagger}_L$, $(m^2_{\tilde{U}})_{RR} = V^u_R \mathbf{m}^{2\,T}_U V^{u\dagger}_R$ and $(m^2_{\tilde{U}})_{LR} = -\frac{v \sin\beta}{\sqrt{2}} V^u_L \mathbf{a}^*_U V^{u\dagger}_R$ *etc.* and where the CKM matrix is given by $V_{KM} = V^u_L V^{d\dagger}_L$.

The SUSY flavor problem

Since the transformation that diagonalizes the quark mass matrices does not simultaneously diagonalize the corresponding squark mass squared matrices, then the off-diagonal mass matrix contributions Δ_{ij}^{f} may contribute to FCNC processes via mass insertions, and furthermore, non-degenerate diagonal terms can also lead to FCNC effects.

In the following figure, the most restrictive limits on several Δ_{ij} quantities arising from Δm_K constraint and also from updated branching fraction limits on $\mu \to e\gamma$ decay: $BF(\mu \to e\gamma) < 4.2 \times 10^{-13}$ at 90% CL are shown .

A Landscape Solution to the SUSY flavor problem

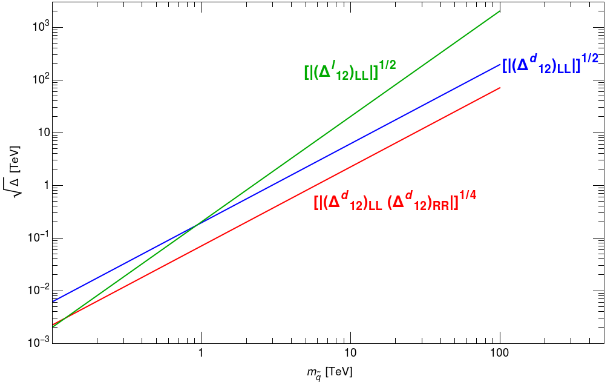


Figure: 25. Upper limits on off-diagonal squark mass terms from Δm_K constraints (blue and red) and off-diagonal slepton masses from $BF(\mu \to e\gamma)$ (green).

A Landscape Solution to the SUSY flavor problem

Along with limits on off-diagonal mass matrix terms, to suppress FCNC effects and hence solve SUSY flavor problem one needs degeneracy on the diagonal. Limits on degeneracy have been computed by Misiak *et al.* in arXiv : hep-ph/9703442. From the Δm_K constraint, for the first two generations of squarks these amount to

$$|m_{\tilde{q}1} - m_{\tilde{q}2}| \leq 2 m_c m_{\tilde{q}}^2 / m_W^2 \tag{26}$$

for both up and down squarks. Thus, for sparticle masses of order m_W, splittings of only a few GeV are allowed and we must be in a state of near degeneracy. As $m_{\tilde{q}}$ increases, then these bounds become much weaker, as can be seen in the following figure.

A Landscape Solution to the SUSY flavor problem

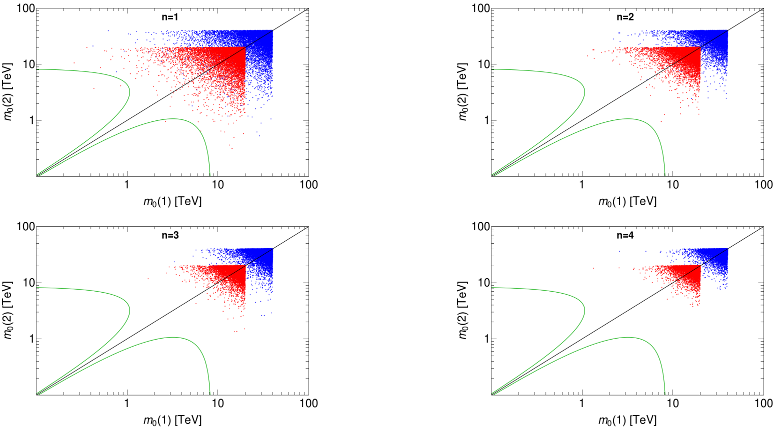


Figure: 26.The values of $m_0(2)$ vs. $m_0(1)$ from an *a*) $n = 1$, *b*) $n = 2$, *c*) $n = 3$ and *d*) $n = 4$, statistical selection of first and second generation matter scalar soft terms. The lower-left of green curves is excluded. The red points denote soft terms scanned up to 20 Tev while blue points show points scanned up to 40 TeV.

A Landscape Solution to the SUSY flavor and CP problems

- As seen in Fig. 23, the landscape pull with $n = 1$ ($n = 2$) results in $m_{\tilde{f}} = 22$ TeV (30 TeV) to be the most probable value with some non-zero probability for $m_{\tilde{f}} = 36$ TeV. This results in decoupling. Because of this decoupling, limits on off-diagonal terms can float as high as tens of TeV, comparable to the tens of TeV for diagonal terms.
- Since this upper bound depends only on gauge quantum numbers, so it is same for both first and second generation. With strong enough pull, this results in quasi-degeneracy, thereby, suppressing FCNC effects.
- This quasi-degeneracy, along with decoupling, helps in solving the SUSY flavor problem.
- The decoupling alone is enough to solve the SUSY CP problem.


Thank you ...

QUESTIONS ?