Small Field Polynomial Inflation

Yong Xu

based on 2104.03977 with Manuel Drees

May 19, PPC2021

UNIVERSITÄT BONN
bctp Bethe Center for Theoretical Physics

Two Points

1. Why polynomial model?
2. Predictions?

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1$ $\Rightarrow$ less steep potentials are favored:
  - fraction power e.g. $V \sim \phi^{2/3}$ (Monodromy inflation [Silverstein,Westphal 08])
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (e.g.Higgs inflation [Bezrukov, Shaposhnikov 07])
  - ...

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^n$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\mathrm{CMB}}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials are favored:
  - fraction power e.g. $V \sim \phi^{2/3}$ (Monodromy inflation [Silverstein,Westphal 08])
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (e.g.Higgs inflation [Bezrukov, Shaposhnikov 07])
  - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials are favored:
    - fraction power e.g. $V \sim \phi^{2/3}$ (Monodromy inflation [Silverstein,Westphal 08])
    - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (e.g.Higgs inflation [Bezrukov, Shaposhnikov 07])
    - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

  - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^n$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials are favored:
    - fraction power e.g. $V \sim \phi^{2/3}$ (Monodromy inflation [Silverstein,Westphal 08])
    - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (e.g.Higgs inflation [Bezrukov, Shaposhnikov 07])
    - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

    - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field
    - Reasonable to insist on renormalizability (in particular ϕ in UV complete theory)

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^n$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials are favored:
  - fraction power e.g. $V \sim \phi^{2/3}$ (Monodromy inflation [Silverstein,Westphal 08])
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (e.g.Higgs inflation [Bezrukov, Shaposhnikov 07])
  - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

  - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field
  - Reasonable to insist on renormalizability (in particular ϕ in UV complete theory)
  - $V(\phi)$: most general renormalizable inflaton potential
  - Has been gaining interests since 1990s [Hodges et.al 90, ... , Musoke & Easther 17,...]

Polynomial Inflation Analysis

- Potential $V(\phi) = \cancelto{\text{negligible}}{Const.} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\cancelto{\text{shifted away}}{\phi^1}.$

Polynomial Inflation Analysis

- Potential $V(\phi) = \cancelto{\text{negligible}}{Const.} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \cancelto{\text{shifted away}}{e\phi^1}.$
    1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
    2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹

Polynomial Inflation Analysis

- Potential $V(\phi) = \cancelto{\text{negligible}}{Const.} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \cancelto{\text{shifted away}}{e\phi^1}.$
  1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  3. Intermediate regime: V can be flat due to negative ϕ^3 term ☺

Polynomial Inflation Analysis

- Potential $V(\phi) = \overset{\text{negligible}}{\cancel{Const.}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \overset{\text{shifted away}}{\cancel{e\phi^1}}$.
    1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
    2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
    3. Intermediate regime: V can be flat due to negative ϕ^3 term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ an inflection-point $\phi_0 = -\frac{3c}{8d}$

Polynomial Inflation Analysis

- Potential $V(\phi) = \overset{\text{negligible}}{\cancel{Const.}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \overset{\text{shifted away}}{\cancel{e\phi^1}}$.
  1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  3. Intermediate regime: V can be flat due to negative ϕ^3 term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ an inflection-point $\phi_0 = -\frac{3c}{8d}$
- Three parameters (d, A, β):

$$V(\phi) = d\left[\phi^4 + \frac{c}{d}(1-\beta)\,\phi^3 + \frac{9}{32}\left(\frac{c}{d}\right)^2\phi^2\right] \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

1. $A \equiv -8/3\phi_0 \longleftrightarrow$ Position of inflection-point

Polynomial Inflation Analysis

negligible　　　　　　　　　　　　shifted away

- Potential $V(\phi) = \cancel{Const.} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \cancel{e\phi^1}$.
    1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
    2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
    3. Intermediate regime: V can be flat due to negative ϕ^3 term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ an inflection-point $\phi_0 = -\frac{3c}{8d}$
- Three parameters (d, A, β):

$$V(\phi) = d\left[\phi^4 + \frac{c}{d}(1-\beta)\,\phi^3 + \frac{9}{32}\left(\frac{c}{d}\right)^2\phi^2\right] \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

1. $A \equiv -8/3\phi_0 \longleftrightarrow$ Position of inflection-point
2. $\beta > 0$: $\longleftrightarrow$ Flatness $V(\phi_0)$

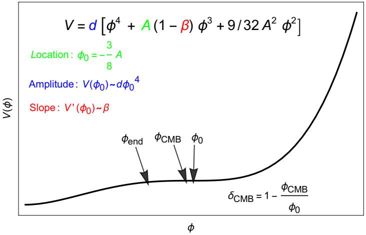
Polynomial Inflation Analysis

- Potential $V(\phi) = \overset{\text{negligible}}{\cancel{Const.}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \overset{\text{shifted away}}{\cancel{e\phi^1}}$.
  1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  3. Intermediate regime: V can be flat due to negative ϕ^3 term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ an inflection-point $\phi_0 = -\frac{3c}{8d}$
- Three parameters (d, A, β):

$$V(\phi) = d\left[\phi^4 + \frac{c}{d}(1-\beta)\,\phi^3 + \frac{9}{32}\left(\frac{c}{d}\right)^2\phi^2\right] \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

  1. $A \equiv -8/3\phi_0 \longleftrightarrow$ Position of inflection-point
  2. $\beta > 0$: $\longleftrightarrow$ Flatness $V(\phi_0)$
  3. d: $\longleftrightarrow$ Amplitude (power spectrum)

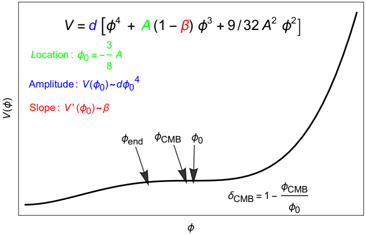
Slow-Roll Predictions



- SR parameters $(M_p \equiv 1)$

$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2 ; \eta_V = V''/V ; \xi_V^2 = V'V'''/V^2$$

Slow-Roll Predictions



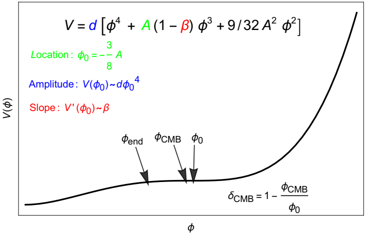
- SR parameters $(M_p \equiv 1)$

$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2 ;\, \eta_V = V''/V ;\, \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\rm CMB} = 1 - \phi_{\rm CMB}/\phi_0$$

Slow-Roll Predictions



- SR parameters $(M_p \equiv 1)$

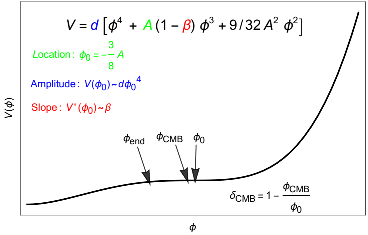
$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2 ; \eta_V = V''/V ; \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\rm CMB} = 1 - \phi_{\rm CMB}/\phi_0$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$

Slow-Roll Predictions



- SR parameters $(M_p \equiv 1)$

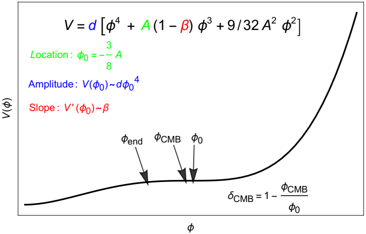
$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2 ; \eta_V = V''/V ; \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\rm CMB}$ $\Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\rm CMB} = 1 - \phi_{\rm CMB}/\phi_0$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left(\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right)$

Slow-Roll Predictions



- SR parameters $(M_p \equiv 1)$

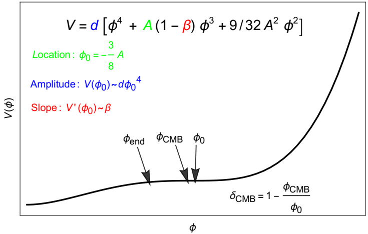
$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2;\ \eta_V = V''/V;\ \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\text{CMB}} \Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\text{CMB}} = 1 - \phi_{\text{CMB}}/\phi_0$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\text{CMB}}/\phi_0^2$
  - $N_{\text{CMB}} \propto \left(\frac{\pi}{2} - \arctan\left(\frac{\delta_{\text{CMB}}}{\sqrt{2\beta}}\right)\right)$
  - $r \propto \left(2\beta + \delta^2\right)^2/\phi_0^2$

Slow-Roll Predictions



- SR parameters $(M_p \equiv 1)$

$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2 ; \eta_V = V''/V ; \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\rm CMB} = 1 - \phi_{\rm CMB}/\phi_0$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left(\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right)$
  - $r \propto \left(2\beta + \delta^2\right)^2 / \phi_0^2$
  - $\alpha \simeq -\frac{576(2\beta+\delta^2)}{\phi_0^4}$
  - $\mathcal{P}_\zeta \simeq \frac{d\,\phi_0^6}{5184\pi^2(\delta^2+2\beta)^2}$
- $n_s = 0.9649$, $N_{\rm CMB} = 65$, $\mathcal{P}_\zeta = 2.1 \cdot 10^{-9}$ $\Rightarrow$ fix parameters:

$$\delta_{\rm CMB} = 7.31 \times 10^{-4}\,\phi_0^2$$

$$\beta = 9.73 \times 10^{-7}\,\phi_0^4$$

$$d = 6.61 \times 10^{-16}\,\phi_0^2$$

- Predictions for r and α?

Predictions for r and α

- r unfortunately not detectable (within my lifetime :)

$$\frac{r}{7.09 \times 10^{-9} \phi_0^6} = 1 - 3.9 \cdot 10^{-2} (65 - N_{\mathrm{CMB}})$$

$$+ 15.0\,(0.9649 - n_s) + 175\,(0.9649 - n_s)^2$$

Predictions for r and α

- r unfortunately not detectable (within my lifetime :)

$$\frac{r}{7.09 \times 10^{-9} \phi_0^6} = 1 - 3.9 \cdot 10^{-2} (65 - N_{\mathrm{CMB}})$$
$$+ 15.0\,(0.9649 - n_s) + 175\,(0.9649 - n_s)^2$$

- α hopefully detectable [S4 CMB]

$$\alpha = -1.43 \cdot 10^{-3} - 5.56 \cdot 10^{-5}\,(65 - N_{\mathrm{CMB}})$$
$$+ 0.02\,(0.9649 - n_s) - 0.25\,(0.9649 - n_s)^2$$

Predictions for r and α

- r unfortunately not detectable (within my lifetime :)

$$\frac{r}{7.09\times 10^{-9}\phi_0^6} = 1 - 3.9\cdot 10^{-2}(65 - N_{\mathrm{CMB}}) + 15.0\,(0.9649 - n_s) + 175\,(0.9649 - n_s)^2$$

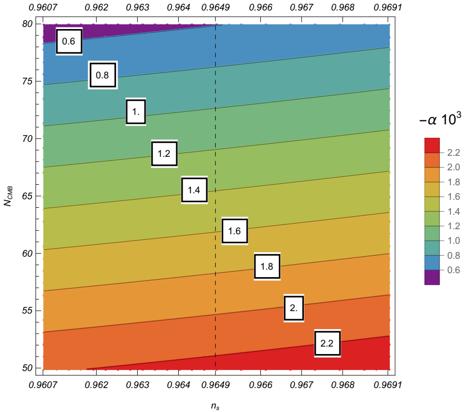
- α hopefully detectable [S4 CMB]

$$\alpha = -1.43\cdot 10^{-3} - 5.56\cdot 10^{-5}\,(65 - N_{\mathrm{CMB}}) + 0.02\,(0.9649 - n_s) - 0.25\,(0.9649 - n_s)^2$$

- Model parameters for central value

$$\delta_{\mathrm{CMB}} = 7.31\times 10^{-4}\phi_0^2;\ \beta = 9.73\times 10^{-7}\phi_0^4;$$

$$d = 6.61\times 10^{-16}\phi_0^2$$



- Inflaton mass: $m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}\Big|_{\phi=0} \simeq 4d\phi_0^2$;

  Inflationary scale: $H_{\mathrm{inf}} = \sqrt{\frac{V(\phi_0)}{3}} \simeq 8.6\cdot 10^{-9}\phi_0^3$

Predictions for r and α

- r unfortunately not detectable (within my lifetime :)

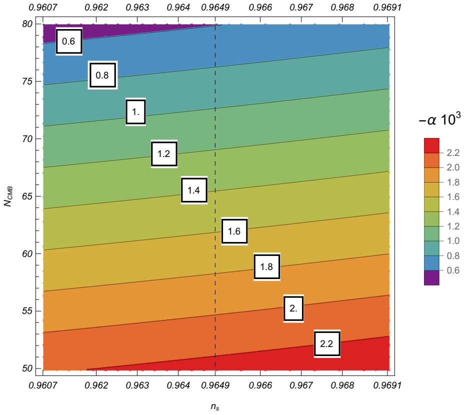
$$\frac{r}{7.09\times10^{-9}\phi_0^6} = 1 - 3.9\cdot10^{-2}(65 - N_{\mathrm{CMB}})$$
$$+ 15.0\,(0.9649 - n_s) + 175\,(0.9649 - n_s)^2$$

- α hopefully detectable [S4 CMB]

$$\alpha = -1.43\cdot10^{-3} - 5.56\cdot10^{-5}\,(65 - N_{\mathrm{CMB}})$$
$$+ 0.02\,(0.9649 - n_s) - 0.25\,(0.9649 - n_s)^2$$

- Model parameters for central value

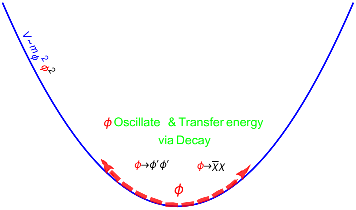
$$\delta_{\mathrm{CMB}} = 7.31\times10^{-4}\,\phi_0^2;\ \beta = 9.73\times10^{-7}\,\phi_0^4;$$
$$d = 6.61\times10^{-16}\,\phi_0^2$$



- Inflaton mass: $m_\phi^2 = \left.\frac{\partial^2 V}{\partial\phi^2}\right|_{\phi=0} \simeq 4d\phi_0^2$;

  Inflationary scale: $H_{\mathrm{inf}} = \sqrt{\frac{V(\phi_0)}{3}} \simeq 8.6\cdot10^{-9}\phi_0^3$
- Question: What is the lower bound for ϕ_0 ?

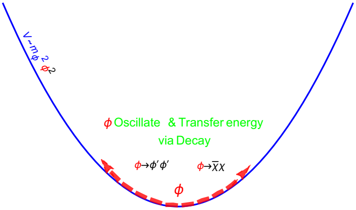
Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

Reheating ⇒ Lower Bound

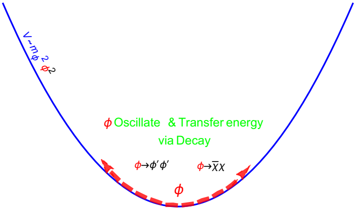


- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)

Reheating ⇒ Lower Bound

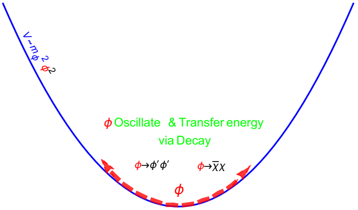


- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$

Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

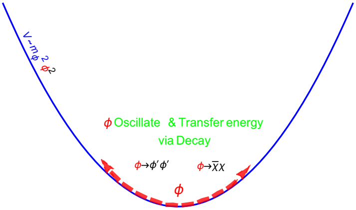
$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

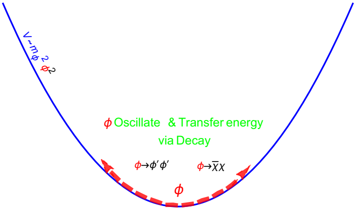
- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4}\Gamma_\phi^{1/2}$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7\times10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4\times10^{-24}}$$

- Remarks:
  1. Ignored Preheating totally!

Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

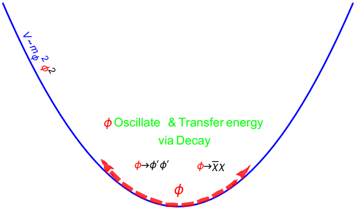
- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

- Remarks:
  1. Ignored Preheating totally!
  2. Though $m_{\phi'} \sim g\phi \Rightarrow$ tachyonic resonance, still ok here, due to (sizeable) self-coupling $\lambda\phi'^4$ ($\Rightarrow$ back-reaction $m_{\phi'} \sim \lambda\langle\phi'^2\rangle$); Pauli blocking for $\chi \Rightarrow$ Preheating not efficient here

Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

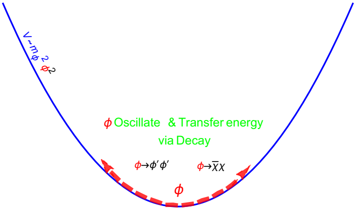
- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7\times10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4\times10^{-24}}$$

- Remarks:
  1. Ignored Preheating totally!
  2. Though $m_{\phi'} \sim g\phi \Rightarrow$ tachyonic resonance, still ok here, due to (sizeable) self-coupling $\lambda\phi'^4$ ($\Rightarrow$ back-reaction $m_{\phi'} \sim \lambda\langle\phi'^2\rangle$); Pauli blocking for $\chi \Rightarrow$ Preheating not efficient here
  3. For more details on (efficient) Preheating: please *see the talk by Qianshu Lu*

Reheating ⇒ Lower Bound



- Decays to Bosons (e.g. SM Higgs) or Fermions

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate: $\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}$; $\frac{y^2}{8\pi} m_\phi$ with $m_\phi \sim \phi_0^2$)
- Reheating Temperature: $T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

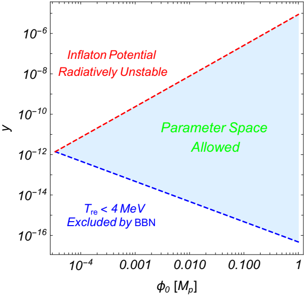
- Remarks:
  1. Ignored Preheating totally!
  2. Though $m_{\phi'} \sim g\phi \Rightarrow$ tachyonic resonance, still ok here, due to (sizeable) self-coupling $\lambda\phi'^4$ (⇒ back-reaction $m_{\phi'} \sim \lambda\langle\phi'^2\rangle$); Pauli blocking for χ ⇒ Preheating not efficient here
  3. For more details on (efficient) Preheating: please *see the talk by Qianshu Lu*
- Question: What are the upper bounds for the couplings? ⇒ Radiative stability

Radiative Stability ⇒ Upper Bound

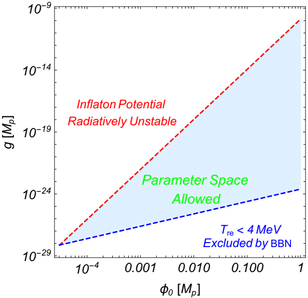
- Require: $\Delta V_{1-\mathrm{loop}}(\phi_0) \ll V(\phi_0)\,; \Delta V'(\phi_0) \ll V'(\phi_0)\,; \Delta V''(\phi_0) \ll V''(\phi_0)$, with

$$\Delta V_{1-\mathrm{loop}} = \frac{1}{64\pi^2} \sum_{\psi=\phi',\chi} (-1)^{2s_\psi} g_\psi \widetilde{m}_\psi(\phi)^4 \left(\ln\left(\frac{\widetilde{m}_\psi(\phi)^2}{Q_0^2} \right) - \frac{3}{2} \right)$$

- Upper bounds for y (coupling $y\phi\bar{\chi}\chi$): $\left|\frac{y^4-3y^4\ln(y^2)}{4\pi^2}\right| < 16d\beta$



- Upper bounds for g (coupling $g\phi|\phi'|^2$): $\frac{g^2}{8\pi^2}\left|\ln\left(\frac{g}{\phi_0}\right)-1\right| < 8d\beta\phi_0^2$



- Radiative Stability + Reheating ⇒ Lower bound $\phi_0 > 3 \cdot 10^{-5} M_p$

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3}$ ⇒ testable in future [S4 CMB] ☺

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3}$ ⇒ testable in future [S4 CMB] ☺
  - Full parameter space: Reheating +Radiative Stability ⇒ $\phi_0 > 3 \cdot 10^{-5} M_p$
  - Implications:
    1. $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p$ ⇒ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3} \Rightarrow$ testable in future [S4 CMB] ☺
  - Full parameter space: Reheating +Radiative Stability $\Rightarrow \phi_0 > 3 \cdot 10^{-5} M_p$
  - Implications:
    1. $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p \Rightarrow$ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    2. $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV!

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0\,;\, \beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4\,;\, d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3} \Rightarrow$ testable in future [S4 CMB] ☺
  - Full parameter space: Reheating +Radiative Stability $\Rightarrow \phi_0 > 3 \cdot 10^{-5} M_p$
  - Implications:
    1. $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p \Rightarrow$ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    2. $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV!
- Well motivated; easy to extend ⇒ Baryogenesis, Strong CP, DM... [work in progress]

Summary

- A polynomial model (d, A, β) can fit data very well:

$$V \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0\,;\, \beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4\,;\, d = 6.61 \times 10^{-16}\phi_0^2/M_p^2.$

  - Predictions:
    - ① $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    - ② $\alpha \simeq -1.43 \cdot 10^{-3}$ ⇒ testable in future [S4 CMB] ☺
  - Full parameter space: Reheating +Radiative Stability ⇒ $\phi_0 > 3 \cdot 10^{-5} M_p$
  - Implications:
    - ① $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p$ ⇒ as light as $\mathcal{O}(100)$ GeV (EW scale);
      For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    - ② $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV!
- Well motivated; easy to extend ⇒ Baryogenesis, Strong CP, DM... [work in progress]

Thank you!