Cosmological PBHs as Dark Matter

Zachary S.C. Picker

XIV INTERNATIONAL CONFERENCE ON INTERCONNECTIONS BETWEEN PARTICLE PHYSICS AND COSMOLOGY
The UNIVERSITY of OKLAHOMA

THE UNIVERSITY OF SYDNEY

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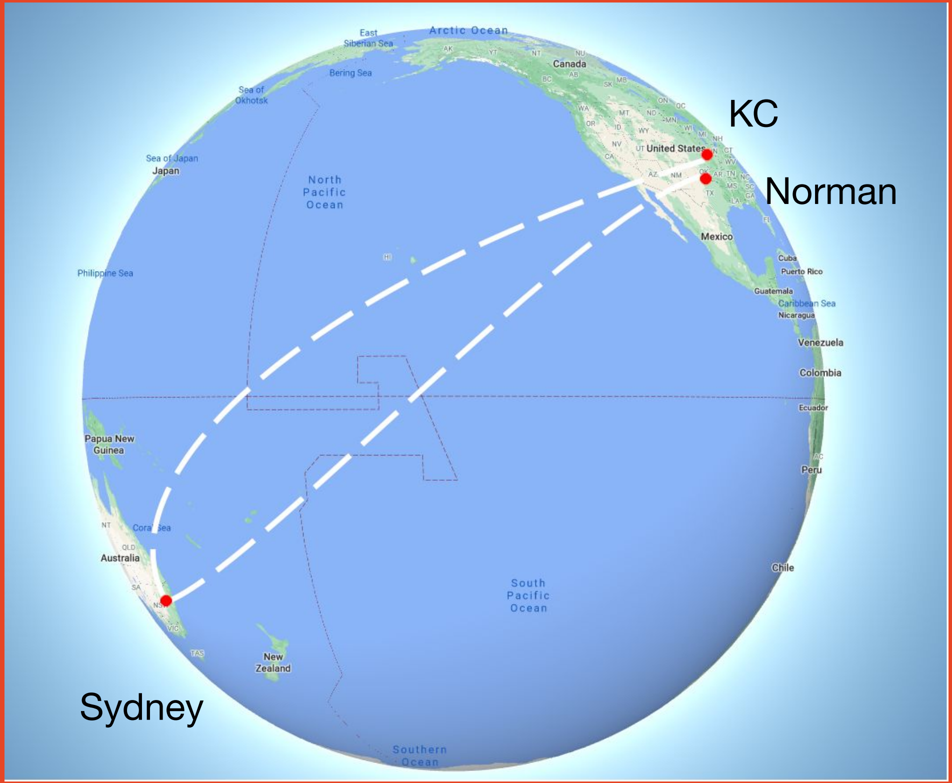

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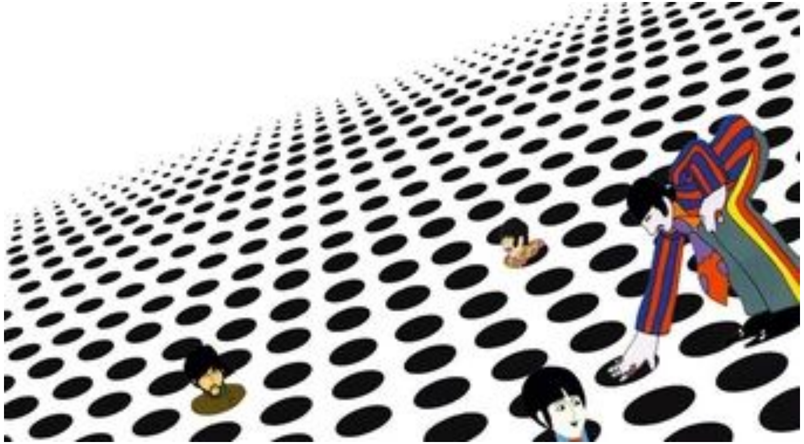
Quick outline

1. Cosmological PBHs
   a. Dark matter considerations
   b. Cosmological black hole solutions
   c. Thakurta solution
2. New phenomenology
   a. LIGO binary abundance constraints
      ⇒ fully lifted
   b. Hawking evaporation
      ⇒ asteroid-mass range constrained
3. Morals and takeaways



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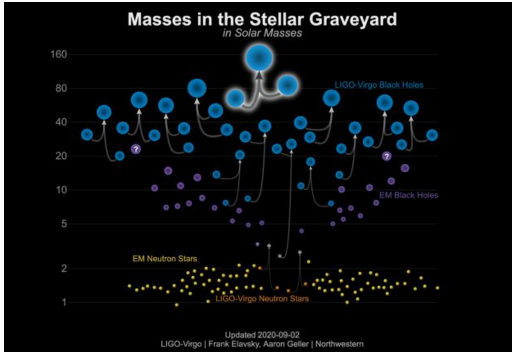
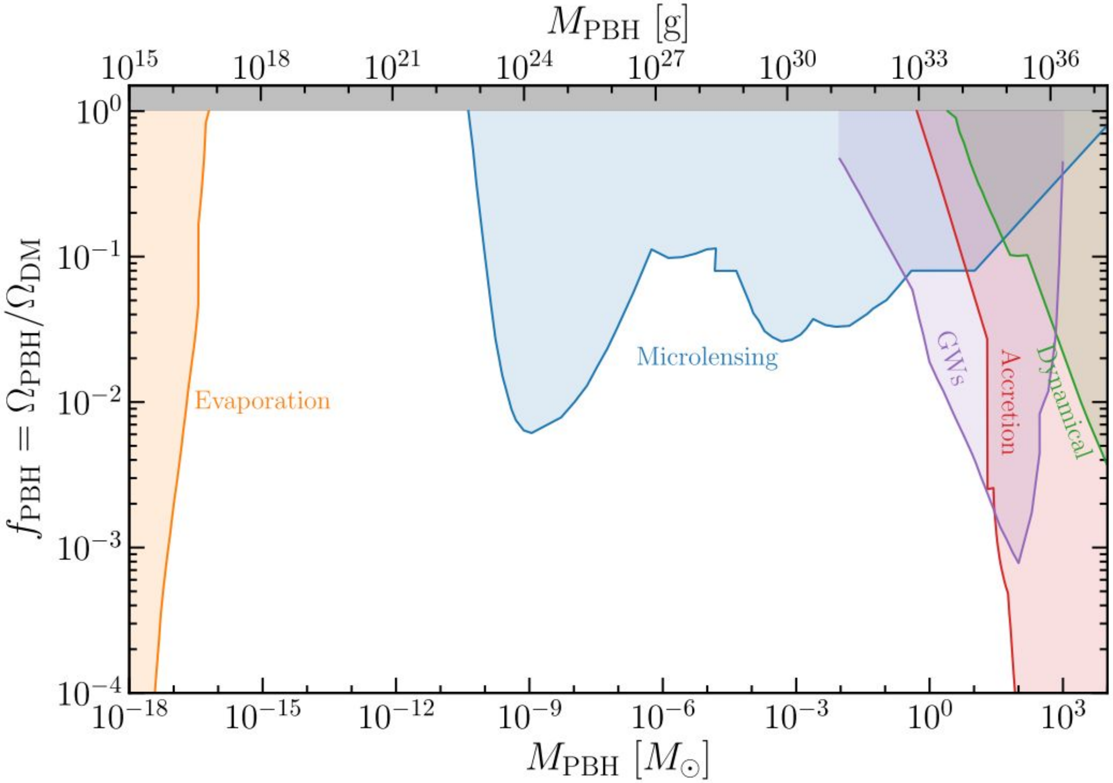
PBHs as dark matter





EHT Collaboration, 2019
LIGO-Virgo/ Northwestern U. / Frank Elavsky & Aaron Geller, 2020

Green & Kavanaugh, 2020

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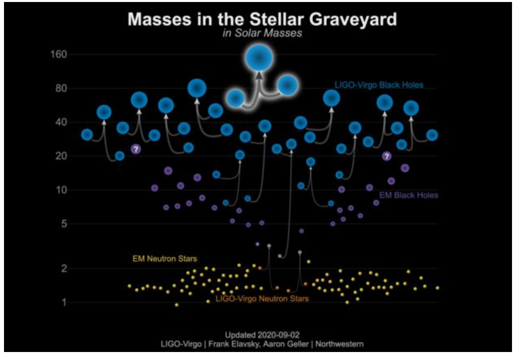
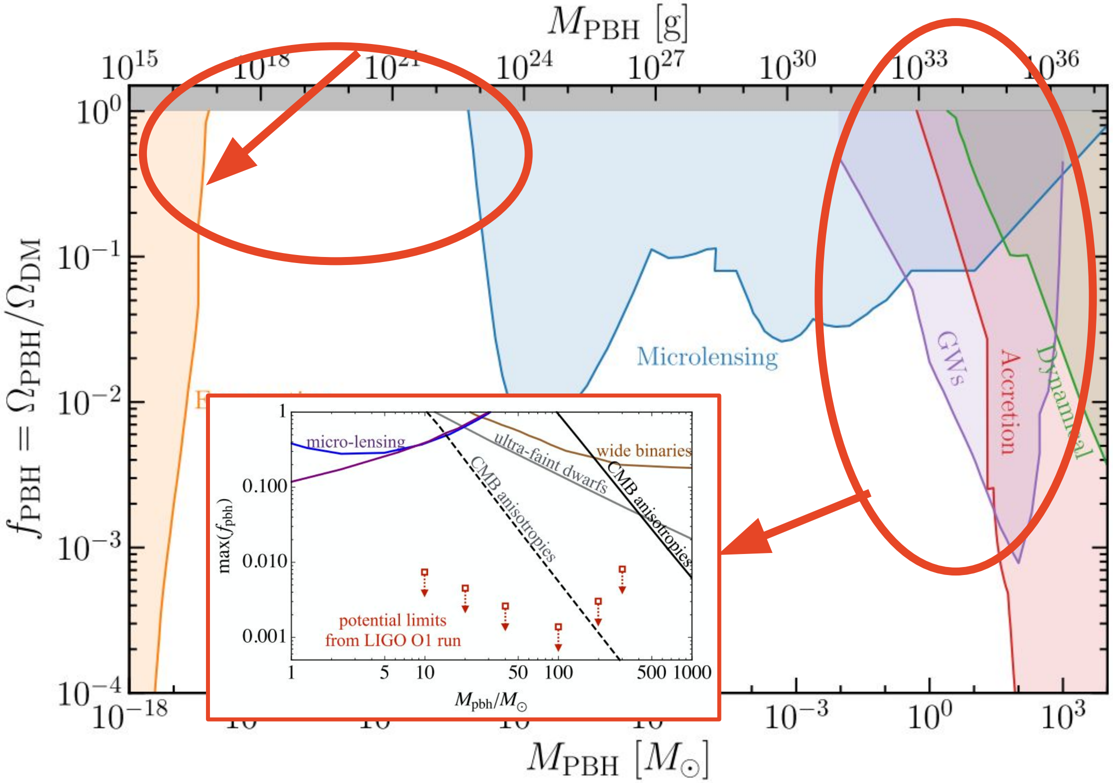
PBHs as dark matter





EHT Collaboration, 2019
LIGO-Virgo/ Northwestern U. / Frank Elavsky & Aaron Geller, 2020

Green & Kavanaugh, 2020
Ali-Haïmoud et al, 2017

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Cosmological PBHs

- Schwarzschild metric:
    - Embedded in flat, empty space
    - Mass is defined at infinity
- Cosmological solution:
    - Should be embedded in cosmological fluid
    - Asymptotically FLRW
    - *Local* mass definition

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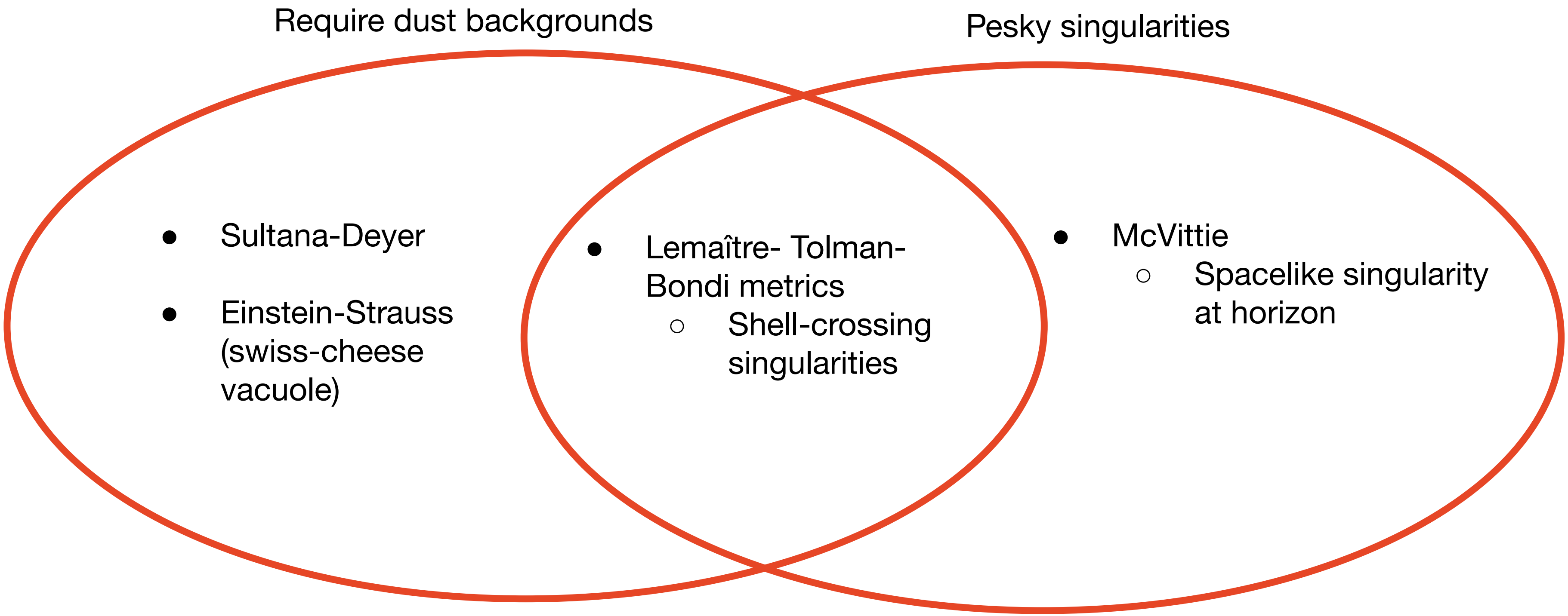
Cosmological PBHs



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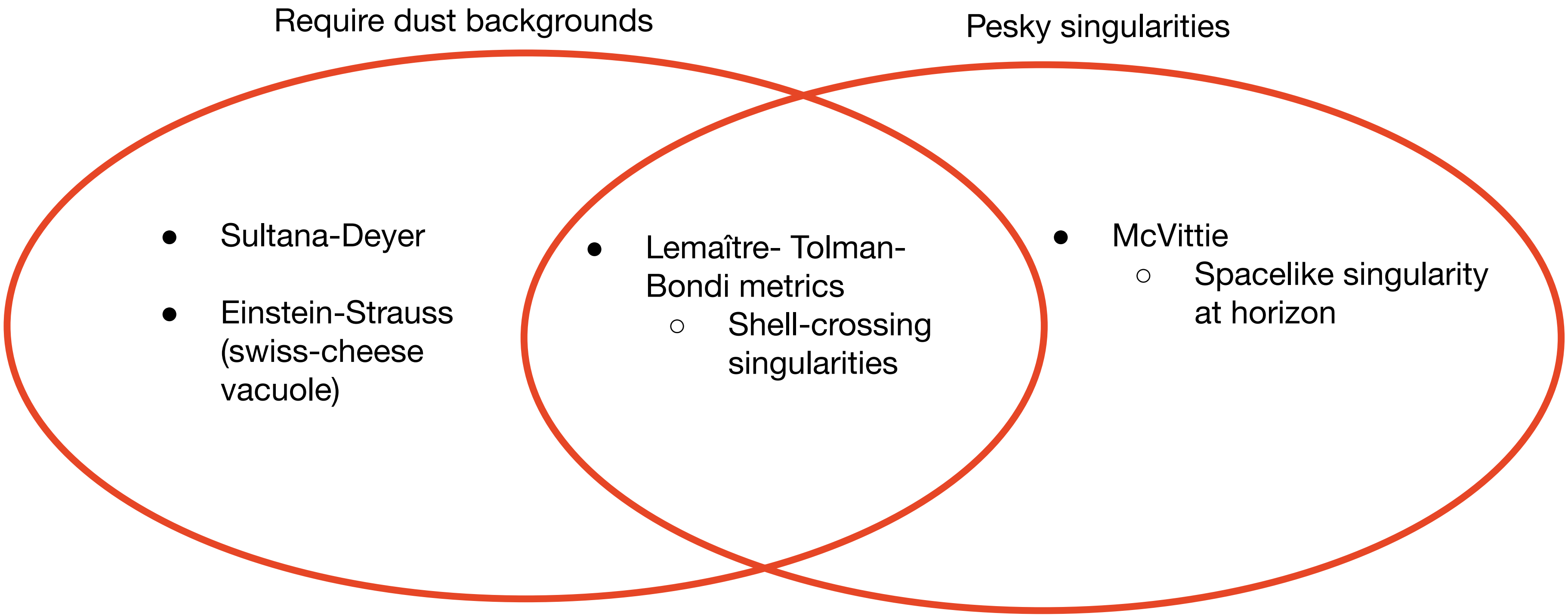
Cosmological PBHs



Generalized McVittie metrics ⇒ Thakurta metric

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Thakurta solution

- Attractor solution of generalized McVittie metrics
- Simple & Elegant: $\mathrm{d}s^2 = a^2 \mathrm{d}s^2_{schw.}$

- $$\mathrm{d}s^2 = f(R)\left(1 - \frac{H^2R^2}{f^2(R)}\right)\mathrm{d}t^2 + \frac{2HR}{f(R)}\mathrm{d}t\,\mathrm{d}R - \frac{\mathrm{d}R^2}{f(R)} - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

- $f(R) = 1 - 2Gma(t)/R$

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- $f(R) = 1 - 2Gma(t)/R$
- Quasi-local Misner-Sharp mass: $m(r,t) = ma(t) + \dfrac{H^2 R^3}{2Gf(R)}$

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Thakurta solution- Kodama time

- No Killing field for dynamic black holes
- Analogous geometrically preferred vector field: Kodama field
  - Can define a 'natural' time coordinate:
- $$ds^2 = \left(1 - \frac{2m(r,t)}{r}\right) dt^2 + \frac{dr^2}{1 - 2m(r,t)/r} + r^2 \left\{d\theta^2 + \sin^2\theta \, d\phi^2\right\}$$
- For observers far from the BH and below cosmological horizon:
  - Kodama time ≈ cosmic time

H. Kodama, *Conserved Energy Flux for the Spherically Symmetric System and the Back Reaction Problem in the Black Hole Evaporation*, Prog. Theor. Phys. **63** (1980) 1217.

G. Abreu and M. Visser, *Kodama time: Geometrically preferred foliations of spherically symmetric spacetimes*, Phys. Rev. D **82** (2010) 044027 [1004.1456].

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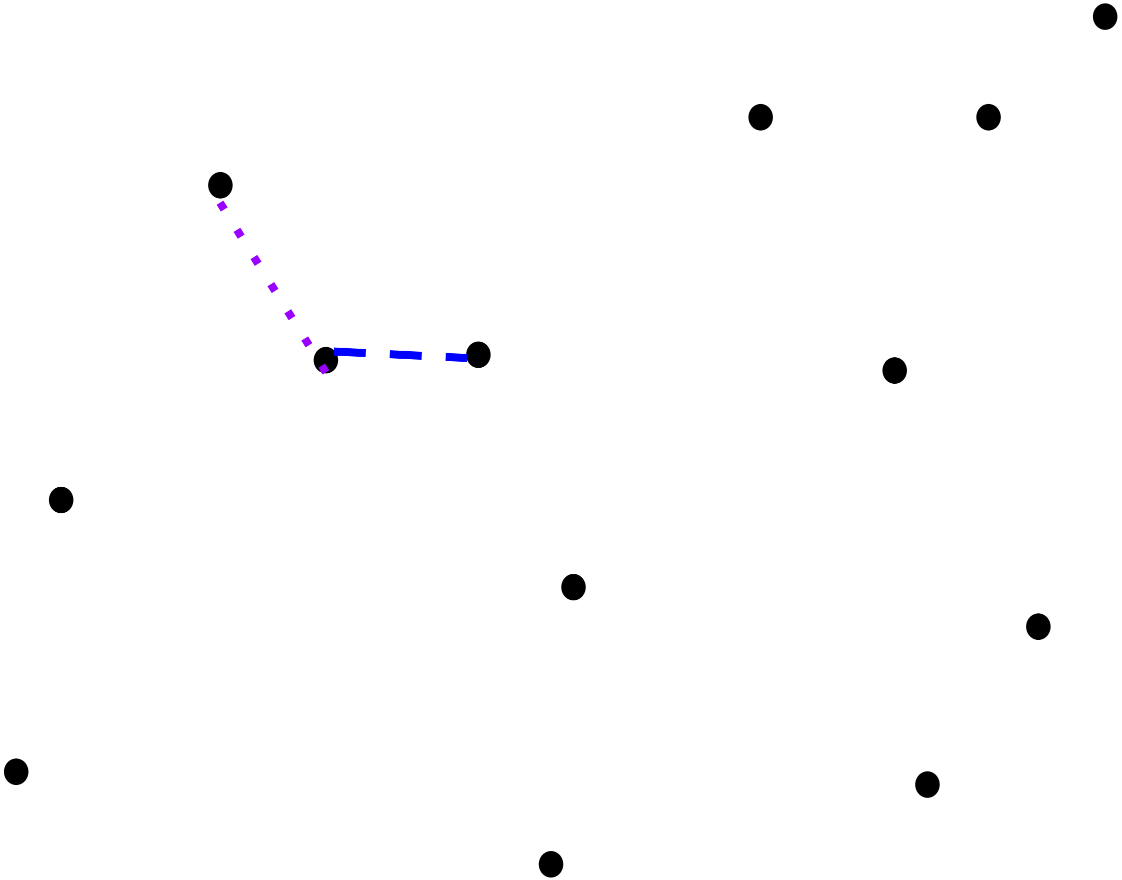
Binary abundance limits



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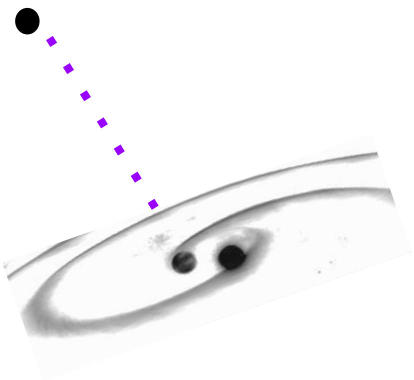
Binary abundance limits - decoupling



Schwarzschild decoupling:

$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{Gm}{R^2}$$

↓

$$\frac{m}{V} \gtrsim \rho$$

Thakurta Decoupling:

$$\ddot{R} = -\frac{Gma}{R^2} + \frac{\ddot{a}}{a}R \qquad E = -GM\mu a^2/(2R)$$

↓

$$\frac{ma(t)}{V} \gtrsim \rho \qquad \dot{R}/R = -\dot{E}/E + 2H$$

↓

$$\dot{E}/E > 2H$$

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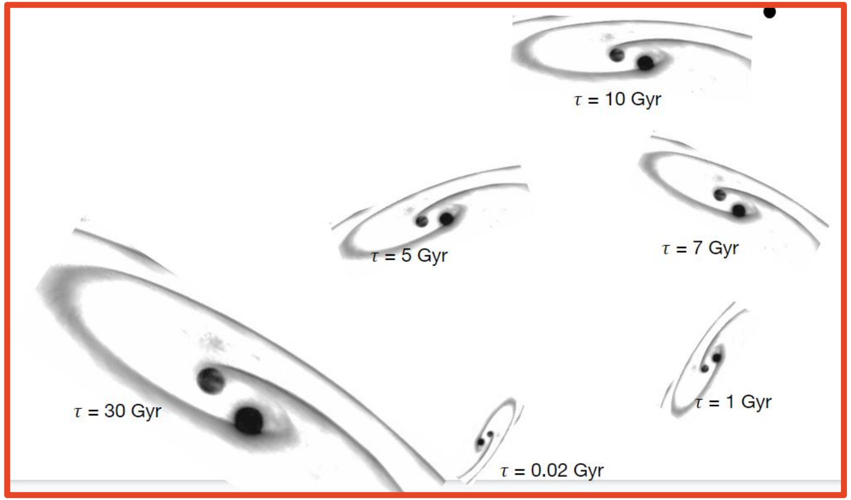
Binary abundance limits - constraints

Schwarzschild PBHs:

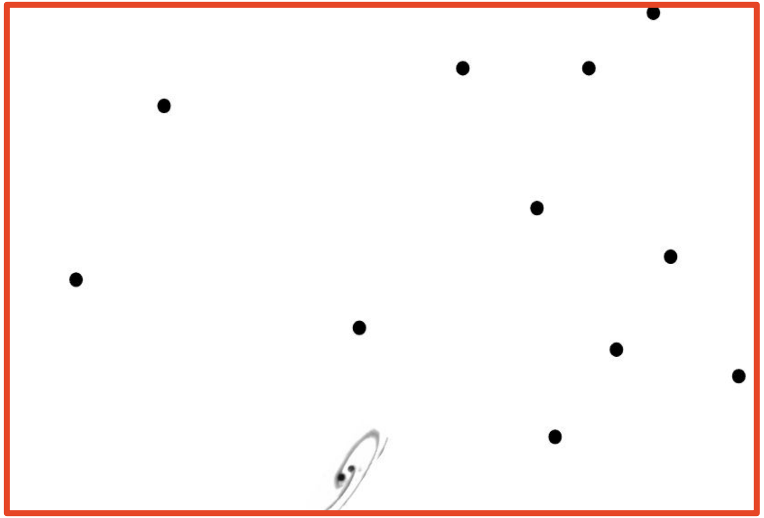
At matter-radiation equality:



Many of these coalesce ~today

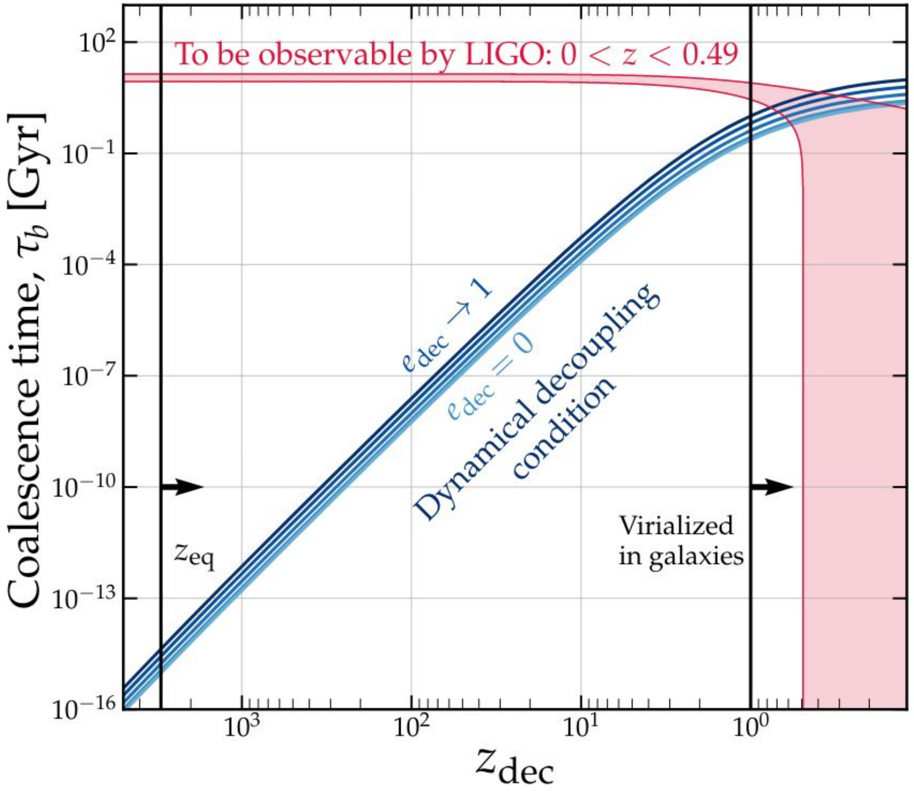
Thakurta PBHs:

At matter-radiation equality:



τ_{max} ~ 100 sec (!)

Binary abundance limits - constraints



arxiv:2008.10743:

Eliminating the LIGO bounds on primordial black hole dark matter

Céline Bœhm,[a] Archil Kobakhidze,[a] Ciaran A. J. O'Hare,[a] Zachary S. C. Picker,[a] Mairi Sakellariadou[b]

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Hawking evaporation

- Interested in smallest PBH which survives until today
  - ‘Critical mass’
  - Smaller PBHs cannot be a dark matter candidate
  - Why?

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Hawking evaporation

- Interested in smallest PBH which survives until today
    - ‘Critical mass’
    - Smaller PBHs cannot be a dark matter candidate
    - Why?
- ‘Stability constraint’:
    - You *could* have smaller PBHs today…
    - Extremely sensitive to PBH mass at formation
    - Cannot populate DM today
- Further constraints from effect of radiation on CMB, BBN, cosmic rays...

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Hawking evaporation - critical mass

- Surface gravity is relatively well-defined in Kodama time
- For the Thakurta metric:
- $$\kappa \approx \frac{1}{2Gma} = 2\kappa_{\mathrm{Schw}}/a\,, \quad T = \kappa/2\pi$$
- $$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{1}{1920\pi G^2 m^2 a^2} = \frac{8}{a^2}\left(\frac{\mathrm{d}m}{\mathrm{d}t}\right)_{\mathrm{Schw}}$$
- New 'critical mass' which evaporates at matter-radiation equality:
- $$m_* \sim 9.6 \times 10^{-13}\ M_\odot \left(\frac{\gamma}{0.2}\right)^{\frac{1}{7}} \left(\frac{h}{0.67}\right)^{-\frac{3}{7}} \left(\frac{\Omega_{\mathrm{r}}}{5.4 \times 10^{-5}}\right)^{-\frac{3}{14}}$$

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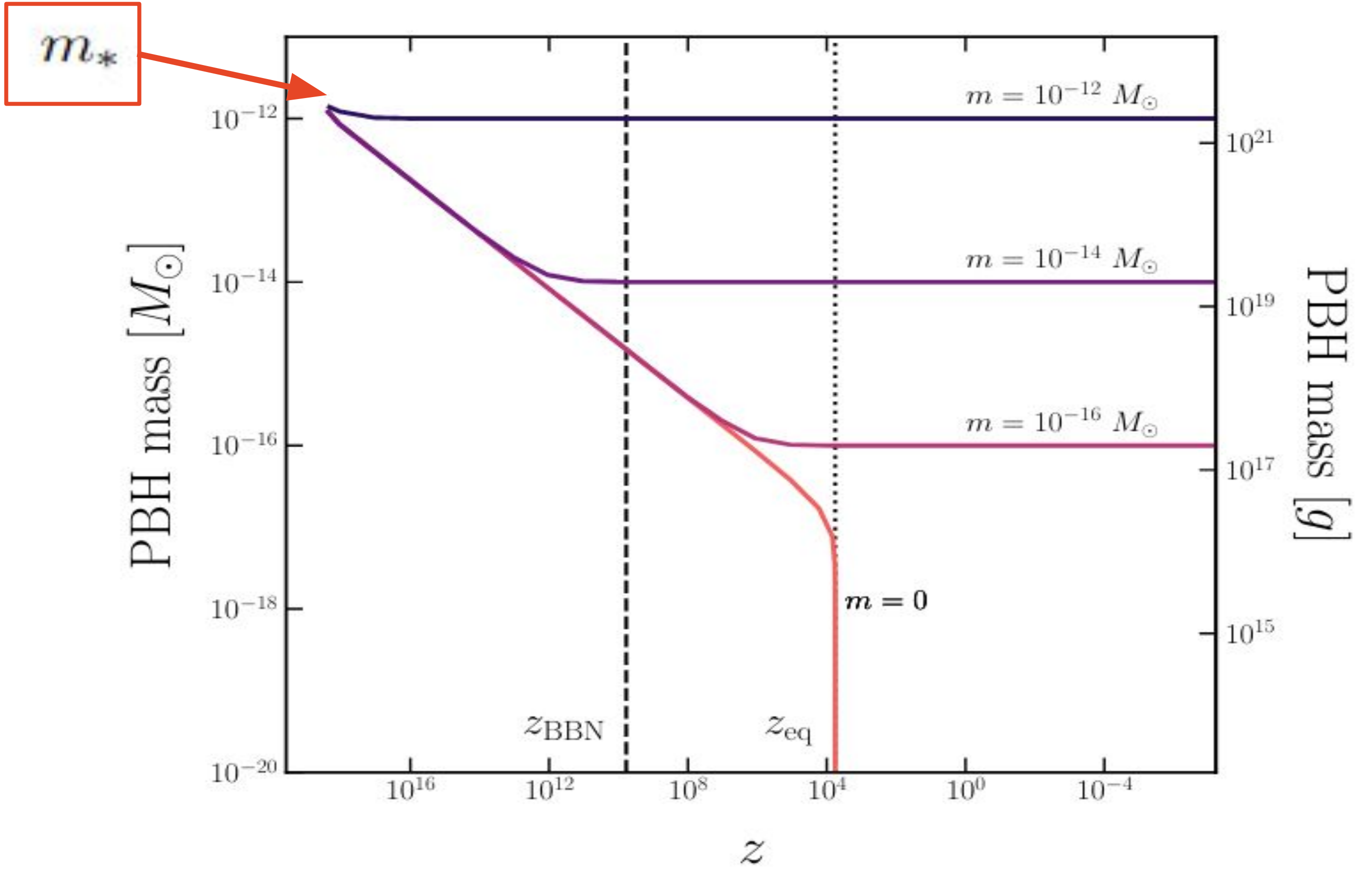
Hawking evaporation - critical mass



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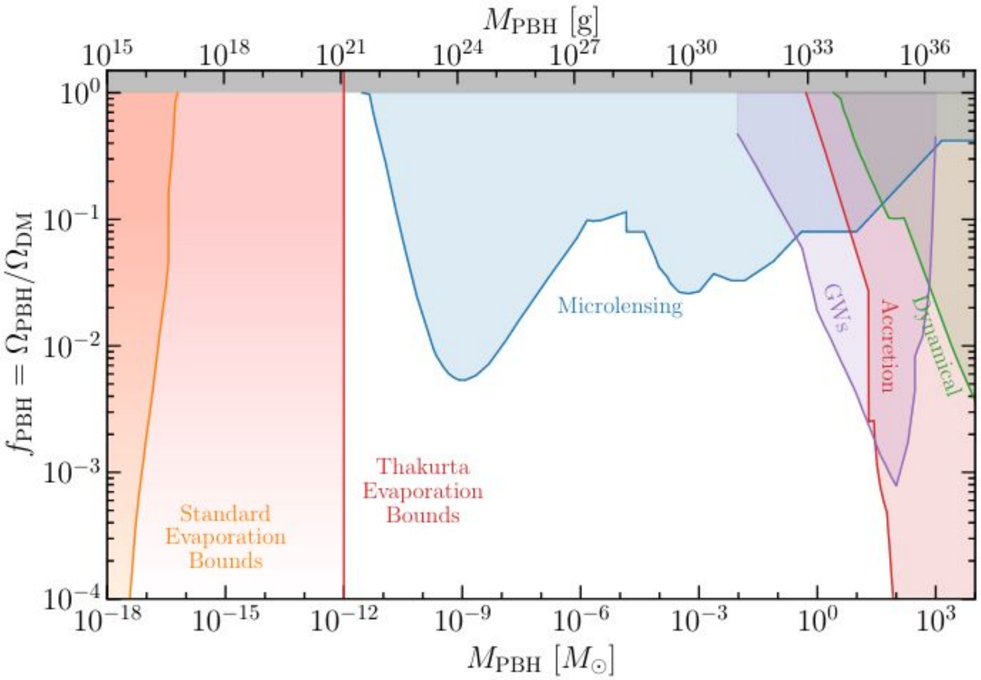
Hawking evaporation - constraints



arxiv:2103.02815:

Navigating the asteroid field: New evaporation constraints for primordial black holes as dark matter

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Morals and takeaways

- Important PBH dark matter calculations take place in the early universe
- Choice of cosmological BH metric is *not* trivial
- In particular, for the Thakurta metric:
    - LIGO binary abundance constraints are lifted
    - Asteroid-mass PBH range is constrained

Main lesson:

Important phenomenological consequences from PBH metric! The story is not yet over...

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Morals and takeaways

- Important PBH dark matter calculations take place in the early universe
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Main lesson:

Important phenomenological consequences from PBH metric! The story is not yet over...

Thanks!

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Extra equations: Thakurta metric

- Misner-Sharp mass definition: $1 - \frac{2M_{MSH}}{R} \equiv \nabla^c R \nabla_c R$
- Thakurta pparent horizon: $R_{\mathrm{b}} = \frac{1}{2H}\left(1 - \sqrt{1 - 8HGma(t)}\right) \approx 2ma(t)R$
- Transformation to Kodama time: $\mathrm{d}\tau = \mathrm{d}t + \frac{HR}{f(R)} \frac{\mathrm{d}R}{1 - \frac{2Gm_{\mathrm{MS}}}{R}}$

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Extra equations: binary abundance

- Exact Schwarzschild decoupling:

$$\frac{ma}{V} \gg \frac{3}{4\pi G}\left|\frac{\ddot{a}}{a}\right|$$
$$= \rho_{\rm cr}\left|-\Omega_m(1+z)^3 - 2\Omega_r(1+z)^4 + 2\Omega_\Lambda\right|$$

- Dominant term in quadrupole energy loss with Misner-Sharp mass:

$$\dot{E} = -\frac{32}{5}\frac{G^4M^3\mu^2a^5}{\mathfrak{a}^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

- Expanded Thakurta decoupling:

$$(1+z_{\rm dec})^3\,H(z_{\rm dec}) < \frac{1}{\tau_b}\frac{96}{425}\left(1+\frac{73}{24}e^2_{\rm dec}+\frac{37}{96}e^4_{\rm dec}\right)$$

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Extra equations: Hawking evaporatin

- Thermodynamic identity: $$\frac{dU}{d\tau} = T\frac{dS}{d\tau} - P\frac{dV}{d\tau} \qquad S = A/4$$
- Artificially add blackbody term to LHS: $$\frac{dU}{d\tau} = -\sigma T^4 A + 2\delta \qquad 1/8 > \delta := HGma$$
- Critical mass relation: $$m_*^3 \sim \frac{z_{eq}}{640\pi G^2 H_0 \sqrt{\Omega_r}}\left(\frac{z_f(m_*)}{z_{eq}} - 1\right)$$
- Redshift at formation ($m = \gamma m_H$): $$z_f(m) = \left(\frac{2GmH_0\sqrt{\Omega_r}}{\gamma}\right)^{-1/2}$$