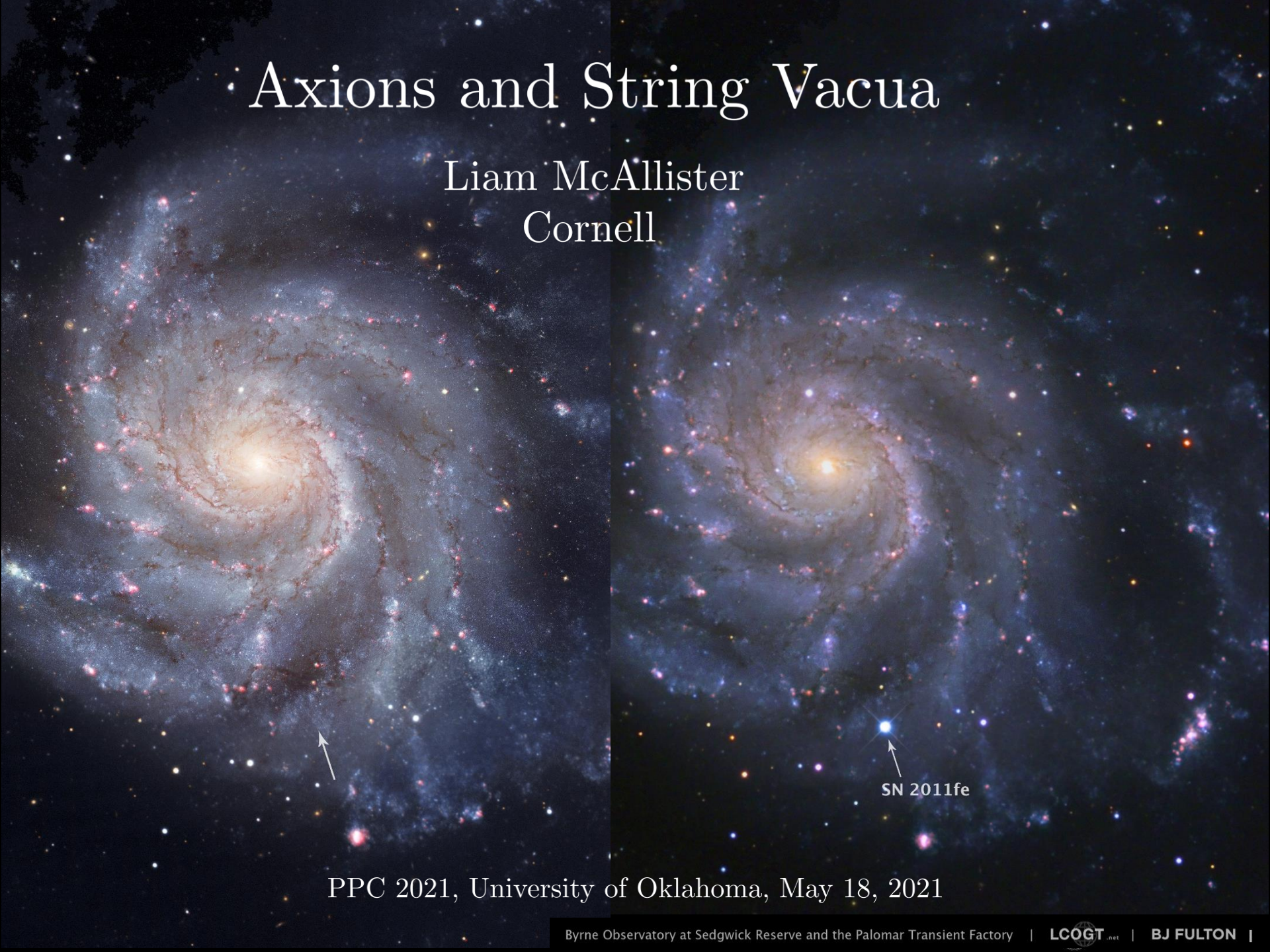


Axions and String Vacua

Liam McAllister
Cornell



PPC 2021, University of Oklahoma, May 18, 2021

Goal

Computation of the String Landscape

Understand *through enumeration* what is possible in quantum gravity

cf. [Vafa 05](#)

Construct ensembles of vacua whose phenomenology can be studied

Develop tools to simplify and automate the computation of vacua

Explore and characterize new classes of solutions

Plan

I. Advances in Constructing Vacua

CYTools: software for computing string data

Demirtas, L.M., Rios-Tascon, 20

Demirtas, L.M., Rios-Tascon, to appear

Demirtas, Kim, L.M., Moritz, Rios-Tascon, work in progress

Vacua with small flux superpotential

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

II. Application to Axion Physics

Superradiance in the Kreuzer-Skarke Axiverse

Mehta, Demirtas, Long, Marsh, L.M., Stott 20

Based on

Work by our group at Cornell:

Mehmet Demirtas, Geoffrey Fatin, Naomi Gendler, Manki Kim, Jakob Moritz, Andres Rios-Tascon
in collaboration with Ben Heidenreich, Cody Long, David J.E. Marsh, Viraf Mehta
Tom Rudelius, Mike Stillman, Matthew Stott

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

Demirtas, L.M., Rios-Tascon, 20

Mehta, Demirtas, Long, Marsh, L.M., Stott 20

Mehta, Demirtas, Long, Marsh, L.M., Stott 20

Demirtas, L.M., Rios-Tascon, to appear

Demirtas, Gendler, Long, L.M., to appear

de Sitter space

Simplest explanation for observed universe.

Should try to construct de Sitter solutions of string theory.

de Sitter solutions are not common.

So a search must survey many candidate geometries.

Not feasible with pen and paper.



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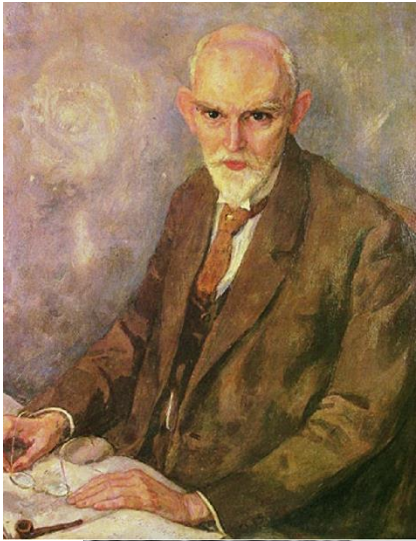
Not feasible with pen and paper.

With partial computerization, human steps are rate-limiting.

Need efficient implementation of every step.

SN 2011fe

Finding de Sitter



Reality is hard

Exact nonsupersymmetric solutions of superstring theories with 4d gravity seem out of reach for the 21st century.

This is not a cosmological issue.

e.g., no derivation of **Standard Model + Einstein gravity** (with no light moduli) to standard demanded for de Sitter.

Non-supersymmetric compactifications are hard, but this does not imply they do not exist.

We need **systematic approximations**.

Fundamental expansions are in α' and g_s .

Strategy for constructing vacua

Conceptually easy, impractical at present:

Find solution preserving $\mathcal{N} = 1$ SUSY, e.g. type II on $CY_3 - \mathcal{O}$.

Directly compute 4d EFT to N^k LO, in α' and g_s .

Exhibit de Sitter solutions in EFT at N^{k-1} LO,
show that N^k LO negligible.

Practical: apply further approximations.

e.g. find parameter regimes where sectors decouple
into **modules** that interact weakly.

Analyze modules in isolation, then weakly couple them.

Final **assembly** is a key challenge.

Our aim

Work under bright lamppost: Calabi-Yau compactifications.

Systematically compute topological and geometric data that enter 4d $\mathcal{N} = 1$ EFT, for a **vast ensemble** of CY_3 .

Automate the construction of vacua.

In time this may reveal controlled de Sitter vacua, or not.

Long, L.M., McGuirk 14

Long, L.M., Stout 16

Braun, Long, L.M., Stillman, Sung 17

Demirtas, Long, L.M., Stillman 18

Demirtas, Kim, L.M., Moritz 19

Demirtas, L.M., Rios-Tascon, 20

Demirtas, Kim, L.M., Moritz 20

Demirtas, L.M., Rios-Tascon, to appear

Demirtas, Kim, L.M., Moritz, Rios-Tascon, to appear

Blumenhagen, Jurke, Rahn, Roschy 10

Braun, Walliser 11

Gao, Shulka 13

Altman, Gray, He, Jejjala, Nelson 14

Cicoli, Muia, Shukla 16

Cicoli, Garcia-Etxebarria, Mayrhofer, Quevedo, Shukla 17

Braun, Lukas, Sun 17

Taylor, Wang 17

Huang, Taylor 18

Halverson, Long 20

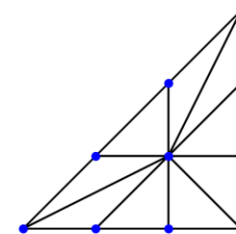
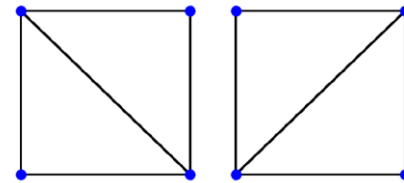
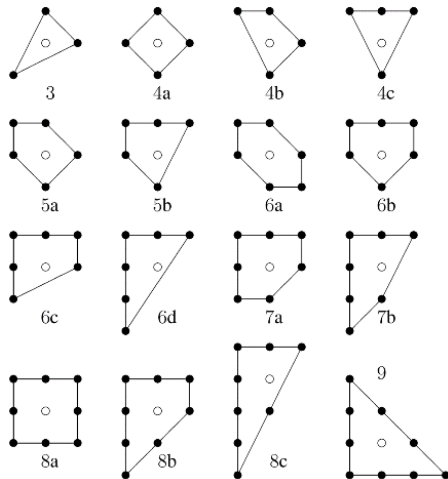
Broeckel, Cicoli, Maharana, Singh, Sinha 20, 21



Ensemble: Hypersurfaces

Largest known set of CY_3 : hypersurfaces in toric varieties.

$\dim_{\mathbb{C}} = 4$ toric varieties are complex manifolds that can be obtained from **triangulations of 4d polytopes**



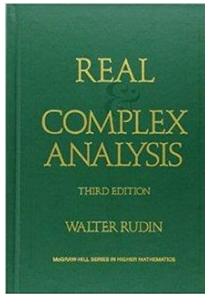
473,800,776 4d reflexive polytopes

Kreuzer and Skarke 2000

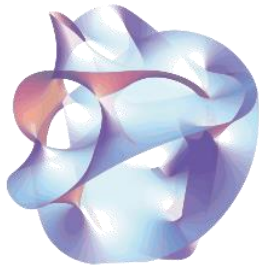
$< 10^{428}$ hypersurfaces

Demirtas, L.M., Rios-Tascon 2020

For hypersurfaces, the task is to triangulate polytopes.
Combinatorial, and amenable to automation.



Analysis



Algebraic geometry



Combinatorics

What to compute?

Needs vary depending on model, so try to get ‘everything’.

At least, everything that follows directly from topological data.

Start: 4d polytope from Kreuzer-Skarke list

Fast
Automatic
Complete

Fine regular star triangulation \Rightarrow toric variety V

Intersection numbers κ_{ijk} of CY_3 $X \subset V$

Kähler cone of V , $\mathcal{K}(V) \subset \mathcal{K}(X)$

Fast
Automatic
Incomplete

True/extended Kähler cone of X

Gopakumar-Vafa invariants of curves in X

Cone of effective divisors on X

Orientifolds of X

Limited

F-theory uplifts Y of orientifolds of X

Divisors supporting $EM5 \subset Y$ or $ED3 \subset X$

Calabi-Yau metric on X

Fatin and L.M., work in progress

Perturbative corrections in α' , g_s

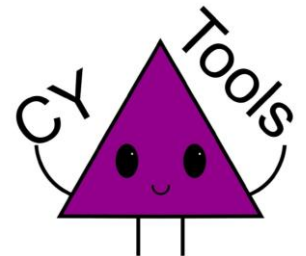
CYTools

A Software Package for Analyzing Calabi-Yau Hypersurfaces

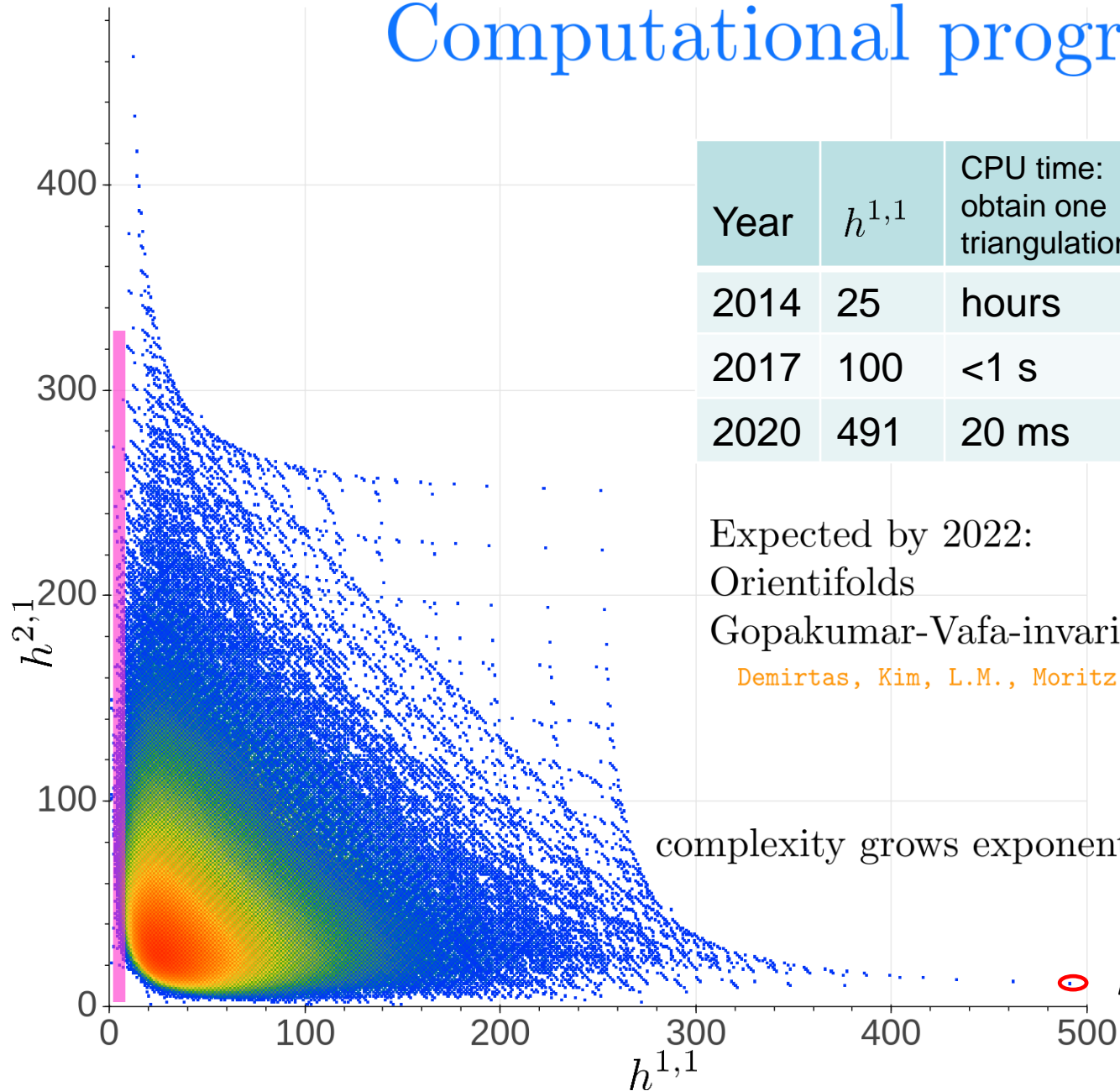
Demirtas, L.M., Rios-Tascon

Purpose-built to construct and analyze triangulations,
and associated Calabi-Yau hypersurfaces in toric varieties.

Can analyze any geometry in the Kreuzer-Skarke list,
orders of magnitude faster than available codes like Sage.



Computational progress



Year	$h^{1,1}$	CPU time: obtain one triangulation	CPU time: intersection numbers
2014	25	hours	hours
2017	100	<1 s	30 min
2020	491	20 ms	3 s

Expected by 2022:

Orientifolds

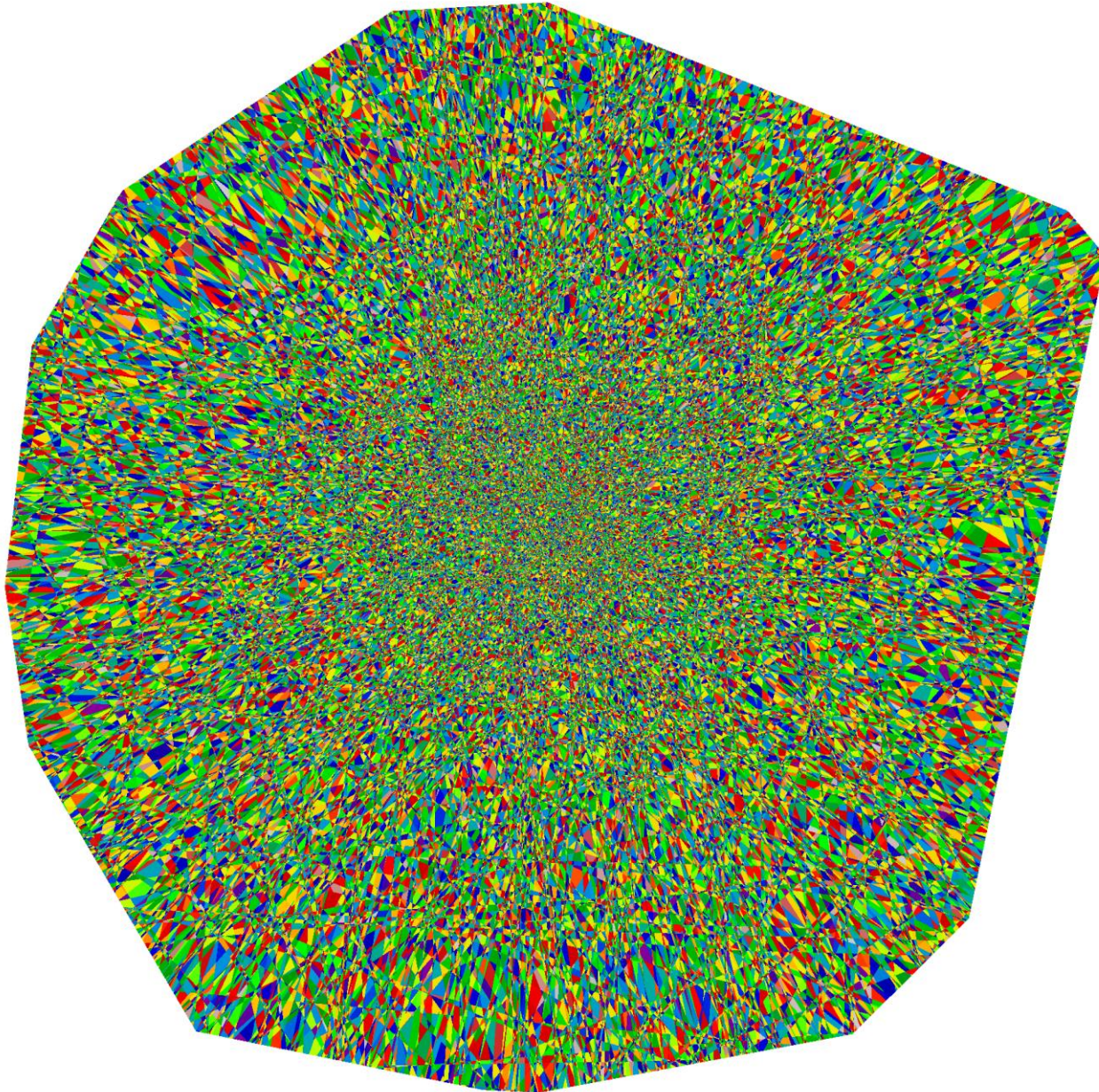
Gopakumar-Vafa-invariants

Demirtas, Kim, L.M., Moritz, Rios-Tascon, work in progress

complexity grows exponentially \rightarrow

$h^{1,1} = 491$

2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold



Application:

Vacua with Small Flux Superpotential

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

Computer-aided discovery of analytic results.

KKLT de Sitter vacua

Compactification of type IIB on an orientifold X of a CY_3 , including:

quantized three-form flux

an $\mathcal{N} = 1$ pure SYM sector on D7-branes

a warped deformed conifold region Klebanov, Strassler 00

containing one or more anti-D3-branes

Claim [KKLT]: in a suitable parameter regime,
these sources can yield metastable dS_4 ,
and corrections to approximations are small.

Kachru, Kallosh, Linde, Trivedi 03

Setup

Type IIB string theory compactified on orientifold, X , of a CY_3 .

Moduli: axiodilaton τ

complex structure z_a , $a = 1, \dots, h^{2,1}$

Kähler: T_i , $i = 1, \dots, h^{1,1}$

Choose $F_3, H_3 \in H^3(X, \mathbb{Z})$. $G_3 := F_3 - \tau H_3$

$4d \mathcal{N} = 1$ supergravity:

$$W_{\text{flux}} = \int_X G_3 \wedge \Omega$$

Gukov, Vafa, Witten 99

For generic G_3 , solutions of $D_\tau W = D_{z_a} W = 0$ are isolated $\Rightarrow \tau, z_a$ fixed.

Giddings, Kachru, Polchinski 01

Setup

Below the scale $m_{z_a} \sim \frac{\alpha'}{\sqrt{\text{Vol}(X)}}$ of the complex structure moduli masses,

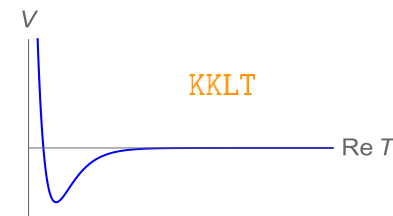
$$\mathcal{K} = -3 \log(T + \bar{T}) \quad W_{\text{flux}} \rightarrow \left\langle \int_X G_3 \wedge \Omega \right\rangle =: W_0$$

Consider a stack of D7-branes that support pure $SU(N_c)$ SYM.

$$W = W_0 + \mathcal{A} e^{-\frac{2\pi}{N_c} T}$$

This supergravity theory has a **SUSY AdS_4 minimum**:

$$D_T W = 0 \Leftrightarrow W_0 = -\mathcal{A} e^{-\frac{2\pi}{N_c} T} \left(1 + \frac{2\pi}{3N_c} (T + \bar{T}) \right)$$



If $W_0 \ll 1$, the minimum is at large volume, $(T + \bar{T})_{\text{min}} \approx -\frac{N_c}{\pi} \ln(|W_0|)$

Statistical argument: $W_0 \ll 1$ is not generic, but should occur for some of the $\mathcal{O}(e^{2b_3})$ choices of G_3 .

One should ask:

MODULI STABILIZATION

Do there exist consistent global models with:

- i. Quantized fluxes giving small classical superpotential **yes**
- ii. *and* a warped conifold region **yes**
- iii. *and* suitable gaugino condensates

Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20

Can one exhibit an explicit and fully-controlled compactification that unifies all necessary components? **TBD**

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Small Flux Superpotentials

$$(T + \bar{T})_{\min} \approx -\frac{N_c}{\pi} \ln(|W_0|) \Rightarrow \text{control requires } |W_0| \ll 1$$

Statistical arguments suggest $|W_0| \ll 1$ should occur,
but exponentially rarely,
and preferentially when moduli space dimension is large.

Our tools permit exploring such moduli spaces.

But brute-force search still very challenging.

Previous record: $\mathcal{O}(0.01)$

Denef, Douglas, Florea 04

Denef, Douglas, Florea, Grassi, Kachru 05

Better to invent and apply a **mechanism**.

Giryavets, Kachru, Tripathy, Trivedi 03

Denef, Douglas, Florea 04

Demirtas, Kim, L.M., Moritz 19



Racetrack Superpotential

$$W = W_{\text{inst}}(z)$$

$$W_{\text{inst}}(z) = \mathcal{A}_1 e^{-p_1 z} + \mathcal{A}_2 e^{-p_2 z} + \dots$$

$$\rightarrow 0 \text{ for } z \rightarrow \infty \quad p_1, p_2 > 0$$

Idea: minimize potential at $z_{\text{min}} \gg 1$ s.t. $|W_{\text{inst}}(z_{\text{min}})| \ll 1$

$$z_{\text{min}} = -\frac{\log\left(-\frac{\mathcal{A}_1 p_1}{\mathcal{A}_2 p_2}\right)}{(p_1 - p_2)}$$

$$W_{\text{inst}}(z_{\text{min}}) = \frac{\mathcal{A}_2 (p_1 - p_2)}{p_1} \left(-\frac{\mathcal{A}_2 p_2}{\mathcal{A}_1 p_1}\right)^{\frac{p_2}{p_1 - p_2}}$$

$$\ll 1 \text{ if } |p_1 - p_2| \ll p_2, \quad \mathcal{A}_2 \ll \mathcal{A}_1$$

Incarnation as Flux Superpotential

Need to show that, in a bona fide solution of string theory:

(1) $W_{\text{flux}}(z) = W_{\text{inst}}(z) = \mathcal{A}_1 e^{-p_1 z} + \mathcal{A}_2 e^{-p_2 z} + \dots$

(2) with $|p_1 - p_2| \ll p_2$ and $\mathcal{A}_2 \ll \mathcal{A}_1$

Then we'll have succeeded:

$$W_0 := \langle W_{\text{flux}} \rangle = W_{\text{inst}}(z_{\text{min}}) = \frac{\mathcal{A}_2(p_1 - p_2)}{p_1} \left(-\frac{\mathcal{A}_2 p_2}{\mathcal{A}_1 p_1} \right)^{\frac{p_2}{p_1 - p_2}} \ll 1$$

(1) we established a sufficient condition on topological data

(2) we found explicit examples

Flux Superpotential in a Calabi-Yau

Let \tilde{X} be the Calabi-Yau threefold hypersurface in $\mathbb{CP}_{[1,1,1,6,9]}$

$X =$ mirror of $\tilde{X} =$ resolution of $\tilde{X} / (\mathbb{Z}_6 \times \mathbb{Z}_{18})$. $(h^{1,1}, h^{2,1}) = (272, 2)$

We find an orientifold of X , and quantized fluxes,

s.t. $(z_1, z_2) = -(\frac{2}{5}, \frac{3}{10}) \cdot z \equiv -(\frac{2}{5}, \frac{3}{10}) \cdot 2\pi i\tau$ is a flat direction, and

$$W_{\text{flux}} = c \left(e^{-\frac{2}{5}z} - \frac{5}{288} e^{-\frac{3}{10}z} \right) + \dots \quad c = -\sqrt{\frac{2}{\pi}} \frac{8640}{(2\pi i)^3}$$

Moduli are stabilized at

$$\langle \tau \rangle = 6.856i, \quad \langle z_1 \rangle = 17.229, \quad \langle z_2 \rangle = 12.925,$$

$$\text{with } |W_0| = 2.037 \times 10^{-8}.$$

$$\sim \left(\frac{5}{288} \right)^{\frac{\frac{3}{10}}{\frac{4}{10} - \frac{3}{10}}}$$

Single-instantons dominate

Totally explicit, no unknown factors in leading-order data.

Just a finite set of computable rational numbers.

Higher orders strongly suppressed:

$$(2\pi i)^3 \mathcal{F}_{\text{inst}} = \underbrace{-540e^{-z_1}}_{\text{leading}} - \underbrace{3e^{-z_2}}_{\mathcal{O}(10^{-5}) \text{ smaller}} - \underbrace{\frac{1215}{2}e^{-2z_1}}_{\mathcal{O}(10^{-5}) \text{ smaller}} + \underbrace{1080e^{-z_1-z_2}}_{\mathcal{O}(10^{-5}) \text{ smaller}} + \underbrace{\frac{45}{8}e^{-2z_2}}_{\mathcal{O}(10^{-10}) \text{ smaller}} + \dots$$

Remark: mass^2 along previously-flat direction is $\mathcal{O}(|W_0|)$

Comments

We easily find many more examples.

Need: knowledge of prepotential

suitable orientifold

some luck with the numbers $p_1, p_2, \mathcal{A}_2/\mathcal{A}_1, Q_{D3}^{\text{flux}}$

By analytic continuation to conifold, mechanism yields vacua

with **small W and a warped conifold**: $e^{2A} \approx |W_0| \ll 1$

Demirtas, Kim, L.M., Moritz 20

Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20

Upshot: **small $|W_0|$ is in the landscape.**

For KKLT we want even more:

Kähler moduli stabilization ($h^{1,1} = 272$ in example!)

anti-D3-brane supersymmetry breaking

Application:

Black Hole Superradiance in the Kreuzer-Skarke Axiverse

Viraf M. Mehta, Mehmet Demirtas, Cody Long, David J. E. Marsh, L.M., Matthew Stott 20

Ultralight Axions in String Theory

Axions almost unavoidable, sometimes numerous.

$$\theta := \int_{\Sigma_p \subset X} C_p$$

In absence of monodromy, perturbative shift symmetry.

Instantons: $S_{\Sigma_p} := \int_{\Sigma_p \subset X} \mathcal{L}_{\text{inst}} \quad e.g. \quad T_p \int_{\Sigma_p \subset X} (\text{Vol} + iC_p)$

$$\mathcal{L}_4 \supset \exp\left(-T_p \text{Vol}(\Sigma_p)\right) \cos(\theta)$$

In geometric regime — cycles large in string units — it is easy to find many ULAs.

No naturalness issues.

An **axiverse** = a theory with many axions.

Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell 09

The **Kreuzer-Skarke axiverse** = ensemble of such theories from type IIB string theory on Calabi-Yau hypersurfaces

We constructed two million geometries with $1 \leq h^{1,1} \leq 491$, and examined 100,000 in detail.

$$\mathcal{L} = \frac{M_{\text{pl}}^2}{2} \mathcal{R} - \frac{1}{2} K_{ij} \partial_\mu \theta^i \partial^\mu \theta^j - \sum_{a=1}^{\infty} \Lambda_a^4 \cos(Q_i^a \theta^i) \quad i = 1, \dots, h^{1,1}$$

$$\text{eig}(K) = \text{diag}(f_i^2) \quad \Lambda_a^4 \propto \exp(-2\pi \text{Vol } \Sigma_a)$$

We computed K_{ij} , Q_i^a , Λ_a in terms of polytope data and moduli vevs.

Demirtas, Long, L.M., Stillman 18

Mehta, Demirtas, Long, Marsh, L.M., Stott 20

Can ask about: axion inflation, quintessence, dark matter;
photon couplings, birefringence;
strong CP quality problem;
black hole superradiance.

Black Hole Superradiance

A Kerr black hole can lose angular momentum to a cloud of axions, spinning down, if there is an axion in the right mass range.

$$\lambda \sim R_S \Leftrightarrow 10^{-14} \text{ eV} \lesssim m \lesssim 10^{-11} \text{ eV} \text{ (stellar)}$$
$$10^{-20} \text{ eV} \lesssim m \lesssim 10^{-16} \text{ eV} \text{ (supermassive)}$$

and provided the axion self-interactions are not strong enough to disrupt the condensate.

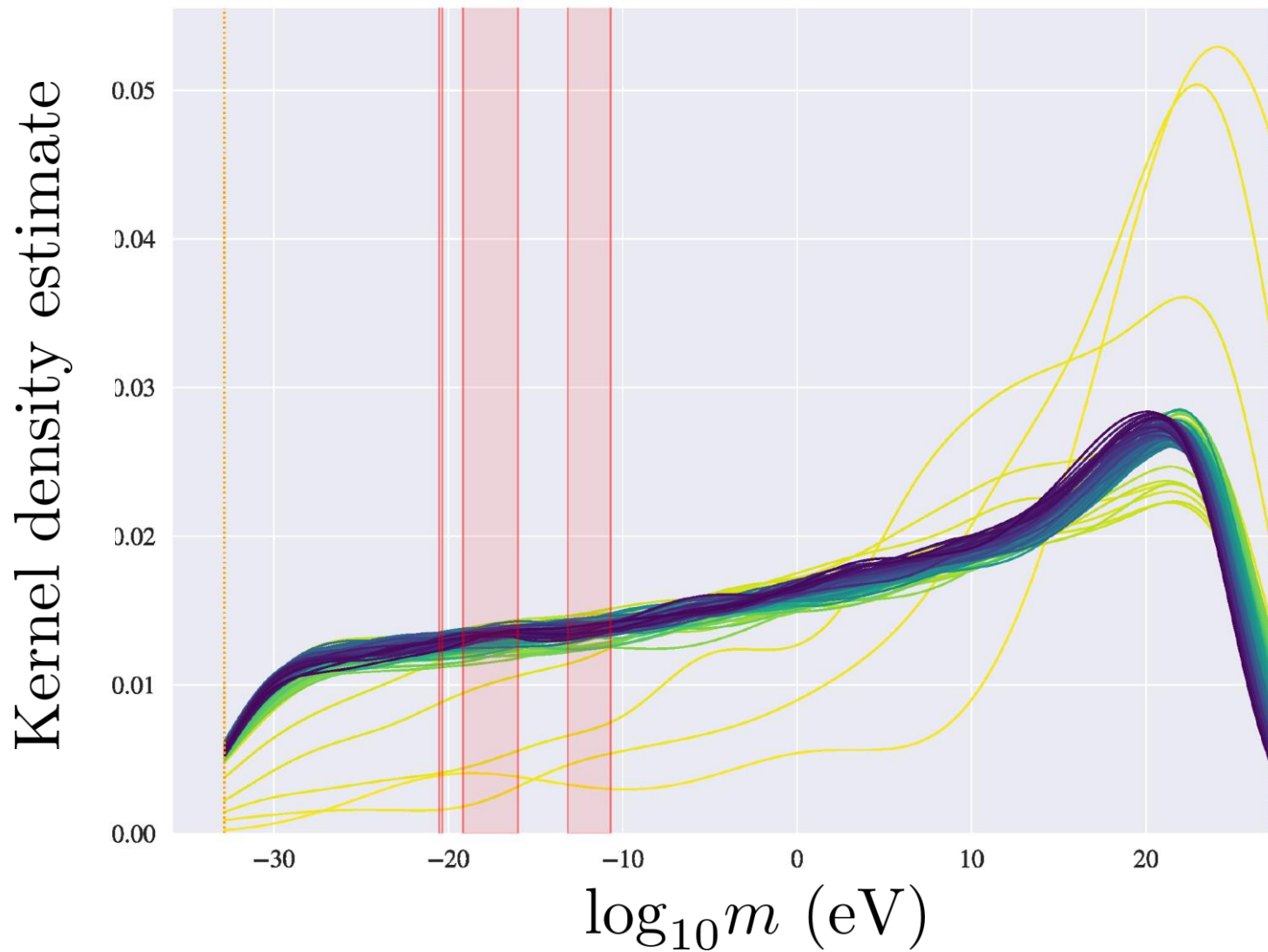
$$f \gtrsim 10^{14} \text{ GeV}$$

Measured spins of black holes constrain axion theories.
Essentially independent of cosmological model.

Only need to know the **masses and decay constants**.

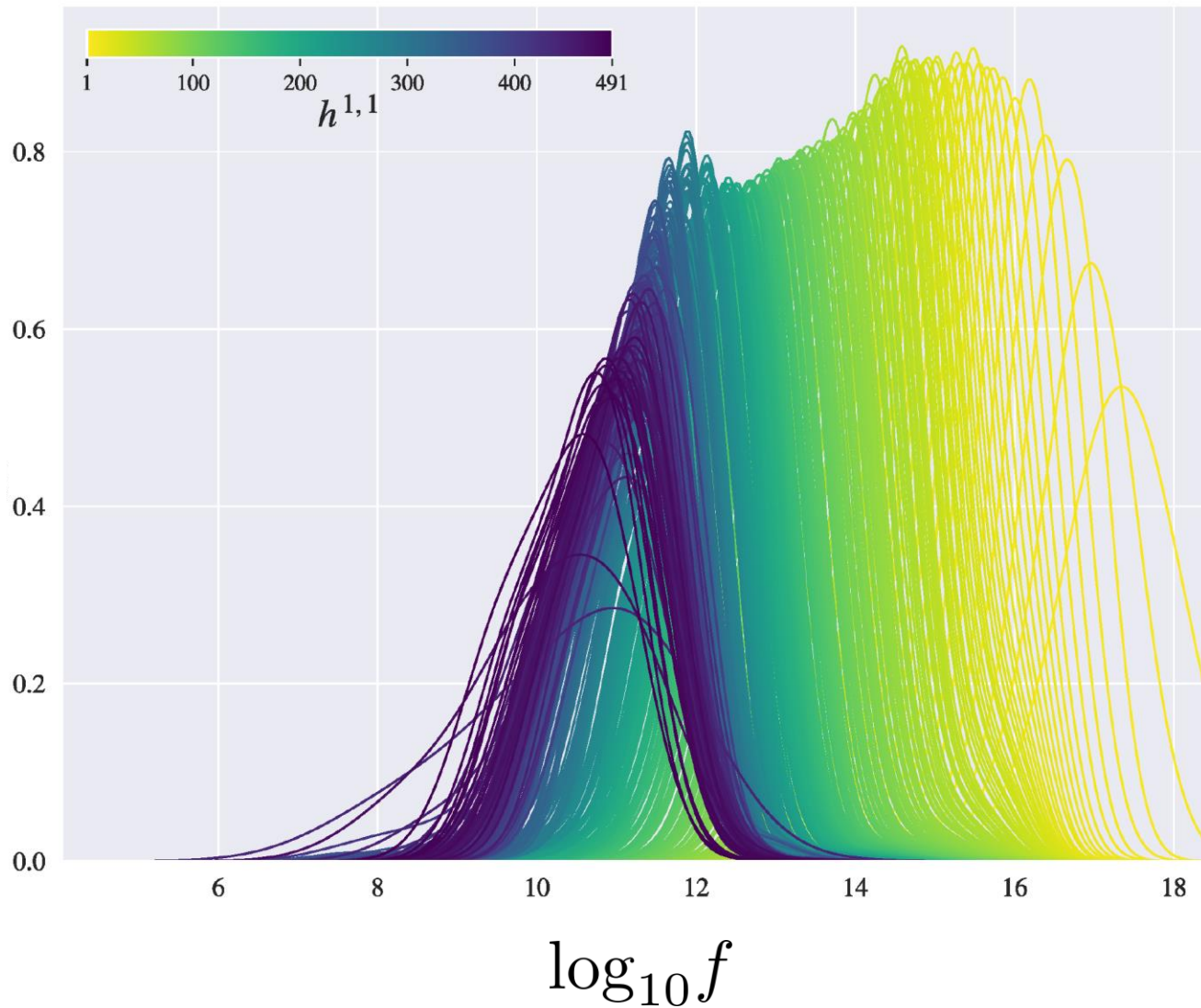
We computed these in our ensemble,
at the extremal point for controlling the α' expansion.

Mass distribution

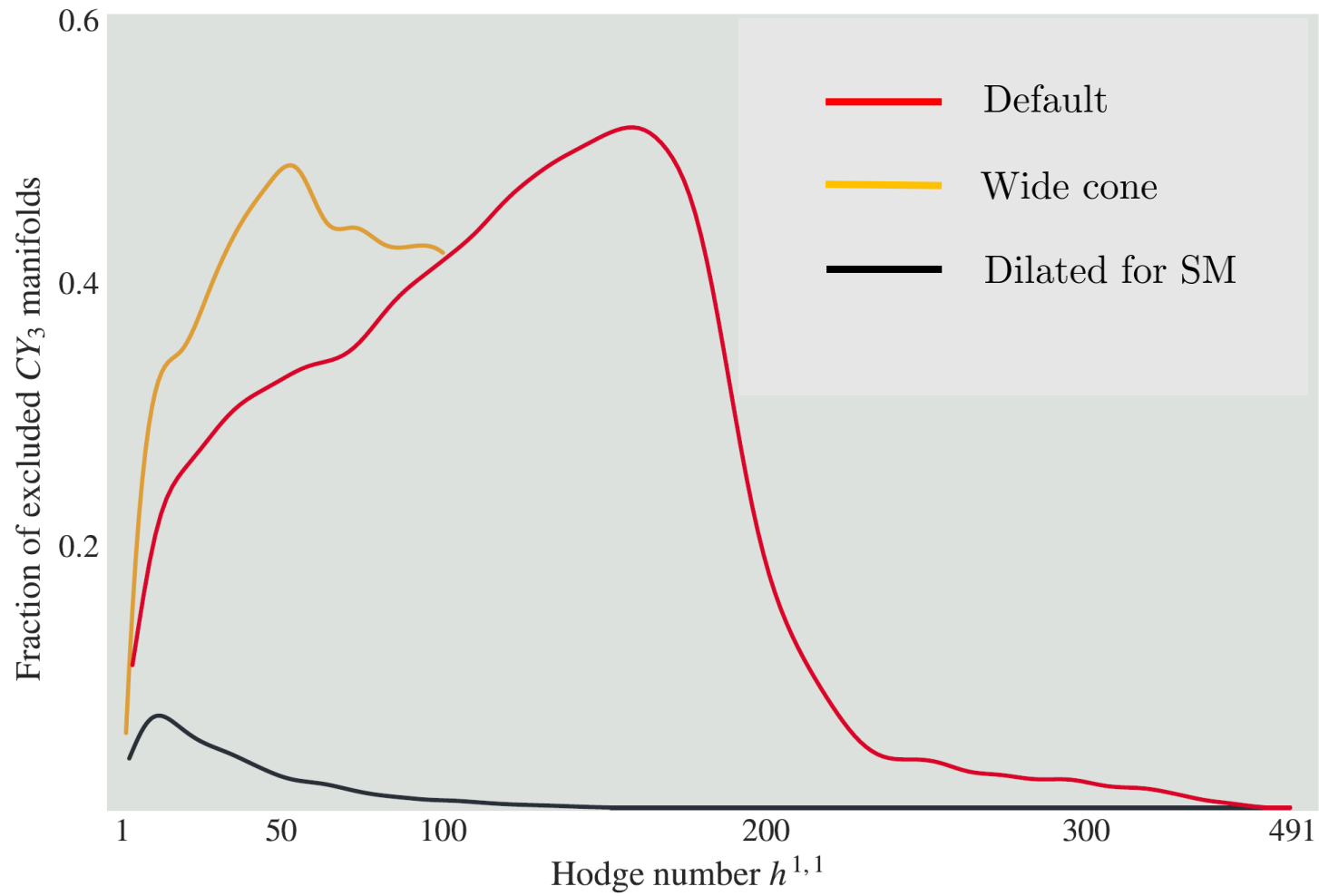


Decay constant distribution

Kernel density estimate



Geometries excluded by superradiance



Conclusions

KKLT scenario for de Sitter vacua requires special structures in classical flux compactification:

exponentially small flux superpotential W_0
warped conifold region

We presented a mechanism for constructing such solutions, via a racetrack of worldsheet instantons.

We gave complete explicit examples.

Small $|W_0|$ is in the landscape.

Our search is automated; large-scale studies possible.

Quantum side of KKLT, and uplift, are works in progress.

Already possible to infer constraints from axions.

Thanks!