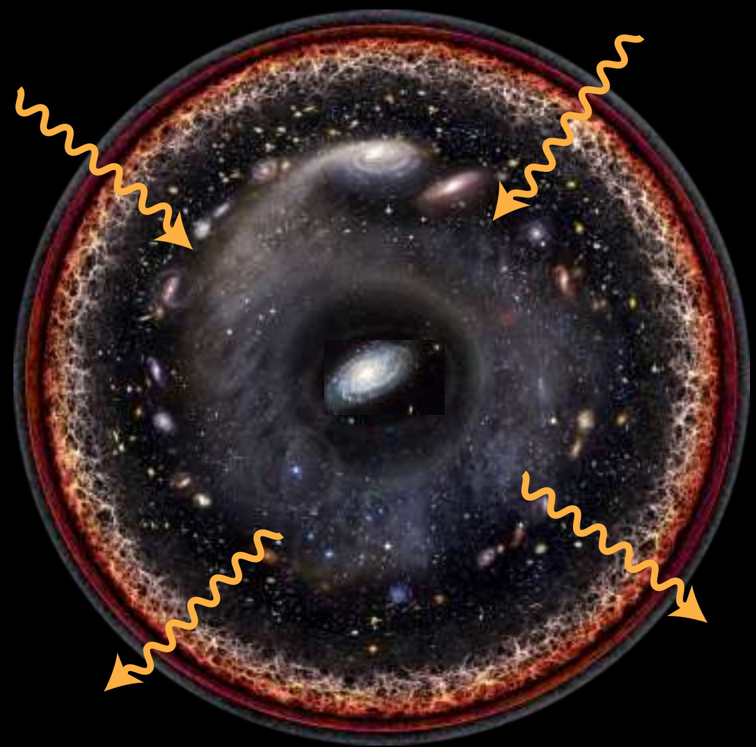
Quantum Information in de Sitter Space



Based on 2002.01326 (JHEP) [L. Aalsma, GS]

+ L. Aalsma, A. Cole, E. Morvan, J.P. van der Schaar and GS, to appear this week

[See also Aalsma's talk on Thursday at the Strings/Formal 2 Session]

Gary Shiu

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

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Motivation

- A lot has been conjectured about dS space and inflation in quantum gravity: (refined) dS conjecture, TCC, no-eternal inflation, quantum break time, …..

- Various timescales have been suggested as the lifetime of dS:

$$t_H \sim \frac{1}{H}\,, \quad t_{\text{scrambling}} \sim \frac{1}{H}\log S_{dS}\,, \quad t_{\text{Page}} \sim \frac{1}{H} S_{dS}\,, \quad t_{\text{Poincare}} \sim \frac{1}{H} e^{S_{dS}}$$

- Can quantum information ideas bound the lifetime of dS space & inflation?

- Information recovery for black holes led to paradoxes (information paradox, no-cloning) which can teach us when EFTs breakdown in gravitational systems.

- For black holes, the studies of entanglement, shockwaves and (traversable) wormholes have been useful to learn about the microphysics.

- Implications of the recent results of island contributions for black holes to dS?

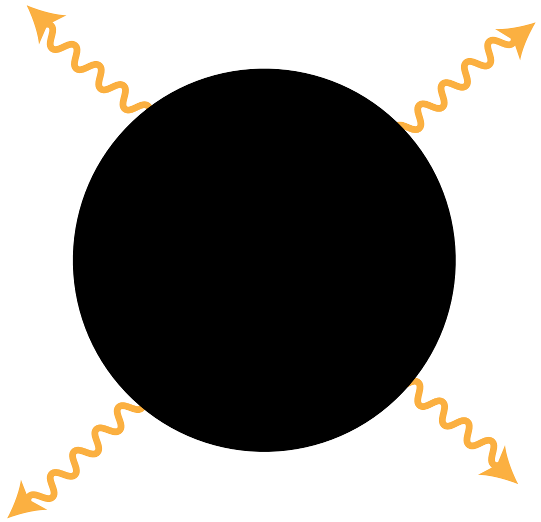
Cosmological Horizon

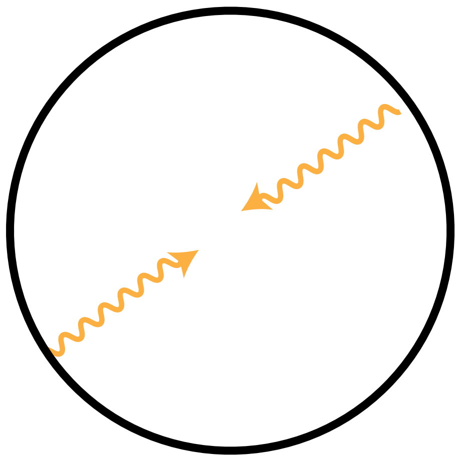
A static observer in dS is surrounded by a cosmological horizon.

The horizon has

- a temperature: $T_{dS} = \dfrac{1}{2\pi\ell}$ [Gibbons, Hawking '77]
- and an entropy: $S_{dS} = \dfrac{\text{Area}}{4\pi G_N}$

and shares many similarities with a black hole horizon.





Quantum Chaos

Quantum chaos provides a measure of how fast perturbations get thermalized. A probe of chaos in quantum systems is the out-of-time-correlator (OTOC).

For black holes: $$\langle V(0)W(t)V(0)W(t)\rangle \simeq 1 - \frac{1}{S_{bh}} e^{\frac{2\pi}{\beta}t} \qquad \beta = T_{BH}^{-1}$$

The time scale when the OTOC drops by an order 1 amount is known as the scrambling time:

$$t_* = \frac{\beta}{2\pi}\log(S_{bh})$$

The (quantum) Lyapunov exponent λ_L determines how fast chaos can grow and it has been argued to obey a universal bound [Maldacena, Shenker, Stanford]:

$$\lambda_L \leq 2\pi/\beta$$

Black holes saturates this chaos bound [Shenker, Stanford];[Maldacena, Shenker, Stanford]; they are fast scramblers [Sekino, Susskind].

Information Scrambling

Given the similarities of the de Sitter horizon with a black hole horizon, it was conjectured that de Sitter space is a fast scrambler [Susskind].

Interestingly, the TCC suggests a lifetime for (quasi) de Sitter space that is parametrically the same as the scrambling time. In 4d:

$$t_{TCC} = \frac{1}{H} \log \frac{H}{M_P} = \frac{1}{2}\frac{1}{H} \log S_{dS}$$

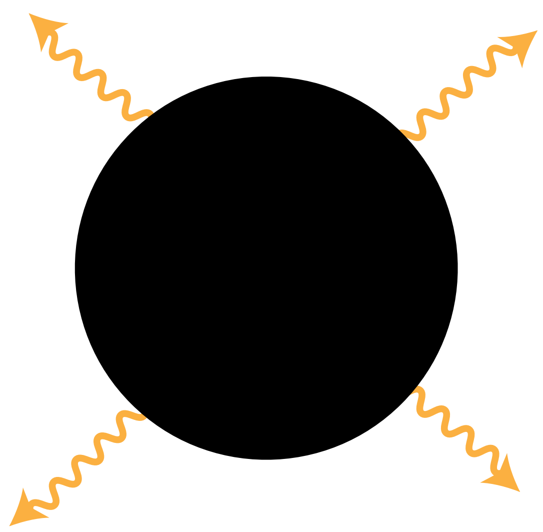
We computed the OTOC for dS [Aalsma, GS] to establish that dS space is maximally chaotic. The scrambling time has the meaning of the fastest information recovery time. Naively, this poses a cloning paradox.

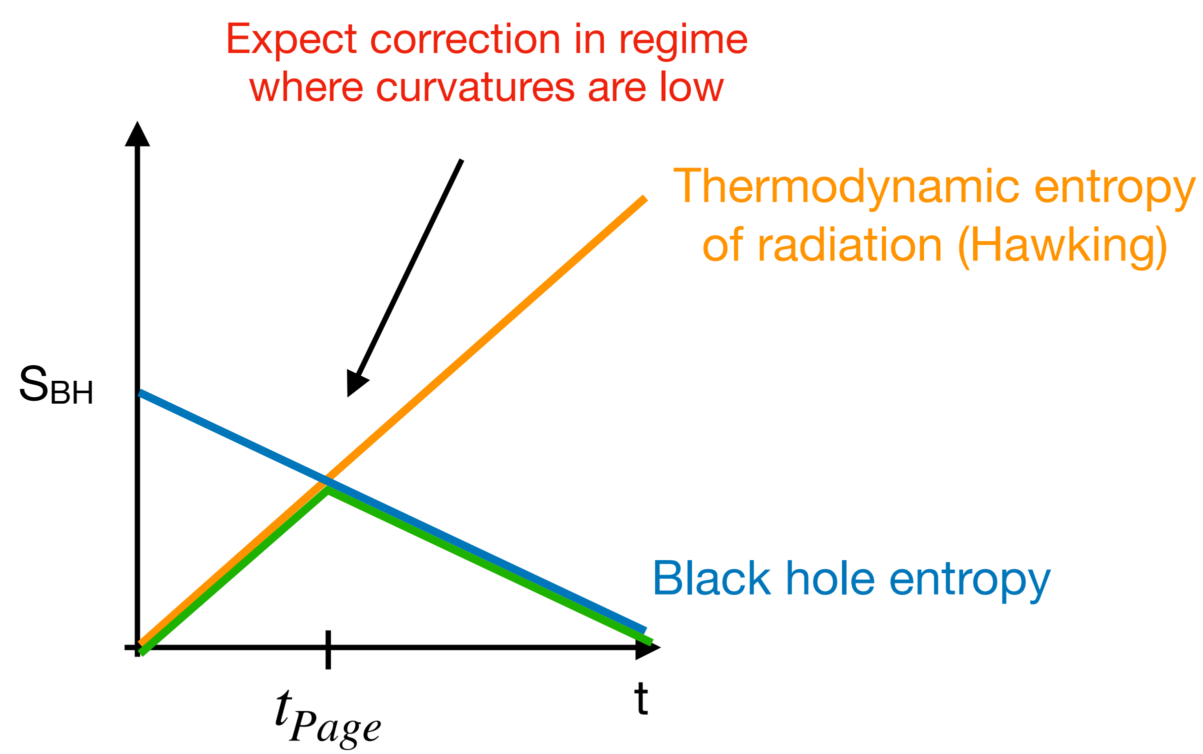
Like for black holes, this is avoided: the decoding of Hawking radiation from the dS horizon can be viewed as info exchanges between different static patches. [Aalsma, GS];[Aalsma, Cole, Morvan, van der Schaar, GS].

Information Bounds

- Because of the finiteness of $S_{dS,}$ we expect an obstruction when trying to access more than S_{dS} bits of info. We found that info transfer is bounded by S_{dS} [Aalsma, GS];[Aalsma, Cole, Morvan, van der Schaar, GS].

- Together with unitarity, finiteness of S_{dS} lead to the BH information paradox:

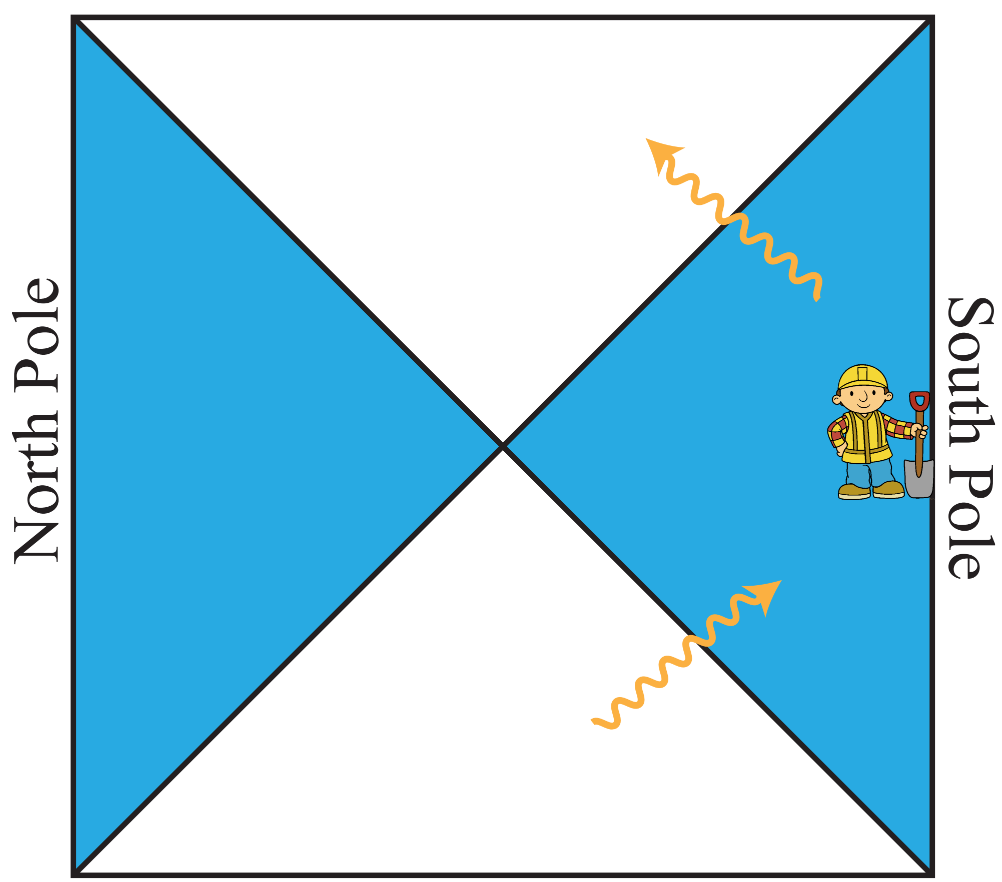




- For black holes, including an island contribution can reproduce the page curve [Almheiri, Hartman, Engelhardt, Maldacena, Marolf, Maxfield, Penington, Shaghoulian, Tajdini + many more works] [See Aalsma's talk and references therein for island contributions to the de Sitter entropy]

Static Patch

- We will consider the static patch of de Sitter space.
- This describes the experience of an observer sitting at the pole.



The horizon has an entropy and temperature.

$$S = \frac{A}{4G_N} \qquad T = \frac{1}{2\pi\ell}$$

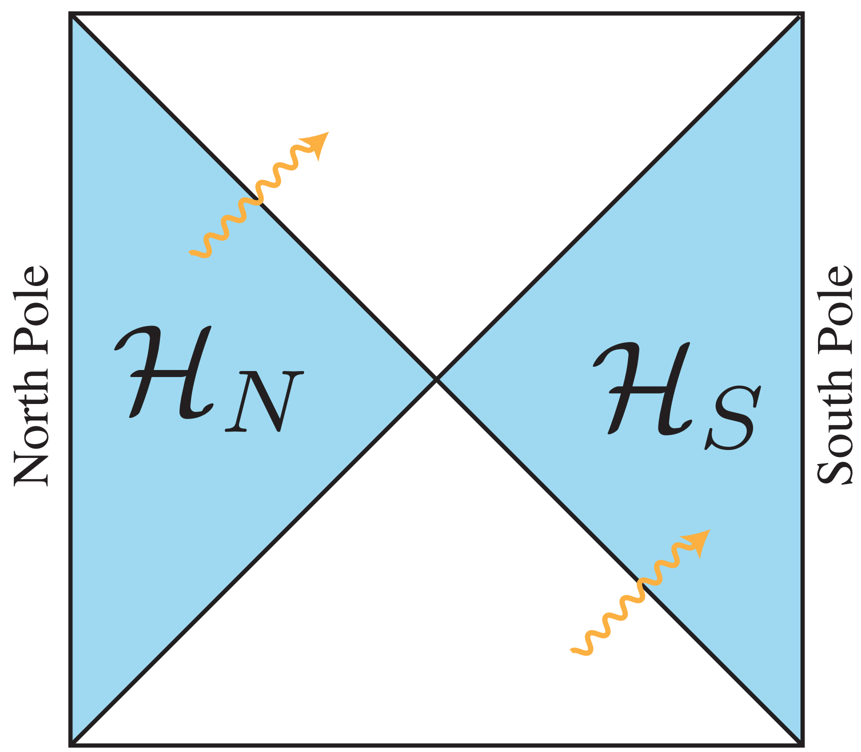
$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\phi^2$$

$$f(r) = 1 - r^2/\ell^2$$

static coordinates

Entanglement in dS Space

The natural vacuum state in dS space is the Bunch-Davis state.



Pure state:

$$\rho = \sum_{ij} e^{-\frac{1}{2}\beta(E_i+E_j)} |E_i\rangle_N |E_i\rangle_S \langle E_j|_N \langle E_j|_S$$

Tracing over the North Pole:

$$\rho_S = \mathrm{tr}_N(\rho) = \frac{1}{Z} \sum_i e^{-\beta E_i} |E_i\rangle_S \, \langle E_i|_S$$

→ maximally entangled state.

Side remark: $\langle \mathcal{O}(x_S) \rangle = \mathrm{Tr}_S \left(\mathcal{O}(x_S) \rho_S \right)$

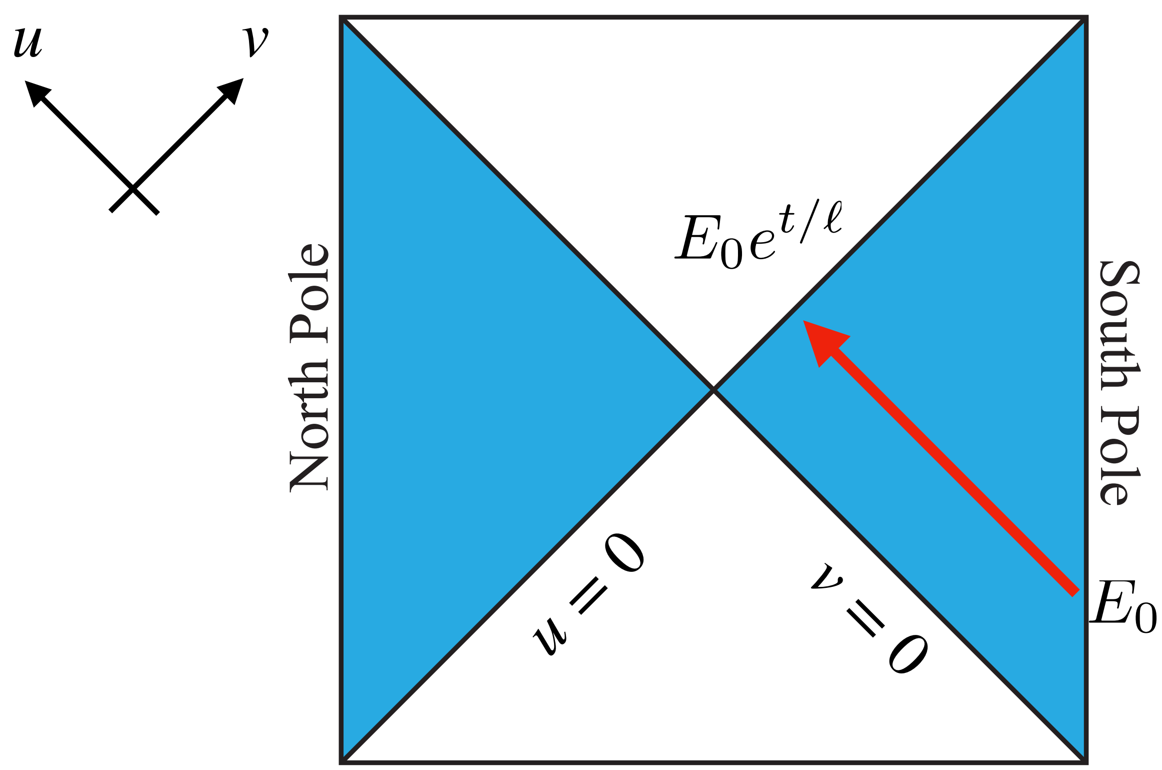
Tracing does **not** break the de Sitter isometries.

Large entanglement suggests the possibility of information recovery.

Shockwave geometry

- Consider a static observer that emits some small amount of energy.

- When measured at a later time t, exponential blueshift in energy. This leads to a shockwave.



Backreacted geometry can be described using Kruskal-like coordinates.

$$u = -\ell e^{-t/\ell}\sqrt{\frac{\ell - r}{\ell + r}}\,, \quad v = \ell e^{t/\ell}\sqrt{\frac{\ell - r}{\ell + r}}$$

$$ds^2 = \frac{4\ell^4}{(\ell^2 - uv)^2}(-dudv) + r^2 \mathrm{d}\phi^2$$

empty de Sitter

Shockwave geometry

[Hotta, Tanaka '93]

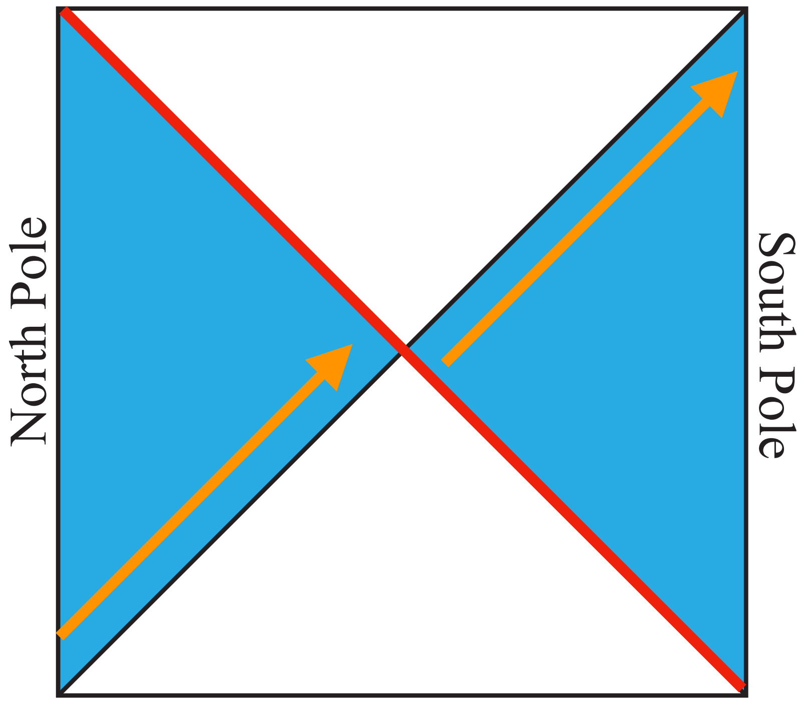
[Gao, Wald ' 00]

- In the infinite boost limit, we get a simple solution.

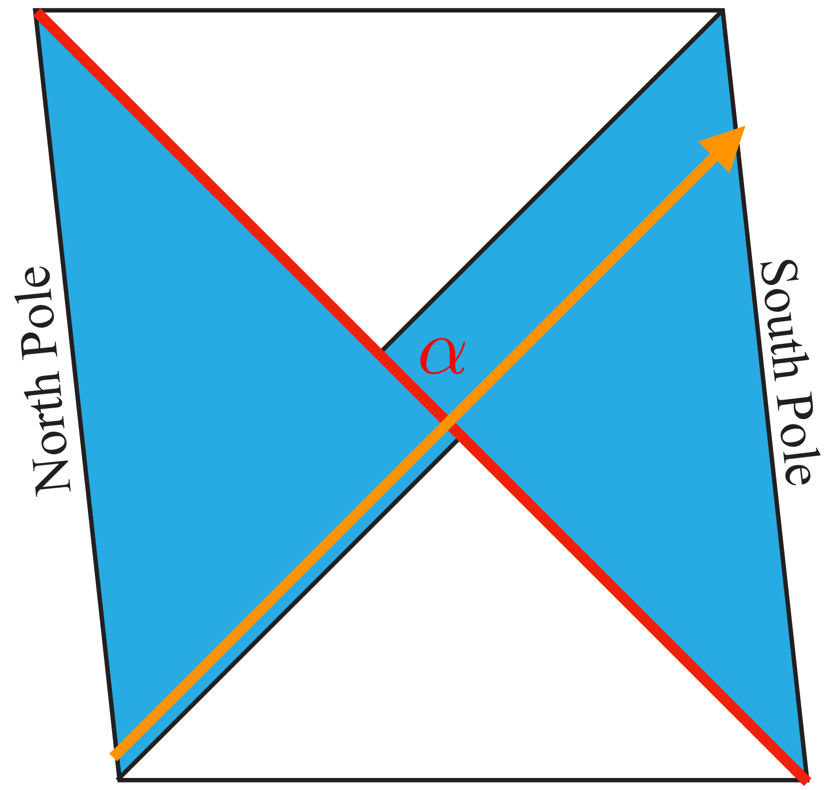
$$ds^2 = \frac{4\ell^4}{(\ell^2 - uv)^2}(-dudv) - 4\alpha\delta(v)dv^2$$

NEC: $\alpha = \pi G_N \ell E_0 \geq 0$

$$T_{vv} = \frac{\alpha}{4\pi G_N \ell^2}\delta(v)$$



coordinate transformation



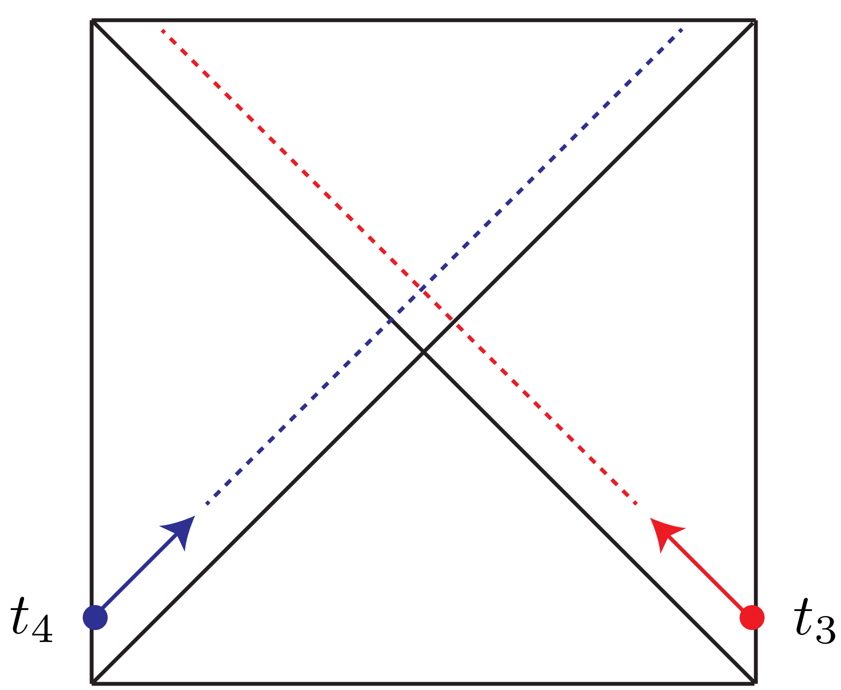
- A positive-energy shockwave opens a wormhole between poles. How much/fast can we transfer information?

Computing the OTOC

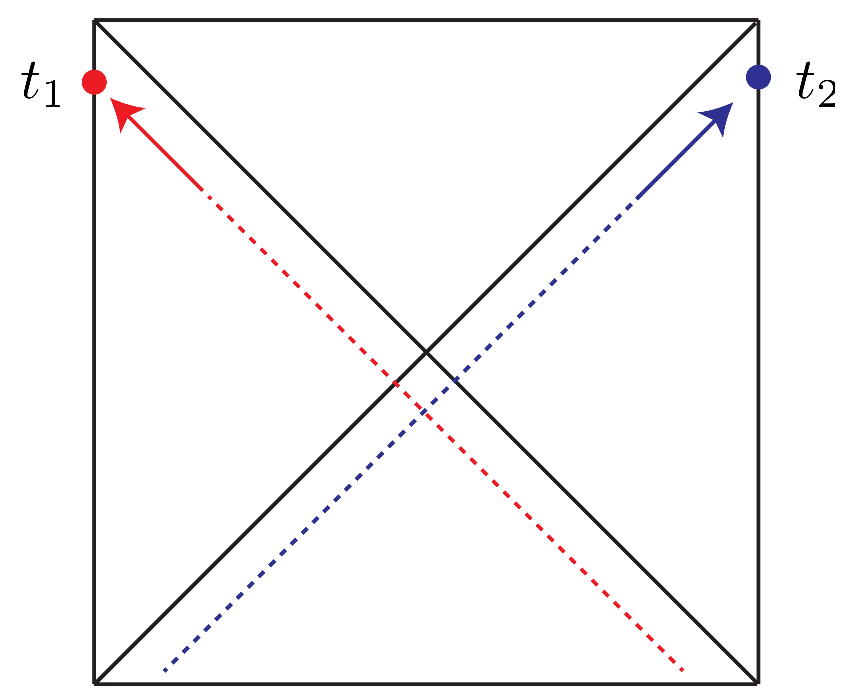
[Aalsma, GS '20]

- The OTOC quantifies the chaotic behavior of the dS horizon. It determines the scrambling time which is the timescale for fastest info transfer/recovery.

As in [Shenker, Stanford '14] we can compute the OTOC by scattering "in" and "out" states.



$$|\Psi\rangle = V_R(t_3)W_L(t_4)\,|\mathrm{TFD}\rangle$$



$$|\Psi'\rangle = W_R(t_2)^\dagger V_L(t_1)^\dagger\,|\mathrm{TFD}\rangle$$

$$\longrightarrow \quad \langle V_L(t_1)W_R(t_2)V_R(t_3)W_L(t_4)\rangle = \langle\Psi'\Psi\rangle$$

OTOC for de Sitter

- Using these ingredients, the OTOC can be computed analytically for conformally coupled fields.

- This gives a result in terms of special functions, expanding we find:

$$\langle V(0)W(t)V(0)W(t)\rangle = 1 - \left(\frac{G_N \pi}{8\ell} e^{t/\ell}\right)^2 + \mathcal{O}\left(\frac{G_N}{\ell} e^{t/\ell}\right)^4$$

[Aalsma, GS '20]

$$t_* = \ell \log\left(S_{dS}\right)$$

De Sitter is also a fast scrambler!

see also [Blommaert '20]

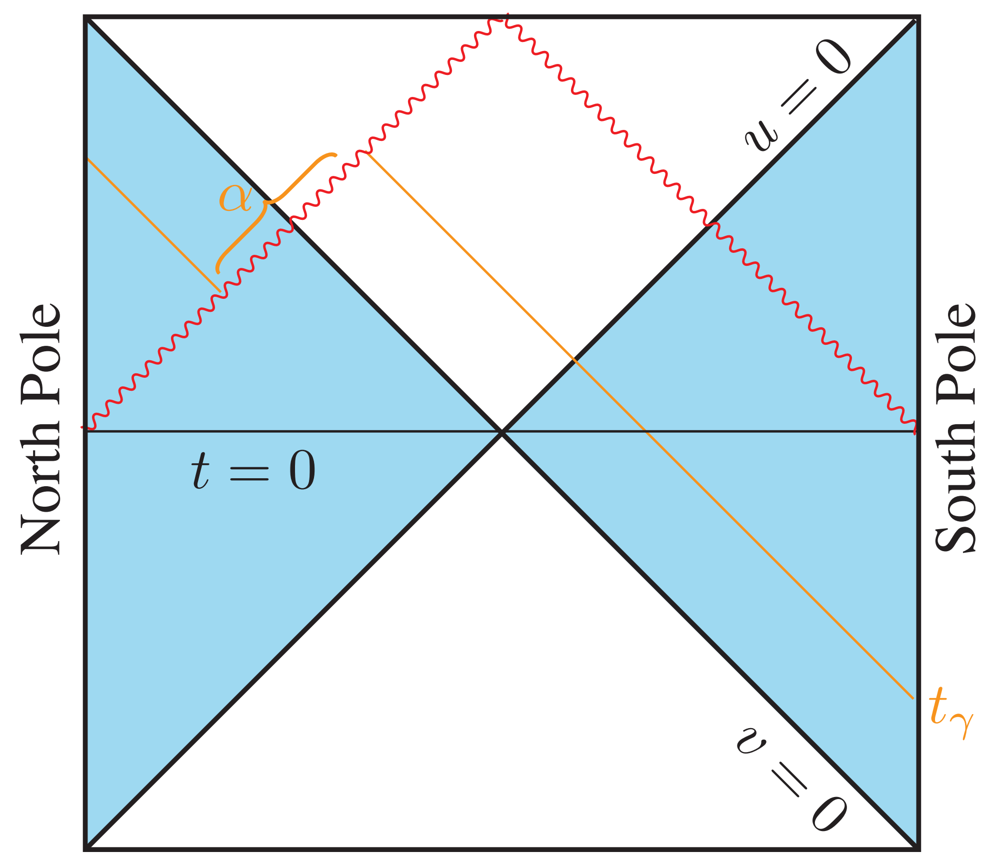
- What does this imply for the information recovery time?

Information flow

[Aalsma, GS ' 20]

[Aalsma, Cole, Morvan, van der Schaar, GS]

The maximum shift $\alpha = 2G_N \mathscr{E} \ell$ on a geodesic crossing an isotropic shockwave is of the order of the Hubble length, when $\mathscr{E}(4\pi\ell) \sim S_{dS}$.



The upper bound on $\mathscr{E}$ corresponds to the largest (Nariai) black hole that fits in dS space. This is a backreaction constraint on the shockwave.

Bound on Information flow

Suppose we send N_{bit} bits of info, with energy ϵ_{bit} in each bit:

$$N_{\text{bit}}\epsilon_{\text{bit}} < \mathcal{E} \lesssim S_{\text{dS}}/(4\pi\ell)$$

The smallest ϵ_{bit} is $T_{dS} = (2\pi\ell)^{-1}$ and thus:

$$N_{\text{bit}} < N_s < \frac{1}{2}S_{\text{dS}} \qquad \text{with} \qquad N_s \equiv \mathcal{E}\,(2\pi\ell)$$

For the message to fit through the wormhole (uncertainty principle):

$$\epsilon_{\text{bit}}\,\ell > \frac{\ell}{\alpha} = \frac{1}{2\mathcal{E}G_N} \sim \frac{1}{N_s}\,S_{\text{dS}}$$

To be consistent with the probe approximation:

$$N_{\text{bit}} < \frac{N_s^2}{4\pi S_{\text{dS}}} \lesssim \ S_{\text{dS}}$$

backreaction becomes large
when N_{bit} exceeds S_{dS}

[Aalsma, GS];[Aalsma, Cole, Morvan, van der Schaar, GS]

de Sitter Complementarity

We will present evidence that information can be recovered in dS space in a scrambling time.

If one only needs to wait a scrambling time to decode a message from the Hawking modes, the no-cloning principle is naively in jeopardy.

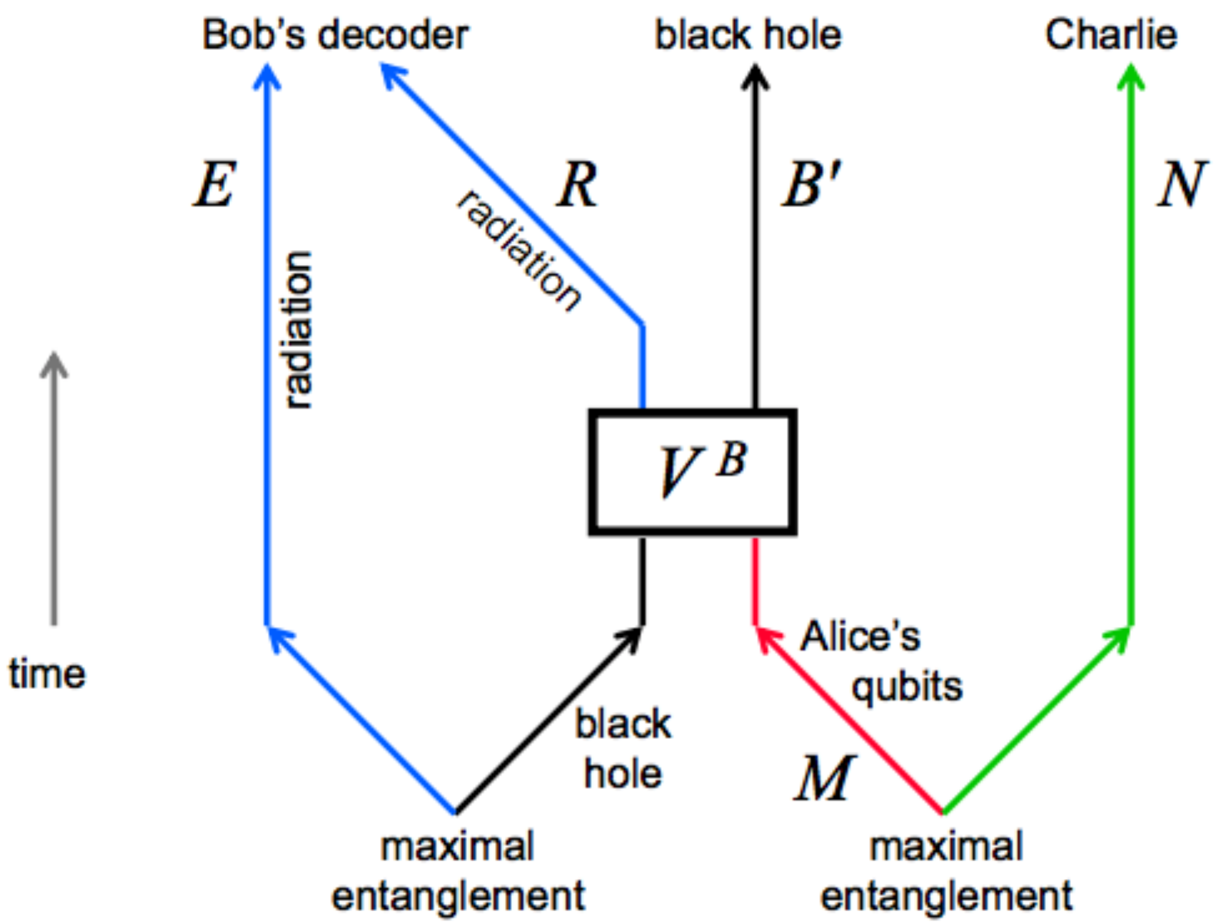
An observer can throw in a (secret) bit of information through the horizon, decode it from the Hawking radiation after $t_{\text{scrambling}}$ and then jump through the horizon and see the same bit twice.

This violation of no-cloning principle does not happen for eternal AdS black holes: due to the entanglement of the interior and exterior regions, information is never copied [Maldacena, Susskind];[Maldacena, Stanford, Yang].

We suggest a similar gravitational description of the Hayden-Preskill protocol for de Sitter space [Aalsma, Cole, Morvan, van der Schaar, GS].

Information retrieval

Bob collects Hawking radiation in order to retrieve information that Alice threw across the BH horizon, when has this info been retrieved?



Hayden-Preskill protocol

When the purification of system N (maximally entangled with system M which contains the message) moves from B' to R.

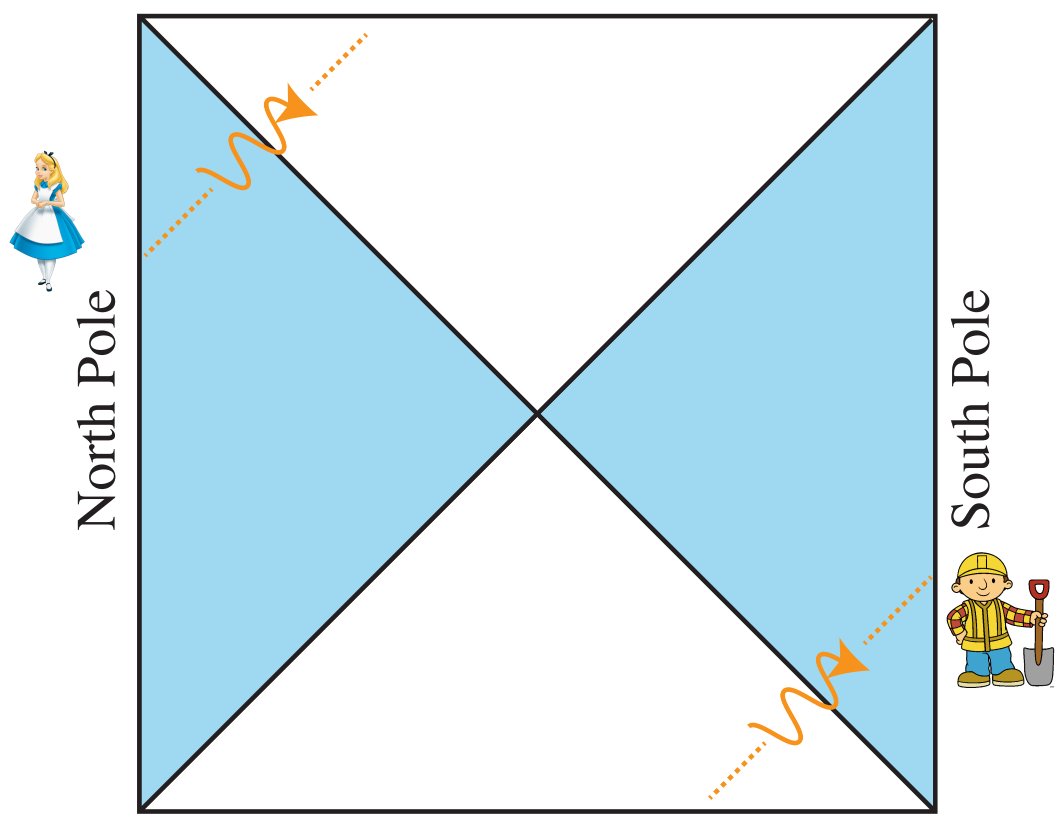
$$||\rho_{B'N} - \rho_{B'} \otimes \rho_N|| \ll 1$$

This happens when Bob receives a few more qubits of info than were in Alice's original message.

Information retrieval in dS

Alice throws system M (which is maximally entangled with reference system N) from the North Pole through the future de Sitter horizon.

If the purification of N transfers to the new radiation (system R), Bob has successfully recovered Alice's information.



This happens when Bob has collected a few more qubits than were in the original message.

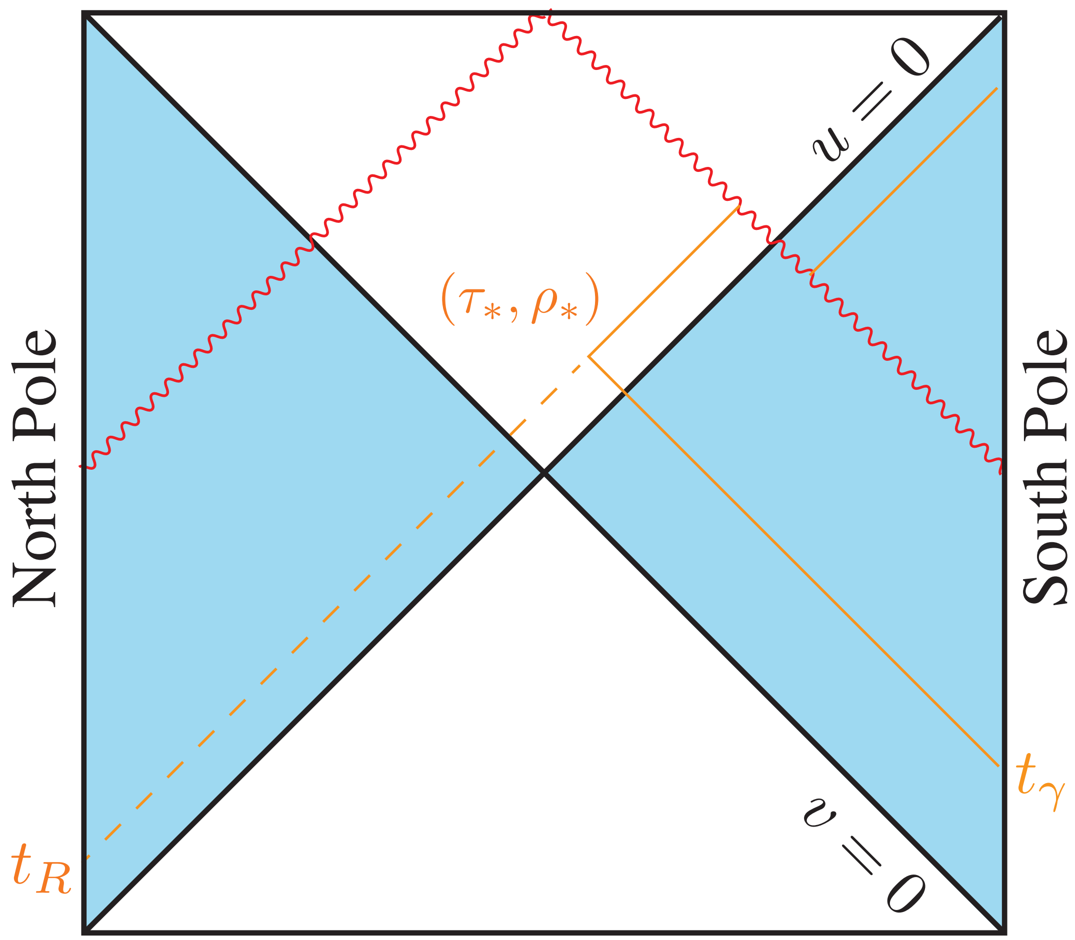
This is given by the scrambling time since it takes longer for system M to thermalize at Bob's horizon than to collect a few Hawking modes.

In our shockwave protocol, this thermalization timescale can be viewed as the the time for antipodal observers to exchange information.

[Aalsma, Cole, Morvan, van der Schaar, GS]

Information recovery

Can either do a doubled-sided or one-sided experiment.



A null ray message is emitted N_γ e-folds before $t = 0$ from the South Pole at a fixed:

$$v_\gamma = \ell e^{-N_\gamma}.$$

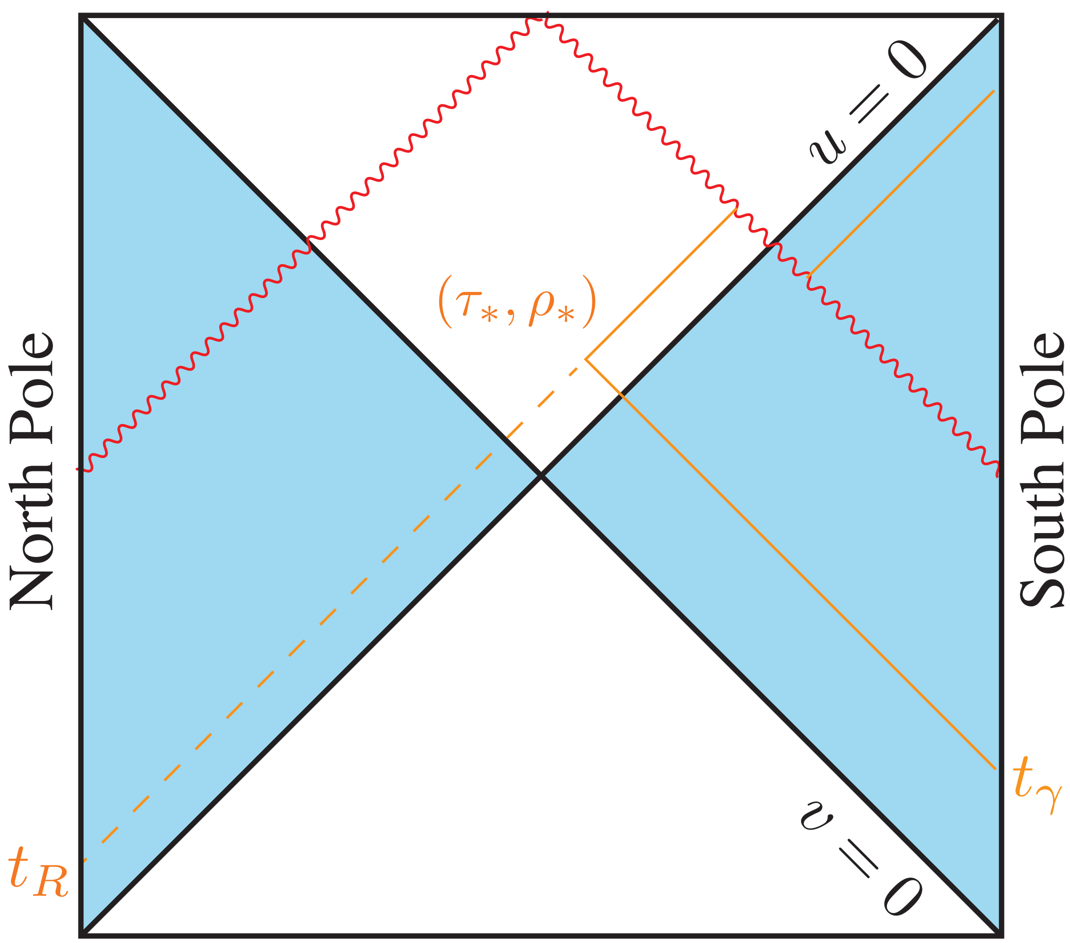
The null ray is reflected after having crossed the horizon to a null ray at fixed (and positive):

$$u_R = \ell e^{-\sigma_R}, \text{ where } \sigma_R = t_R/\ell$$

As long as $u_R \neq \ell$, the reflected null ray will be intercepted by the shockwave.

[Aalsma, Cole, Morvan, van der Schaar, GS]

Information recovery



In planar coordinates, the reflection occurs at:

$$e^{-\tau_*/\ell} = \frac{1}{2} e^{-\sigma_R} \left(e^{N_\gamma + \sigma_R} - 1 \right)$$
$$\rho_*/\ell = \frac{1}{2} e^{-\sigma_R} \left(e^{N_\gamma + \sigma_R} + 1 \right)$$

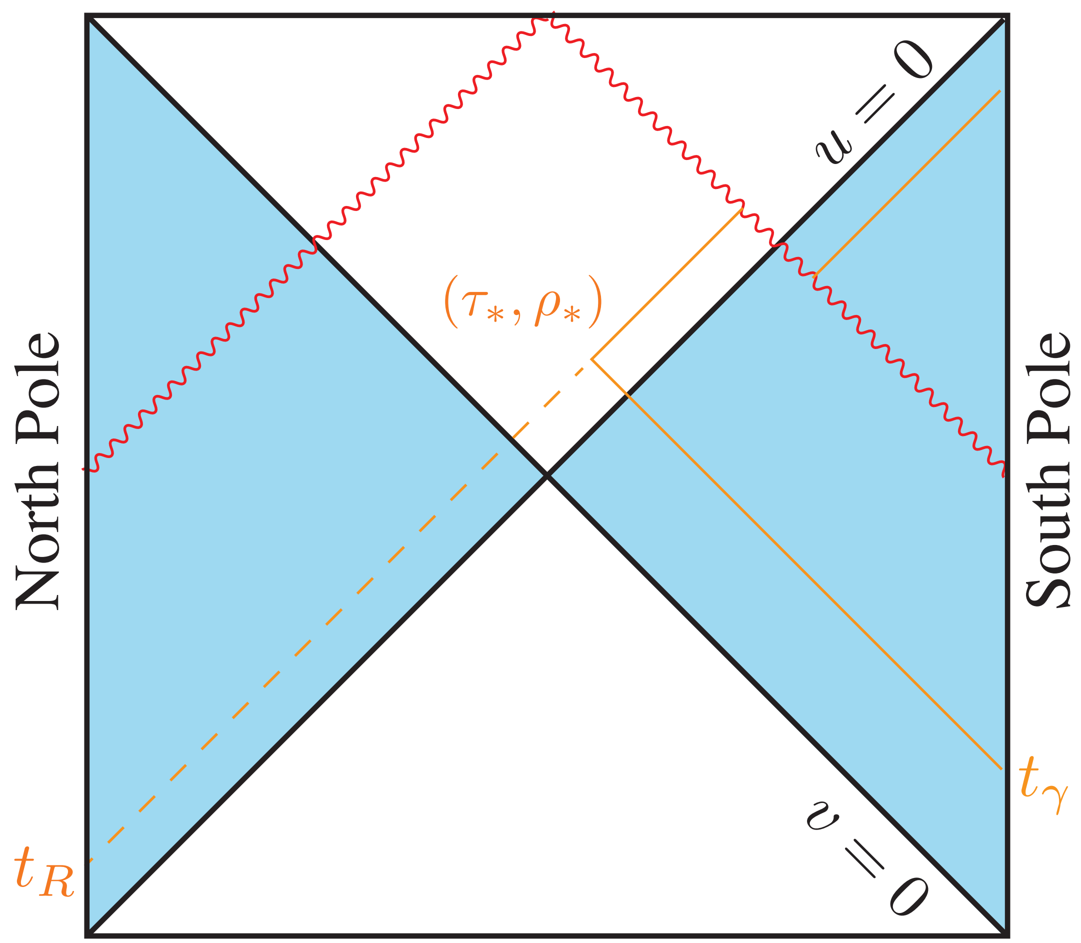
The time difference between emission from the South Pole and the reflection is:

$$e^{\Delta N_*} = \frac{2e^{(\sigma_R + N_\gamma)}}{e^{(\sigma_R + N_\gamma)} - 1}$$

The typical number of e-folds after which the message is reflected is approximately log 2, when $e^{\sigma_R + N_\gamma} \gg 1$.

[Aalsma, Cole, Morvan, van der Schaar, GS]

Information recovery



After the message hits the shockwave, the null curve will be shifted to:

$$u_\gamma{}' = u_\gamma - \alpha = \ell\left(e^{-\sigma_R} - \frac{N_\gamma}{2S_{\rm dS}}\right)$$

For the message to return to the South Pole,

$$\sigma_R > \log\left(\frac{2S_{\rm dS}}{N_\gamma}\right)$$

More than sufficient if the reflection occurs at a distance of the order of a Planck length away from the South Pole horizon, i.e., $\sigma_R \sim \log S_{dS}$.

[Aalsma, Cole, Morvan, van der Schaar, GS]

Information recovery

The time for the message to leave the South Pole and return to Bob:

$$\Delta N_{\text{recover}} = N_\gamma - \log\left(\frac{N_\gamma}{2S_{\text{dS}}} - e^{-\sigma_R}\right)$$

For Bob to get back the message as fast as possible, he can extremize with respect to N_γ:

$$\Delta N_{\text{recover}} = 1 + 2e^{-\sigma_R} S_{\text{dS}} + \log(2S_{\text{dS}})$$

Information recovery happens as fast as possible when $\sigma_R \gg 1$, in which case $\Delta N_{\text{recover}} = \log\left(2S_{dS}\right)$, the scrambling time.

We interpret this as the decoding protocol: when successful, removes the message from behind the horizon back into Bob's causal region.

[Aalsma, Cole, Morvan, van der Schaar, GS]

Bounds from backreaction

We have found earlier an information bound $N_{\mathrm{bit}} \leq S_{dS}$ with the inequality saturated when the shockwave energy is of order S_{dS}.

The only states that Bob and Alice have access to are the Hawking modes emitted from the de Sitter horizon, which allows them to collect energy at an average rate of one unit of T_{dS} per e-fold.

The shockwave energy $\mathscr{E}$ collected in N_γ e-folds:

$$N_s \equiv \mathscr{E}(2\pi\ell) = N_\gamma \leq S_{dS}$$

Backreaction becomes large at $Ht_{\mathrm{Page}} \equiv S_{dS}$ though it may happen earlier [See Aalsma's talk].

Bounds from backreaction

The scrambling time is the shortest time for information recovery obtained by extremizing $\Delta N_{\text{recover}}$ over N_γ. In general,

$$\Delta N_{\text{recover}} = N_\gamma - \log\left(\frac{N_\gamma}{2S_{\text{dS}}} - e^{-\sigma_R}\right)$$

The recovery time and the number of bits of info transfer depends on N_γ (or equivalently, the shockwave energy):

	Small shockwave energy	**Large shockwave energy**
Energy:	$E_{\text{shock}} = 1/\ell$	$E_{\text{shock}} = 1/\ell_p$
Max # of bits:	$N_{\text{bit}} = 1$	$N_{\text{bit}} = S_{\text{dS}}$
Recovery time:	$t = \ell \log(\ell/\ell_p)$	$t = \ell S_{\text{dS}}$
	Scrambling time	Page time

Conclusions

- Using quantum information ideas, we provide a bulk gravitational description of $t_{\text{scrambling}}$ [this talk] and t_{Page} [Aalsma's talk] for dS space.

- de Sitter space is a fast scrambler; the scrambling time is the shortest time for a static observer to decode info from the horizon.

- Our findings suggest a bulk decoding protocol for de Sitter space: information is not lost but there is a bound on its amount.

- Using shockwaves to recover information, backreaction is small for at most t_{Page}, but possibly shorter.

- We consider a static observer here, but perhaps similar effects limit the lifetime of inflation, cf. [Arkani-Hamed, Dubovsky, Nicolis, Trincherini, Villadoro].