Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors

Keith R. Dienes
University of Arizona

Tucson, Arizona

Work in collaboration with
- Fei Huang
- Jeff Kost
- Kevin Manogue
- Shufang Su
- Brooks Thomas

arXiv: 2001.02193
arXiv: 2101.10337

This work was supported in part by the National Science Foundation through its employee IR/D program. The opinions and conclusions expressed herein are those of the speaker, and do not necessarily represent the National Science Foundation.

PPC Conference
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See back-to-back talks in parallel session immediately after lunch for more details.

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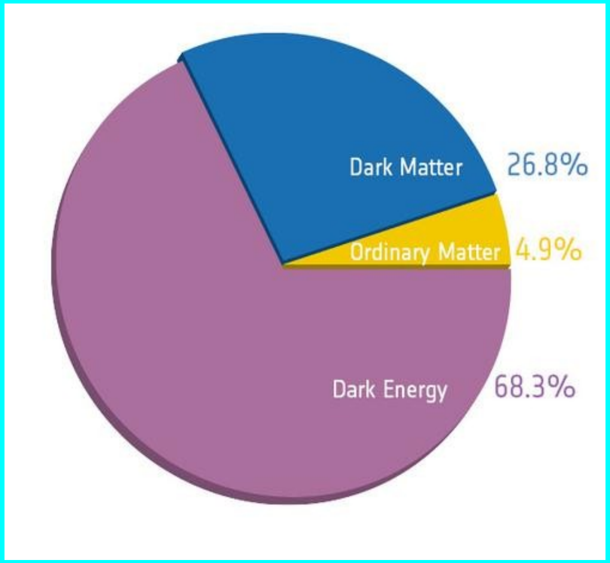
Dark Matter = ??

- Situated at the nexus of particle physics, astrophysics, and cosmology
- Dynamic interplay between theory and current experiments
- Of fundamental importance: literally 23% of the universe!
- Necessarily involves physics beyond the Standard Model

One of the most compelling mysteries facing physics today!

This is important, since the total energy density of the universe coming from dark matter is **at least five times** that from visible matter!



Physics from **visible sector**

Physics from the **dark sector (dark matter)**

Dark energy

- Indeed, it is primarily the "dark" physics which drives the evolution of the universe through much of cosmological history... cannot be ignored!
- Moreover, thanks to advances in observational cosmology over the past two decades (COBE, Planck, etc.), we are rapidly gaining data concerning the nature and properties of the dark sector!

This is thus a ripe area for study!

Unfortunately, very little is known about the dark sector.

- What is the production mechanism?   Is it thermal or non-thermal?
- Does the dark sector contain one species, or are there many different components?   What are the interactions between these components?
- What kinds of phase transitions or non-trivial dynamics might be involved in establishing the dark matter that we observe today?

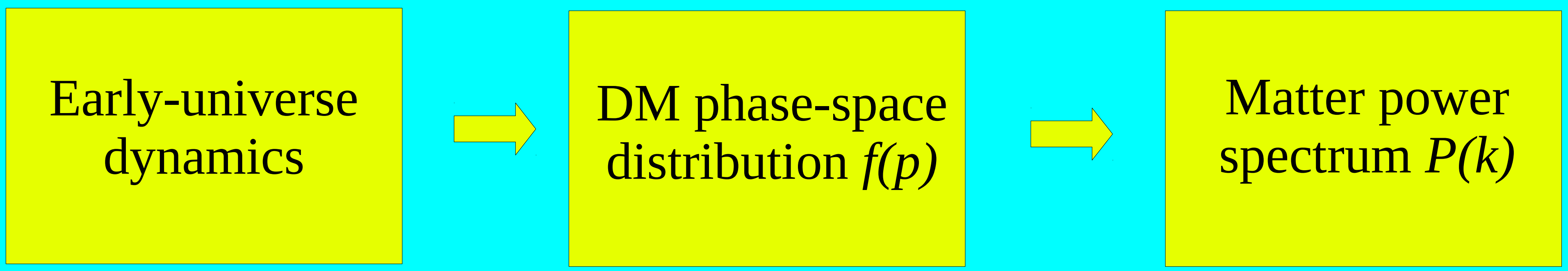
This is important because dark matter is critical for many aspects of cosmological evolution, e.g.,

- The dark sector drives cosmological expansion
- The dark sector seeds structure formation.

This then leads to two critical questions ---

- **What imprints might non-trivial dark-sector dynamics leave in the present-day universe?**
- **To what extent can we decipher the archaeological record, exploiting information about the present-day universe in order to learn about / constrain the properties of the dark sector?**

In this talk we shall concentrate on one aspect of the present-day universe: **the matter power spectrum** ***P(k)***, which tells us about structure formation. This depends on the **dark-matter phase-space distribution** ***f(p)***, which in turn is highly sensitive to the early-universe dynamics we wish to constrain.



Clearly a given dynamics leads to a unique *f(p)* and then to a unique *P(k)*. However, this process is not invertible.

Nevertheless, we can ask: **To what extent can we find *signatures* or *patterns* in *f(p)* and *P(k)* which might tell us about early-universe dynamics that produced the dark matter? What *can* we learn?**

In general, once the dark matter is produced in the early universe, its properties can be described through its phase space distribution *f(p,t)*:

$$f(\vec{x},\vec{p},t) \approx f(p,t)$$

homogeneity, isotropy

number density
$$n(t) \equiv g\int \frac{d^3p}{(2\pi)^3} f(p,t)$$

energy density
$$\rho(t) \equiv g\int \frac{d^3p}{(2\pi)^3} E f(p,t)$$

where
$$E = \sqrt{p^2+m^2}$$

pressure
$$P(t) \equiv g\int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p,t)$$

equation of state
$$w(t) \equiv \frac{P(t)}{\rho(t)}$$

***f(p,t)* is therefore the central quantity in understanding the cosmological properties of the dark sector**

- *e.g.*, cold or hot, thermal or non-thermal, *etc*.

It is important to understand how *f(p)* evolves with time.

In an FRW universe,

$H \equiv \dot{a}/a$

$$x(t) \;=\; x(t')\frac{a(t)}{a(t')} \quad\Rightarrow\quad p(t) \;=\; p(t')\frac{a(t')}{a(t)} \quad\Rightarrow\quad \frac{d\log p}{dt} \;=\; -H(t)$$

Thus time evolution corresponds to *additive shifts in log(p)*.

physical number density

$$n(t) \;\sim\; \int d^3p\, f(p,t) \;\sim\; \int dp\, p^2 f(p,t) \;\sim\; \int d\log p\; p\, p^3 f(p,t)$$

comoving number density

$$N(t) \;\sim\; na^3 \;\sim\; \int d\log p\, (ap)^3 f(p,t)$$

Therefore define

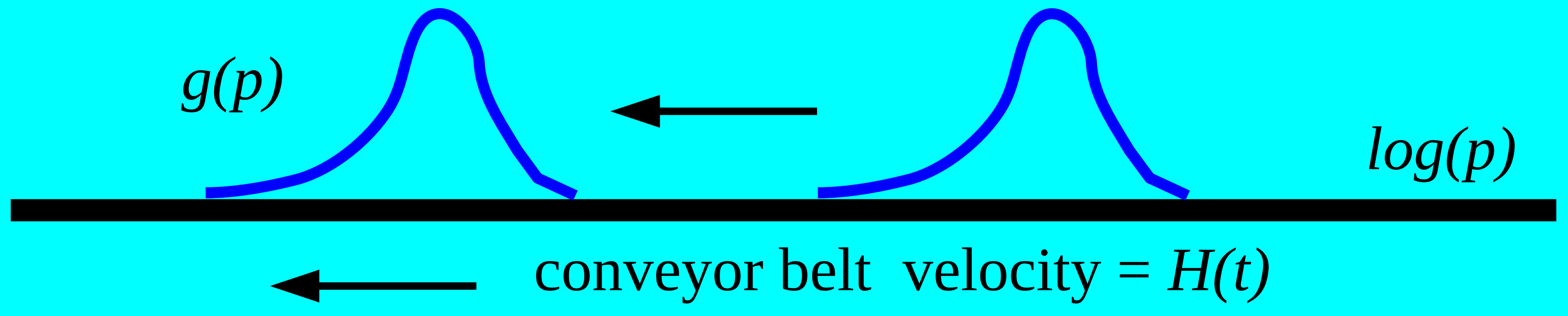
$$g(p,t) \;\equiv\; a(t)^3 p^3 f(p,t)$$

Thus, once the dark matter is produced, $g(p,t)$ evolves with time according to

$$g(p(t),t) \;=\; g(p(t'),t')$$

Comoving $N(t)$ → No overall rescaling.

Thus, if we plot $g(p)$ versus $log(p)$, the total area under the curve is proportional to the (fixed!) comoving particle number density $N \sim na^3$. Under subsequent time evolution, the curve for $g(p)$ merely slides towards smaller values of $log(p)$ without distortion, as if carried along a cosmological "conveyor belt" moving with velocity $H(t)$.



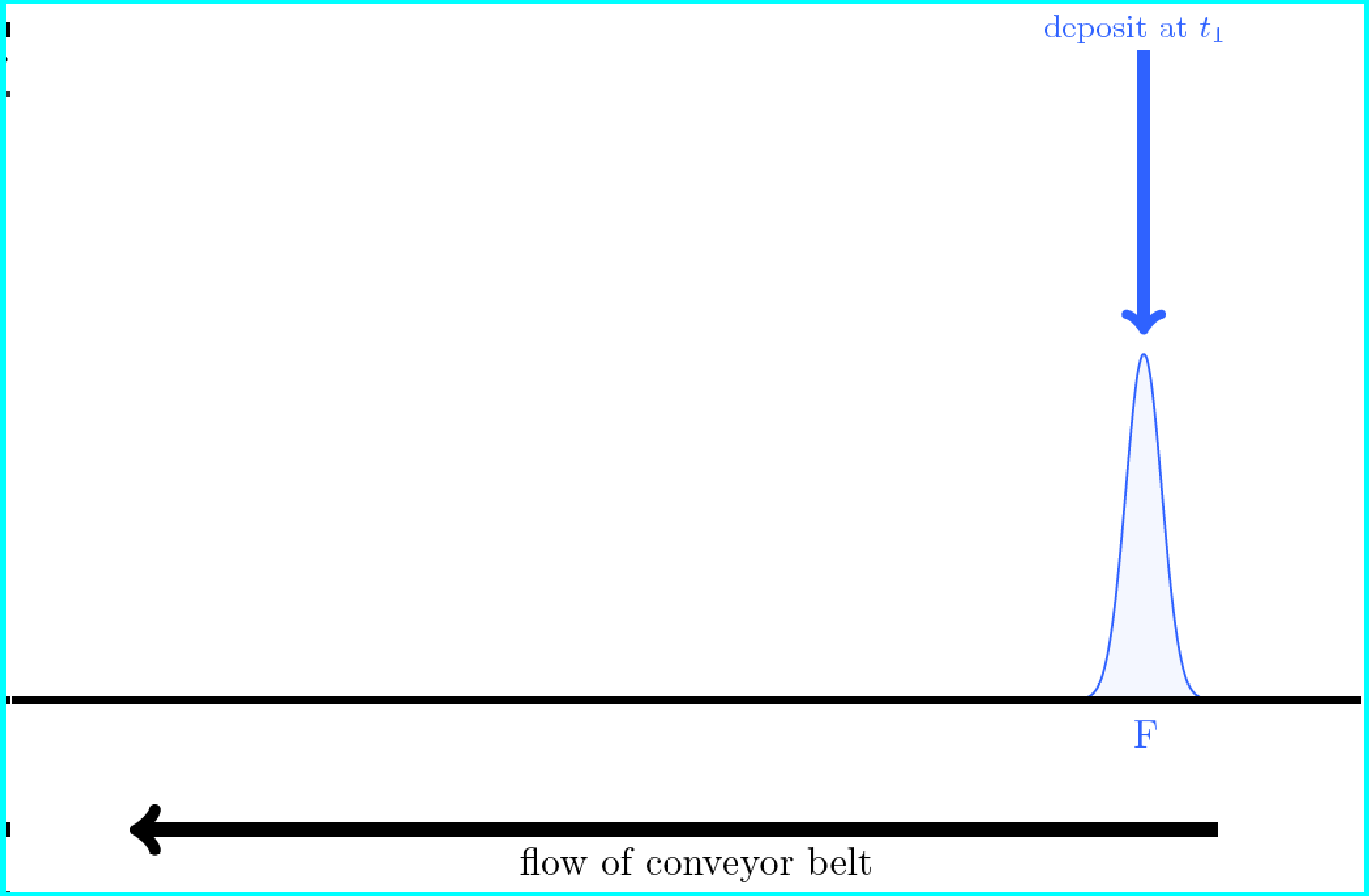
For a *minimal* dark sector, regardless of the particular production mechanism, we expect that *g(p)* appears on the cosmological conveyor belt when the dark matter is produced and then simply redshifts towards smaller *log(p)*.

By contrast, for a *non-minimal* dark sector, it is possible that dark-matter production may be more complicated, with <u>different "deposits" onto the cosmological conveyor belt occurring at different moments in cosmological history</u>.

<u>Non-minimal</u> dark sector:

- Dark sector containing an *ensemble* of particle species instead of a single DM component.
- Phenomenology of dark sector is not determined by the properties of any individual constituent alone, but instead determined *collectively* across all components.

For example, let us consider packets deposited at different times during cosmological history…



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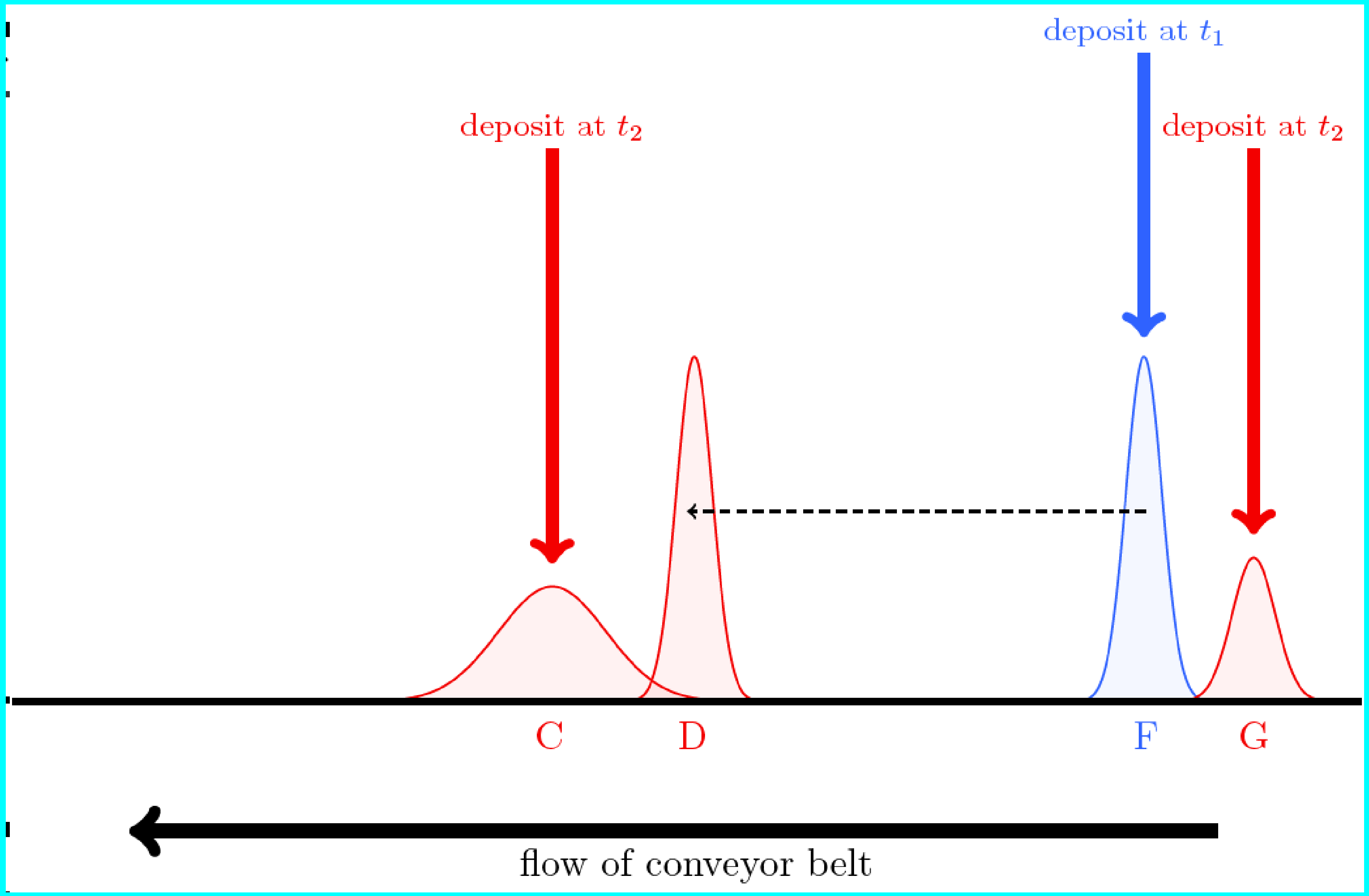

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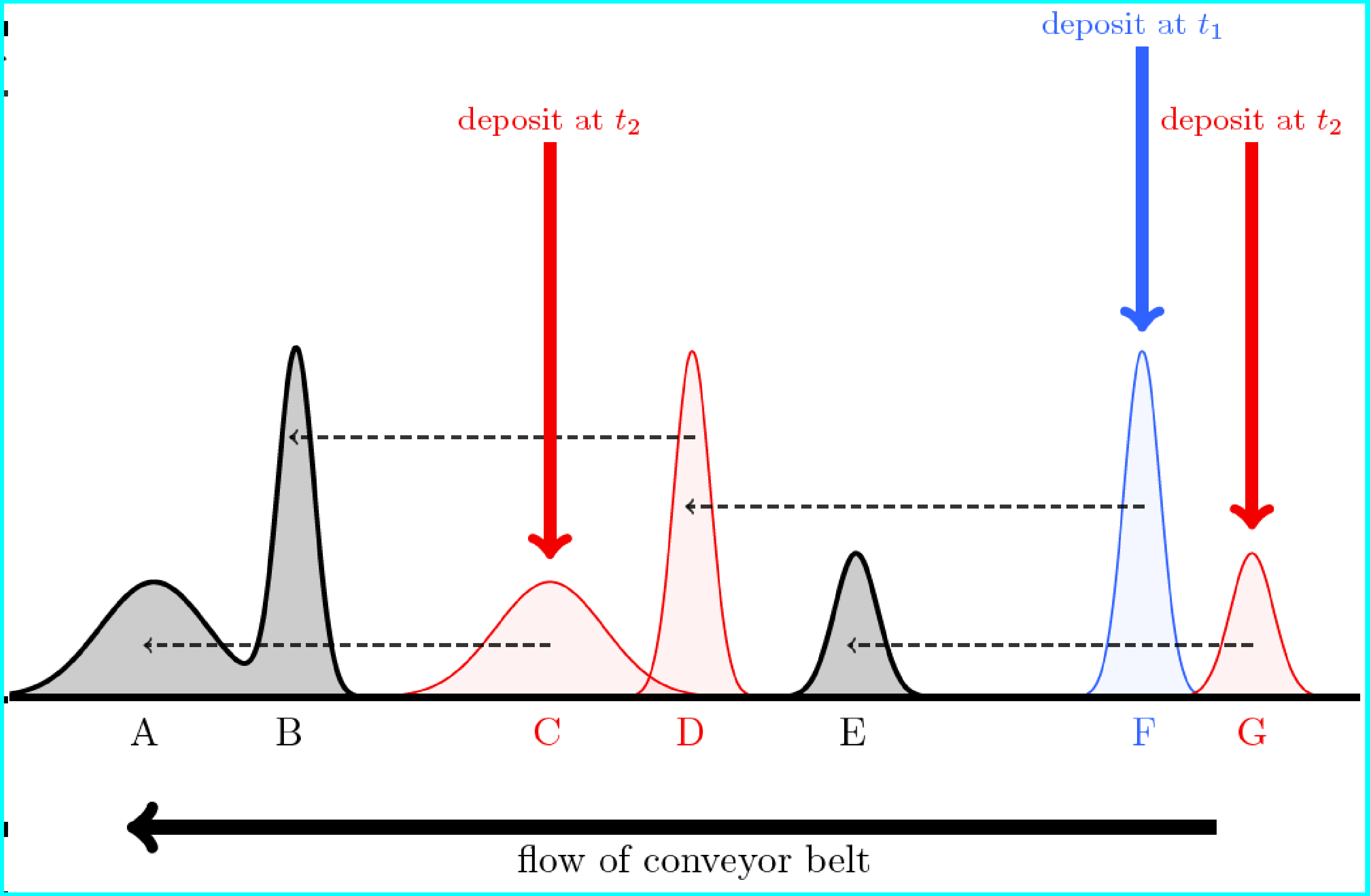


Final result is highly non-trivial, can even be multi-modal!

In general, the final *g(p)* is realized as the accumulation **of all previous deposits** occurring at all previous times during cosmological history.

Let $\Delta(p,t)$ = the profile of the dark-matter deposit rate at time *t*.
Then at any time *t* we have

$$g(p) = \int^t dt' \, \Delta\left(p\frac{a(t)}{a(t')}, t'\right)$$

If the deposits occur at discrete times t_i, then

$$\Delta(p,t') = \sum_i \Delta_i(p)\delta(t'-t_i)$$

$$g(p) = \sum_i \Delta_i\left(p\frac{a(t)}{a(t_i)}\right)$$

Thus, *g(p)* reflects a particular cosmological history.

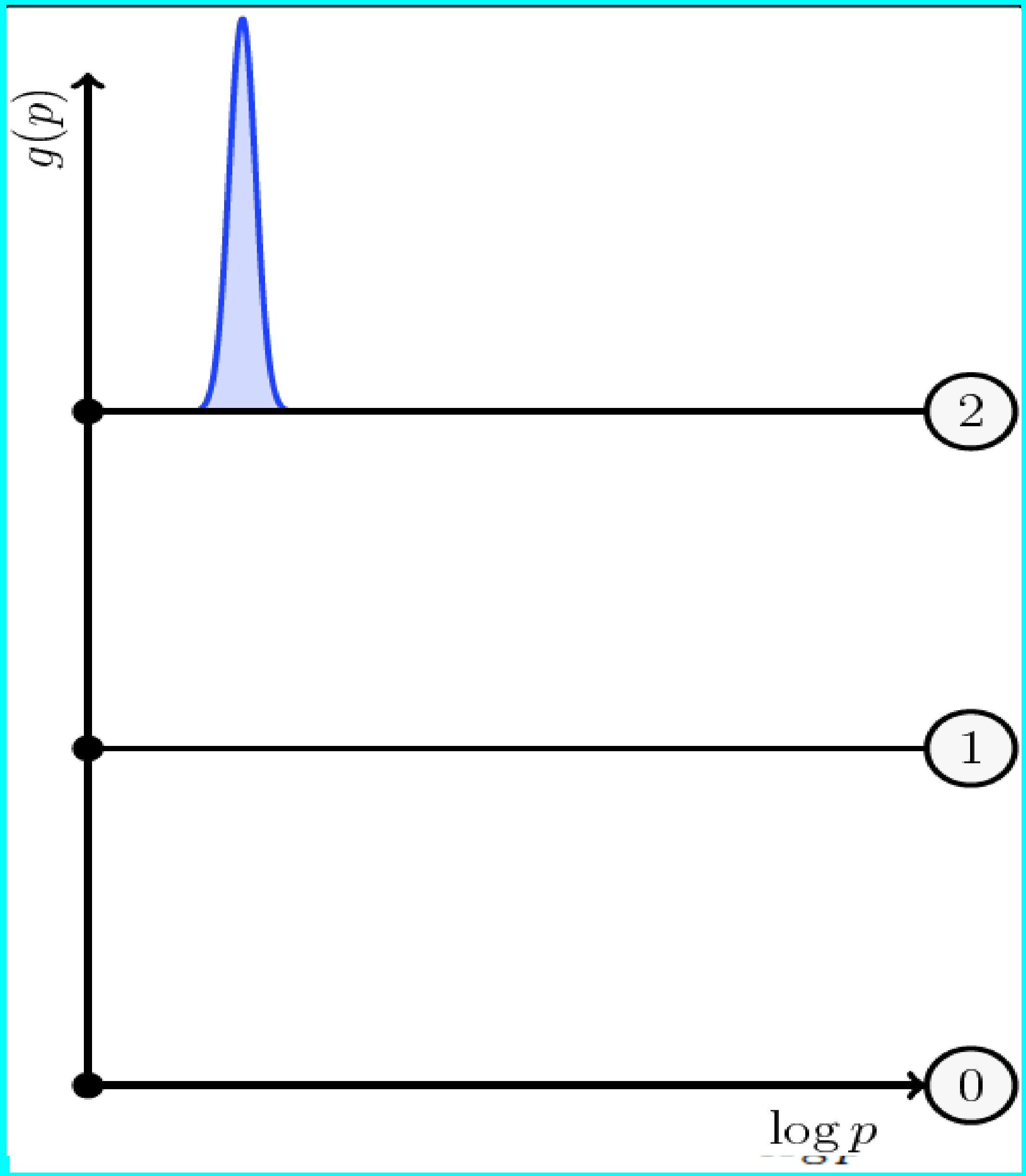
Archaeological question: To what extent can we use *g(p)* to *resurrect* this history? We can only determine sums along backward "FRW lightcones"!

We have already seen that multi-modality suggests that separate deposits occurred at different moments in cosmological history.

- Is such a pattern of deposits natural?
- What kinds of non-minimal dark sectors can give rise to such deposit patterns?

If our non-minimal dark sector contains an ensemble of states with different masses, lifetimes, and cosmological abundances, then <u>intra-ensemble decays</u> (*i.e.*, decays from heavier to lighter dark-sector components) will naturally give rise to such scenarios!

To see this, consider a three-state system with only the heaviest state initially populated. For simplicity, assume only a single unimodal packet –- can even be thermal!



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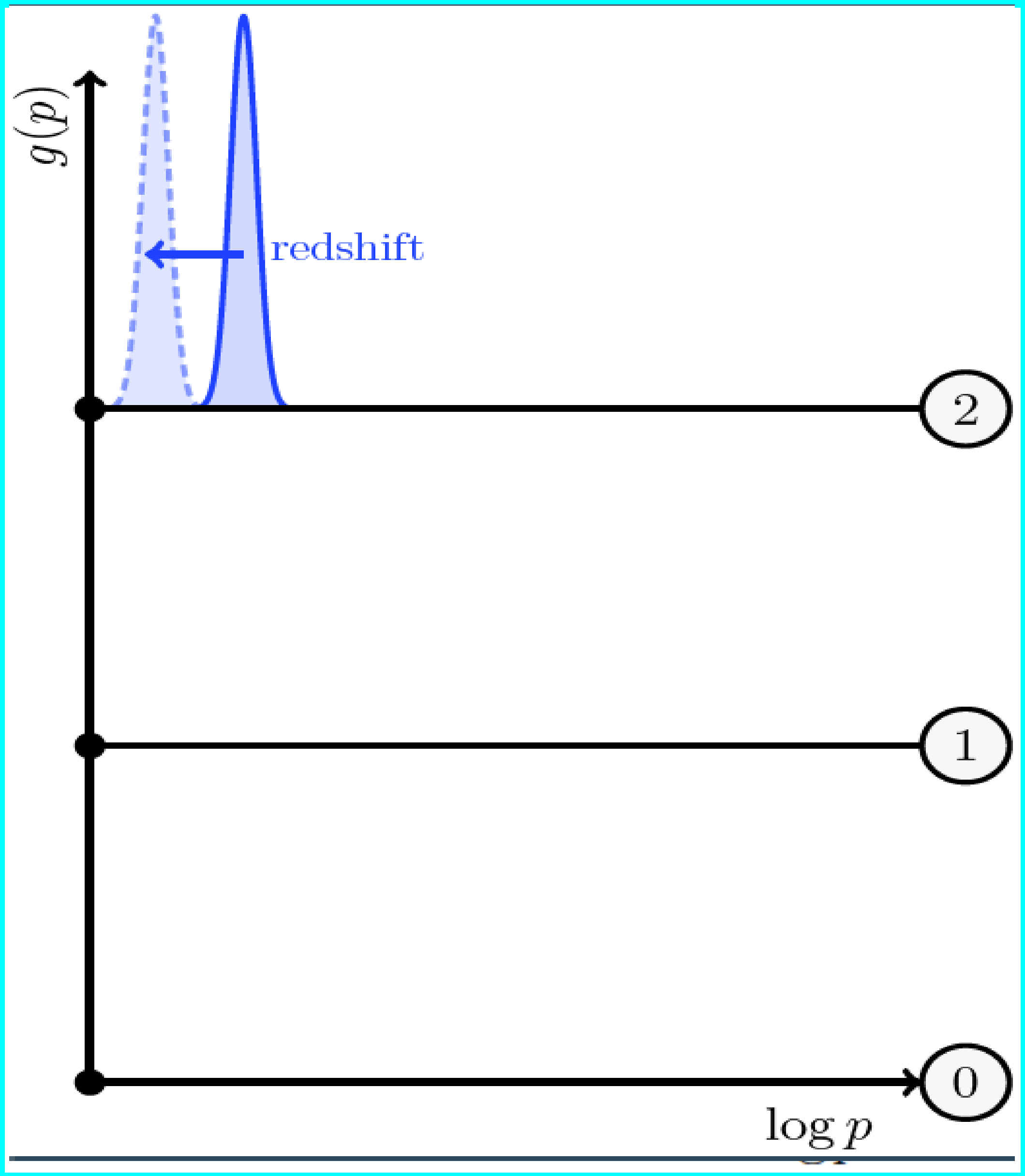

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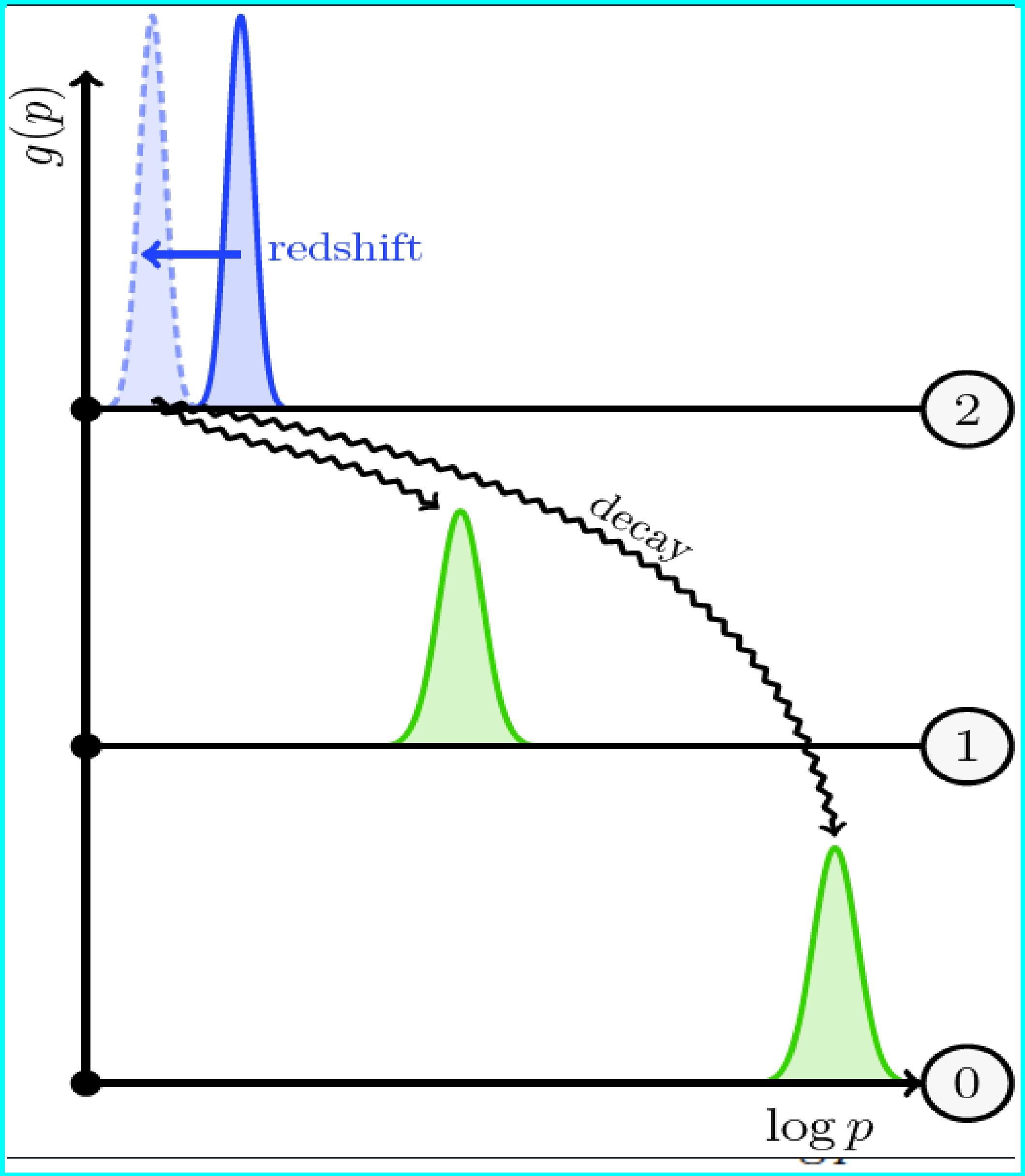

- **<u>2→1+0</u>**: Daughters have extra kinetic energy (higher p) and also are wider (larger Δp) than the parent.

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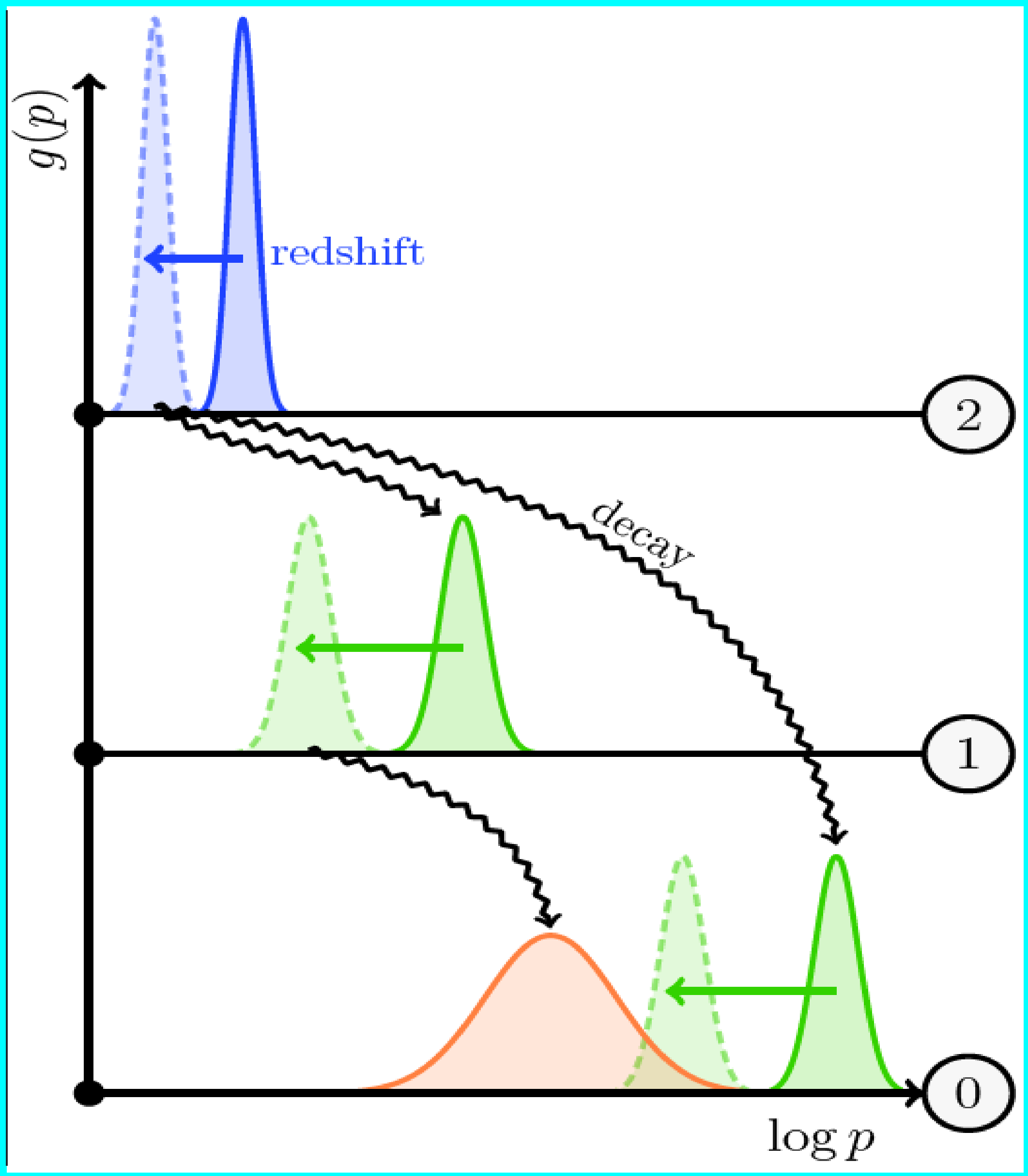

- **<u>2→1+0</u>**: Daughters have extra kinetic energy (higher *p*) and also are wider (larger Δp) than the parent.
- **<u>1→0+0</u>**: Decay produces two identical superposed daughter packets (hence twice the area), again wider and at higher *p* than parent.

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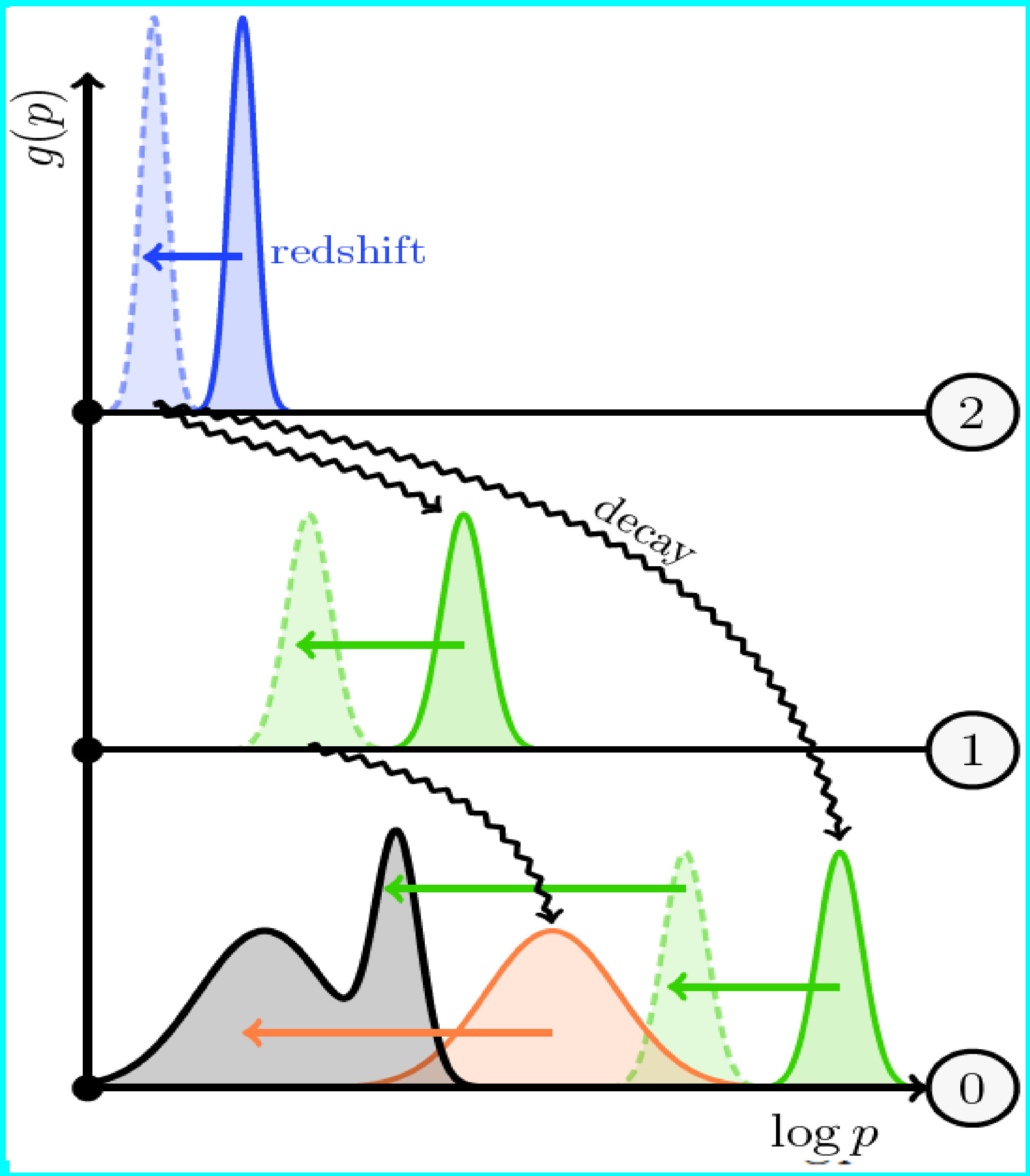
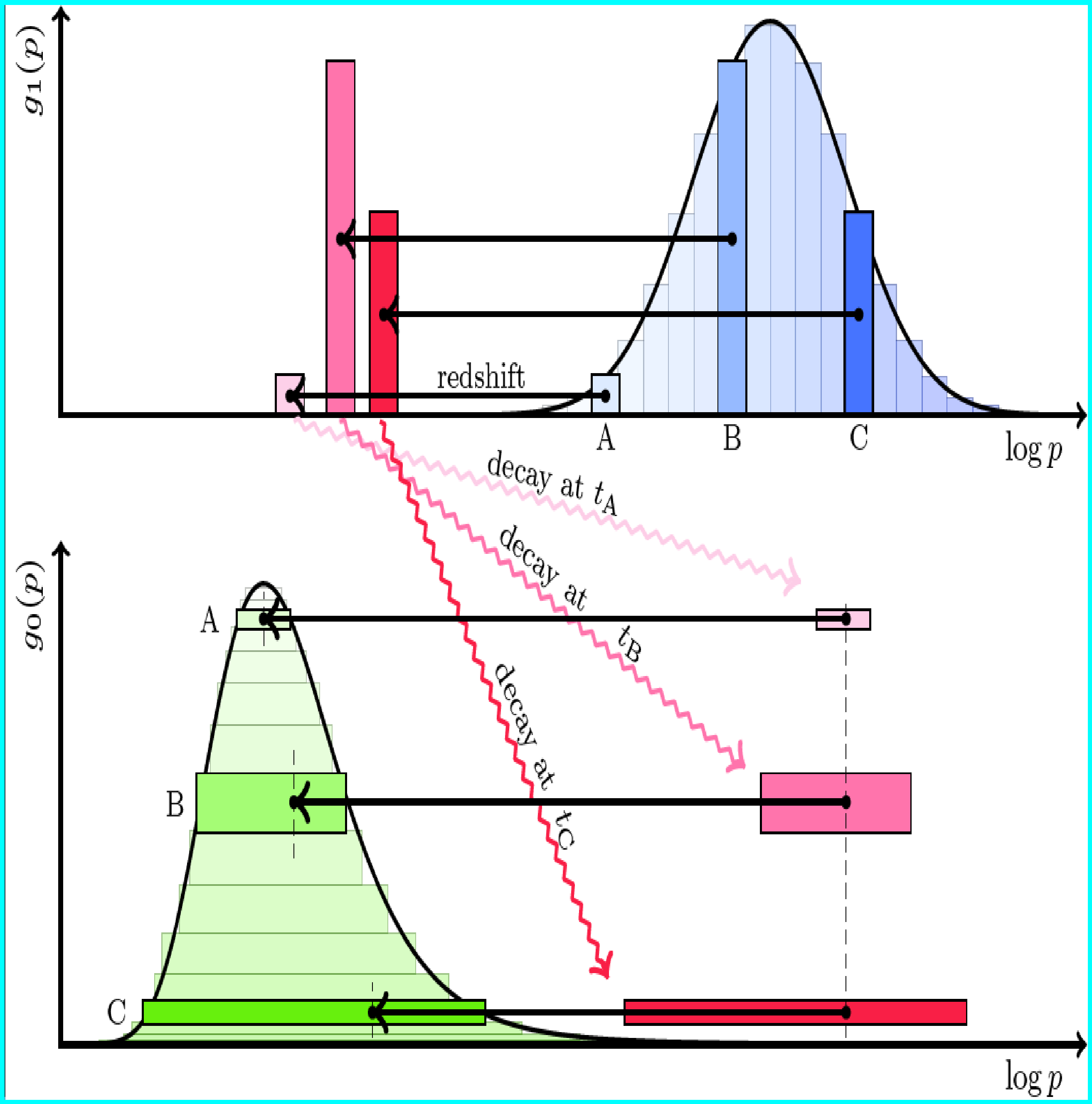


- **$2 \rightarrow 1+0$**: Daughters have extra kinetic energy (higher p) and also are wider (larger Δp) than the parent.
- **$1 \rightarrow 0+0$**: Decay produces two identical superposed daughter packets (hence twice the area), again wider and at higher p than parent.
- **Resulting *g(p)* is a non-trivial superposition of packet deposits from 2 independent decay chains, thus carries an imprint of the early complex decay dynamics.**

But even the process of decay from a parent packet to a daughter packet is highly non-trivial.

To what extent does the daughter packet contain generic information about the parent?

Study the decay process in detail.
Start with the parent....



- Redshifted daughter contributions combine to produce daughter packet.
- *Leftward tilt* of daughter packet is relativistic effect stemming from parent momenta.
- *Vertical* momentum slices of parent packet become *horizontal* building blocks of daughter packet.
- Maximum/minimum widths of daughter packet indicate maximum/minimum momenta of parent packet.
- Rising/falling slopes of daughter packet carry information about decay kinematics.

Through these sorts of analyses, we can learn many things about the parent packet simply by studying the properties of the daughter packet.

For example,

at the time of production

relativistic parent $\Longleftrightarrow \left\{ \begin{array}{l} \text{leftward contribution} \\ \text{to daughter tilt.} \end{array} \right.$

Very useful result! *For example...*

In principle, a relativistic daughter packet which is narrow, with $\Delta p << m$ as well as $\Delta p << \langle p \rangle$, could be the result of either

- a relativistic parent experiencing a close-to-marginal decay, or
- a non-relativistic parent experiencing a far-from-marginal decay.

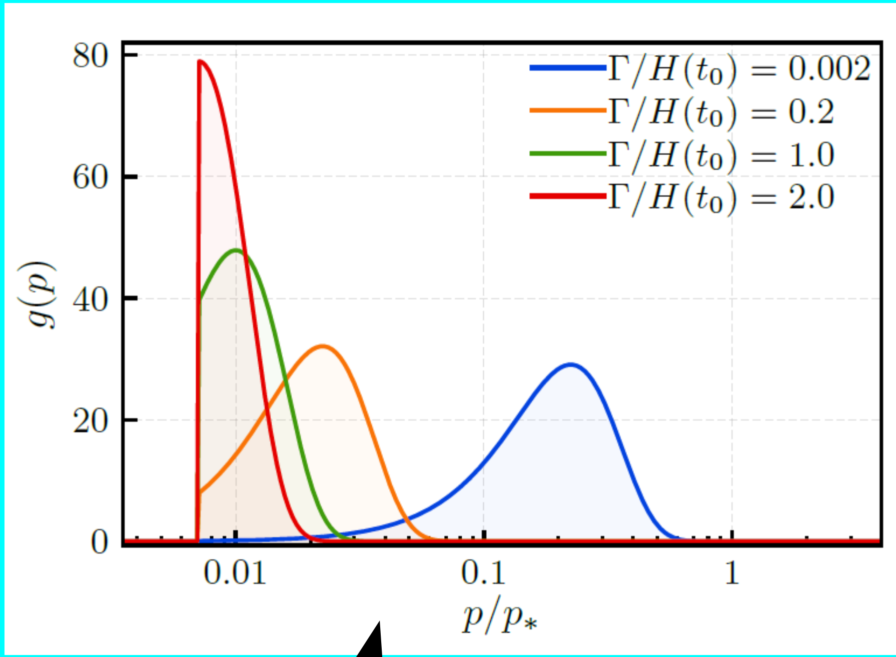
The tilt of the daughter packet may allow us to distinguish between these two possibilities!

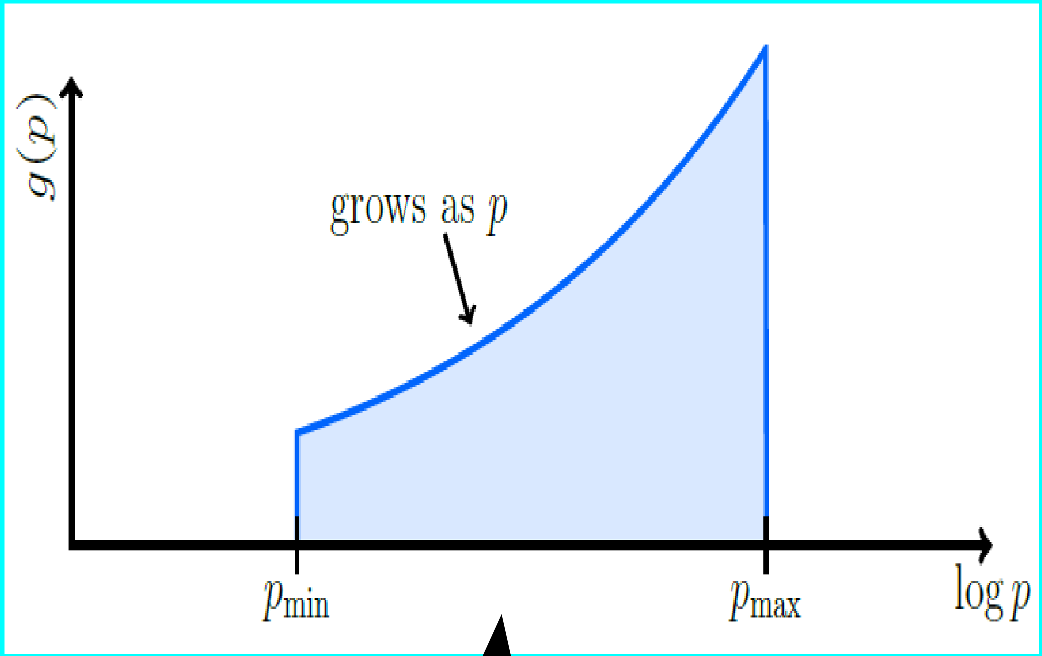
Combining these effects, we obtain the full daughter distributions!

<table>
<tr><th rowspan="2">Case</th><th colspan="3">Daughter distribution</th><th>Parent</th><th colspan="2">Decay</th></tr>
<tr><th>rel? $\langle p \rangle$</th><th>width $\Delta p/m$</th><th>relative width $\Delta p/\langle p \rangle$</th><th>rel at decay?</th><th>near absolute marginality?</th><th>near relative marginality?</th></tr>
<tr><td>A</td><td rowspan="2">$p \ll m$</td><td rowspan="2">narrow</td><td rowspan="3">$\mathcal{O}(1)$</td><td rowspan="3">non-rel</td><td rowspan="2">near</td><td>far</td></tr>
<tr><td>B</td><td>$\mathcal{O}(1)$</td></tr>
<tr><td>C</td><td rowspan="3">$p \sim m$</td><td>$\mathcal{O}(1)$</td><td>$\mathcal{O}(1)$</td><td>far</td></tr>
<tr><td>D</td><td>narrow</td><td>narrow</td><td rowspan="2">$\text{rel}_{\sim}$</td><td>near</td><td>near</td></tr>
<tr><td>E</td><td>$\mathcal{O}(1)$</td><td rowspan="3">$\mathcal{O}(1)$</td><td>$\mathcal{O}(1)$</td><td>$\mathcal{O}(1)$</td></tr>
<tr><td>F</td><td rowspan="6">$p \gg m$</td><td rowspan="2">wide</td><td rowspan="2">non-rel</td><td rowspan="2">far</td><td>far $(p_{\text{parent}} \ll m_{\text{daughter}})$</td></tr>
<tr><td>G</td><td>far $(p_{\text{parent}} \sim m_{\text{daughter}})$</td></tr>
<tr><td>H</td><td>$\mathcal{O}(1)$</td><td>narrow</td><td>$\text{rel}_{\gg}$</td><td>near</td><td>near</td></tr>
<tr><td>I</td><td rowspan="3">wide</td><td rowspan="3">$\mathcal{O}(1)$</td><td>non-rel</td><td>far</td><td>far $(p_{\text{parent}} \gg m_{\text{daughter}})$</td></tr>
<tr><td>J</td><td>$\text{rel}_{\sim}$</td><td>far</td><td>$\mathcal{O}(1)$</td></tr>
<tr><td>K</td><td>$\text{rel}_{\gg}$</td><td>$\mathcal{O}(1)$ or far</td><td>near</td></tr>
</table>

- In many cases we can even ***invert*** this process and ***reconstruct*** the decay momentum of the parent and the marginality of the decay process directly from the shape of the daughter packet!
- In some cases (*e.g.*, Cases D and H), the reconstruction is unique, while in other cases several possible reconstructions exist.
- Such results even hold for packets which are part of multi-modal distributions.

Along the way, we also found other useful results ---
e.g., *universal* functional forms for certain daughter packets!





Holds for all daughter packets in the limit that the parent is extremely non-relativistic.

Holds for all daughter packets emerging from two-body decays in the limit that the parent is extremely relativistic while the daughter is extremely *non*-relativistic within the rest frame of the parent.

[details in 2001.02193]

Such non-trivial DM phase-space distributions *f(p)* have non-trivial effects on structure formation in the early universe (clusters, galaxies, etc.)

→ **Specifically, they produce non-trivial deviations in the present-day matter power spectrum *P(k)* relative to what would have been expected for CDM.**

Note ---

- Studying the connection between ***f(p)*** and ***P(k)*** provides a way of learning about dark matter from its ***gravitational interactions*** **only!**
- This therefore provides a way of learning about the dark sector ***even if the dark sector has no direct connection to the SM***.

Recall basic point: **Cold DM helps to seed and promote structure formation. However, if DM has a non-negligible velocity, then this over-abundance diffuses outward, leaving to a *suppression* of structure relative to what occurs for CDM.**

Thus, over a fixed time interval (to present), greater DM velocity (momentum) ⇨ greater length scale (smaller *k*) over which diffusion can occur.

- A conservative estimate for k simply calculates the (free-streaming) "horizon" size associated with such diffusion...

$$k \sim \frac{1}{d_{\rm hor}} \sim \frac{1}{vt} \sim \frac{\sqrt{p^2+m^2}}{p}\frac{1}{t}$$

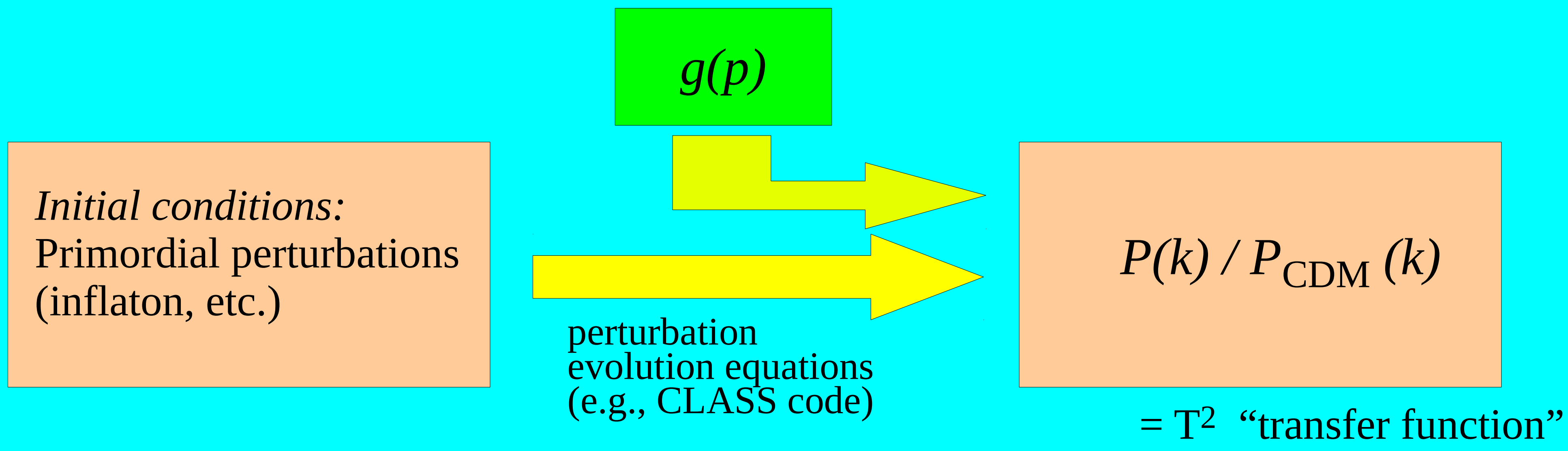
- More properly, we define

$$k_{\rm FSH}(p) \equiv \xi \left[\int_{t_{\rm prod}}^{t_{\rm now}} \frac{p/a(t)}{\sqrt{p^2/a(t)^2+m^2}}\frac{dt}{a(t)}\right]^{-1} = \xi\left[\int_{a_{\rm prod}}^{1}\frac{da}{Ha^2}\frac{p}{\sqrt{p^2+m^2a^2}}\right]^{-1}$$

For any p, defines the *minimum* k that could be affected.

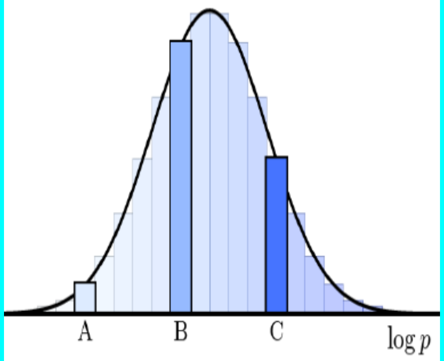
O(1) coefficient

Given *g(p)* , we then proceed to calculate the corresponding suppression fraction ("transfer function") $T^2(k) = P(k) / P_{\rm CDM}(k)$ for the matter power spectrum as a function of *k* ...



In general, the connection between *g(p)* and *P(k)* is highly non-trivial. However, we would like to understand this relationship with an eye towards developing some rough procedures towards inverting it...

Our approach



- We begin by considering *momentum slices* through our dark-matter packet, relating each slice of momentum p to a corresponding value $k_{\rm FSH}$.
- Normally, $k_{\rm FSH}$ would be interpreted as defining the ***minimum*** value of k which can be affected by dark matter in that slice.
- However, we shall instead **take the defining relation for $k_{\rm FSH}(p)$ as defining a *mapping* between the p-variable of $g(p)$ and the k-variable of $P(k)$.** In other words, we shall **identify $k_{\rm FSH}(p)$ with k** and thereby consider $g(p)$ as having a corresponding profile in k-space:

$$\widetilde{g}(k) \equiv g(k_{\rm FSH}^{-1}(k)) \, |\mathcal{J}(k)|$$

inverse of $k_{\rm FSH}(p)$ relation

corresponding Jacobian

Indeed, it then follows that

$$N(t) \sim \int d\log p \; g(p) = \int d\log k \; \widetilde{g}(k)$$

Thus the *k*-profile describes a dark-matter distribution in *k*-space!

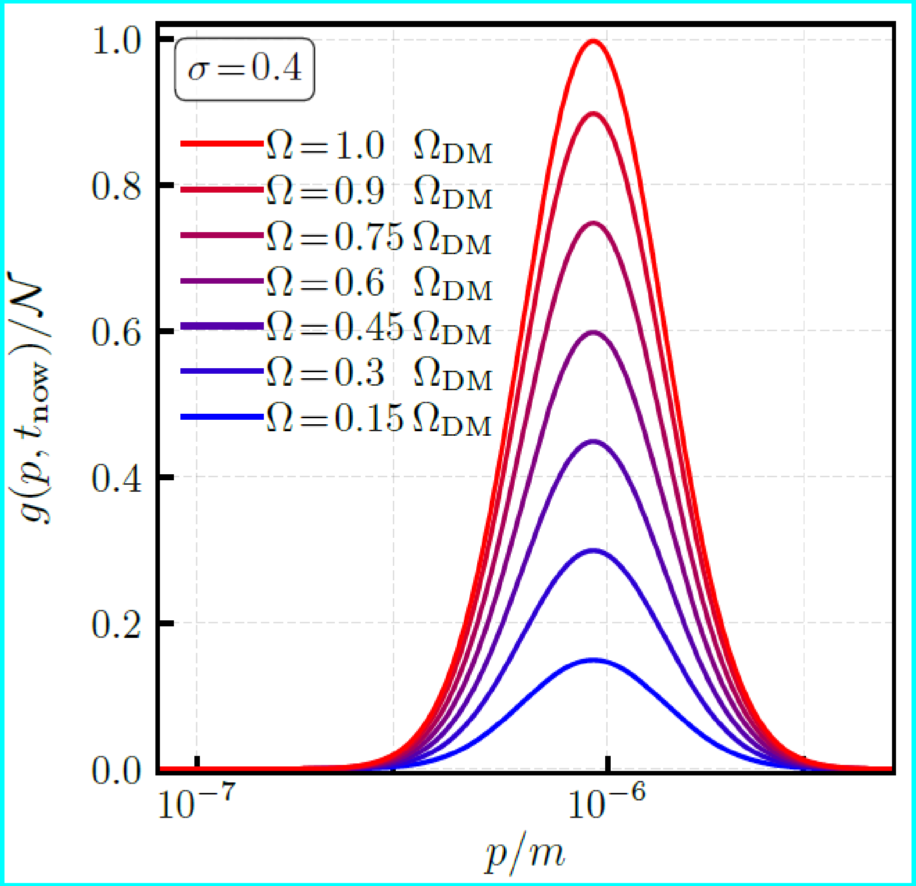
Moreover, because this *k*-profile lives in the same space as *P(k)*,
these two functions can even be plotted together along the ***same*** axis!

Now it makes sense to ask:

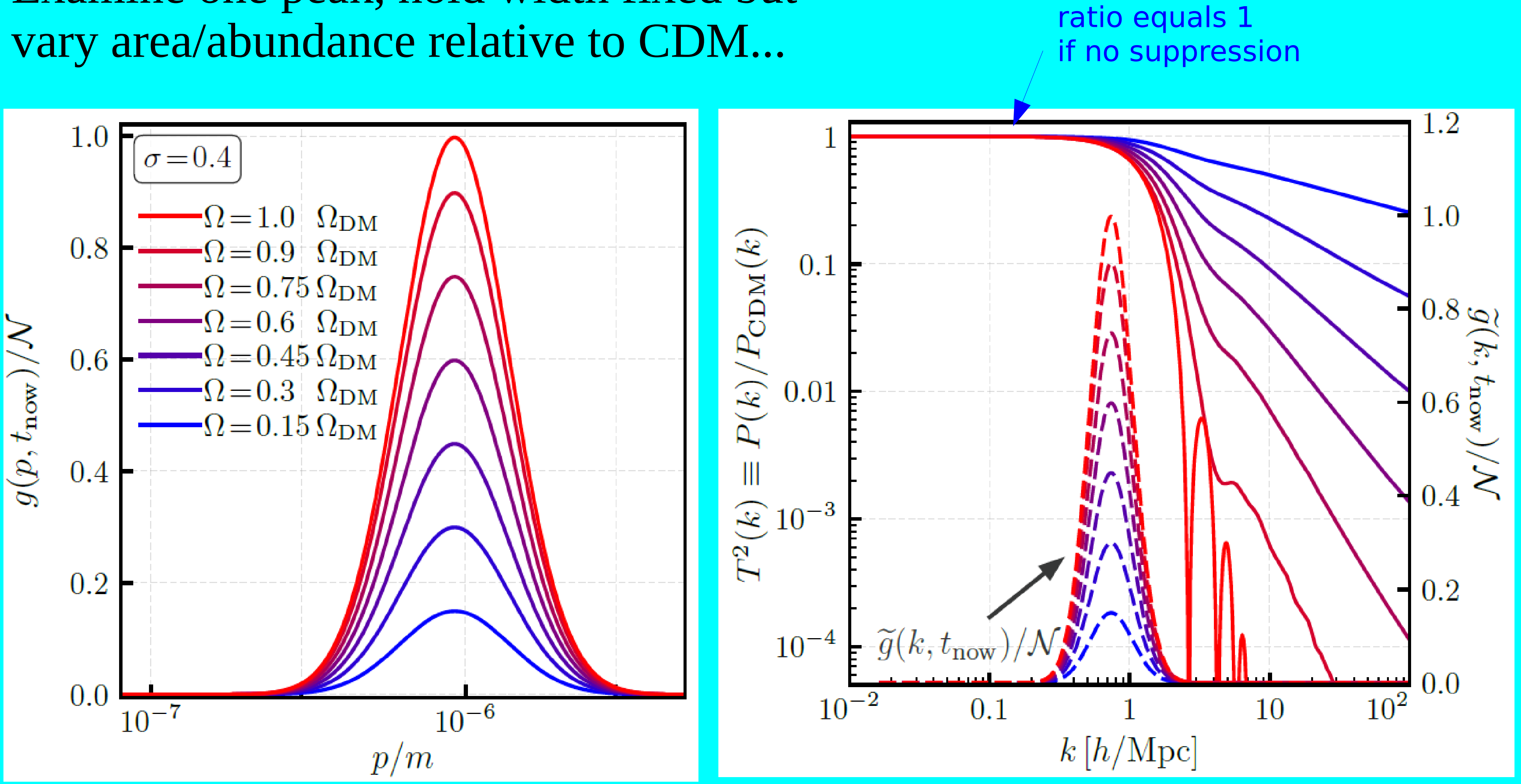
Can we discover/conjecture any relation between *these* two functions?

So let's explore...

Examine one peak, hold width fixed but vary area/abundance relative to CDM...

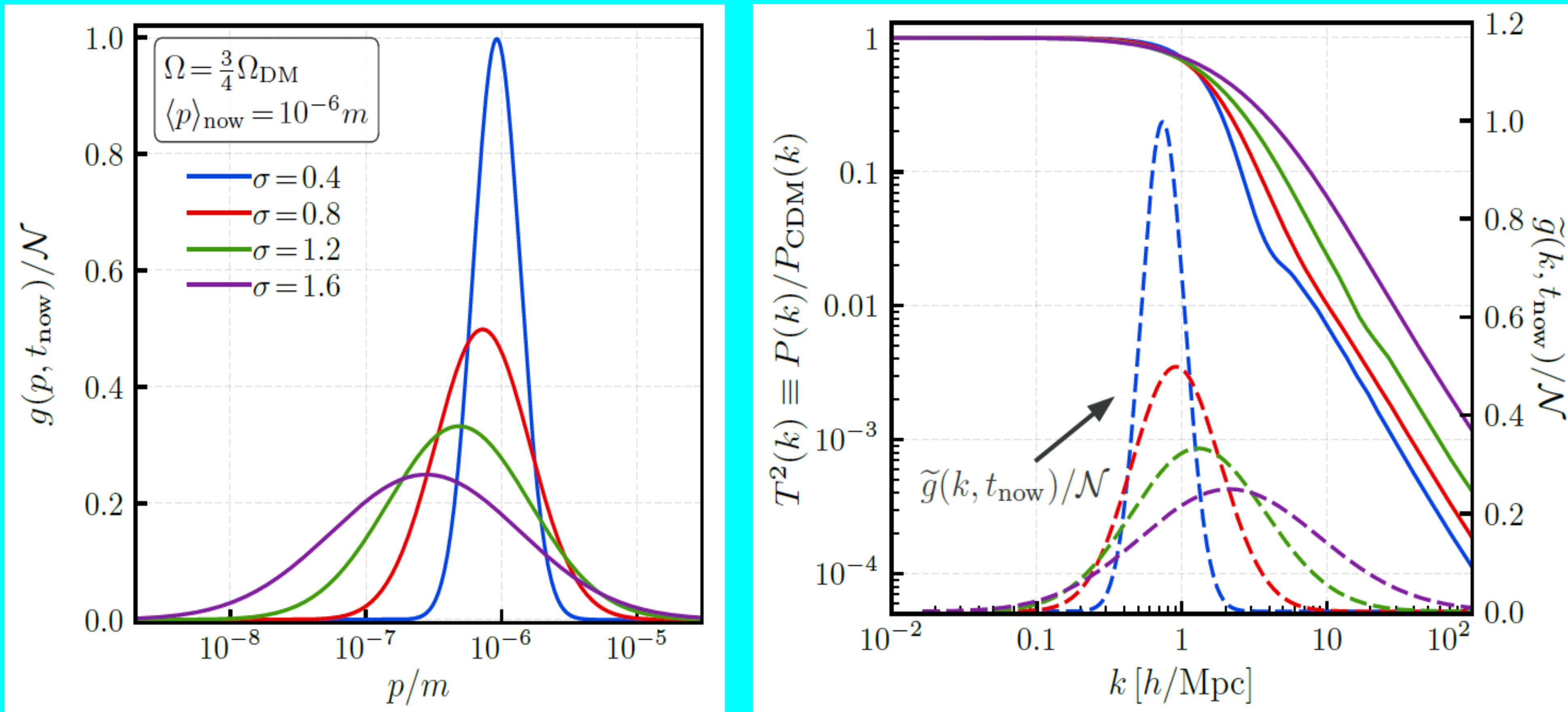


Examine one peak, hold width fixed but vary area/abundance relative to CDM…



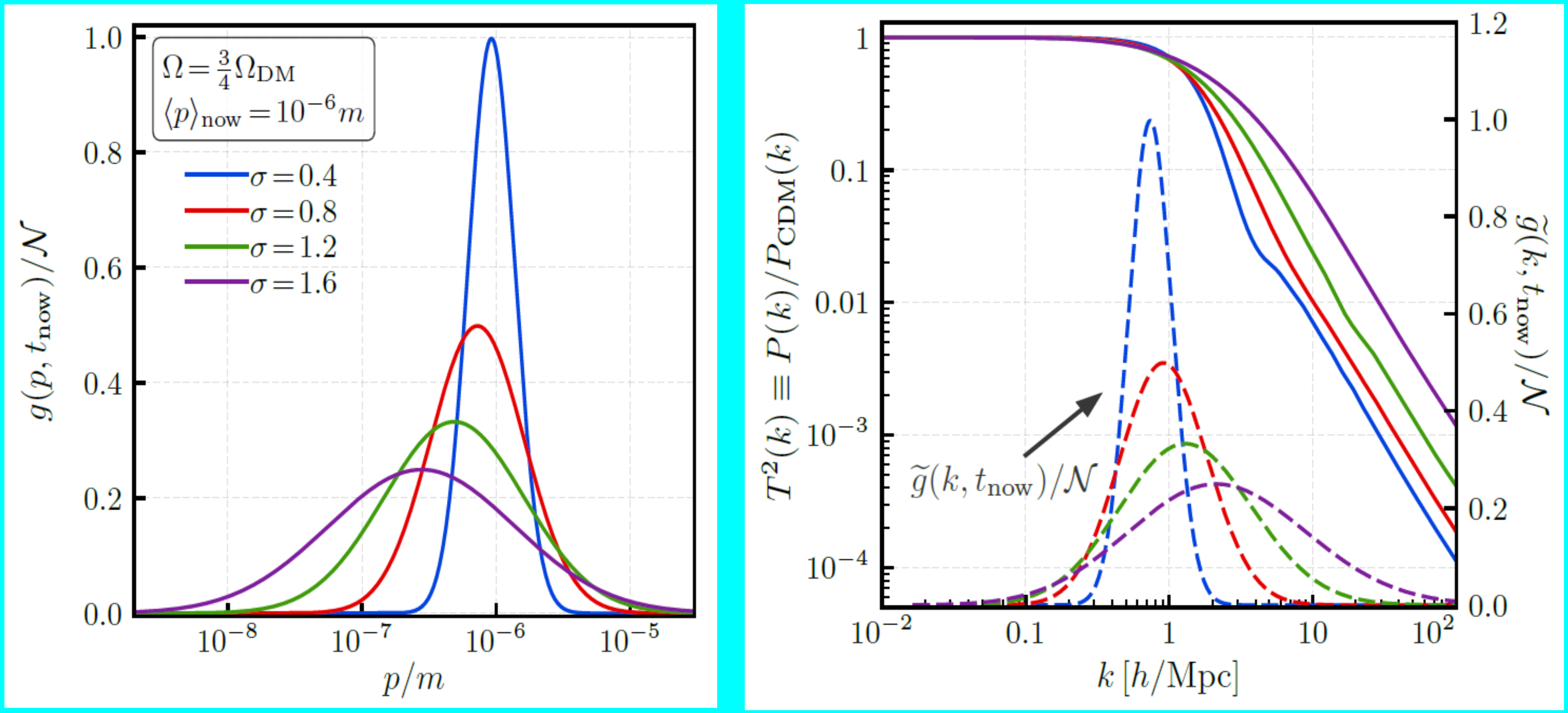
- Wiggles from DM acoustic oscillations emerge more dramatically as suppression is enhanced ⇨ irrelevant for us.
- More abundance ⇨ stronger **suppression** at larger *k*
  ⇨ steeper **slope** at larger *k*.

Examine one peak, now hold abundance and
<*p*> fixed relative to CDM but vary width...



- *Note*: Holding <*p*> fixed, vary width ⇨ <log *p*> shifts (as above)
- Increasing width ⇨ **slower** change in slope
  ⇨ **less suppression** at large *k*
  ⇨ **BUT slope at large *k is the same!!***

Examine one peak, now hold abundance and
<*p*> fixed relative to CDM but vary width...

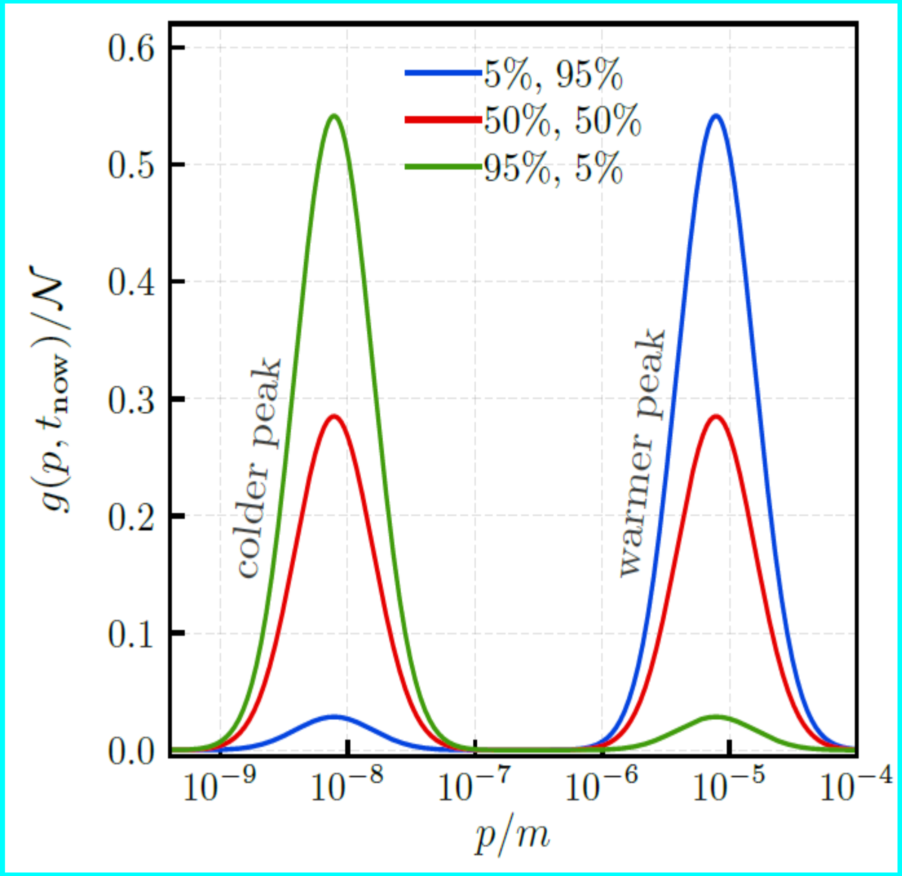


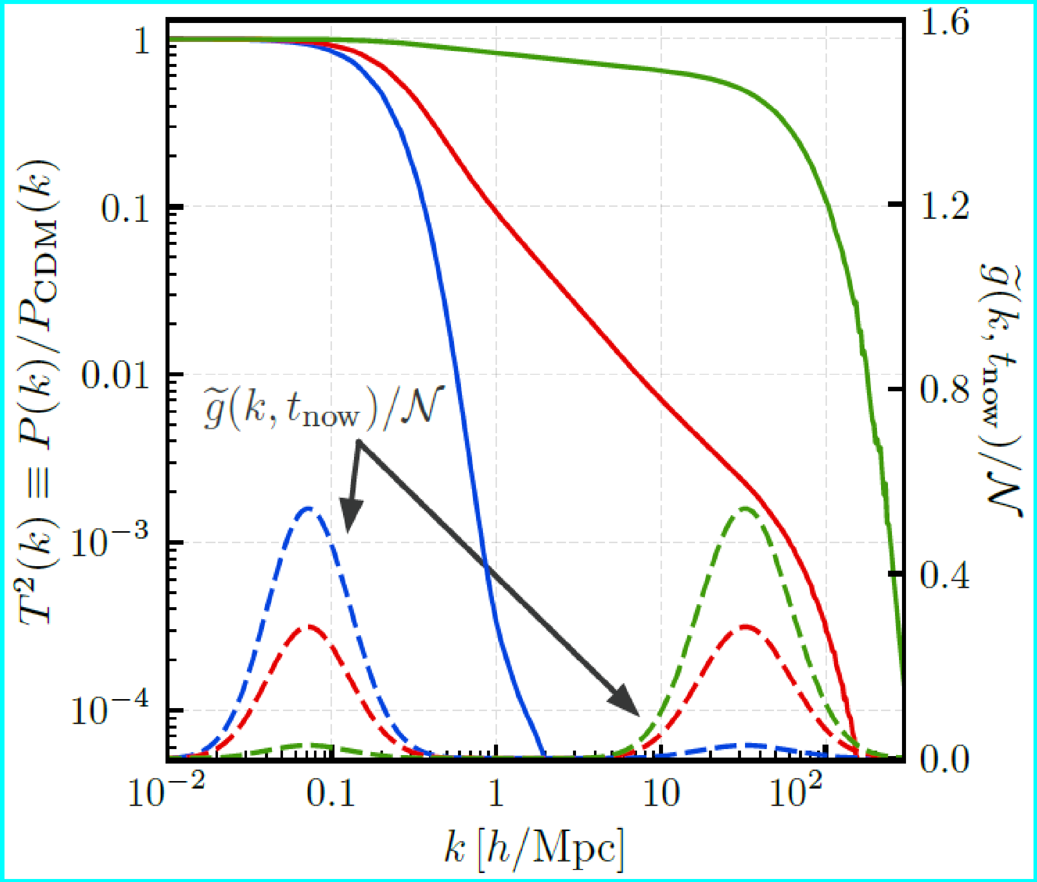
- Suggests that accumulated abundance ⟺ <u>slope</u> of transfer function!
- Indeed, as we sweep left to right in *k*-space,
  more accumulated abundance ⟹ slope increasingly steep.
- *Note*: at large *k*, same accumulated abundance but different *suppression*!

Abundance correlates <u>not</u> with net suppression, but with its <u>slope</u> !!

Does this behavior survive for more complex *g(p)*?

Examine *two* peaks, vary relative abundances between them...





- As we sweep left to right in *k*-space,
  - *within* peaks: accumulated abundance increases ⇨ slope increases!
  - *between* peaks: no accumulation of abundance ⇨ slope approximately constant!
- Thus, still find **accumulated abundance ⟺ slope!**

Let's formalize this quantitatively.

At any value of *k*, the total accumulated abundance is

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \widetilde{g}(k')\, d\log k'}{\int_{-\infty}^{+\infty} \widetilde{g}(k')\, d\log k'}$$

Indeed, for any value of *k*, this is the **fraction of the dark-matter number density which is effectively "hot"** (*i.e.*, free-streaming) relative to the corresponding value of $p = k_{\mathrm{FSH}}^{-1}(k)$!

inverse of the free-streaming relation

We shall therefore refer to ***F(k)*** as the ***hot fraction function*****.**

Our claim, then, is that the *slope* of the transfer function at any value of k *is directly related to* $F(k)$!

$$F(k) \;\approx\; \eta\left(\left|\frac{d\log T^2}{d\log k}\right|\right)$$

some as-yet unknown function η

Equivalently, taking derivative of both sides,

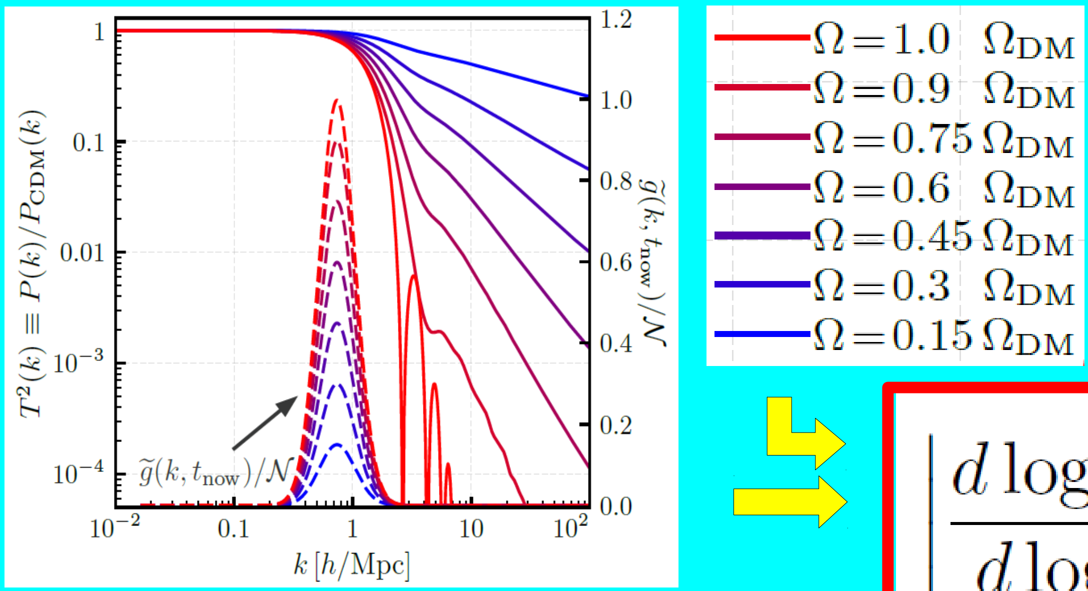
$$\frac{\tilde{g}(k)}{\mathcal{N}} \;\approx\; \eta'\left(\left|\frac{d\log T^2}{d\log k}\right|\right)\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

DM phase-space distribution!

first derivative of T^2

second derivative of T^2

Pushing this further,
we can even conjecture a specific function η !



$$\left|\frac{d\log T^2}{d\log k}\right| \approx [F(k)]^2 + \frac{3}{2}F(k)$$

approximate relation holds to very high precision!

Our conjecture then takes the non-trivial form

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16} + \left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2} \left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

This would allow us to "resurrect" *g(k)* from the transfer function *T²(k)*!

Technical point...

- This conjecture assumes/requires that the transfer function has a *negative-semidefinite second derivative* (i.e., constant slope or concave-down).

- Generally, this tends to occur in situations in which our dark-matter distributions –- no matter how complex in shape –- are relatively "clustered" in *k*-space.

- If there are widely separated clusters in the DM distribution, then our conjecture is expected to hold within each cluster individually.

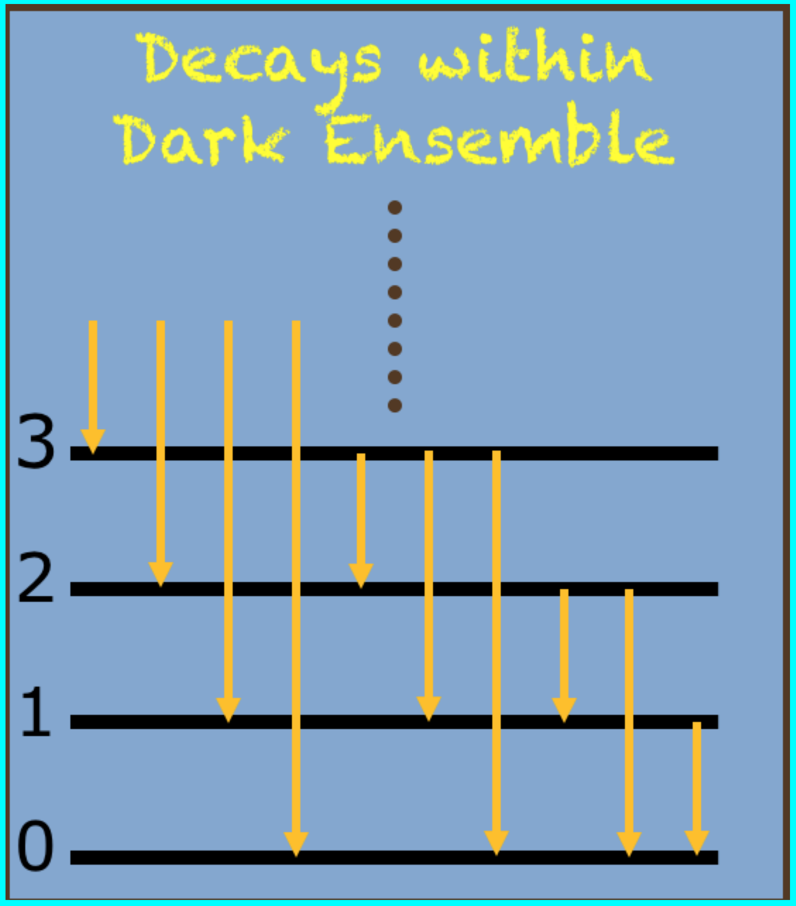
- As we shall see, this restriction to clusters is not severe, and still allows us to resurrect *g(p)* for a wide variety of models of non-trivial early-universe dynamics.

<u>Rest of talk</u>:

Let's now see how these ideas play out in practice!

In general, the dark sector can contain *many* components with many different masses and many possible decay chains.

How robust are our observations?



Let's consider a toy model...

Dark ensemble consists of *N+1* real scalars ϕ_j with $j = 0,1, \ldots N$, and a mass spectrum:

$$m_j = m_0 + j^\delta \Delta m$$

In our analysis we will consider ***10 distinct*** levels…

Lagrangian:

$$\mathcal{L} = \sum_{\ell=0}^{N} \left(\frac{1}{2} \partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell} \sum_{j=0}^{i} c_{\ell ij}\, \phi_\ell \phi_i \phi_j \right) + \cdots$$

The trilinear coupling:

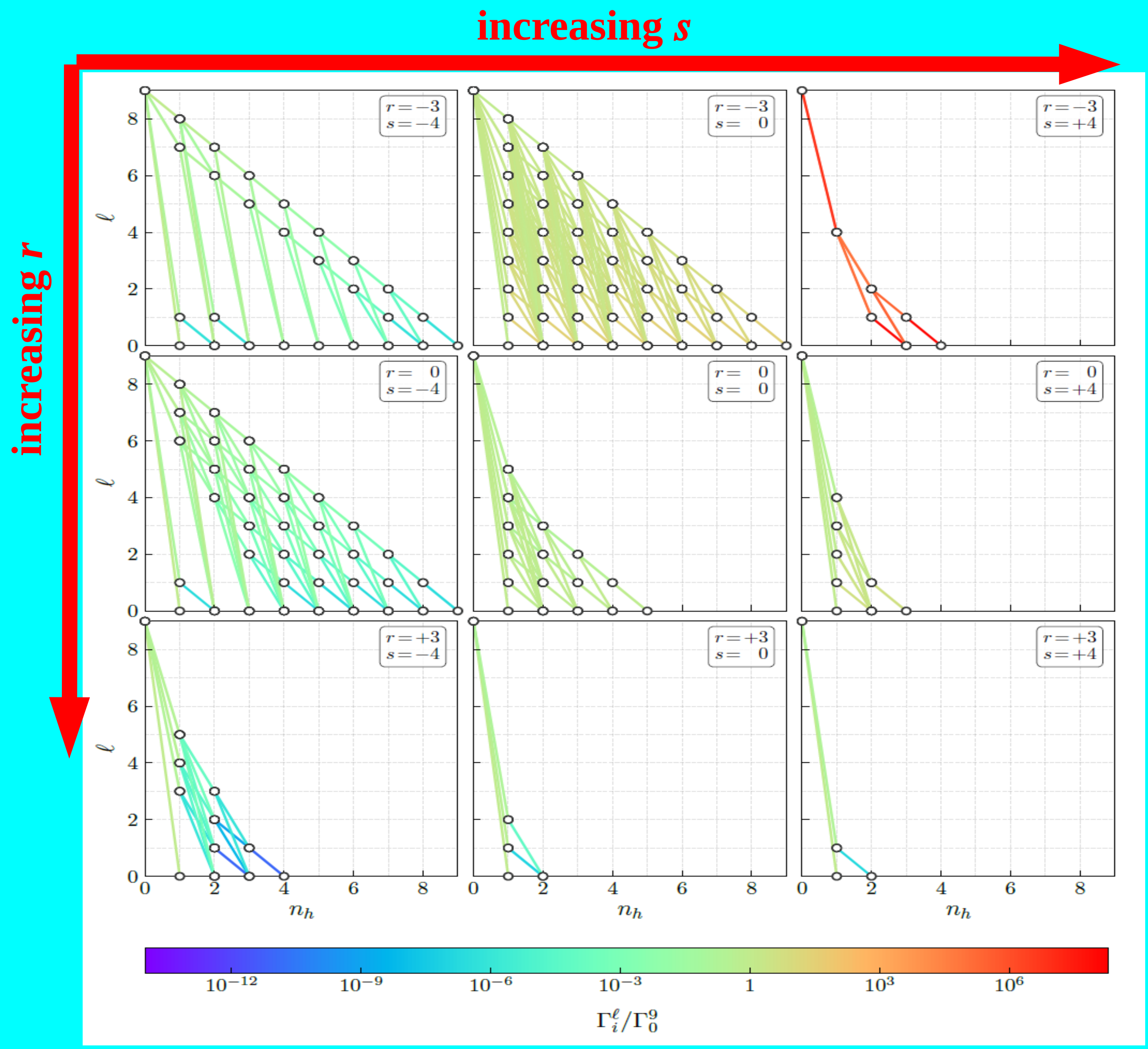
mass difference between parent and daughters

mass difference between daughters

$$c_{\ell ij} = c_0 \mu R_{\ell ij} \left(\frac{m_\ell - m_i - m_j}{\Delta m} \right)^r \left(1 + \frac{m_i - m_j}{\Delta m} \right)^{-s}$$

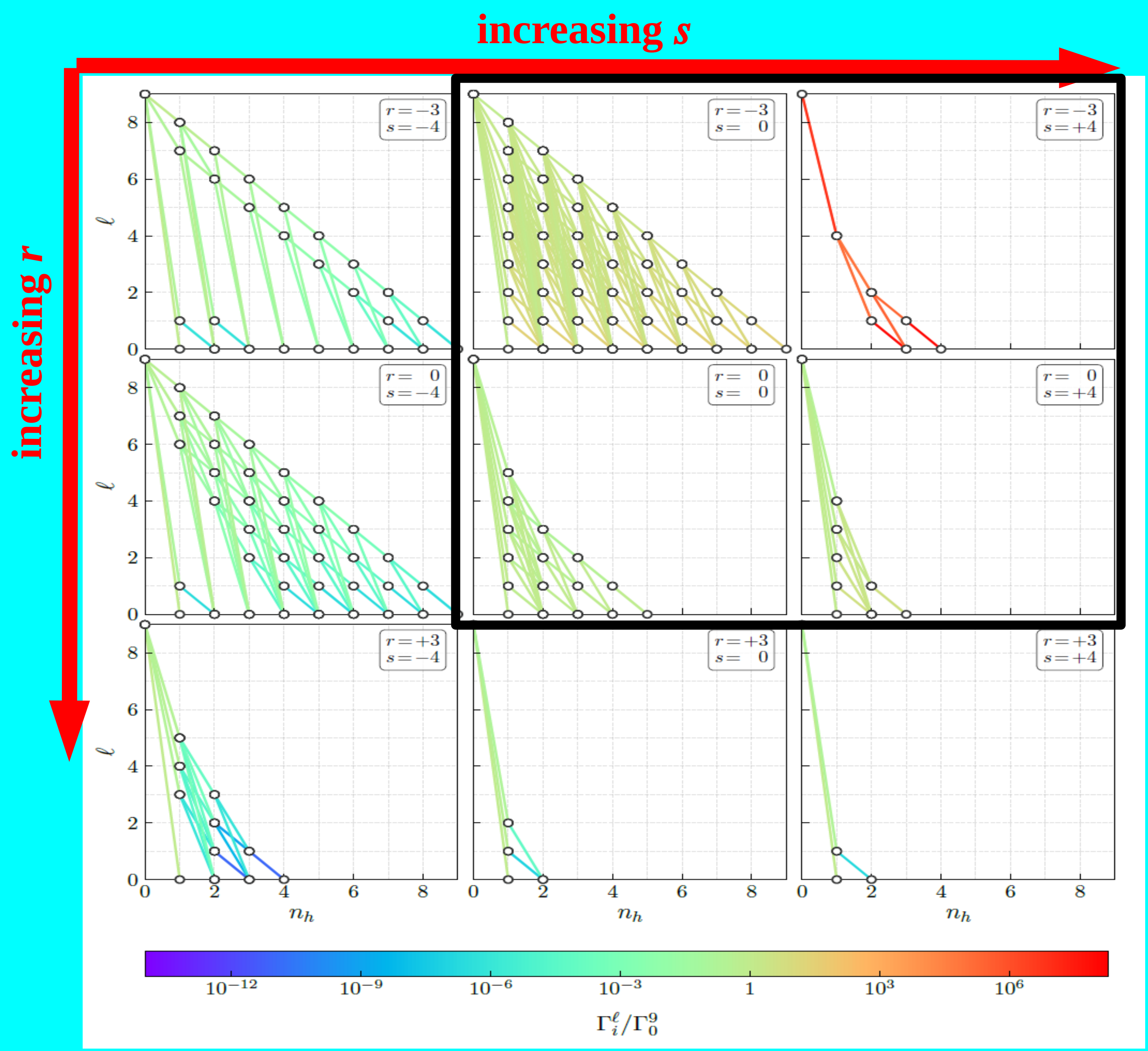
- **Larger *r* :** prefers decays yielding more "<u>**r**</u>adiation" (big mass jumps)
- **Larger *s* :** prefers decays with more <u>**s**</u>ymmetry between daughters

Many possible patterns of decay chains, depending on (*r*,*s*)...



Color indicates normalized decay rate

Many possible patterns of decay chains, depending on (*r*,*s*)...

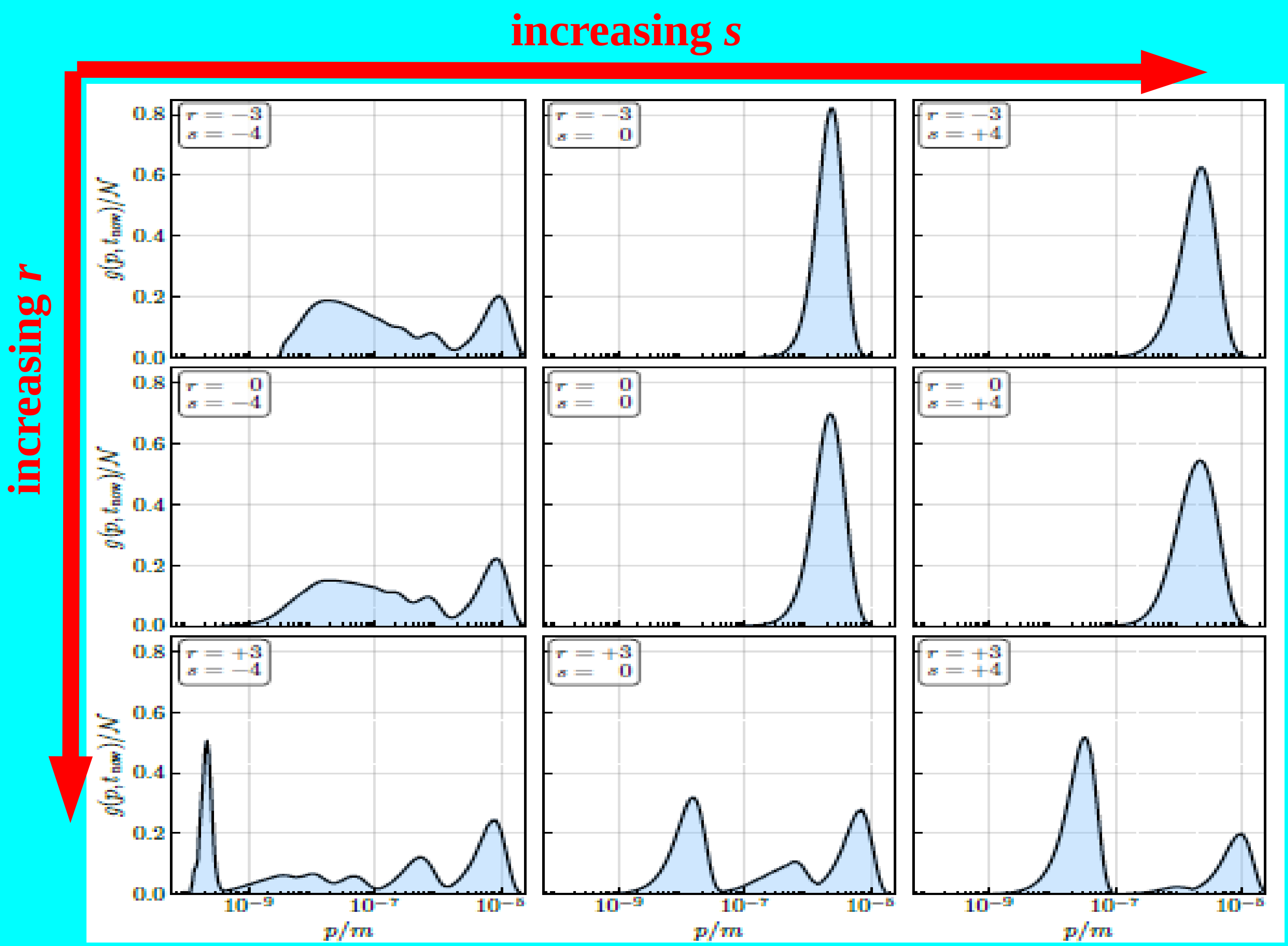


Color indicates normalized decay rate

Deposits to the ground state tend to happen around the same time

Deposits to the ground state tend to happen at different times

Finally, obtain resulting phase-space distributions *g(p)* after all decays have concluded.



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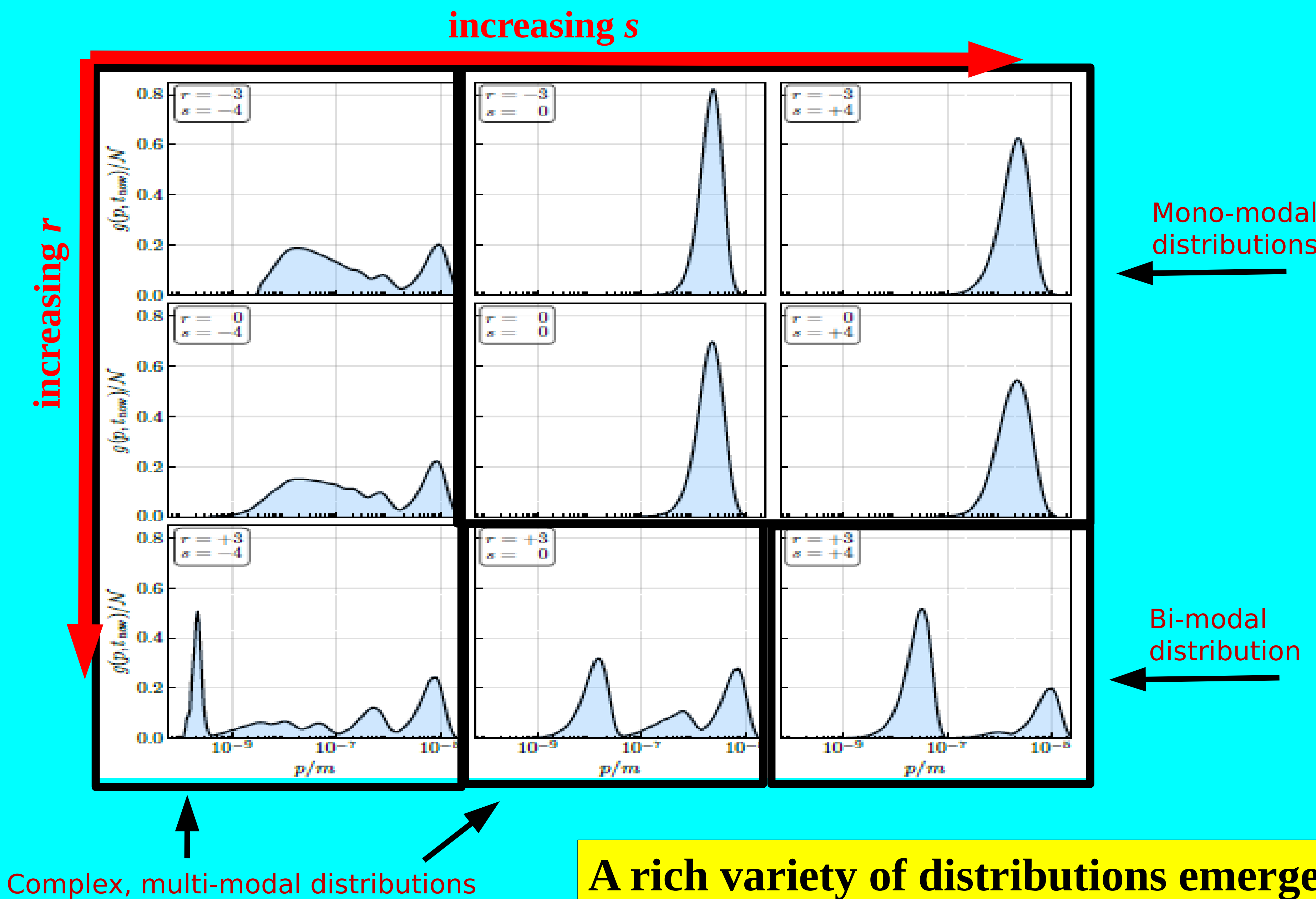

A rich variety of distributions emerges!

Finally, to what extent can we "resurrect" the dark-matter phase-space distribution from the transfer function?

Recall our conjecture....

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16}+\left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2}\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

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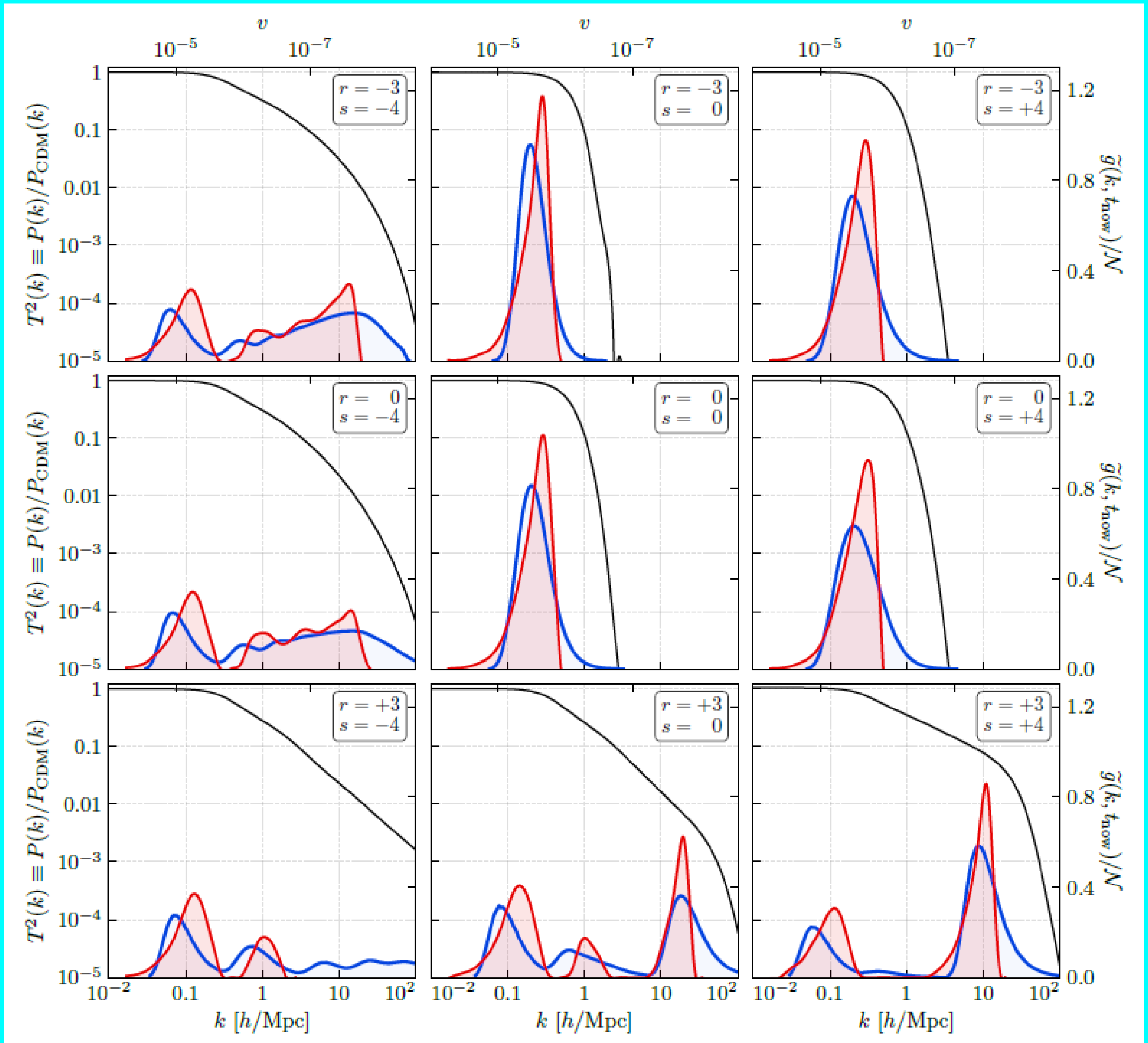

Blue outline = original k-space DM distribution

Pink shaded = reconstruction directly from transfer function

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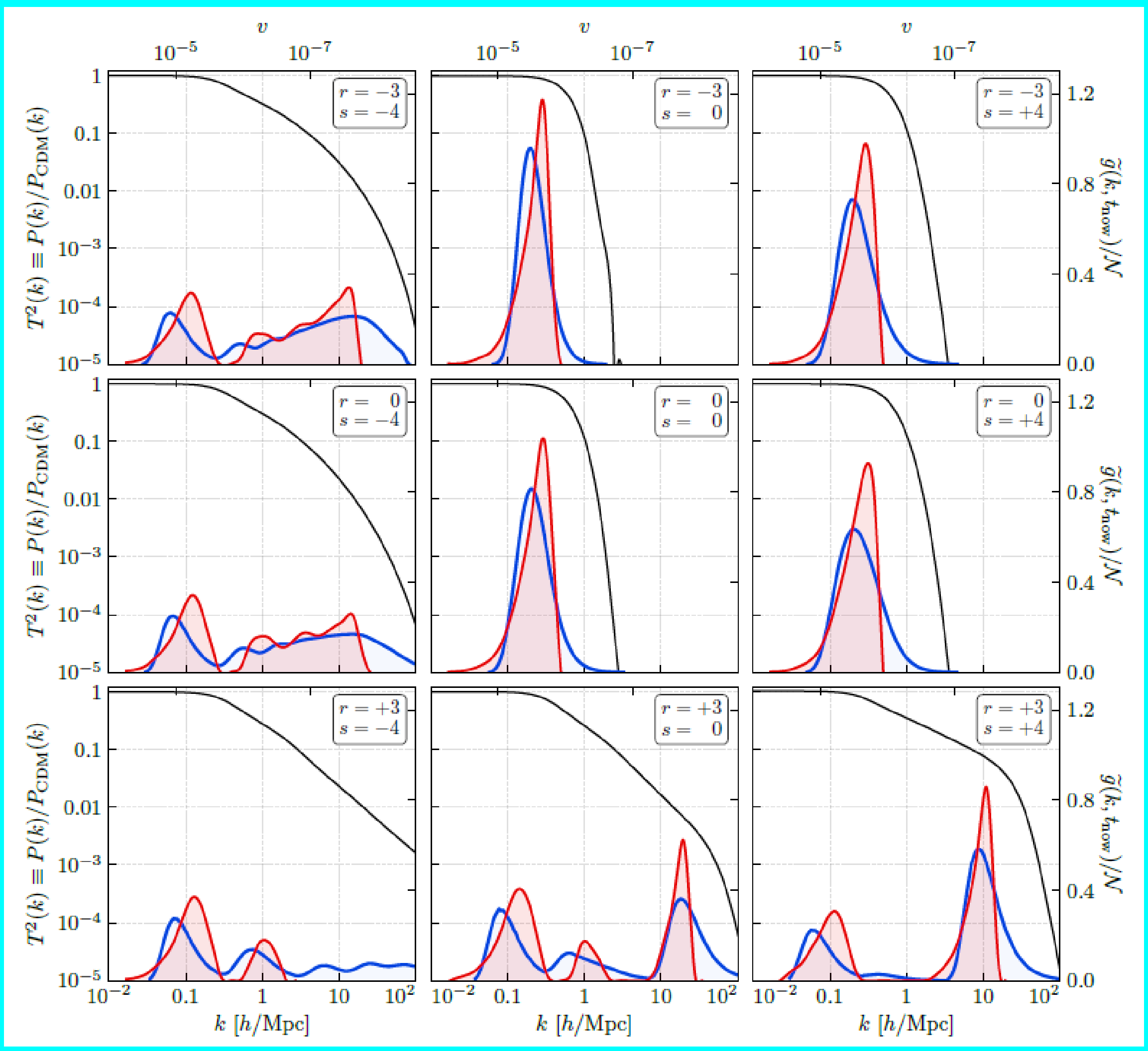


Blue outline = original *k*-space DM distribution

Pink shaded = reconstruction directly from transfer function

Archaeological reconstruction is surprisingly accurate for a **variety** of possible DM distribution shapes (thermal, non-thermal, uni-modal, multi-modal, *etc*.)!

Pushing these ideas one step further...

The power spectrum *P(k)* tells us about the spectrum of relative matter overdensities when they are still small ($\delta \ll 1$).

- Over time, each matter overdensity δ will grow. Once δ enters the non-linear regime ($\delta \sim 1$), gravitational interactions within the overdensity become important.
- Eventually the non-linearities take over, and the overdense region experiences **gravitational collapse**.
- The end result a **virialized dark-matter halo with a mass *M***.

Since *P(k)* describes the spectrum of initial matter overdensities, *P(k)* should also determine the spectrum of virialized dark-matter halos:

$$P(k) \longrightarrow dn/d\log M$$

Differential number density of halos per unit log(mass).

$P(k)$? $dn / d \log M$

Highly non-linear process! No rigorous analytical method exists.
State of the art: **Press-Schechter formalism**. *Ingredients*:

- Effective mass/radius relation $M \equiv \frac{4\pi}{3}\bar{\rho}(c_W R)^3$ with $c_W \sim 2.5$
- Assume linear growth in overdensity δ until critical value δ_c, then sudden and instantaneous gravitational collapse.

$$\delta_c = \delta(t_c) = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$$

t_c = time of collapse from spherical-collapse model
$\delta(t)$ functional form from linear-growth model

- Assume Prob (collapse has occurred producing halo of mass $m>M$) = **2** x Prob($\delta_M > \delta_c$, where δ_M = average over volume of radius $R(M)$)

Effects of gravitational collapse drawing in mass from *outside* the critically overdense regions, so that most mass is eventually pulled into halos

So need to know $P(\delta_M > \delta_c)$. Model this as

$$P(\delta_M > \delta_c) = \int_{\delta_c}^{\infty} d\delta_M \; f(\delta_M)$$

where we choose

$$f(\delta_M) = A\sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma}\left[1+\left(\frac{\sigma^2}{\delta_c^2}\right)^p\right]\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$$

Sheth-Tormen function

$A \sim 0.3222$, $p \sim 0.3$

and where

$$\sigma^2 = \int_0^{\infty} d\log k \; W^2(k,R)\,\frac{k^3 P(k)}{2\pi^2}$$

variance of density fluctuations

window function – *e.g.*, $W(k,R) = \Theta(1-kR)$

Matter power spectrum enters here!

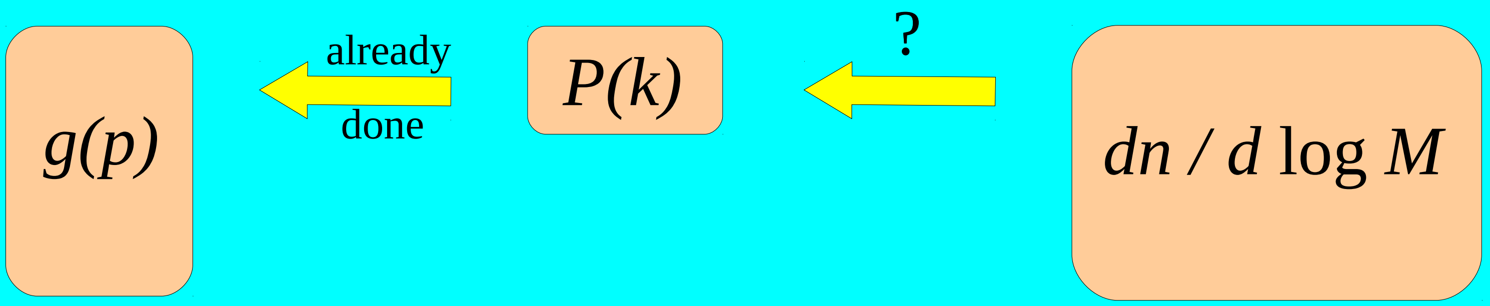
We can then pull all the pieces together to obtain *dn*/*d*log*M*:

$$\frac{dn}{d\log M} = 2\left(\frac{\bar{\rho}}{M}\right)\frac{\partial P(\delta_M > \delta_c)}{\partial \log M}$$

number density of halos of mass *M*

In general, the connection *P(k)* and *dn/d*log*M* is highly non-trivial, including all the non-linearities associated with gravitational collapse.

- However, is there a way of using *dn/d*log*M* to reconstruct *P(k)*?



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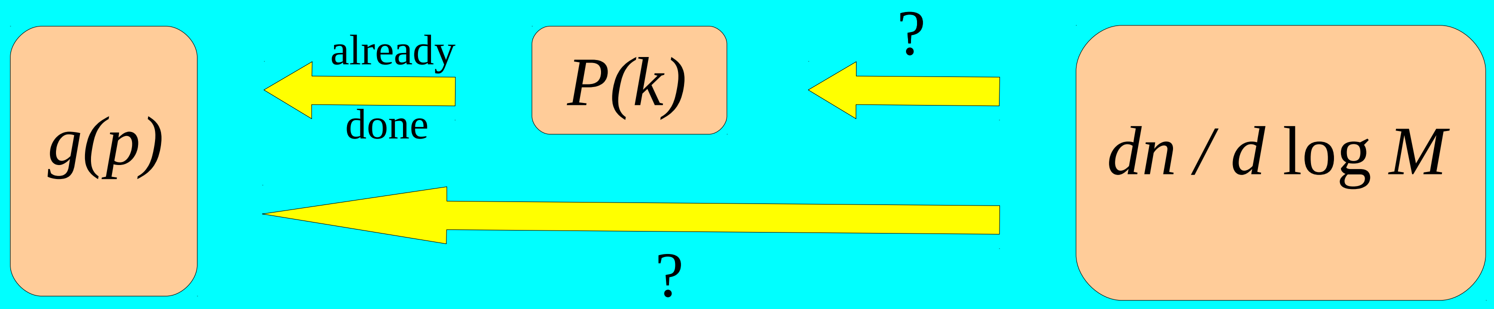


- **Even more ambitiously**, is there a way of using *dn/d*log*M* to go all the way back in one step and reconstruct the **primordial dark-matter velocity distribution *g(p)*** directly?

Once again, our approach is highly unorthodox and similar to what we did for the previous reconstruction!

- Recall effective mass/radius relation and window function

$$M \equiv \frac{4\pi}{3}\overline{\rho}(c_W R)^3 \qquad W(k,R) = \Theta(1-kR)$$

⇨ for any *k*, corresponding *M* can be *as large as* ~ $1/k^3$.

- Let us instead take these relations as defining a *functional map* between the previous variable *k* and a new variable *M*:

$$M(k) \equiv \frac{4\pi}{3}\overline{\rho}\left(\frac{c_W}{k}\right)^3$$

- ***Now we can express all of our quantities in (p, k, or M)-space and compare them on the same axes!***

Define hot-fraction function *F(M)* in *M*-space:

$$F(M) \equiv \frac{1}{\mathcal{N}} \int_{\log M}^{\infty} d\log M' \, g_M(M')$$

Define halo-mass suppression function:

$$S(M) \equiv \sqrt{\frac{dn/d\log M}{(dn/d\log M)_{\rm CDM}}}$$

analogue of transfer function *T(k)*

We then find

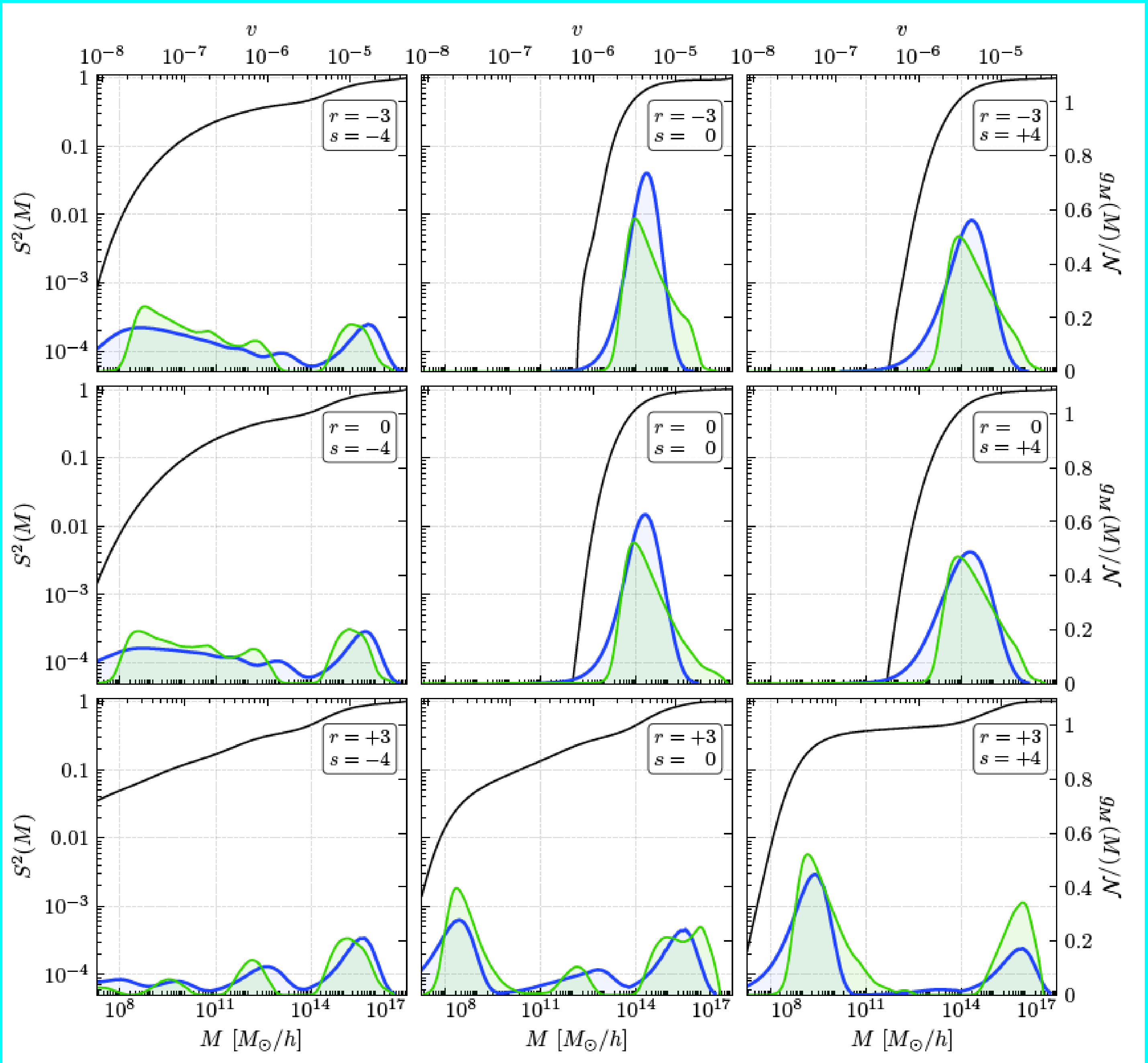
$$\frac{d\log S^2(M)}{d\log M} \approx \frac{7}{10} F^2(M)$$

Direct relationship between hot-fraction function and slope of suppression function!

$$\frac{g_M(M)}{\mathcal{N}} \approx \sqrt{\frac{5}{14}} \left(\frac{d\log S^2(M)}{d\log M}\right)^{-1/2} \left|\frac{d^2 \log S^2(M)}{(d\log M)^2}\right|$$

Can reconstruct DM phase-space distribution *g(M)* directly from $S^2(M)$!!

Finally, how well does it work?



Blue outline = original DM distribution

Green shaded = reconstruction directly from halo-mass function

Again surprisingly accurate for all DM distribution shapes (thermal, non-thermal, uni-modal, multi-modal, *etc*.)!

Conclusions

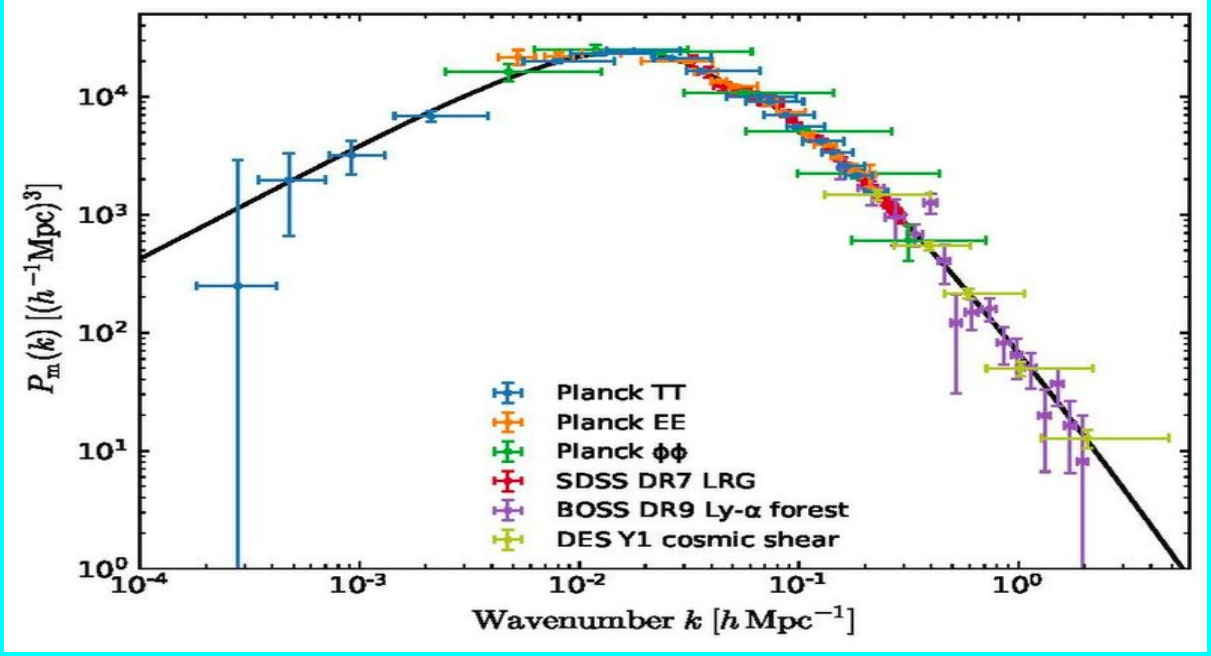
- Early-universe processes such as decays in non-minimal dark sectors can leave identifiable imprints in *f(p)* and *P(k)*, certain features of which may allow us to go backwards and archaeologically reconstruct the early-universe dynamics.
  - ***Useful tools are possible multi-modality of f(p) and hot fraction function F(k).***
  - ***We even conjectured a relation which enables us to "resurrect" f(p), given P(k).***
- Such approaches may ultimately be the *only* way of learning about dark-sector dynamics if the dark sector has no direct couplings to the SM.
- The dark sectors of string theory generically include unstable KK towers of the form we have discussed here. **Thus string theory generically leads to multi-modal *f(p)* distributions and non-trivial *P(k)* spectra.** This provides motivation to measure *P(k)* with increased precision, even beyond current experimental limits.

Yet to explore...

- How to incorporate effects that might come from couplings to SM? Could potentially affect evolution of phase-space distributions in additional subtle ways.
- Incorporation of observational bounds and constraints (Lyman α, *etc.* – w/Haibo Yu)
- Do these kinds of transfer functions fall within the general forms expected from effective theories of structure formation?
- We have thus far studied only the *linear* power spectrum. Can this analysis be extended to the *non-linear* regime (even higher *k*)?

The matter power spectrum *P(k)* at large *k*

Our ability to perform this kind of archaeological reconstruction is predicated on our ability to measure *P(k)* at large *k*. Understanding how well we can probe *P(k)* at large *k* is therefore critical for our work.



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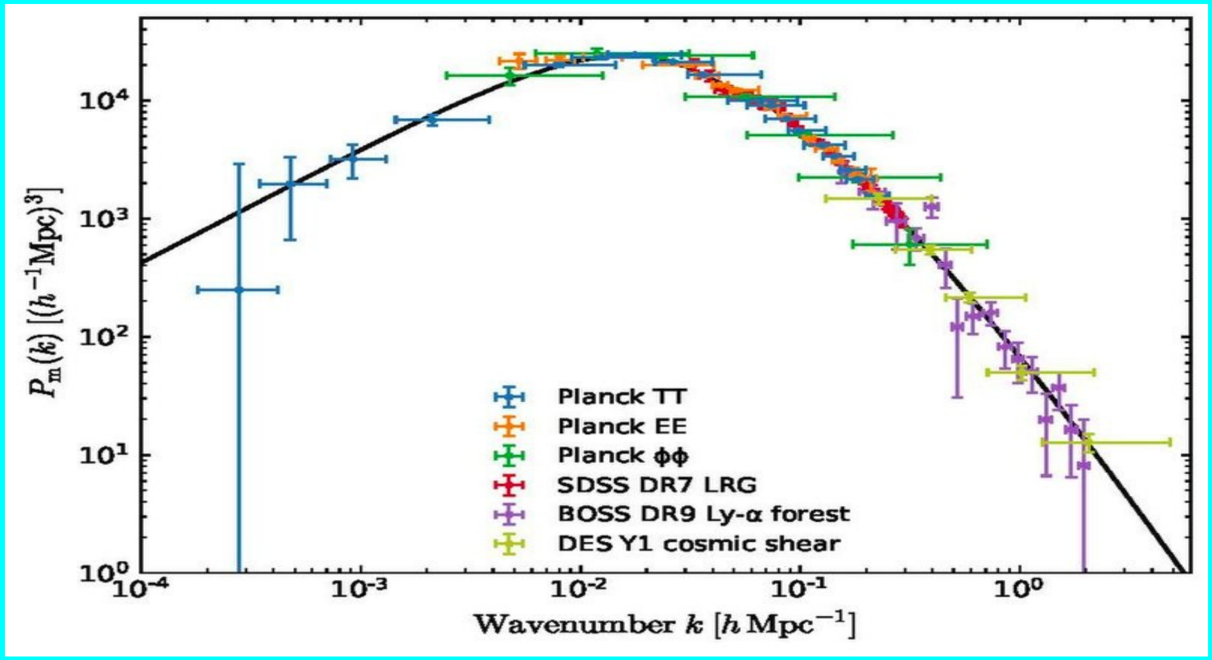

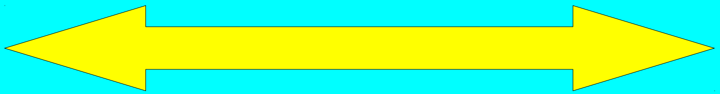

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- Linearized time evolution valid

 *P(k)* reliable!

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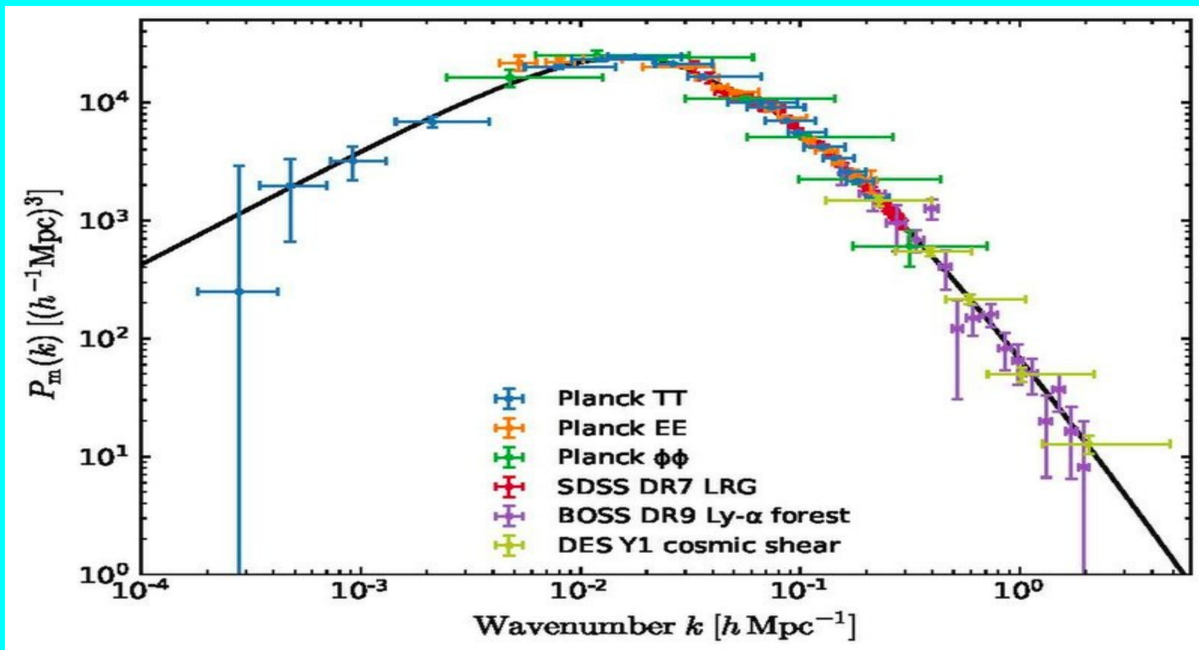

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- Lyman-α data up to *z*~5
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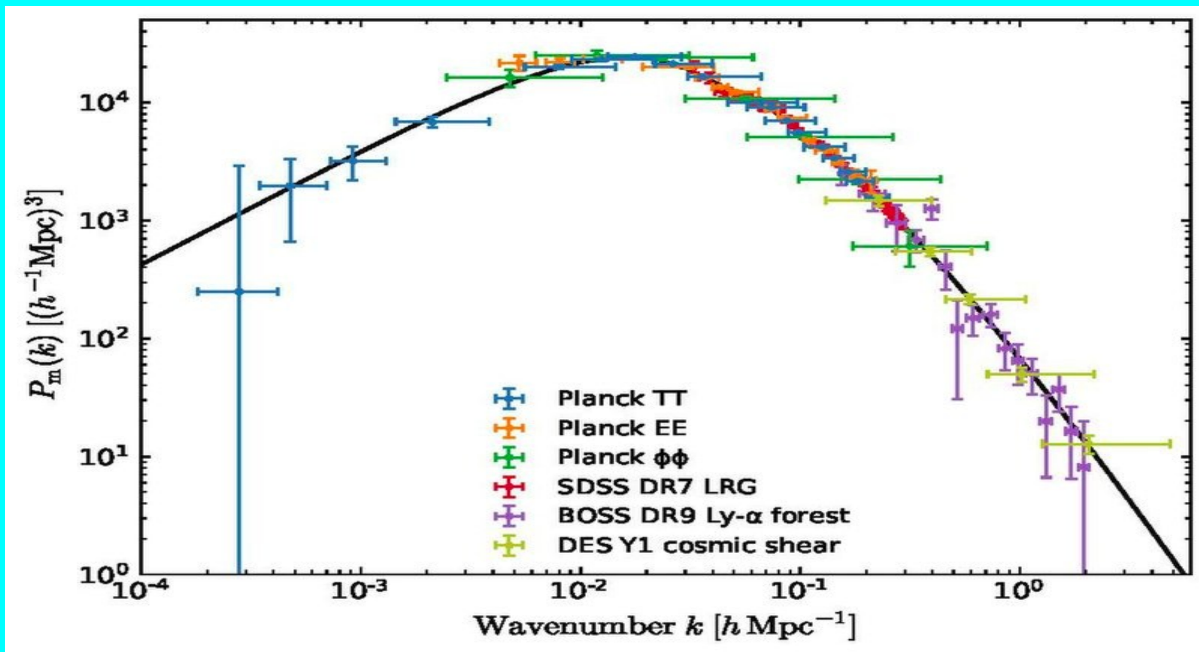

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- Can in principle probe *z*~6 to *z*~30.
- *Challenging*: would require extreme sensitivity, large collection area, but may be feasible [S. White]

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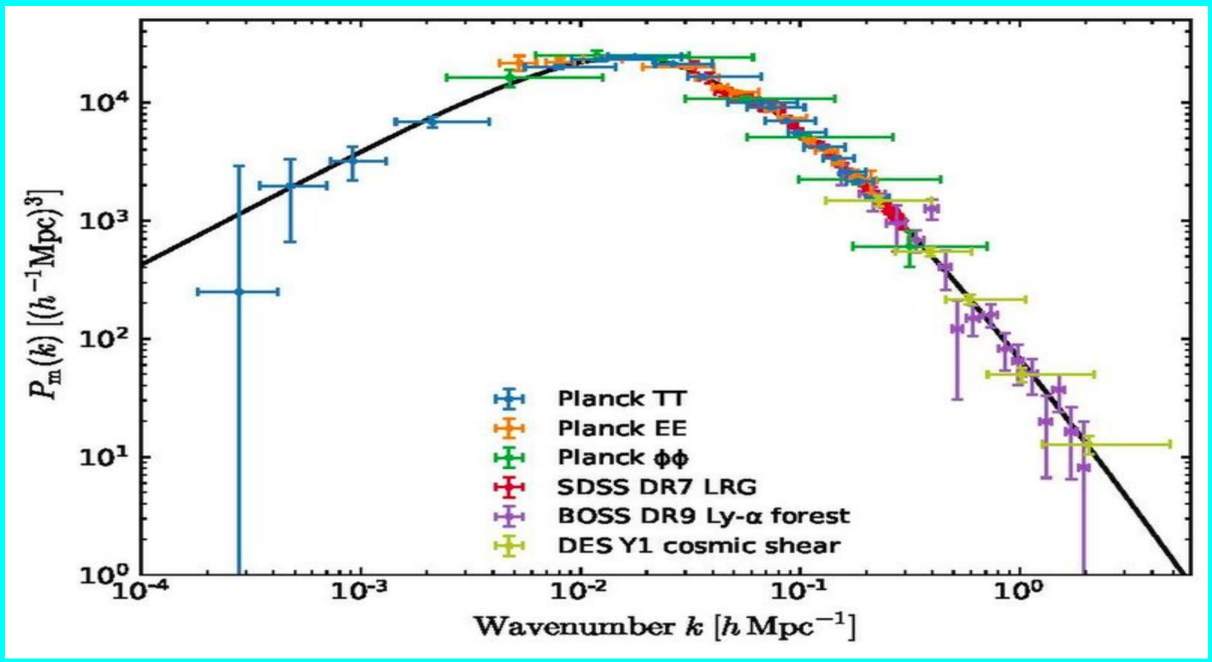

Beyond this, higher-*k* physics enters the non-linear regime. Would need to analyze the *non-linear P(k)*! Do similar reconstructions exist for non-linear *P(k)*? [work in progress]

- Computationally intensive
- Important for small-scale anomalies

?

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Final Comment

In this talk we have concentrated on situations in which the decays of the ensemble constituents have occurred long before the present time.

→ Thus, the higher components have long since been completely depopulated, and the dark matter today consists of only the lightest constituent.

However, what if our timescales are different, and these sorts of decays are continuing to occur, with many ensemble constituents still carrying sizable cosmological abundances and decaying even today?

Is this a logical possibility?
Is this a viable framework for dark-matter physics?

Dynamical Dark Matter (DDM)
an alternative framework for dark-matter physics

DDM originally proposed in 2011 with **Brooks Thomas...**

- 1106.4546
- 1107.0721
- 1203.1923

and then further developed in many different directions with many additional collaborators...

- 1204.4183 (also w/ S. Su)
- 1208.0336 (also w/ J. Kumar)
- 1306.2959 (also w/ J. Kumar)
- 1406.4868 (also w/ J. Kumar, D. Yaylali)
- 1407.2606 (also w/ S. Su)
- 1509.00470 (also w/ J. Kost)
- 1601.05094 (also w/ J. Kumar, J. Fennick)
- 1606.07440 (also w/ K. Boddy, D. Kim, J. Kumar, J.-C. Park)
- 1609.09104 (")
- 1610.04112 (also w/ F. Huang and S. Su)
- 1612.08950 (also w/ J. Kost)
- 1708.09698 (also w/ J. Kumar, D. Yaylali)
- 1712.09919 (also w/ J. Kumar, J. Fennick)
- 1809.11021 (also w/ D. Curtin)
- 1810.10587 (also w/ J. Kumar & P. Stengel)
- 1909.07981 (also w/ A. Desai)
- 1910.01129 (also w/ D. Kim, H. Song, S. Su & D. Yaylali)
- 1912.10588 (also w/ Y. Buyukdag & T. Gherghetta)
- **… plus ongoing collaborations with many others...!**