Lorentz Invariance from Locality of Massless Spin 2

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A few fundamental principles

1. Rotation and translation symmetries
2. Locality
3. Unitarity

There is an additional symmetry under Lorentz boosts.
Fundamental, or derivable?

Otherwise, are there theories satisfying 1 – 3 that are not Lorentz invariant?

Outline

General theory of spin 1 without assuming Lorentz invariance

Consistent picture of the spin 1 case:

- The Standard Model does not require Lorentz boost invariance. E.g. the speed c could be different in every sector: c_γ, $c_e, \ldots$

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- Additionally, requiring locality in leading order interactions without Lorentz invariance $\Longrightarrow$
  1. The photon sector separately must be fully Lorentz invariant, i.e. is standard.
  2. The matter sector need not be Lorentz invariant, though must have a conserved charge: $\dot{Q} = \partial_t \int d^3x\, J_0 = 0$.

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More interesting with spin 2?

Outline

Standard Lorentz invariant theory

$$\mathcal{L}_{\mathrm{GR}} = \frac{1}{2}\left(\eta^{\alpha\beta}\partial_\alpha h^{\mu\nu}\partial_\beta h_{\mu\nu} - \eta^{\mu\nu}\partial_\mu h^{(4)}\partial_\nu h^{(4)}\right) + \partial_\mu h^{\mu\nu}\partial_\nu h^{(4)} - \partial_\mu h^{\mu\alpha}\partial_\nu h^{\nu}_{\alpha} - \kappa\, h_{\mu\nu} T^{\mu\nu} + \ldots$$

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The leading order interaction is manifestly local:

$$\begin{aligned}\frac{\mathcal{L}_{\mathrm{ex}}}{\kappa^2} &= T_{\mu\nu}\frac{T^{\mu\nu}}{\Box} - \frac{T^{(4)}}{2}\frac{T^{(4)}}{\Box} \\ &= T_{ij}\frac{T_{ij}}{\Box} - \frac{T}{2}\frac{T}{\Box} + \frac{T_{00}}{2}\frac{T_{00}}{\Box} - 2T_{0i}\frac{T_{0i}}{\Box} + T_{00}\frac{T}{\Box}\end{aligned}$$

Here $T^{(4)} = \eta^{\mu\nu}T_{\mu\nu}$ and $T = \delta_{ij}T_{ij}$.

Non-Lorentz-invariant theory

Construction:

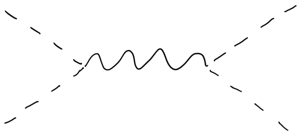
- Assume rotation invariance in order to *define* spin.
- Couple to some (not necessarily conserved) matter sector, $\mathcal{T}^{\mu\nu}[\Psi_m]$.
- Write down all the leading order operators:

$$\mathcal{L} = \Big[- A \; \dot{h}_{0i}\partial_j h_{ij} + B \; \dot{h}\partial_i h_{0i} - \frac{1}{2}\Big(C \; \partial_i h_{00}\partial_i h - D\partial_i h_{00}\partial_j h_{ij} + E \; \partial_i h \partial_j h_{ij} + F \, (\partial_i h_{0i})^2 - G \; \partial_j h_{ij}\partial_k h_{ik} - H \; \partial_j h_{0i}\partial_j h_{0i} \Big) - \frac{1}{4}\Big(I \; \dot{h}^2 - J \; \partial_i h \partial_i h - K \; \dot{h}_{ij}\dot{h}_{ij} + L \; \partial_k h_{ij}\partial_k h_{ij} \Big) \Big] - \kappa \, h_{\mu\nu}\mathcal{T}^{\mu\nu}$$

where $h = \delta_{ij} h_{ij}$.

$A = B = \ldots = L = 1$ is GR, while otherwise $\mathcal{L}$ breaks Lorentz invariance.

Tree-level scattering



E.g. the simplest 2 d.o.f. theory is immediately non-local:

$$\frac{\mathcal{L}_{\rm ex}}{\kappa^2} = \mathcal{T}_{ij}\frac{\mathcal{T}_{ij}}{\Box} - \frac{\mathcal{T}}{2}\frac{\mathcal{T}}{\Box} + \frac{\mathcal{T}_{00}}{2}\left[2a_5\nabla^2\partial_t^2 + a_6\nabla^4 + a_7\partial_t^4\right]\frac{\mathcal{T}_{00}}{\Box\nabla^4}$$
$$- 2\mathcal{T}_{0i}\left[\frac{a_3\nabla^2 + a_4\partial_t^2}{\Box\nabla^2}\right]\mathcal{T}_{0i} + \mathcal{T}_{00}\left[\frac{a_1\nabla^2 + a_2\partial_t^2}{\Box\nabla^2}\right]\mathcal{T}$$

where $a_{1,\dots,7}$ are polynomials of the coefficients $A,\dots,L$. E.g.
$a_1 \equiv (E - J + L)/D$,
$a_5 \equiv (2JK + E(3I - K) - 3IJ + L(2I - K - 2B^2/H))/D^2$.

Constraints from locality

Enforcing no-instantaneous-signaling in leading order gravitational interactions $\implies$

1. For 2 degrees of freedom all (5) graviton mass terms must vanish.
2. $A, B, \ldots, L$ organize such that the graviton sector is fully Lorentz invariant.
3. The matter sector is also Lorentz invariant, i.e. $\mathcal{T}^{\mu\nu} \equiv T^{\mu\nu}$ is the conserved energy-momentum tensor.

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Locality requires:

Spin 1 (single photon) $\rightarrow$ internal $U(1)$.
Spin 2 (gravity) $\rightarrow$ Lorentz symmetry.

Lorentz invariance is thus *required* when coupling to gravity, as compared to spin 1 where only the photon needed to be Lorentz invariant.

Time translations

If we also break time translation symmetry:
$\{A, B, \ldots, L\} \to \{A(t), B(t), \ldots, L(t)\}$

- Restricting to a flat background: locality $\implies$ Lorentz boost invariance.

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- Recover some additional terms that prevent restoration of time-translation invariance:

$$\mathcal{L} = \mathcal{L}_{GR} + \alpha t \left(\frac{1}{2} \partial_i h \partial_i h - \frac{1}{2} \partial_k h_{ij} \partial_k h_{ij} + \partial_j h_{ij} \partial_k h_{ik} - \partial_i h \partial_j h_{ij} \right)$$

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At the leading order to which we are working, this is the Gauss-Bonnet term

$$\Delta\mathcal{L}_{GB} = f(t)\sqrt{-g}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)$$

with $\ddot{f} = -\alpha t$. (JL, Hertzberg [2008.06510])

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Including dimension 3 terms ($\sim h\,\partial h$) allows one to possibly recover cosmological backgrounds when requiring locality.

- Computationally challenging, and on non-trivial backgrounds causality itself may become more subtle.