Phase diagram and transport coefficients from the Polyakov Nambu Jona-Lasinio Lagrangian

in collaboration with
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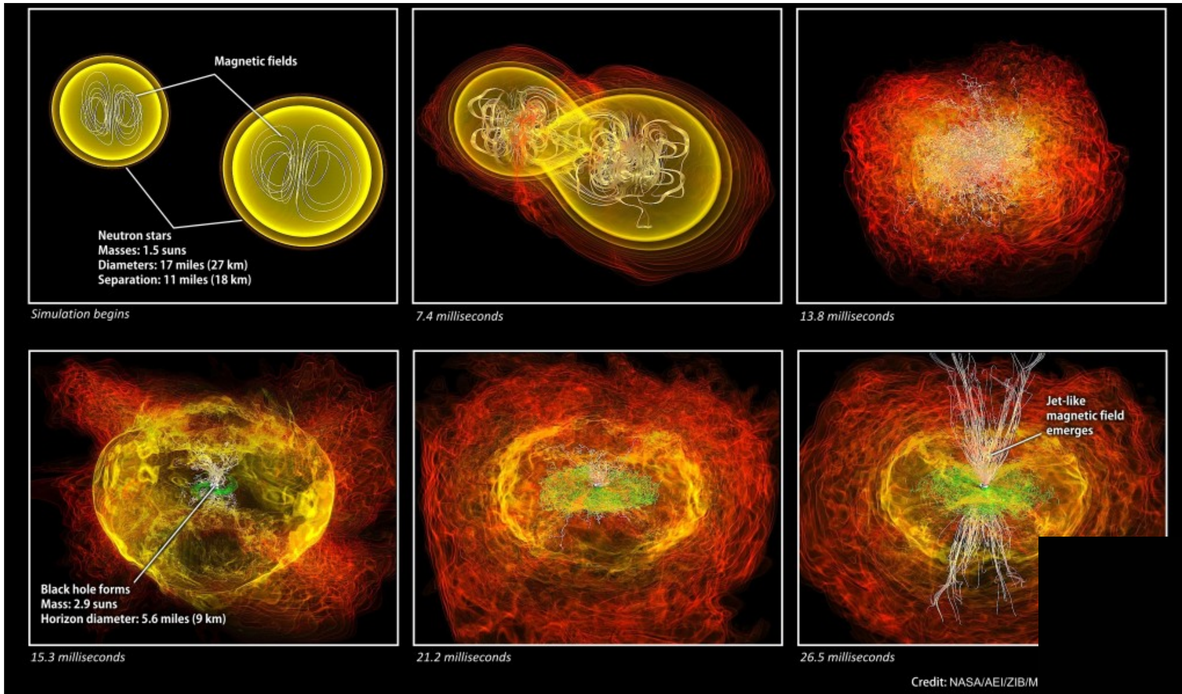
Simulations of Neutron Stars, Neutron Star Collisions and Heavy Ion Collisions

need the same input

PHASE DIAGRAMM OF STRONGLY INTERACING MATTER s(T,μ), ε(T,μ)

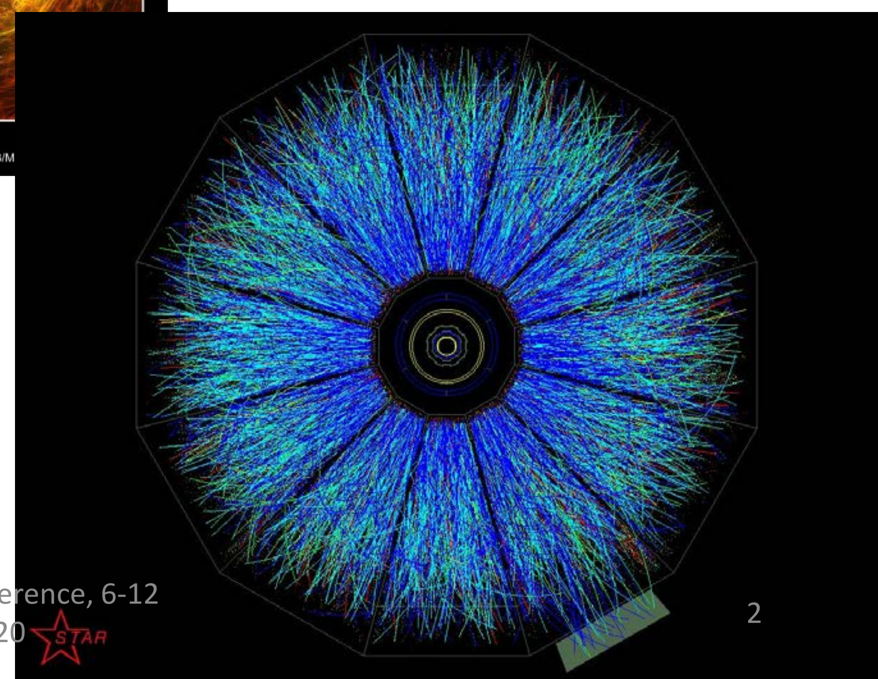


Heavy ion collision:

symmetric nuclear matter

d = u

$0 < \rho < 4\rho_0$



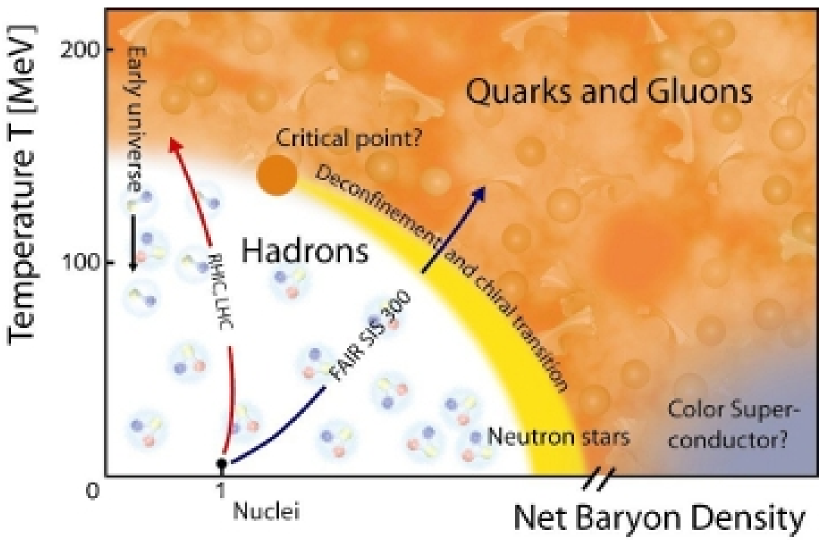
Neutron Star collisions

asymmetric matter

d > u

$0 < \rho < 8\rho_0$

What are the problems?



Why not calculate simply?
Quantumchromodynamics (QCD) can be calculated on a lattice
but only for μ=0 (same number of quarks and antiquarks)

Taylor expansion or similar expansion methods allows for calculations for $\mu/T \ll 1$

Neutron Stars as well as heavy-ion collisions need calculations at
chemical potentials $\mu/T \gg 1$

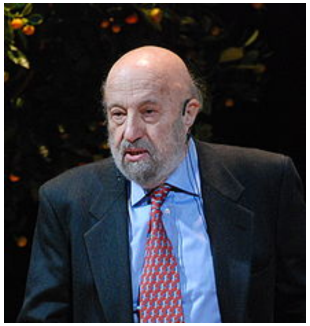
- ❑ either assumptions about continuation to finite μ
- ❑ or effective theories which cover the whole (T,μ) plane

Effective Lagrangian
to study phase phase diagram and phase transitions at finite chemical potential (NICA,FAIR, neutron stars)

The Nambu Jona-Lasinio Lagrangian is such an effective field theory

Nambu

- allows for predictions for finite T and μ
- needs as input only vacuum values + (YM Polyakov loop)
- shares the symmetries with the QCD Lagrangian
- can be « derived » from QCD Lagrangian



Jona-Lasinio

quark 4-point interaction

$$\mathcal{L}_{NJL} = \bar{\Psi}_i(i\gamma_\mu\partial^\mu - M_0)\Psi_i - G_c^2[(\bar{\Psi}_i\Psi_i)^2 + (\bar{\Psi}_i\gamma_5\lambda_{ij}\Psi_j)^2]$$
$$+ H \det_{ij}[\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H \det_{ij}[\bar{\psi}_i(1+\gamma_5)\psi_j] + \sum_{ij} \bar{\psi}_i\mu_{ij}\gamma_0\psi_j$$

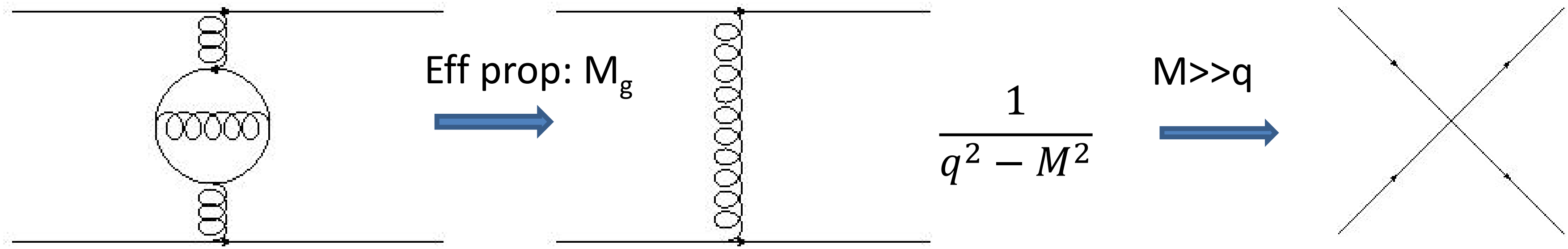
Ψ_i quark fields (u,d,s)

5 parameters
Λ = upper cut off of the internal momentum loops
G_c = coupling constant
M_0 = bare mass of u,d and s quarks
H= coupling constant 't Hooft term

Fixed by vacuum values
m_π, m_K,
η-η' mass splitting
π decay constant
chiral condensate (-241 MeV)^3

NJL Lagrangian

⇒ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom as QCD can be obtained by discarding the gluon dynamics completely.

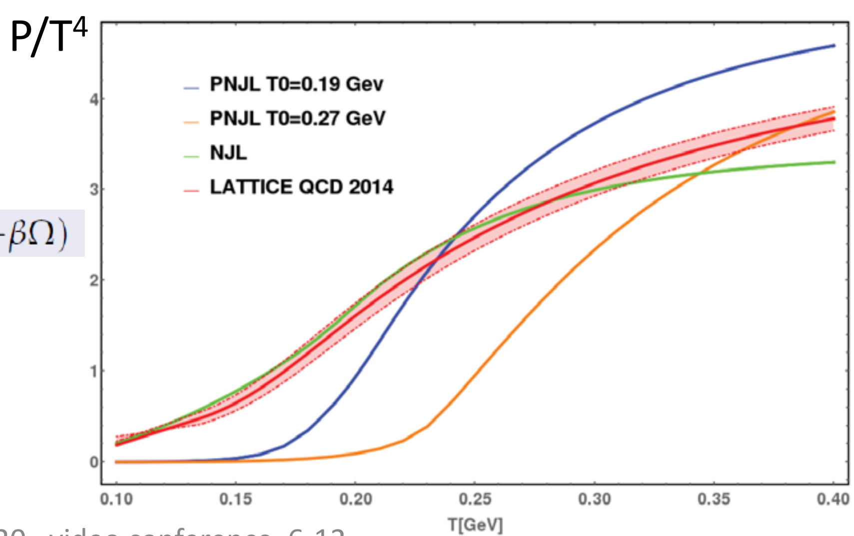


Problem:

Thermodynamic properties obtained by partition sum $Z = Tr[\exp -\beta(H - \mu N)] = \exp(-\beta\Omega)$

- pressure
- entropy density
- energy density

do not coincide with lattice calculations



1^{st} improvement: PNJL: gluons on a static level

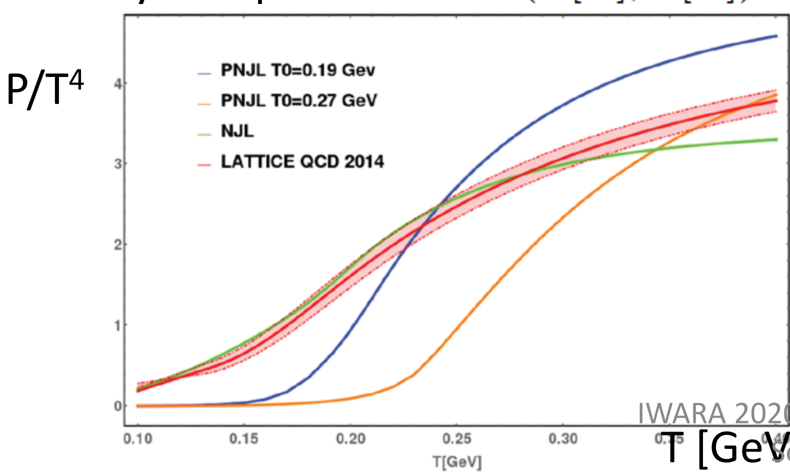
Eur.Phys.J. C49 (2007) 213-217

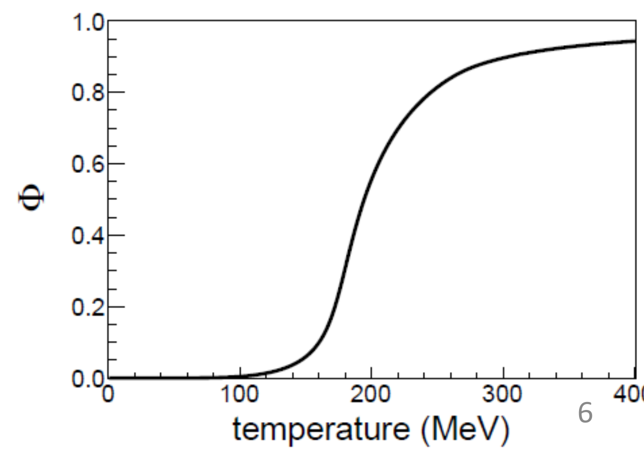
$$
\begin{aligned}
\mathcal{L}_{PNJL} \; = \; & \bar{\Psi}_i(i\gamma_\mu D^\mu - M_0)\Psi_i - G_c^2[\,(\bar{\Psi}_i\Psi_i)^2 + (\bar{\Psi}_i\gamma_5\lambda_{ij}\Psi_j)^2] \\
& + H \det_{ij}[\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H \det_{ij}[\bar{\Psi}_i(1+\gamma_5)\Psi_j] + \sum_{ij} \bar{\Psi}_i\mu_{ij}\gamma_0\Psi_j \\
& - \mathcal{U}(\Phi[A], \bar{\Phi}[A])
\end{aligned}
$$

$$D^\mu = \partial^\mu - iA^\mu \quad ; \quad A^\mu = \delta_{\mu 0} A^0$$

- Gluons introduced as static degree of freedom which couples to the quarks
- Static gluon potential gives the right pressure for the gluons for T -> ∞
- Polyakov loop $\Phi = \frac{1}{N_c} \mathrm{Tr}_c \langle \mathbf{P} \exp\left(-\int_0^\beta d\tau A_0(x,\tau)\right)\rangle$: order parameter of the

  deconfinement transition in Yang Mills
- Polyakov potential $\mathcal{U}(\Phi[A], \bar{\Phi}[A])$ fitted to the pure gauge sector



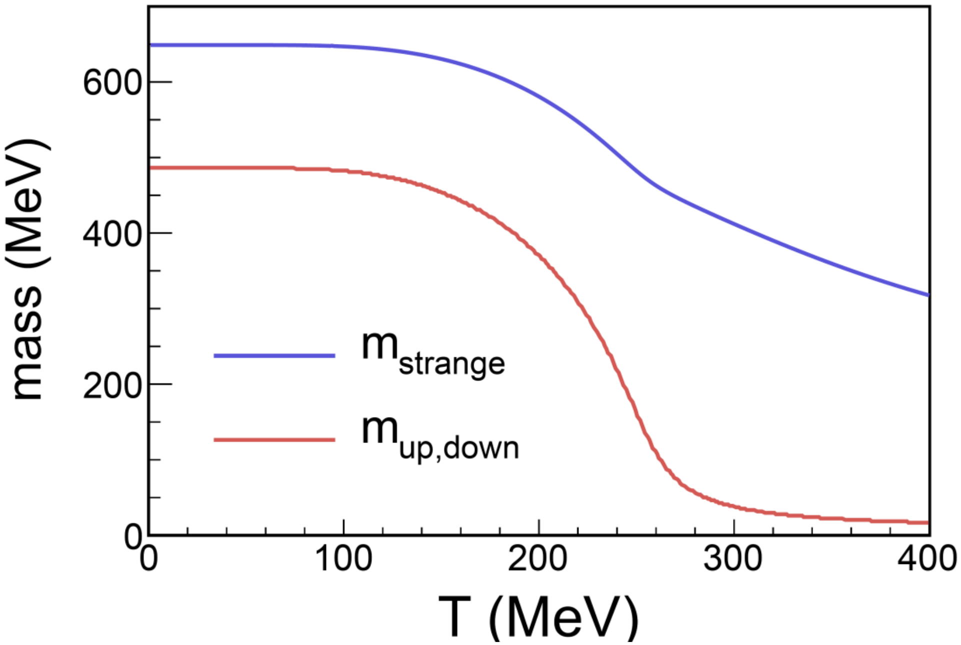


Quark Masses in NJL and PNJL

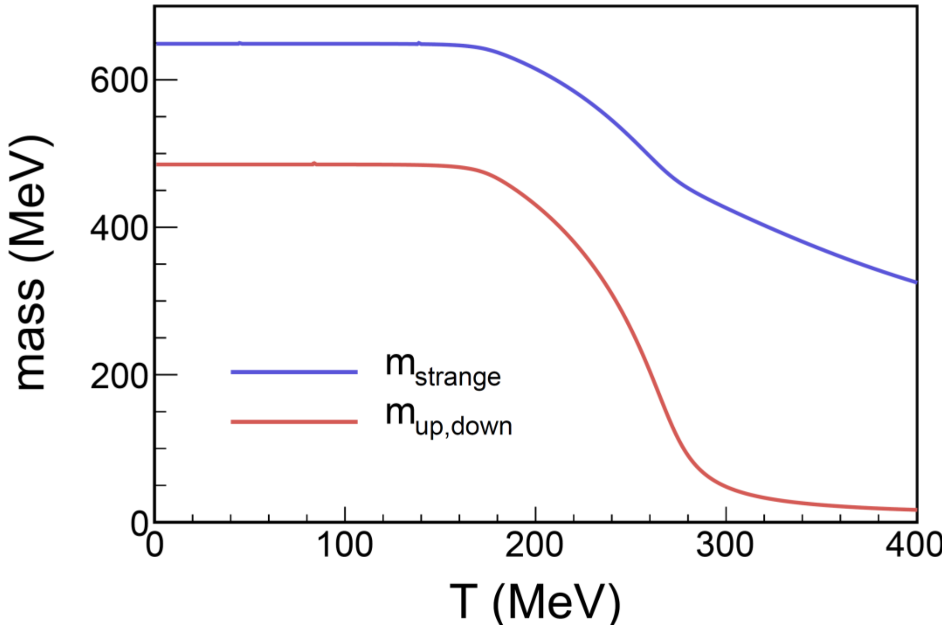
Quark masses are obtained by minimizing the partition sum

$$\mathbf{M} = \hat{\mathbf{M}}_0 - 4\mathbf{G} < \bar{\psi}\psi > +2\mathbf{H} < \bar{\psi}'\psi' >< \bar{\psi}''\psi'' >$$

NJL



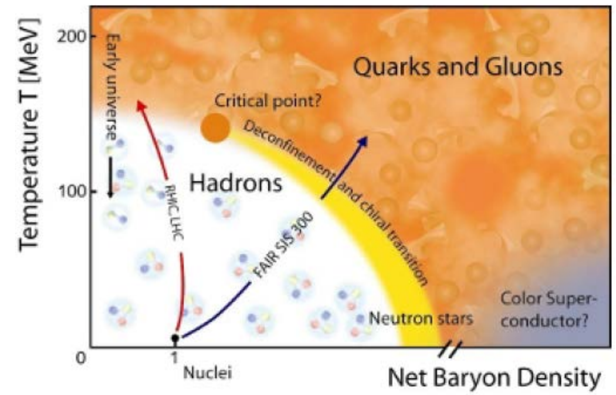
PNJL



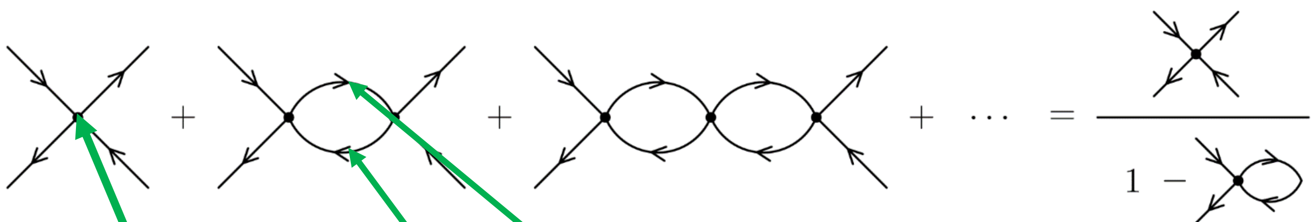
In PNJL the transition is steeper than in NJL

How can we get mesons?

At low temperature/density strongly interacting matter consists of hadrons, but the PNJL Lagrangian contains quarks (and mean field gluons) only. How to get mesons?



4-point $\mathscr{K}$ interaction can be kernel for a Bethe-Salpeter equation



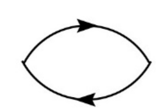
which is formally identical to a meson propagator

$$\mathscr{K} = \Omega \; 2G_{eff} \; \bar{\Omega}$$

$$T(p) = \mathscr{K} + i \int \frac{d^4k}{(2\pi)^4} \mathscr{K} \, S\left(k + \frac{p}{2}\right) S\left(k - \frac{p}{2}\right) T(p) \qquad \Omega = 1\!\!1_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$

$$T(p) = \frac{2G_{eff}}{1 - 2G_{eff}\Pi(p)}, \qquad \Pi(p_0, p) = -\frac{1}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \Omega \, S\left(k + \frac{p}{2}\right) \Omega \, S\left(k - \frac{p}{2}\right)$$

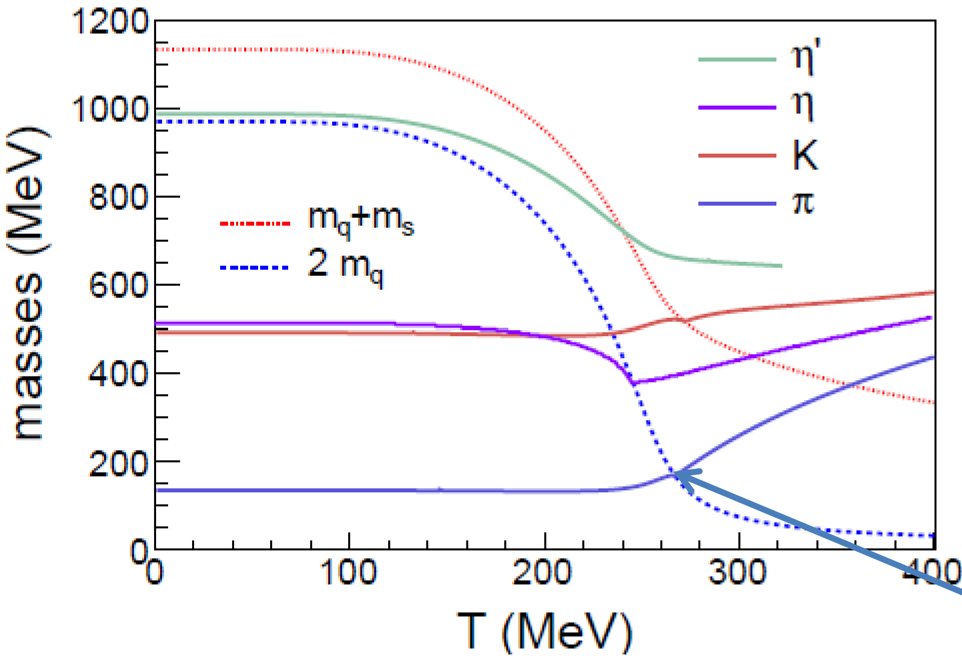
In (P)NJL one can sum up this series analytically;

 $\equiv \Pi$

How to get mesons?

The meson pole mass and the width one obtains by solving:

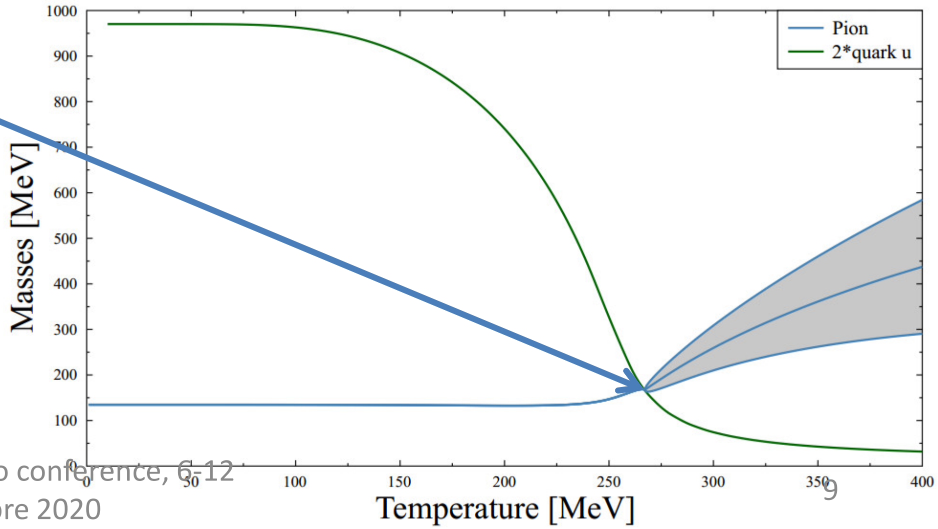
$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$



masses of pseudoscalar mesons and of quarks at μ = 0

At T=0 physical and calculated masses agree quite well

When mesons become unstable they develop a width



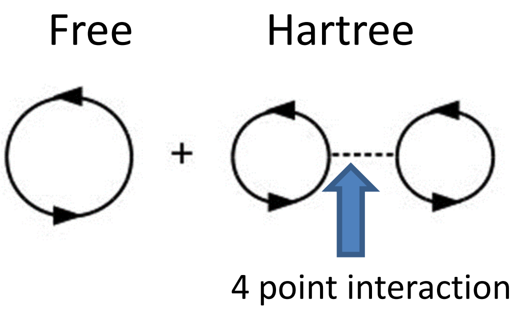
How to improve the agreement with lattice calculations

Up to now:

partition function

$$Z = Tr[\exp -\beta(H - \mu N)] = \exp(-\beta\Omega)$$

in leading order of N_c , $(1/N_C)^{-1}$, the number of colors:



$$\begin{aligned}
&\Omega_q^{(-1)}(T,\mu_i;\langle\bar\psi_i\psi_i\rangle,\Phi,\bar\Phi)\\
=\ &\ln(Tr[\exp(-\beta\int dx^3(-\bar\psi(i\partial\!\!\!/ - m)\psi - \mu\bar\psi\psi))])\\
+\ &2G\sum_k < \bar\psi_k\psi_k >^2 - 4K\prod_i < \bar\psi_k\psi_k > + U_{PNJL}
\end{aligned}$$

Two extensions:

- Going beyond leading order in N_c . This includes directly the mesonic degrees of freedom
- Modification of the gluon potential due to the presence of the quarks

Modification of the gluon potential U due to the presence of quarks

$$\frac{\mathbf{U(T,\Phi,\bar{\Phi})}}{\mathbf{T^4}} = -\frac{\mathbf{b_2(T)}}{\mathbf{2}}\bar{\Phi}\Phi - \frac{\mathbf{b_3}}{\mathbf{6}}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{\mathbf{b_4}}{\mathbf{4}}(\bar{\Phi}\Phi)^3$$

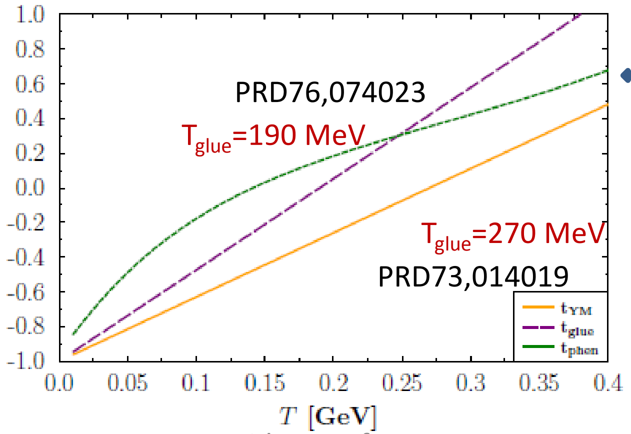
$b_2(T) = a_0 + (\frac{a_1}{1+\tau}) + \frac{a_2}{(1+\tau)^2} + \frac{a_3}{1+\tau)^3}$ $\tau = f\frac{T - T_{glue}}{T_{glue}}$ $T_{glue} = a + bT + cT^2 + dT^3 + e\frac{1}{T}$

a_0	a_1	a_2	a_3	b_3	b_4	a	b	c	d	e	f
6.75	-1.95	2.625	-7.44	0.75	7.5	0.086	0.36	0.57	-1.15	-0.0005	0.57

$$t_{YM} = \frac{T - T_{YM}^{cr}}{T_{YM}^{cr}} = 0.57\,\frac{T - T_{glue}^{cr}}{T_{glue}^{cr}} = 0.57 t_{glue}\,.$$

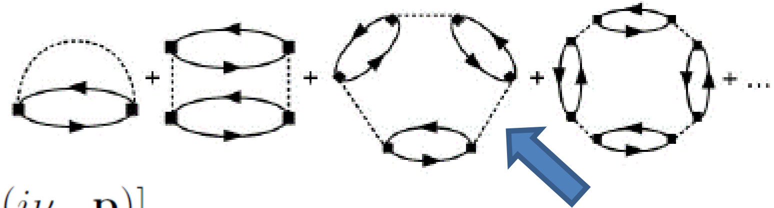
$$\tau_{phen} = 0.57\frac{T - T_{phen}(T)}{T_{phen}(T)}.$$

$$T_{phen}(T) = a + bT + cT^2 + dT^3 + e\frac{1}{T}.$$



our parametrization right asymptotic limit

O(N_c =0) for the partition sum



4 point interaction
PRC96,045205

$$\Omega_q^{(0)}(T,\mu_i) = \frac{1}{2}\int_0^1 \frac{d\lambda}{\lambda}\int \frac{d^3p}{(2\pi)^3} T\sum_n \mathrm{Tr}\,[S^\lambda(i\nu_n,\mathbf{p})\Sigma^\lambda(i\nu_n,\mathbf{p})]$$

$$\Sigma^\lambda(i\nu_n,\mathbf{p}) = \sum_M T\sum_m \int \frac{d^3q}{(2\pi)^3}\,\Omega\; S_H(i\omega_m,\mathbf{q})\;\bar{\Omega}\; t_M^\lambda(i\nu_n - i\omega_m,\mathbf{p}-\mathbf{q})$$

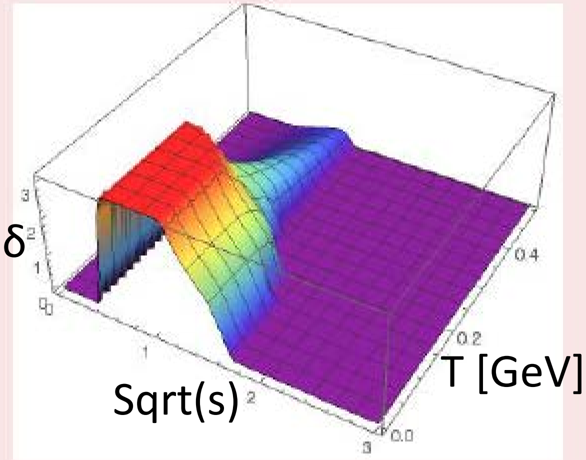
$$t_M^\lambda(i\nu_n - i\omega_m,\mathbf{p}-\mathbf{q}) = \frac{2\lambda\mathcal{K}_M}{1-2\lambda\mathcal{K}_M\Pi(i\nu_n - i\omega_m,\mathbf{p}-\mathbf{q})}$$

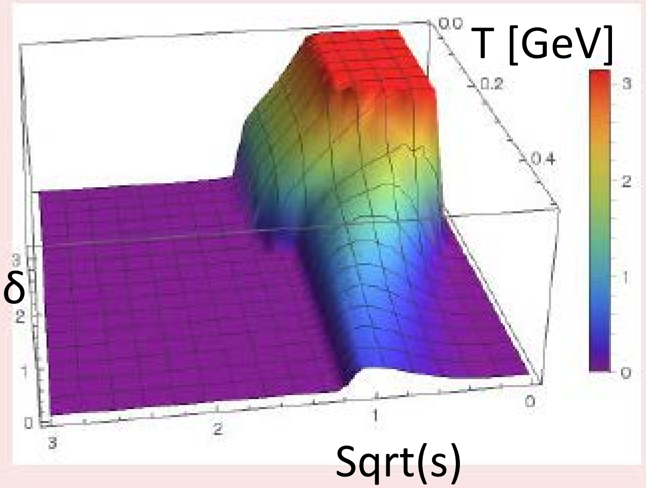
meson loops contribute to the grand potential because :

$$\Omega_q^{(0)}(T,\mu_i) = \sum_{M\in J^\pi=\{0^+,0^-\}} \Omega_M^{(0)}(T,\mu_M(\mu_i))$$

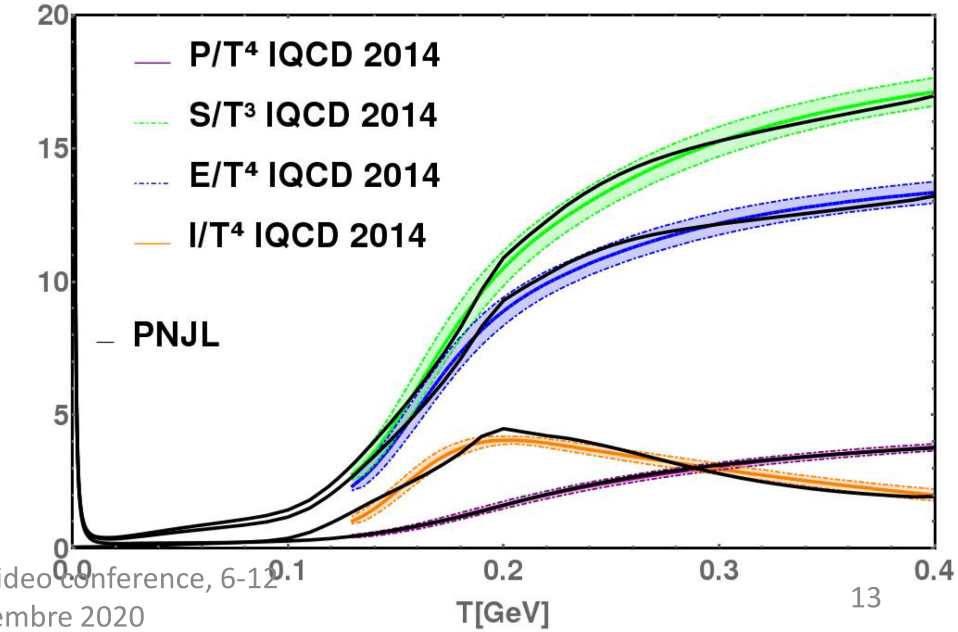
$$\Omega_M^{(0)}(T,\mu_M) = -\frac{g_M}{2\pi}\int \frac{d^3p}{(2\pi)^3}\int_0^{+\infty} d\omega \left[\frac{1}{e^{\beta(\omega-\mu_M)}-1} + \frac{1}{e^{\beta(\omega+\mu_M)}-1}\right]\,\delta(\omega,\mathbf{p};T,\mu_M)$$

with the phase shifts δ
$$\delta_M(\omega,\mathbf{p};T,\mu_M) = \frac{1}{2i}\log\frac{1-2\mathcal{K}_M\Pi_M(\omega-\mu_M-i\epsilon,\mathbf{p})}{1-2\mathcal{K}_M\Pi_M(\omega-\mu_M+i\epsilon,\mathbf{p})}$$





These two modifications allow to reproduce
pressure P,
entropy density s,
energy density E
interaction measure I
of the lattice calculations at μ=0.

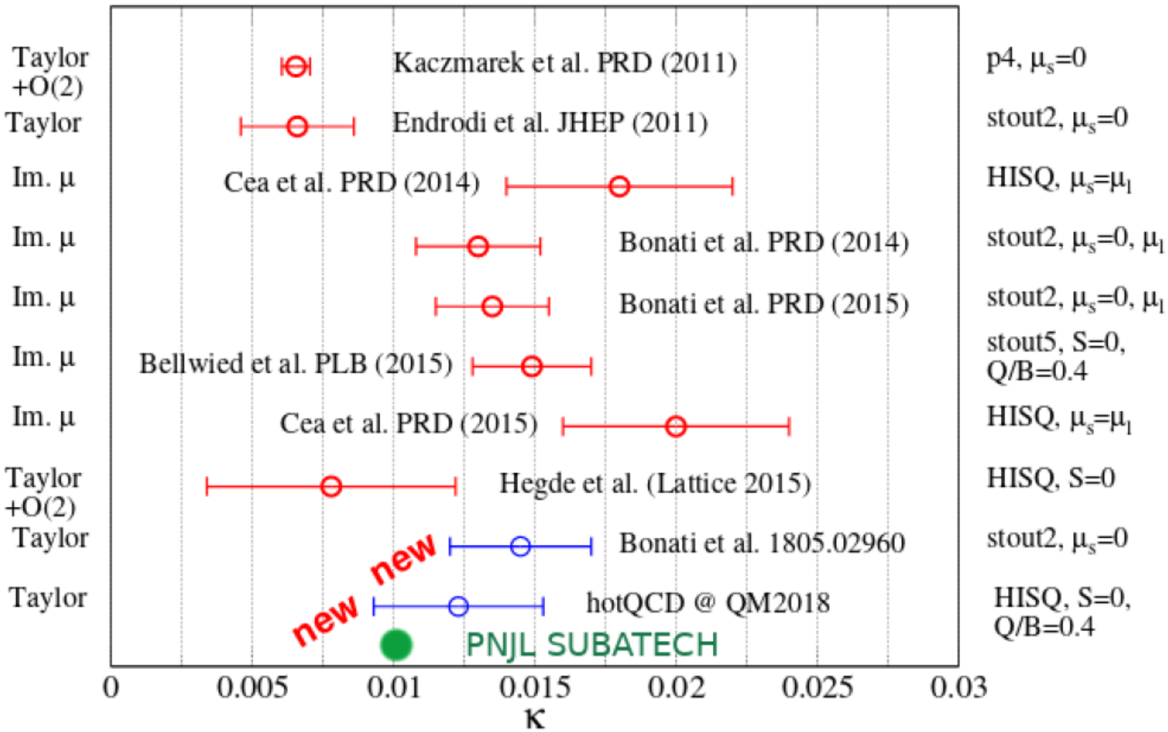


Good starting point to explore the phase diagram in the whole T,μ plane

Extension to small but finite chemical potential

Like in lattice calculation: Taylor expansion around μ = 0

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \dots \qquad \kappa = \left. \frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2} \right|_{\mu_B = 0}$$



We find

T_c = 138 MeV
κ = 0.01

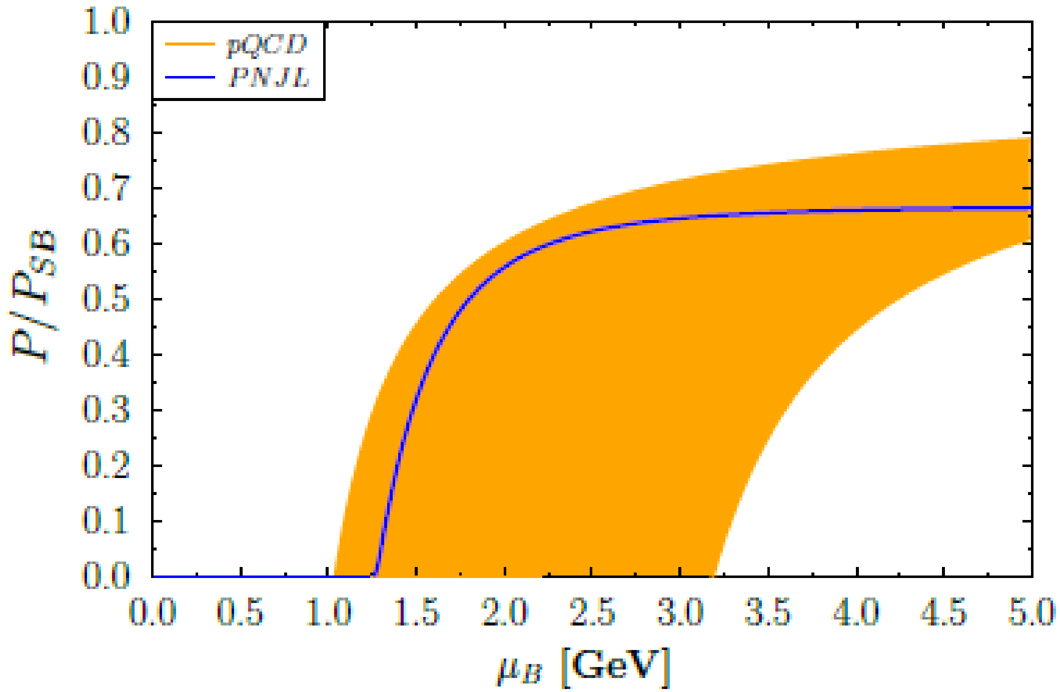
Very similar to lattice calculations

Also for small but finite μ we reproduce the lattice results: confidence for larger μ

Limit of (very) large chemical potential

For (very) large μ (T=0) contact with perturbative QCD calculations (PRL 117 042501)

PNJL in the error bars of pQCD



Having verified that at small μ and at very large μ the thermodynamical quantities of PNJL agree with QCD based approaches

we can explore the finite μ region

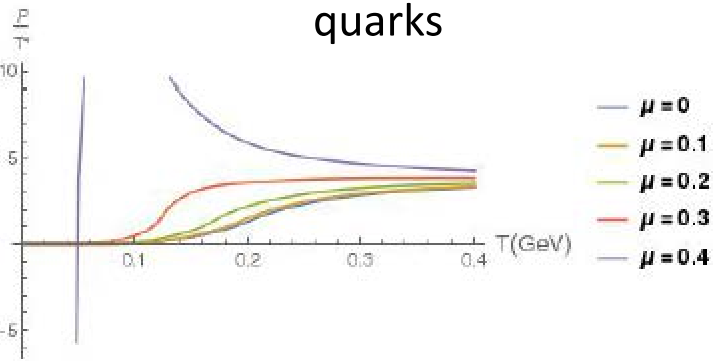
The PNJL equation of state for finite μ

Calculation of thermal quantities at finite μ is straight forward in PNJL

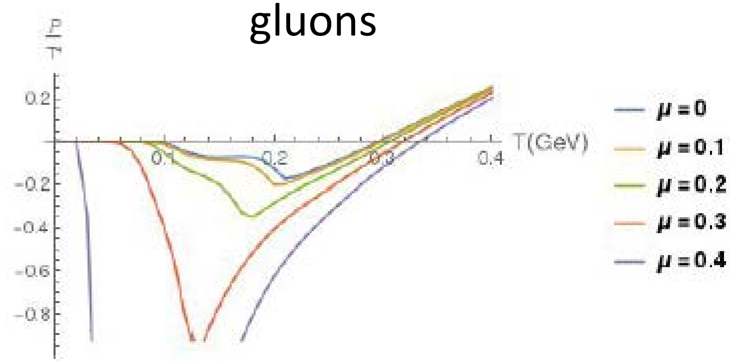
Contribution to the pressure of the different particles

quarks

P/T^4

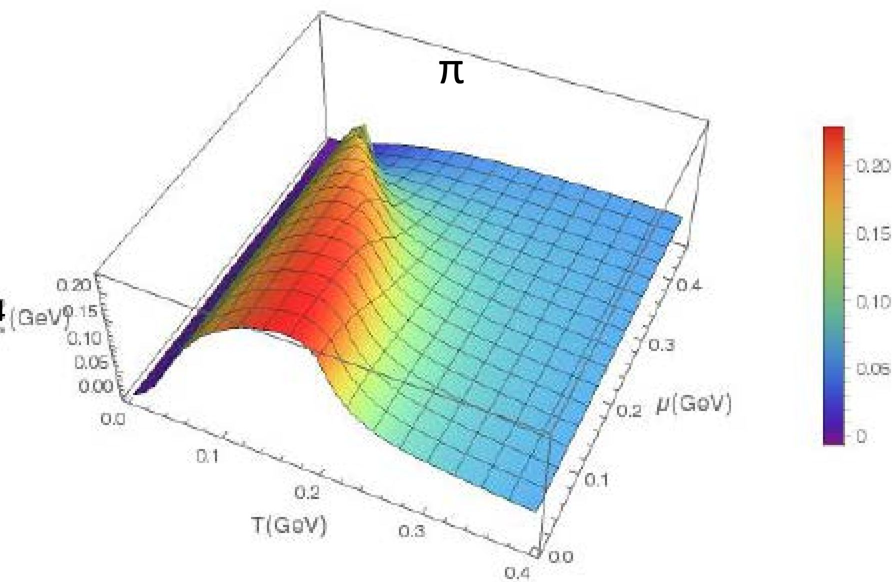


gluons



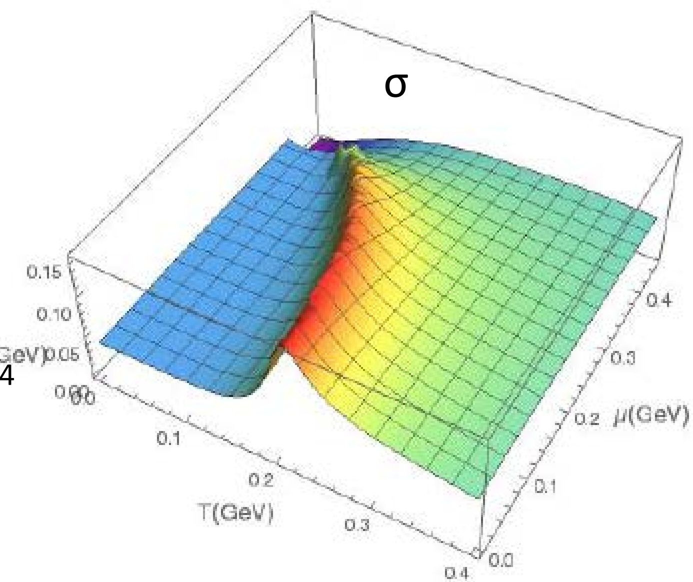
π

P/T^4



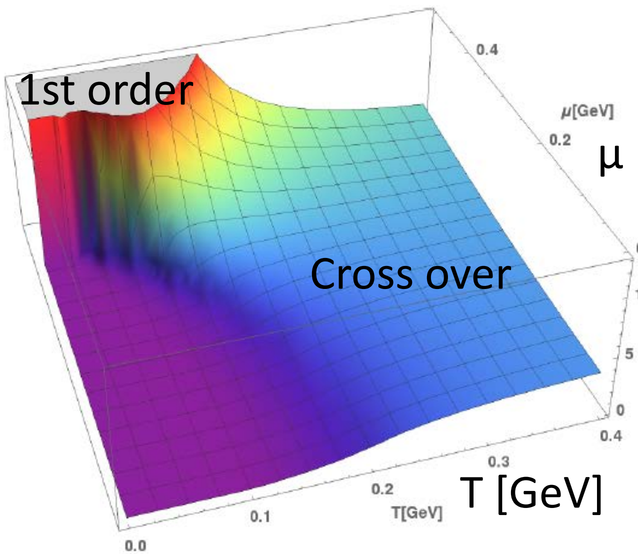
σ

P/T^4



Phase diagram at finite μ

Total P(T,μ)



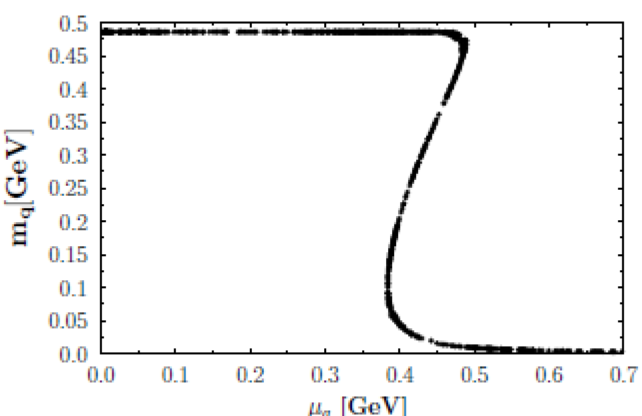
Cross over at $\mu = 0$

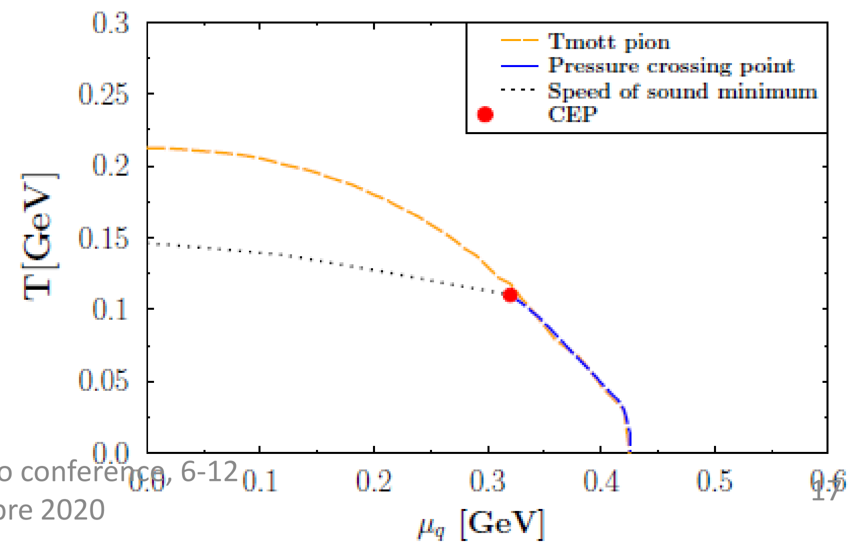
1st order phase transition for $\mu >> 0$ with a CEP

$T^{CEP} = 110$ MeV

$\mu_q^{CEP} = 320$ MeV

For small temperatures the equation of state shows a first order phase transition with the quark mass (chiral condensate) as order parameter





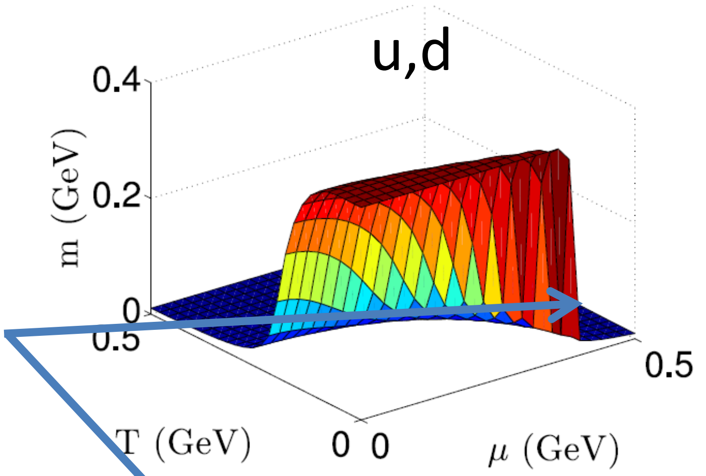
Masses close to the tricritical point

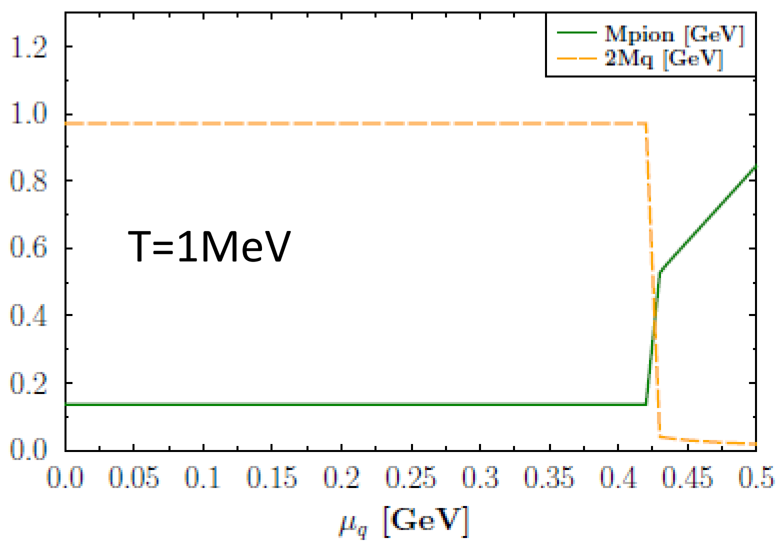
PNJL Lagrangian:
transition between quarks and hadrons

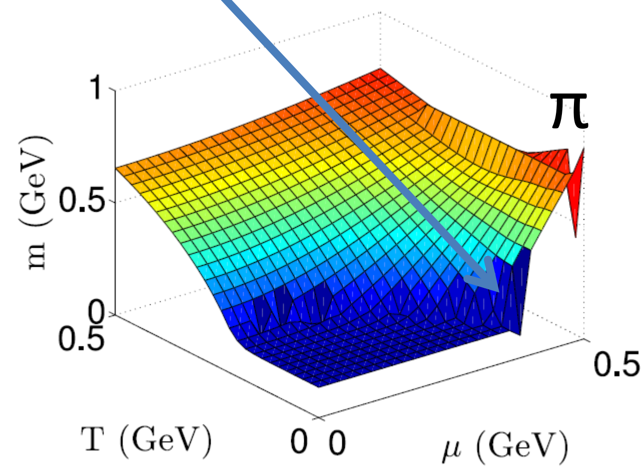
Cross over at μ = 0

1st order transition μ >>0

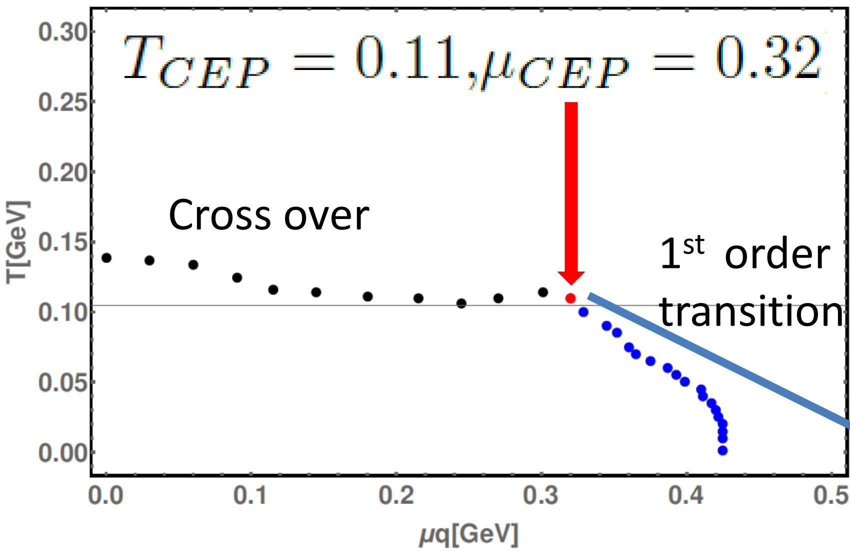
sudden change of q and meson mass





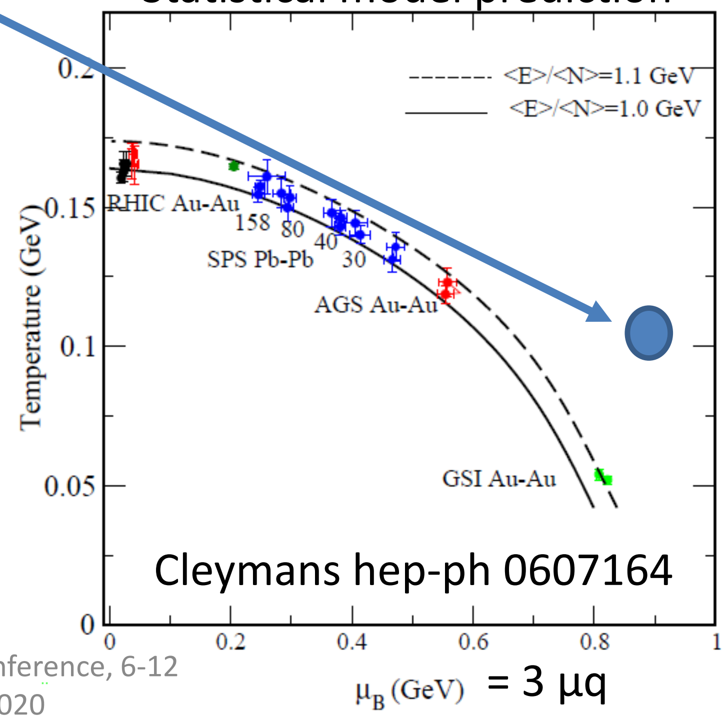


Borderline between quark gluon plasma and hadrons



Freeze out condition

Statistical model prediction



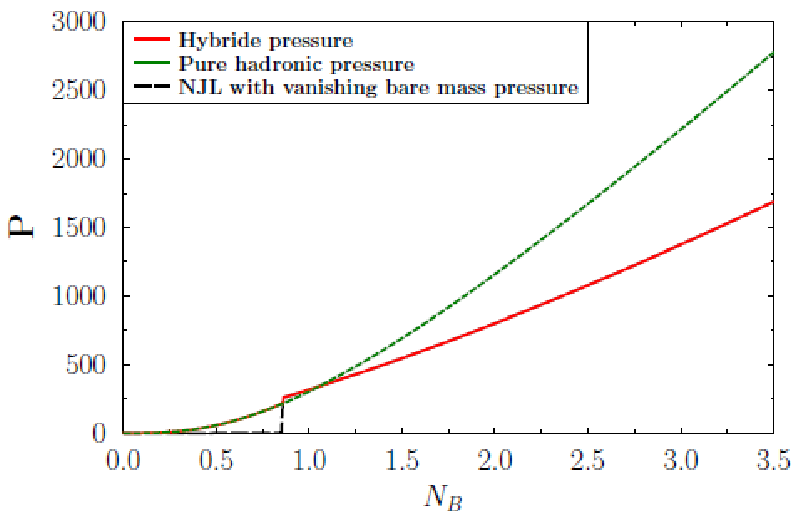
= 3 μq

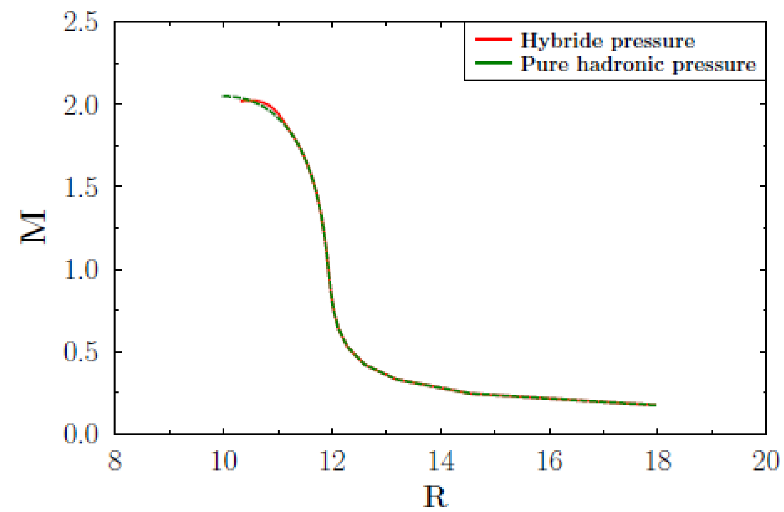
Phase transition point may be reachable in experiments at NICA and FAIR

Neutron star matter

In order to have a realistic EOS in the hadron sector we have to add hadrons

Presently we use a Douchin Haensel type hadronic EOS
to improve on the hadronic sector *Astron. Astrophys.*, 380:151, 2001





In future : combination with more sophisticated hadronic model (Schramm)

Transport coefficients in a QGP plasma at finite temperatures and densities

Key quantity: relaxation time $$\frac{1}{\tau_a(T,\mu_q)} = \sum_{b=q,\bar{q}} n_b(T,\mu)\bar{W}_{ab}$$

transition rate $$\bar{W}_{ab} = \frac{1}{n_a n_b}\int \frac{d^3 p_a}{(2\pi)^3}\int \frac{d^3 p_b}{(2\pi)^3} f_a^{(0)}(p_a) f_b^{(0)}(p_b)\; v_{\mathrm{rel}}\sigma_{ab\to cd}(s,T,\mu_q)$$

$f_a^{(0)}$ are the Polyakov loop modified thermal distributions

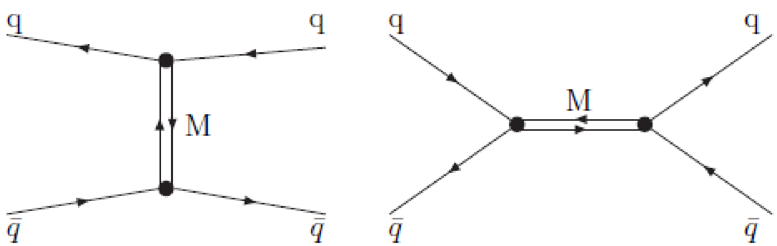
cross sections in Born approximation

4-point interaction -> meson exchange

elastic scattering in s,t,u channel

PNJL ingredients:

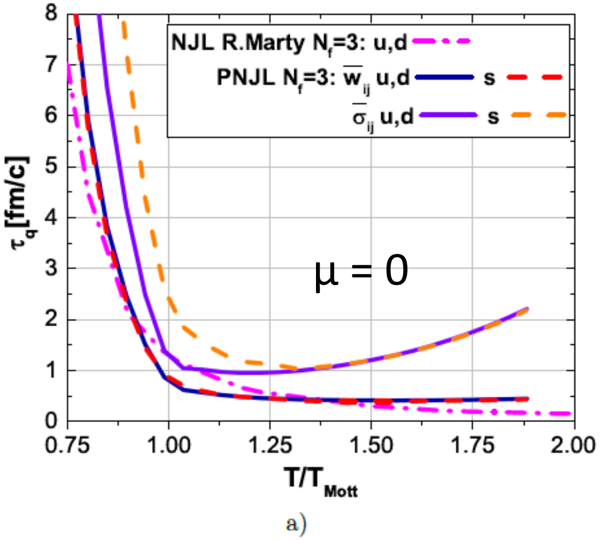
example

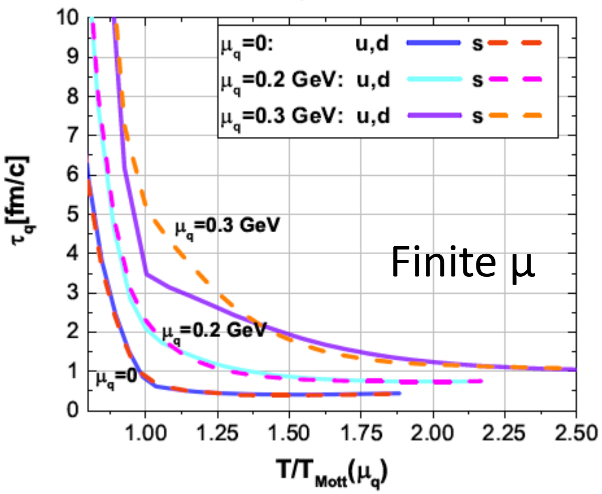


$q + \bar{q} \to q + \bar{q}$

- ❑ exchanged mesons
- ❑ masses of incoming quarks
- ❑ Polyakov loop modified thermal distributions

relaxation time:
u,d,s, quarks



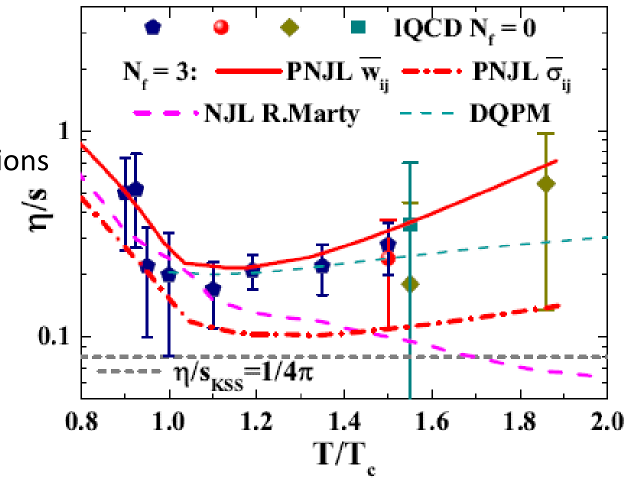


Shear viscosity:

$$\eta^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{15T}\sum\int\frac{d^3p}{(2\pi)^3}\frac{\mathrm{p}^4}{E_i^2}\tau_i(T,\mu_B)\; d_i f_i^{\phi}$$

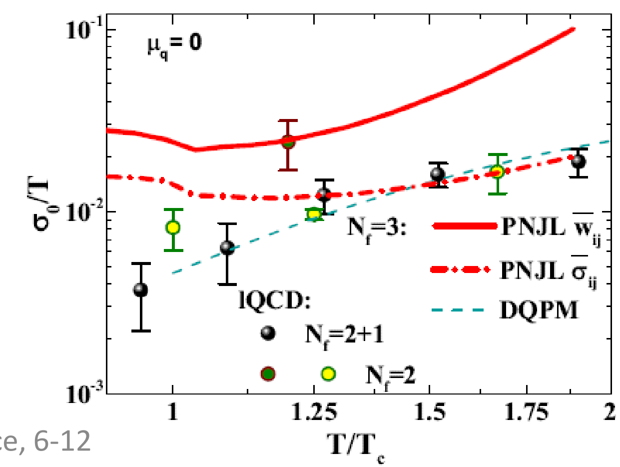
$$d_q = d_{\bar{q}} = 2N_c = 6$$

f_i^{ϕ} Polyakov loop modified thermal distribution functions



Electric conductivity:

$$\sigma_0^{\mathrm{RTA}}(T,\mu_B) = \frac{e^2}{3T}\sum_{i=q,\bar{q}} q_i^2 \int\frac{d^3p}{(2\pi)^3}\frac{\mathrm{p}^2}{E_i^2}\;\tau_i(T,\mu_B) d_i f_i^{\phi}$$



Summary

(P)NJL Lagrangian: same symmetries as QCD and allows calculations at finite μ.
-> study of the equation of state in the (T,μ) plane possible

Bethe Salpeter equation in $q\bar{q}$ channel → mesons as pole masses

Going to next to leading order in N_c and introducing an effectivequark gluon interaction (guided by more fundamental approaches) we can reproduce
lattice equation of state at μ=0 ,
lattice expansion coeff for finite μ
pQCD calculations at very large μ
This makes extension to finite T and μ meaningful (without any new parameter)

We obtain the equation of state and the phase diagram in the (T, μ) plane necessary for neutron star, neutron star collisions and heavy ion physics

We find a first order phase transition for finite μ.
and can explore now the consequences by employing
transport approaches for heavy ion and neutron star collisions

PNJL allows as well to calculate transport coefficients which are compatible with lattice results.
Next step: to put all this in a transport approach (heavy ion and neutron star collisions)

backup

Baryons

Phys.Rev. C91 (2015) 065206

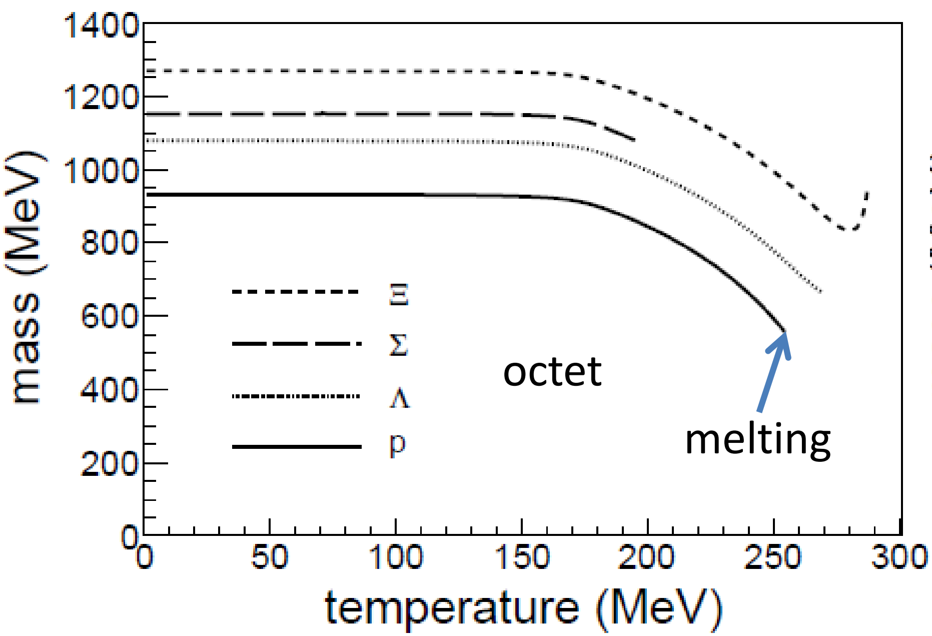
Omitting Dirac and flavor structure :

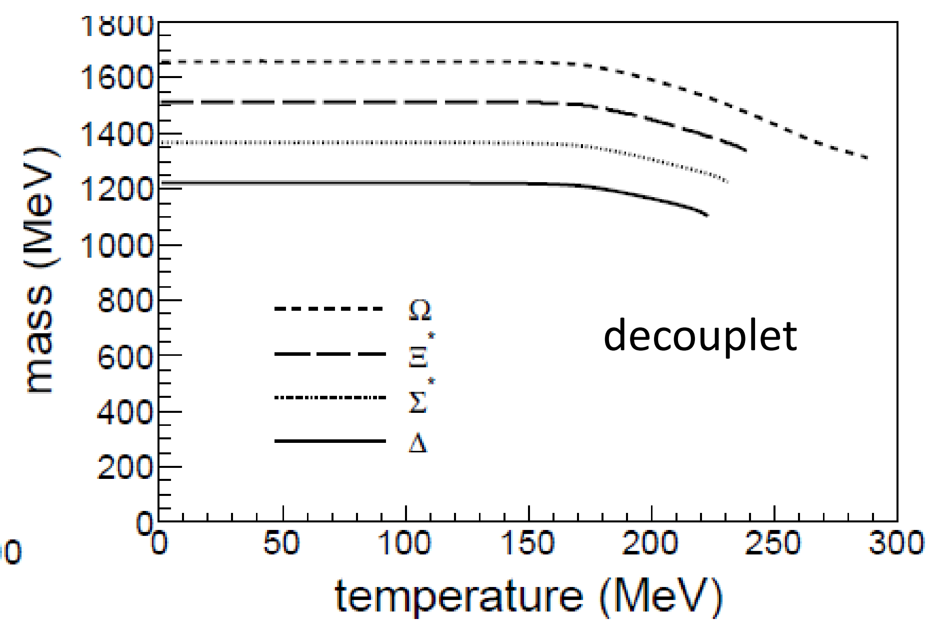
$$\left[1 - \frac{2}{m_{quark}} \frac{1}{\beta} \sum_n \int \frac{d^3q}{(2\pi)^3} S_q(i\omega_n, q)\; t_D(i\nu_l - i\omega_n, -q)\right]\Bigg|_{i\nu_l \to P_0 + i\epsilon = M_{Baryon}} = 0$$

where we approximated the quark propagator for the exchanged quark by:

$$S_q(q) = \frac{1}{\not{q} - m_{quark}} \to -\frac{\mathbb{1}_{Dirac}}{m_{quark}}$$

5% error (Buck et al. (92))

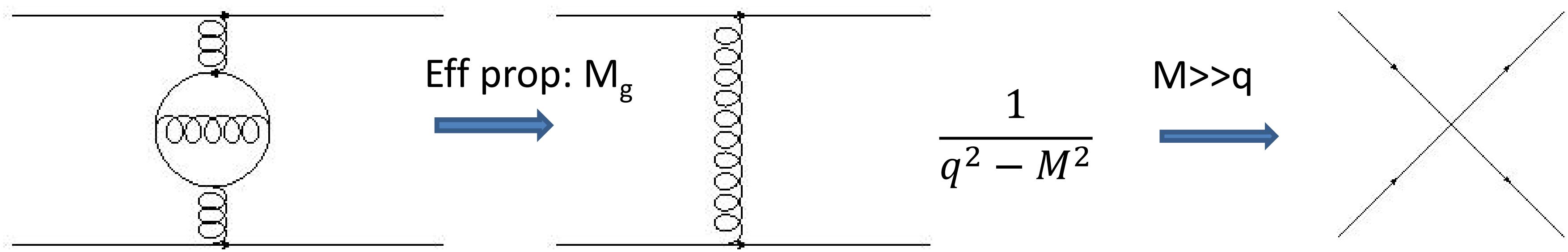




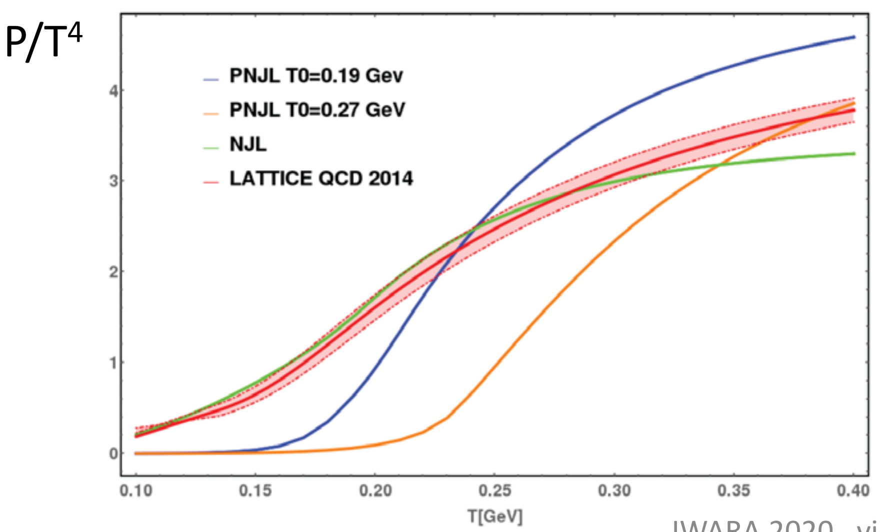
The more strange quarks the higher the melting temperature

NJL Lagrangian

$\Rightarrow$ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom as QCD can be obtained by discarding the gluon dynamics completely.



$$\frac{1}{q^2 - M^2}$$



Renewed interest because

Going beyond leading order in N_c + including a gluon mean field potential brings PNJL energy density and entropy density closer to lattice results

Phys.Rev. C96, 045205

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How can we get mesons?

Quarks are the degrees of freedom of the Lagrangian

To study the phase transition we need mesons

Use a Trick : **Fierz transformation** of the original Lagrangian

Fierz Transformation allows for a reordering of the field operators in 4 point contact interactions. It is simultaneously applied in Dirac, color and flavor space

Example in Dirac space:

$$(\bar{\chi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\chi) = (\bar{\chi}\chi)(\bar{\psi}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\chi)(\bar{\psi}\gamma_{\mu}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\gamma_5\chi)(\bar{\psi}\gamma_{\mu}\gamma_5\psi) - (\bar{\chi}\gamma_5\chi)(\bar{\psi}\gamma_5\psi)$$

Scalar vector peudovector pseudoscalar