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From Quarks to Cosmos

The electrical conductivity, magnetic parameter, plastic flow and toroidal magnetic field decay in magnetars

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Abstract

Magnetars are a kind of pulsars powered mainly by superhigh magnetic fields. They are popular sources with many unsolved issues in themselves, but also linked to various high energy phenomena, such as QPOs, giant flares, fast radio bursts and super-luminous supernovae. In this talk, we first review the relation between braking index and the magnetic field evoluton of pulsars including magnetars. Combining with the latest EoSs, we then introduce the eigen equations of Ohmic dissipation of toroidal magnetic fields in general relativity, and calculate the electical conductivity, give a specific relation between the magnetization parameter, and magnetic field in the magnetar crust. We investigate the temperature-dependent shear rates as well as temperature-dependent (shear) viscosity considering magnetically driven plastic flows in the crust of cmagnetars, the onset of the soft gamma repeater outburst is controlled by magnetospheric dissipation induced by the plastic motions of the crust, according to our results and analysis of relevant energy scales.

Main issues and methods

- **Main issues:**
- ✓ **How to constrain magnetar braking indexes and explain them?**
- ✓ **How to deduce an eigenvalue equation of Ohmic dissipation for toroidal field in a curved space and to calculate the decay rate of crust magnetic field?**
- ✓ **How to study electrical conductivity and magnetization parameters of NS crust?**
- ✓ **How to study plastic flow in magnetars?**

Methods:

- ✓ **Based on the estimated ages of their associated supernova remnants (SNRs), we estimate the values of braking indices of eight magnetars with SNRs. Five magnetars have smaller mean braking indices $n < 3$, and we interpret them within a combination of magneto-dipole radiation and wind-aided braking. The larger mean braking indices $n > 3$ for the other three magnetars are attributed to magnetic field decay.**

- ✓ **For the sake of simplicity, we only consider dipole magnetic field (corresponding to $l = 1$ and $m = 0$). The magnetic field is confined in the crust in force-free model. Through the adoption of Legendre polynomial, spherical Bessel function, Maxwell's equations in the covariant form, and tensor analysis, we deduce an eigenvalue equation of toroidal field in a curved space. To calculate the decay rate of toroidal magnetic field, we must investigate the crust conductivity and combine with the EoS and the magnetic field configuration.**

- ✓ **We chose two fiducial dipole fields of 10^13 G and 10^14 G, four different temperatures, and two different impurity concentration parameters in studying NS crustal conductivity and magnetization parameters.**

- ✓ **We presumed a NS crust responds in an elastic, reversible manner to stresses below certain yield value and plastically above it. To construct an useful mathematical formulation we describe the mechanism of slow plastic flow together with a general slip boundary condition using the simplest configuration where one characteristically tiny element in the crust of neutron stars is approximately a slab configuration.**

Main conclusions

☐ Constraining magnetars' braking indice

$$n \approx 1 - \frac{\nu}{\dot{\nu} t_{\rm SNR}} = 1 + \frac{P}{\dot{P} t_{\rm SNR}}.$$

- ✓ Five magnetars with 1<n<3, attributed to a combination of MDR and wind-aided braking
- ✓ Three with n>3 , attributed to a combination of MDR and dipolar magnetic field decay

☐ Toroidal magnetic field decay

$$B_\varphi = \frac{1}{2}\mu x R B \sum_n A_n \frac{j_1(n\pi x)}{x R^2} e^{\frac{-c^2 \lambda_n t}{4\pi\sigma R^2}} \sin\theta,$$

$$\frac{4\pi\sigma}{c}\frac{\partial H(r,t)}{\partial x^0} = \left(1 - \frac{2M(r)}{r}\right)^{\frac{1}{2}} \frac{1}{r^2}\frac{\partial}{\partial r} \times \left[Z\left(1 - \frac{2M(r)}{r}\right)^{\frac{1}{2}} \frac{\partial(r^2 H(r,t))}{\partial r}\right] - \frac{2ZH(r,t)}{r^2}.$$

☐ Magnetization parameter and apply to high-*n* PSR J1640-4631

$$\omega_B \tau = \frac{\tau_{Ohm}}{\tau_{Hall}} = (1-50) \times \frac{B_0}{10^{13}\mathrm{G}}. \quad (1)$$

$$B_p(t) = B_0 \frac{\exp(-Zt/\tau_{Ohm})}{1 + \frac{\tau_{Ohm}}{\tau_{Hall}}[1 - \exp(-Zt/\tau_{Ohm})]} \quad (2)$$

$$\frac{\ddot{P}(t)}{\dot{P}(t)} + \frac{\dot{P}(t)}{P(t)} = \frac{-2Z\left(1 + \frac{1}{\omega_B \tau}\right)}{\left(1 + \frac{1}{\omega_B\tau} - \exp(-Zt/\tau_{Ohm})\right)\tau_{Ohm}}. \quad (3)$$

$$(n-3)\frac{\dot{P}(t)}{P} = \frac{-2Z\left(1 + \frac{1}{\omega_B \tau}\right)}{\left(1 + \frac{1}{\omega_B\tau} - \exp(-Zt/\tau_{Ohm})\right)\tau_{Ohm}} \quad (4)$$

☐ Plastic flow in magnetars

- ✓ A magnetic field with an unbalanced radial component induces an azimuthal plastic slip flow, whose velocity depends on how much one exceeds the elastic part. This also implies that field evolution along the flow provides an ever-increasing twist to the corona.
- ✓ This twist could be relieved slowly through visco-plastic dissipation, or suddenly in coronal flare events. The temperature dependent shear rate in the magnetar crust will be further studied in the future.