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# Consequences of Length Discretization on Metric Tensor and Geodesic Equation

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**Abstract:**
Generalized Uncertainty Principle suggests length discretization. Its possible impacts on Einstein Field Equations shall be studied. We focus on metric tensors, line metric, and geodesic equation.

# Questions

- Gravity Quantization requires spacetime quantization or fundamental limits on length and momentum.

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8 \pi G}{c^4} T_{\mu\nu}$$

- Various models predict minimum measurable length (space discretization).
- Minimum measurable length and/or maximal momentum => generalized uncertainty principle (GUP)

$$\Delta p \, \Delta q \geq \frac{1}{2}[1 + \langle \hat{p}^2 \rangle] = \frac{1}{2}[1 + (\Delta p)^2 + \langle \hat{p} \rangle^2]$$

- $T_{\mu\nu}$ in EFE (rhs) is quantizable!
- Would GUP suggest quantization for lhs of EFE?!
- How look like the metric tensor, line metric and geodesic equation?

# Results

**GUP**

$$[\hat{x}_i\,, \hat{p}_j] \geq \delta_{ij} i\,\hbar\left(1 + \beta p^2\right),$$

$$\hat{x}_i = \hat{x}_{0i}(1 + \beta p^2),$$
$$\hat{p}_j = \hat{p}_{0j},$$

where $p^2 = g_{ij} p^{0i} p^{0j}$ and $g_{ij}$ is the Minkowski spacetime metric tensor

**Metric tensor**

$$\tilde{g}_{\mu\nu} = g_{AB}\frac{\partial x^A}{\partial \zeta^\mu}\frac{\partial x^B}{\partial \zeta^\nu} \simeq g_{ab}\left[\frac{\partial x^a}{\partial \zeta^\mu}\frac{\partial x^b}{\partial \zeta^\nu} + \beta\frac{\partial \dot{x}^a}{\partial \zeta^\mu}\frac{\partial \dot{x}^b}{\partial \zeta^\nu}\right] \simeq \left(1 + \beta \ddot{x}^\lambda \ddot{x}_\lambda\right) g_{\mu\nu},$$

**Line metric**

$$d\tilde{s}^2 = g_{\mu\nu}\left(dx^\mu dx^\nu + \beta^2 d\dot{x}^\mu\, d\dot{x}^\nu\right) = \left(1 + \beta^2 \ddot{x}^\lambda \ddot{x}_\lambda\right) ds^2$$

**Geodesic eq.**

$$\ddot{x}^\delta + \beta g_{\mu\nu,\alpha}\left|\ddot{x}^\delta\right|^2 = \left(-\Gamma^\delta_{\mu\nu} + \beta g_{\mu\nu} x^{(4)\delta}\right)\dot{x}^\mu \dot{x}^\nu,$$

where $x^{(4)\delta} = (\ddddot{x})^\delta$, $\left|\ddot{x}^\delta\right|^2 = g_{\delta\alpha}\ddot{x}^\delta \ddot{x}^\alpha$ and $g_{\mu\nu,\alpha} = \partial g_{\mu\nu}/\partial x^\alpha$.