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From Quarks to Cosmos

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Real scalar field stars of the EKG equations including matter

It is argued that static (not oscillatory) solutions of the Einstein-Klein-Gordon (EKG) equations including matter, exist for real scalar fields, whenever the field interaction with matter is included. Then, the coupling allows the existence of static solutions in contraposition with the case of the pure EKG equations. Surprisingly, when the considered matter is a photon-like gas, it turns out that the gravitational field intensity at large radial distances becomes nearly constant, exerting an approximately fixed force to bodies. The effect is clearly related with the mass less character of the photon-like matter, and suggests a link with the forces defining velocity curves for galaxies.

- The search for boson and fermion-boson stars is a subject of current interest in modern Astrophysics . As it is known, the Einstein-Klein-Gordon (EKG) equations have no static solutions for real scalar fields [1]. Also, although the complex EKG equations show centrally symmetric solutions, they need to be time dependent in a stationary form (harmonic).
- In two classic references on fermion-boson stars [2,3], it was investigated the existence and stability of such systems. The discussion in these works defined conditions for the existence of these objects for a general case. However, the interactions between the scalar fields and matter were not considered .
- In the present study, we relax the assumption of the lack of interaction between the scalar field and matter for the considered system. This is done in order to study the possibility for the appearance of static solutions (and not only the oscillatory ones) when the scalar field is real.
- In order to simplify the discussion, we write the EKG equations by considering matter described by simple constitutive relations including the photon gas one.
- The interaction between the field and the gas is introduced by assuming that the scalar field source *j(r)* is proportional to the matter energy density $\varepsilon(r)$.
- The constitutive relations for matter were considered in two cases: One reflecting a nearly pressureless gas and the second associated to photon like particles.

Spherically symmetric solutions were searched. The equations reduced to a set of differential equations in the radial axis, which were solved by assuming initial conditions for the scalar field and the matter at the origin.
The use of the Bianchi identity, third equation at the wright, which resulted in a first order equation, simplified the solution.

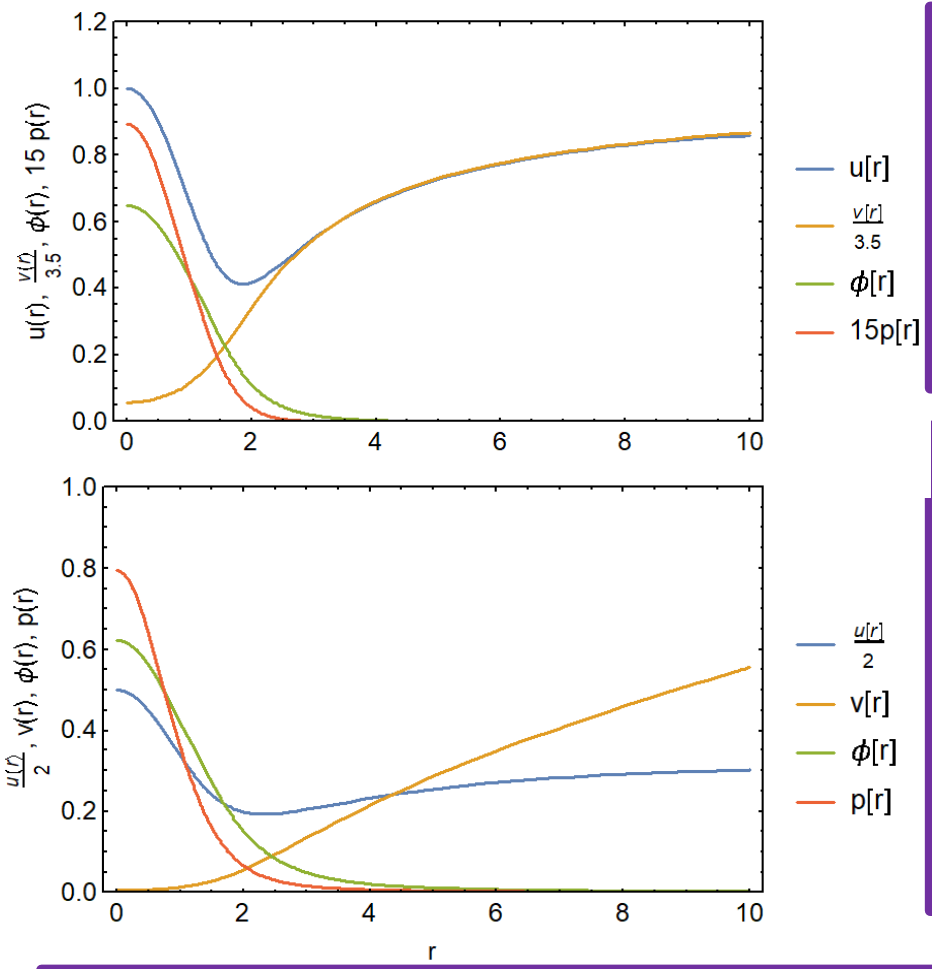
The EKG equations for the diagonal temporal component of the covariant metric *v(r)*, the radial component of the inverse metric *u(r)* and the scalar field $\phi(r)$ took the form

$$\frac{u'(r)}{r} - \frac{1-u(r)}{r^2} = -\frac{1}{2}(u(r)\phi'(r)^2 + \phi(r)^2 + 2j(r)\phi(r)) - \epsilon(r),$$

$$\frac{u(r)}{v(r)}\frac{v'(r)}{r} - \frac{1-u(r)}{r^2} = -\frac{1}{2}(-u(r)\phi'(r)^2 + \phi(r)^2 + 2j(r)\phi(r)) + p(r),$$

$$p'(r) + (\epsilon(r) + p(r))\frac{v'(r)}{2v(r)} - \phi(r)\ j'(r) = 0,$$

$$j(r) + \phi(r) - u(r)\ \phi''(r) = \phi'(r)(\frac{u(r)+1}{r} - r\ (\frac{\phi(r)^2}{2} + j(r)\phi(r) + \frac{\epsilon(r)-p(r)}{2}))$$



The solutions were found by firstly giving at the origin r=0, the values of the matter energy density *e(0)*, and the value of *u(0)=1* (which is forced if the solution should be regular at the symmetry axis). Further, a temptative initial value of $\phi(0)$ was assigned in addition to its required vanishing radial derivative $\phi'(0)=0$. The resulting solution for $\phi(r)$ became singular at some radial distance, tending to be, either positive or negative. Then, by iteratively reducing or agumenting the initial value for $\phi(0)$, the repetitive solutions were approaching a decaying value of the scalar field at large radii.

- The figure at the top left, shows the solution of the EKG equations corresponding with a nearly pressureless matter with a constitutive relation *e(r)=40 p(r)*. The interaction between matter and field was described by *J(r)= g e(r)* with *g=0.9*. The initial conditions for all the variables at zero radius are defined in the plots.
- The figure at botton left, illustrates the solution for photon like matter: *e(r)=3 p(r)* . Note that in this case the "gravitational potential" v(r) increases nearly linearly in the farway regions. This implies that an also nearly constant attractive force is there exterted on small bodies, suggesting a mechanism similar to that one determining the velocity curves of stars in galaxies.

Main Conclusions

Static solutions of the EKG equations are obtained, thanks to the interaction between the scalar field and matter. For pressurless matter the solutions are normal stars in which the space time tends to be the Minkowski one at infinity. They also satisfy the stability crititerium, its mass increases when the density of matter at the origin also increases. When the matter is photon like, a nearly constant force acts on farway laying small bodies. Mechanisms allowing for the decay of such forces at large distances are preliminarily explored.

[1] P. Jetzer, Phys. Rep. 220, 161-227 (1992).
[2] A. B. Henriques, A. R. Liddle and R.G. Moorhouse. Lett. B 233, 99 (1989).
[3] A. B. Henriques, A. R. Liddle and R.G. Moorhouse. Nucl. Phys. B 337, 737 (1990).