Poisson type conformastat spherically symmetric spacetimes

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Abstract

We construct conformastat spherically symmetric spacetimes from given solutions of Poisson's equation. As a simple application, we present an exact perfect fluid solution starting with the seed potential-density pair corresponding to a massive spherical dark matter halo with a logarithm potential.

Poisson type spacetimes

The metric for a conformastat spherically symmetric spacetime is given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2). \tag{1}$$

By setting $e^{\lambda} = \left(1 - \frac{\phi(r)}{2}\right)^4$, we get for the energy density the following nonlinear Poisson type equation

$$\nabla^2\phi = 4\pi G\rho\left(1 - \frac{\phi}{2}\right)^5. \tag{2}$$

A simple particular form of ρ is

$$\rho = \frac{\rho_0}{\left(1 - \frac{\phi}{2}\right)^5}. \tag{3}$$

Replacing this expression in (2) one finds that the pair (ϕ, ρ_0) is a solution of the Poisson's equation. Hence $(\phi, \rho_0) = (\Phi, \rho_N)$ for any physical system. To obtain the other metric function ν an additional assumption must be imposed.
For a perfect fluid source

$$T^{ab} = (\rho + p)U^a U^b + p g^{ab}, \tag{4}$$

such assumption is the condition of pressure isotropy which can be cast as

$$L\mathcal{G}_{,xx} = 2\mathcal{G}L_{,xx}, \quad L \equiv e^{-\lambda}, \quad \mathcal{G} \equiv Le^{\nu}, \quad x \equiv r^2. \tag{5}$$

Logarithmic potential type perfect fluid spheres

Dark matter haloes can be modeled in Newtonian theory with a logarithmic potential of the form

$$\Phi = \frac{1}{2} v_0^2 \ln(\frac{r^2 + a^2}{b}), \tag{6}$$

where a and b are constants, and v_0 is circular speed at large radii, also a constant. The mass density distribution is

$$\rho_N = \frac{v_0^2 (r^2 + 3a^2)}{4\pi G (r^2 + a^2)^2}. \tag{7}$$

Solving (5), a particular solution is

$$e^{\frac{\nu}{2}} = C\left(r^2 + a^2\right)\left[1 - \frac{1}{4}\frac{v_0^2}{c^2}\ln\left(\frac{r^2 + a^2}{b}\right)\right]^6 \text{(with } C \text{ constant)}, \tag{8a}$$

$$e^{\frac{\lambda}{2}} = \left[1 - \frac{1}{4}\frac{v_0^2}{c^2}\ln\left(\frac{r^2 + a^2}{b}\right)\right]^4. \tag{8b}$$