Kinematic Constraints on Spatial Curvature from Supernovae Ia and Cosmic Chronometers

Cosmology

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Abstract

In this work an interesting approach to estimate the spatial curvature Ω_k from data independently of dynamical models is suggested. It was done through three kinematic parametrizations of the comoving distance ($D_C(z)$) with second degree polynomial, of the Hubble parameter ($H(z)$) with a second degree polynomial and of the deceleration parameter ($q(z)$) with first order polynomial. All these parametrizations are done as function of redshift z. We used SNe Ia dataset from Pantheon compilation with 1048 distance moduli estimated on the range $0.01 < z < 2.3$ with systematic and statistical errors and a compilation of 31 $H(z)$ data estimated from cosmic chronometers. The spatial curvature found for $D_C(z)$ parametrization was $\Omega_k = -0.03^{+0.24+0.56}_{-0.30-0.53}$. The parametrization for deceleration parameter $q(z)$ resulted in $\Omega_k = -0.08^{+0.21+0.54}_{-0.27-0.45}$. The $H(z)$ parametrization had incompatibilities between $H(z)$ and SNe Ia data, so these analyses were not combined. Both $D_C(z)$ and $q(z)$ parametrizations are compatible with the spatially flat Universe as predicted by many inflation models and data from CMB. This type of analysis may be interesting as it avoids any bias because it does not depend on assumptions about the matter content for estimating Ω_k.

Introduction

The evidence that the Universe is accelerating comes from observations of SNe Ia, CMB, BAO and data from $H(z)$ ([1] and [2]). The acceleration phase of the universe can be theoretically supported by the constant Λ plus the Cold Dark Matter component. This model has cosmological parameters that can be constrained by observational data [2].

Parametrizations help to reconstruct the evolution of the Universe without considering dynamics, that is: regardless of dynamics. However, using the FLRW metric, we can report that these parametrizations ($H(z)$, $q(z)$)) for the spatial curvature and cosmology distances: luminosity distance (d_L) and angular distance d_A). All this using distance data from SNe Ia that must restrict the curvature parameter without assuming any Cosmology dynamics.

In this present work, we study the spatial curvature with third order parametrizations in the comoving distance, second order in $H(z)$ and first order in $q(z)$. We combined the distance-luminosity data of SNe Ia [3] and measures of $H(z)$ [4], being possible to estimate the values of Ω_k.

In order to do that, we have assumed the Cosmological Principle with the FLRW line element and the basic equations:

$$D_C(z) = \int_0^z \frac{dz'}{E(z')} \tag{1}$$

$$q(z) = -\frac{\ddot{a}}{aH^2} = \frac{1+z}{H}\frac{dH}{dz} - 1 \tag{2}$$

where $D_C(z) \equiv \frac{H_0}{c} d_c(z)$ is dimensionless comoving distance and $E(z) \equiv \frac{H(z)}{H_0}$ is dimensionless Hubble parameter. Luminosity distance D_L relates with transverse comoving distance D_M as

$$D_L(z) = (1+z) D_M(z) \tag{3}$$

where $D_M = \mathrm{sinn}(D_C, \Omega_k)$ and

$$\mathrm{sinn}(x, \Omega_k) \equiv \begin{cases} \frac{1}{\sqrt{\Omega_k}} \sinh\left[x\sqrt{\Omega_k}\right] & \text{for } \Omega_k > 0, \\ x & \text{for } \Omega_k = 0, \\ \frac{1}{\sqrt{-\Omega_k}} \sin\left[x\sqrt{-\Omega_k}\right] & \text{for } \Omega_k < 0, \end{cases} \tag{4}$$

Main Objectives

1. This work uses parametrizations through z on $H(z)$, $q(z)$ and d_c.
2. When these parametrizations were made using polynomial approach on z the estimative from Ω_k can be done without considering Cosmology dynamics.

Ω_k from line-of-sight comoving distance, $D_C(z)$

In order to put limits on Ω_k by considering the line-of-sight comoving distance, we can write $D_C(z)$ as a second degree polynomial such as:

$$D_C = z + d_2 z^2 + d_3 z^3, \tag{5}$$

where d_2 and d_3 are free parameters, then we can write:

The dimensionless luminosity distance is

$$D_L(z) = (1+z)\,\mathrm{sinn}\,(z + d_2 z^2 + d_3 z^3, \Omega_k). \tag{6}$$

Now, these equations shall be compared with $H(z)$ measurements and luminosity distances from SNe Ia, respectively, in order to determine d_2 and Ω_k.

Ω_k from $H(z)$

In order to assess Ω_k by means of $H(z)$ we need an expression for $H(z)$. If one wants to avoid dynamical assumptions, one must resort to kinematical methods which use an expansion of $H(z)$ over the redshift.

Let us try a simple $H(z)$ expansion, namely, the quadratic expansion:

$$\frac{H(z)}{H_0} = E(z) = 1 + h_1 z + h_2 z^2. \tag{7}$$

In order to constrain the model with SNe Ia data, we obtain the luminosity distance from Eqs.(3), (1) and (7). We have

$$D_C = \int_0^z \frac{dz'}{E(z')} = \int_0^z \frac{dz'}{1 + h_1 z' + h_2 z'^2}, \tag{8}$$

which gives three possible solutions, according to the sign of $\Delta \equiv h_1^2 - 4h_2$, such as

$$D_C = \begin{cases} \frac{2}{\sqrt{-\Delta}}\left[\arctan\left(\frac{2h_2 z + h_1}{\sqrt{-\Delta}}\right) - \arctan\frac{h_1}{\sqrt{-\Delta}}\right], & \Delta < 0, \\ \frac{2z}{h_1 z + 2}, & \Delta = 0, \\ \frac{1}{\sqrt{\Delta}} \ln\left|\left(\frac{\sqrt{\Delta} + h_1}{\sqrt{\Delta} - h_1}\right)\left(\frac{\sqrt{\Delta} - h_1 - 2h_2 z}{\sqrt{\Delta} + h_1 + 2h_2 z}\right)\right|, & \Delta > 0, \end{cases} \tag{9}$$

from which follows the luminosity distance $D_L(z) = (1+z)\,\mathrm{sinn}\,(D_C, \Omega_k)$.

Ω_k from $q(z)$

Now we can analyze Ω_k by parametrizing $q(z)$. From (2) one may find $E(z)$ as

$$E(z) = \exp\left[\int_0^z \frac{1 + q(z')}{1 + z'} dz'\right]. \tag{10}$$

If we assume a linear z dependence in $q(z)$, as

$$q(z) = q_0 + q_1 z, \tag{11}$$

which is the simplest $q(z)$ parametrization that allows for an acceleration transition as required by SNe Ia data, one may find

$$E(z) = e^{q_1 z}(1+z)^{1+q_0-q_1}, \tag{12}$$

while the line-of-sight comoving distance $D_C(z)$ (1) is given by

$$D_C(z) = e^{q_1} q_1^{q_0 - q_1}\left[\Gamma(q_1 - q_0, q_1) - \Gamma(q_1 - q_0, q_1(1+z))\right], \tag{13}$$

where $\Gamma(a, x)$ is the incomplete gamma function defined as $\Gamma(a, x) \equiv \int_x^\infty e^{-t} t^{a-1} dt$, with $a > 0$, from which follows the luminosity distance as $D_L(z) = (1+z)\,\mathrm{sinn}\,(D_C, \Omega_k)$, which can be constrained from observational data.

Results

The outcomes from parametrization are shown on

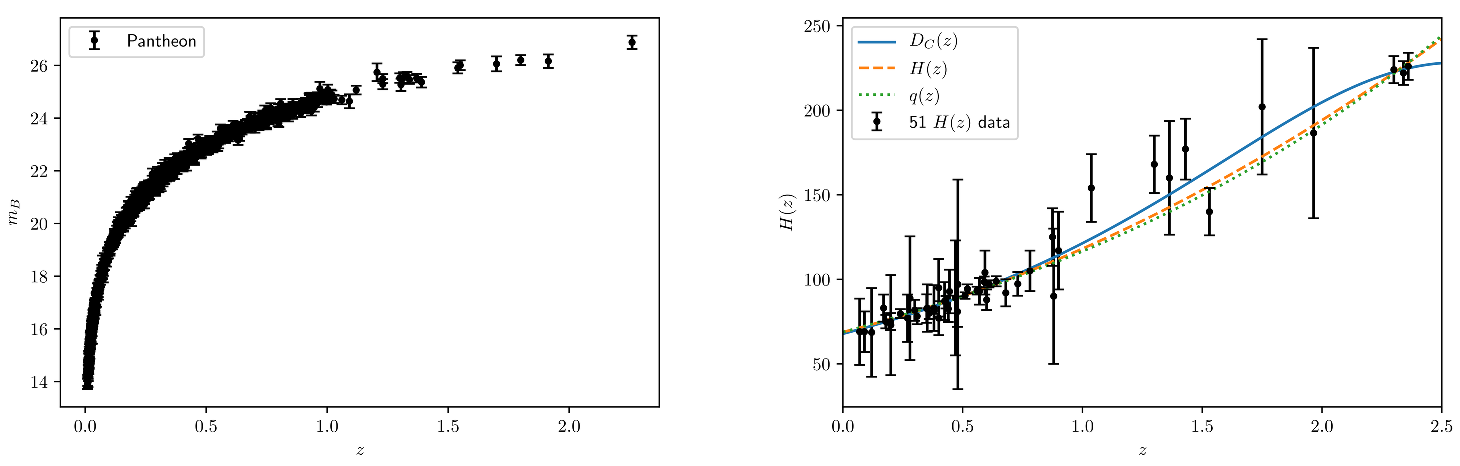

Figure 1: a) SNe Ia apparent magnitude m_B from Pantheon. The error bars shown correspond only to statistical errors, but we use the full covariance matrix (statistical+systematic errors) in the analysis. **b)** 51 $H(z)$ data compilation. The lines represent the best fit from SNe+$H(z)$ data for each model.

The table shows results from parameterization

Treatments	Response 1	Response 2
Parameter	$D_C(z)$	$q(z)$
H_0	$69.0 \pm 2.4 \pm 4.9$	$69.3 \pm 2.4^{+4.8}_{-4.7}$
Ω_k	$-0.03^{+0.24+0.56}_{-0.30-0.53}$	$-0.08^{+0.21+0.54}_{-0.27-0.45}$
d_2	$-0.255 \pm 0.030^{+0.059}_{-0.061}$	–
d_3	$0.029 \pm 0.011^{+0.023}_{-0.022}$	–
q_0	–	$-0.536 \pm 0.085 \pm 0.17$
q_1	–	$0.73 \pm 0.15 \pm 0.30$

Table 2: Constraints from Pantheon+$H(z)$ for $D_C(z)$ and $q(z)$ parametrizations. The central values correspond to the mean and the 1 σ and 2 σ c.l. correspond to the minimal 68.3% and 95.4% confidence intervals.

Conclusions

- This work showed the comoving distance D_C, the Hubble parameter $H(z)$ and the deceleration parameter $q(z)$ as third, second and first degree polynomials on z, respectively, and obtained, for each case, the Ω_k value.
- Supernovae type Ia data and Hubble parameter measurements were combined nice constraints are found over the spatial curvature, without the need of assuming any particular dynamical model. Results were showed can be in table above.
- The values obtained for the spatial curvature in each case were $\Omega_k = 0.11^{+0.21}_{-0.24}$, $-0.03^{+0.21}_{-0.24}$ and $-0.05^{+0.21}_{-0.25}$ at 1σ c.l., respectively (see table 2), all compatible with a spatially flat Universe, as predicted by most inflation models and confirmed by CMB data, in the context of ΛCDM model.

References

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