Fusion dynamics of $^{12}C+^{12}C$ reaction: An astrophysical interest within the relativistic mean-field approach

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Introduction

- The carbon and oxygen burning reactions i.e. $^{12}C+^{12}C$, $^{12}C+^{16}O$ and $^{16}O+^{16}O$ play a crucial role in the later phases of stellar evolution and explosions [1, 2].
- The study of fusion dynamics and astrophysical S-factor of these reactions holds a paramount significance in the astrophysics.
- The relativistic mean field (RMF) formalism has been applied successfully in the description of finite nuclei as well as the infinite nuclear matter [3, 4, 5].
- It is of interest to assess the microscopic R3Y nucleon-nucleon (NN) potential derived from RMF approach along with the RMF density distributions to study the dynamics of the above fusion reactions of astrophysical interest.
- In this work, the fusion cross-section of $^{12}C+^{12}C$ reaction is investigated using microscopic R3Y and well known M3Y NN interaction and the results are then compared with the experimental data [1, 2].

Theoretical Formalism

The present study of fusion cross-section mainly consists of following steps:

- Calculation of nuclear interaction potential using the well-known M3Y and microscopic R3Y nucleon-nucleon (NN) potential along with the relativistic mean-field (RMF) densities in double folding integral.
- Calculation of fusion cross-section using ℓ- summed Wong Formula.
- Then finally calculations of the astrophysical S-factor.

Interaction Potential

The total interaction potential V_T (R) between two interacting nuclei is given as:

$$V_T(R) = V_C(R) + V_n(R) + V_\ell(R). \tag{1}$$

Here, $V_C(R)(= Z_pZ_te^2/R)$ and $V_\ell(R)$ are the well known Coulomb and centrifugal potentials, and $V_n(R)$ is nuclear potential, which we have calculated here using the double folding integral [6]

$$V_n(\vec{R}) = \int \rho_p(\vec{r}_p)\rho_t(\vec{r}_t)V_{eff}\left(|\vec{r}_p - \vec{r}_t + \vec{R}| \equiv r\right) d^3r_p d^3r_t, \tag{2}$$

In this equation V_{eff} is the effective nucleon nucleon (NN) interaction and can be evaluated using widely known M3Y interaction [7] given as

$$V_{eff}^{M3Y}(r) = 7999\frac{e^{-4r}}{4r} - 2140\frac{e^{-2.5r}}{2.5r} + J_{00}(E)\delta(r). \tag{3}$$

The last term is a pseudo-potential which includes the effect of single nucleon-exchange.

Relativistic Mean Field Formalism

In RMF theory the nucleons are considered to be point-like particles denoted by Dirac spinors ψ interacting through the exchange of mesons and photons. It describes the meson-nucleon many body system phenomenologically by a Lagrangian density of the form [3, 4, 5]

$$\begin{aligned}\mathcal{L} = {} & \overline{\psi}\{i\gamma^\mu\partial_\mu - M\}\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ & - g_s\overline{\psi}\psi\sigma - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_w^2\omega^\mu\omega_\mu - g_w\overline{\psi}\gamma^\mu\psi\omega_\mu - \frac{1}{4}\vec{B}^{\mu\nu}.\vec{B}_{\mu\nu} \\ & + \frac{1}{2}m_\rho^2\vec{\rho}^\mu.\vec{\rho}_\mu - g_\rho\overline{\psi}\gamma^\mu\vec{\tau}\psi\cdot\vec{\rho}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\psi}\gamma^\mu\frac{(1-\tau_3)}{2}\psi A_\mu.\end{aligned} \tag{4}$$

Here M, m_σ, m_ω and m_ρ denote the masses of nucleon, σ, ω and ρ- mesons respectively. g_σ, g_ω, g_ρ denote linear coupling constants for respective mesons. The parameter set NL3* is used here to obtain the masses and coupling constants of the mesons.

The microscopic R3Y NN potential analogous to M3Y potential have been derived recently by solving the field equation for mesons in limit of one-meson exchange [3, 4, 5] and is given by

$$\begin{aligned}V_{eff}^{R3Y}(r) = {} & \frac{g_\omega^2}{4\pi}\frac{e^{-m_\omega r}}{r} + \frac{g_\rho^2}{4\pi}\frac{e^{-m_\rho r}}{r} - \frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g_2^2}{4\pi}re^{-2m_\sigma r} \\ & + \frac{g_3^2}{4\pi}\frac{e^{-3m_\sigma r}}{r} + J_{00}(E)\delta(r).\end{aligned} \tag{5}$$

Using the above mentioned double folding procedure for M3Y, the nuclear interaction potential can be obtained for the R3Y interactions, along with nuclear matter densities derived from the RMF-Lagrangian.

Results: Nuclear matter Densities and Interaction Potential

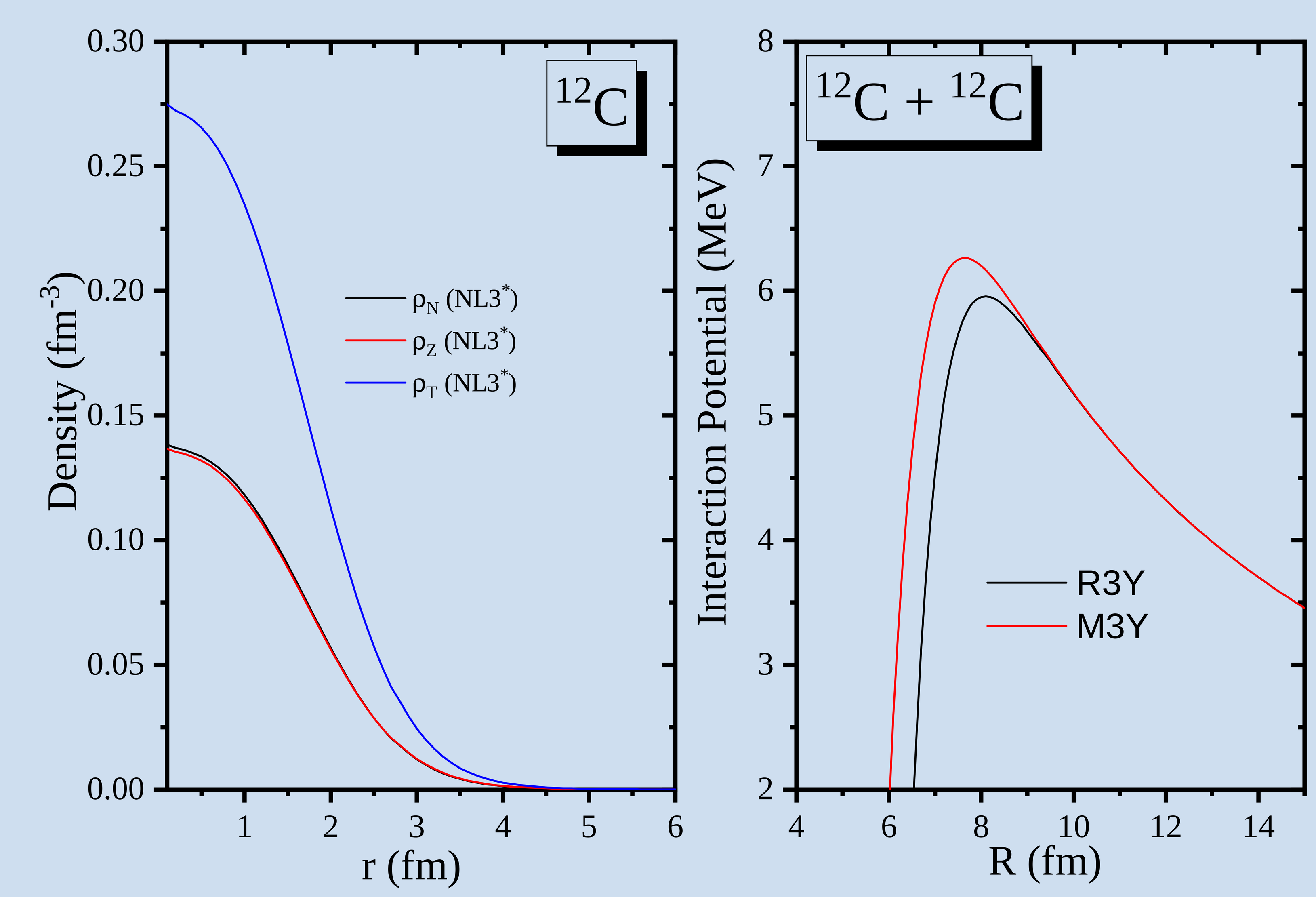

Figure 1:The RMF (NL3*) neutron (ρ_N), proton (ρ_Z) ant total (ρ_T) density distributions for ^{12}C (left) and interaction potential ($V_T(R) = V_C(R) + V_n(R)$) as a function of radial separation R for $^{12}C+^{12}C$ system using R3Y (black line) and M3Y (red line) NN interactions (right).

$\ell-$ summed Wong Formula

The fusion cross section in terms of partial wave is given as [8],

$$\sigma(E_{c.m.}) = \frac{\pi}{k^2}\sum_{\ell=o}^{\ell_{max}}(2\ell+1)P_\ell(E_{c.m}). \tag{6}$$

Here, $E_{c.m}$ is the center-of-mass energy of two colliding nuclei and P_ℓ is known as the transmission coefficient for ℓ^{th} partial wave which is generated by using hill-Wheeler approximation. For details see [3, 4, 8] and references there in. Wong [8] replaced the ℓ summation in Eq. (6) by an integral using the following approximations:

(i) $\hbar\omega_\ell \approx \hbar\omega_0$, and

(ii) $V_B^\ell \approx V_B^0 + \frac{\hbar^2\ell(\ell+1)}{2\mu R_B^{0\,2}}$, assuming $R_B^\ell \approx R_B^0$. After integration, the $\ell = 0$ barrier-based simple Wong formula [8] is given as:

$$\sigma(E_{c.m.}) = \frac{R_B^{0\,2}\hbar\omega_0}{2E_{c.m.}}.ln\left[1 + exp\left(\frac{2\pi}{\hbar\omega_0}(E_{c.m.} - V_B^0)\right)\right]. \tag{7}$$

However the ℓ-summation procedure introduced by Wong using only $\ell = 0$ barrier, excludes the actual modifications entering the potential due to its angular momentum dependence and thus overestimates the cross-section at above barrier energies. To overcome this problem, Wong formula is further extended by Gupta and collaborators [8] in terms of summation over ℓ-partial waves. Details can be found in Refs. [3, 4, 8].

Astrophysical S-factor

At energies far below the barrier, the Coulomb force dominates. In order to remove most of the Coulomb barrier penetration effect, the astrophysical S-factor was introduced and is given by [1, 2]

$$S = \sigma E_{c.m.}exp(87.21E^{-1/2} + 0.46E) \tag{8}$$

The calculation of S-factor is done for $^{12}C+^{12}C$ reaction.

Results: Fusion Cross-Section and S-Factor

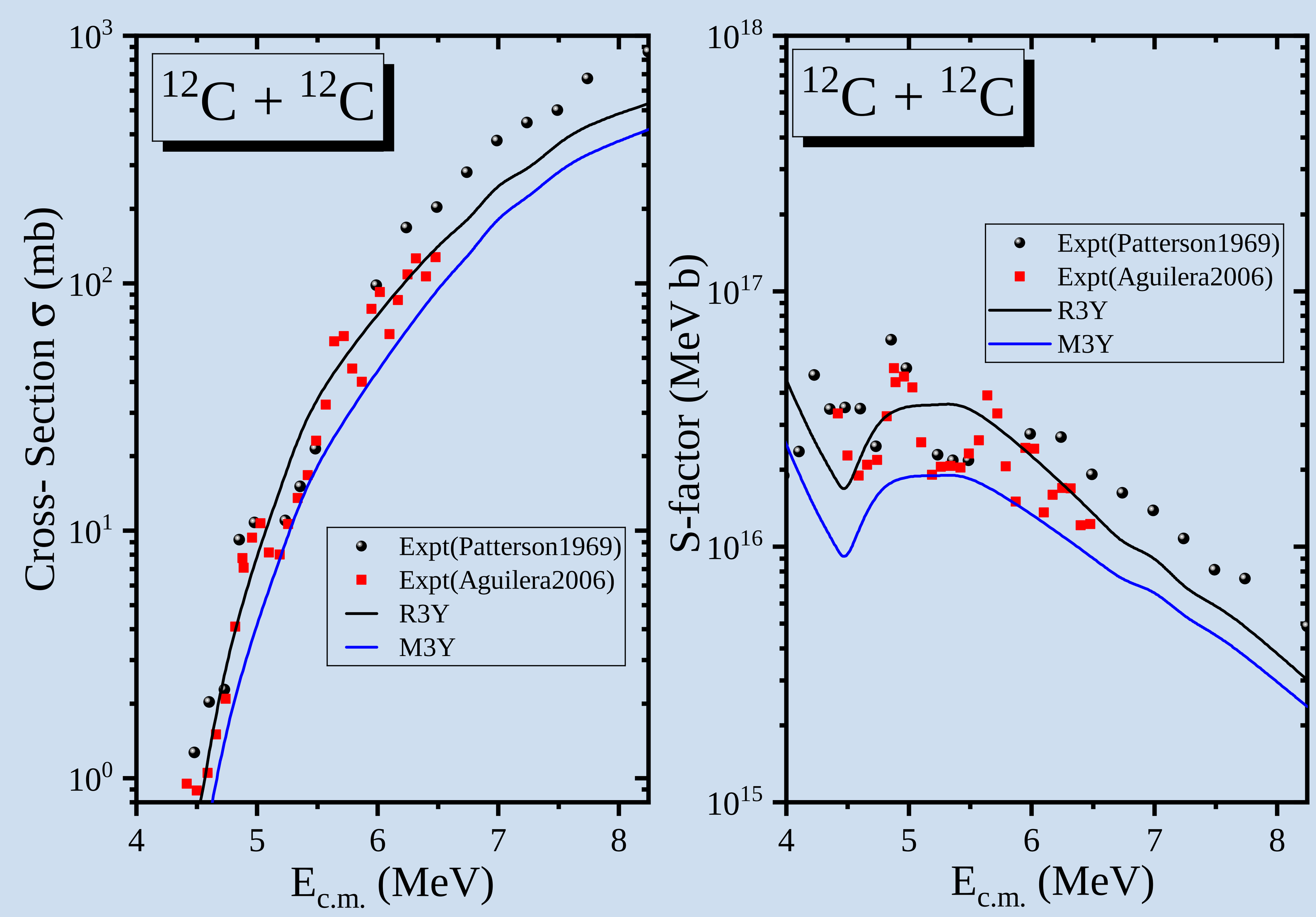

Figure 2:Fusion cross-section (σ) (left) and astrophysical S-factor (right) as function of center of mass energy ($E_{c.m.}$) calculated using the ℓ-summed Wong formula for R3Y (black line) and M3Y (blue line). The experimental data is taken from references [1, 2].

Remarks and Conclusion

- The height of barrier obtained for M3Y potential is little bit higher than that of R3Y (NL3*) NN potential.
- The microscopic R3Y potential gives comparatively better fit to the experimental data of fusion cross section for $^{12}C+^{12}C$ reaction than the M3Y potential.
- For S-factor also the R3Y potential has better agreement with the experimental data in comparison to the M3Y potential.
- It will be of further interest to study the reactions like $^{12}C+^{16}O$ and $^{16}O+^{16}O$ and so on, which are of astrophysical significance within this microscopic approach.

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