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From Quarks to Cosmos

# Triaxially-deformed Freely-precessing Neutron Stars: Continuous electromagnetic and gravitational radiation

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## Introduction

★ A rapidly rotating, asymmetric neutron star (NS) in the Milky Way undergoes **free precession**, making it a potential source for **multimessenger** observation. The free precession could manifest in (i) the spectra of continuous **gravitational waves** (GWs), and (ii) the timing behavior and pulse-profile of radio and/or X-ray **pulsars**.

★ We extend previous work and investigate in great detail the free precession of a **triaxially deformed** NS with **analytical and numerical** approaches. In particular, its associated continuous GWs and pulse signals are derived. Explicit examples are illustrated for the continuous GWs, as well as timing residuals in both **time and frequency domains**.

## Methods

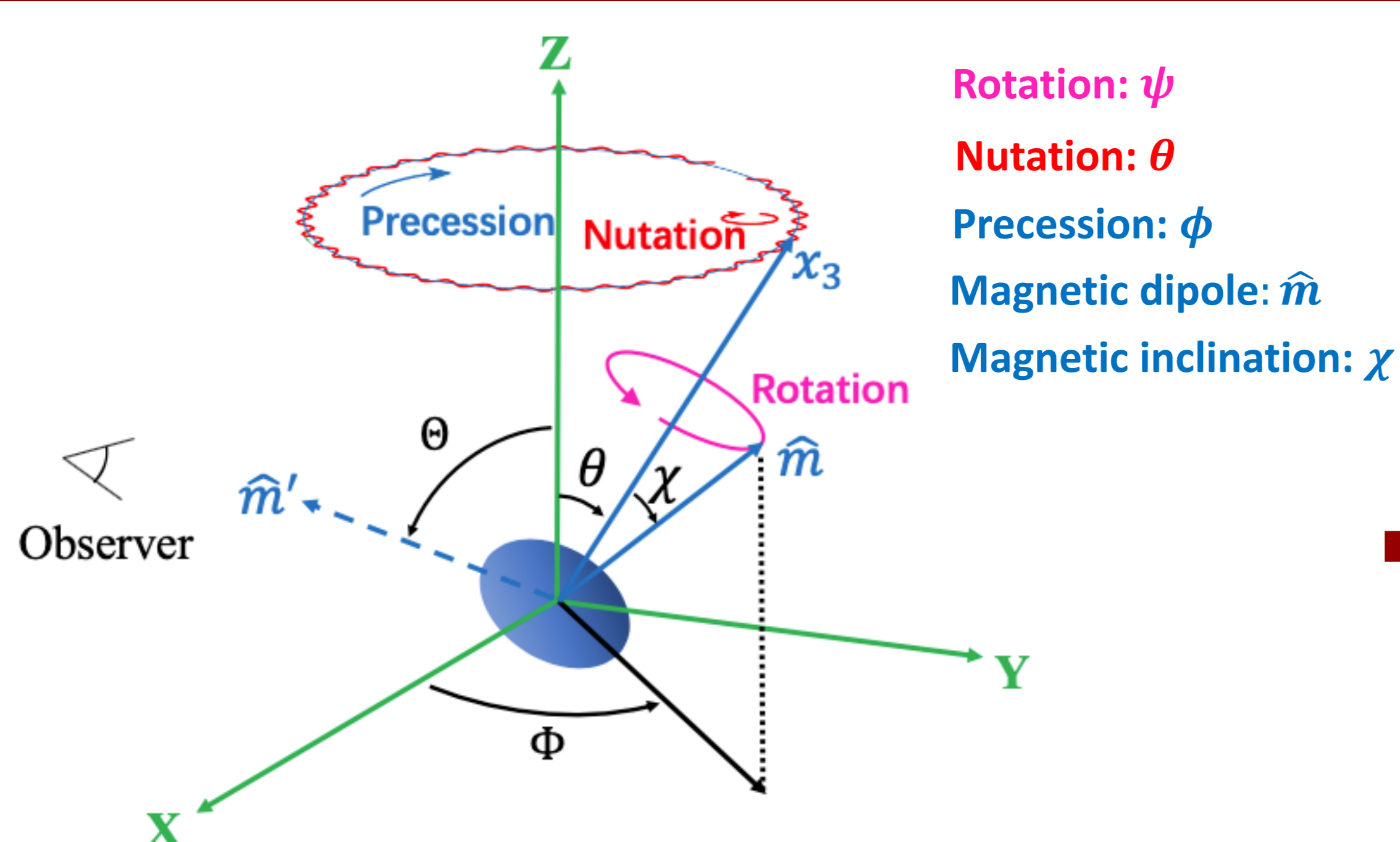


Figure 1. Configuration and kinematics of freely-precessing triaxially-deformed NS.

### Assumptions:

- the moments  $I_i$  ( $i = 1, 2, 3$ ) are constant
- the NS undergoes free precession

### Time evolution of the configuration:

- Analytical solution with **elliptic functions**
- Numerical approach using **quaternions**

### Parameterized description of precessing NS:

$$\epsilon \equiv \frac{I_3 - I_1}{I_3} \quad \delta \equiv \frac{I_2 - I_1}{I_3 - I_2} \quad \gamma \equiv \tan \theta_{\min}$$

**Oblateness****Nonaxisymmetry****Wobble**

### Modulated timing signals of pulsar:

$$\Delta\Phi = F(\psi, \theta, \phi)$$

Phase residual = Function of NS configuration

### Modulated pulse profile:

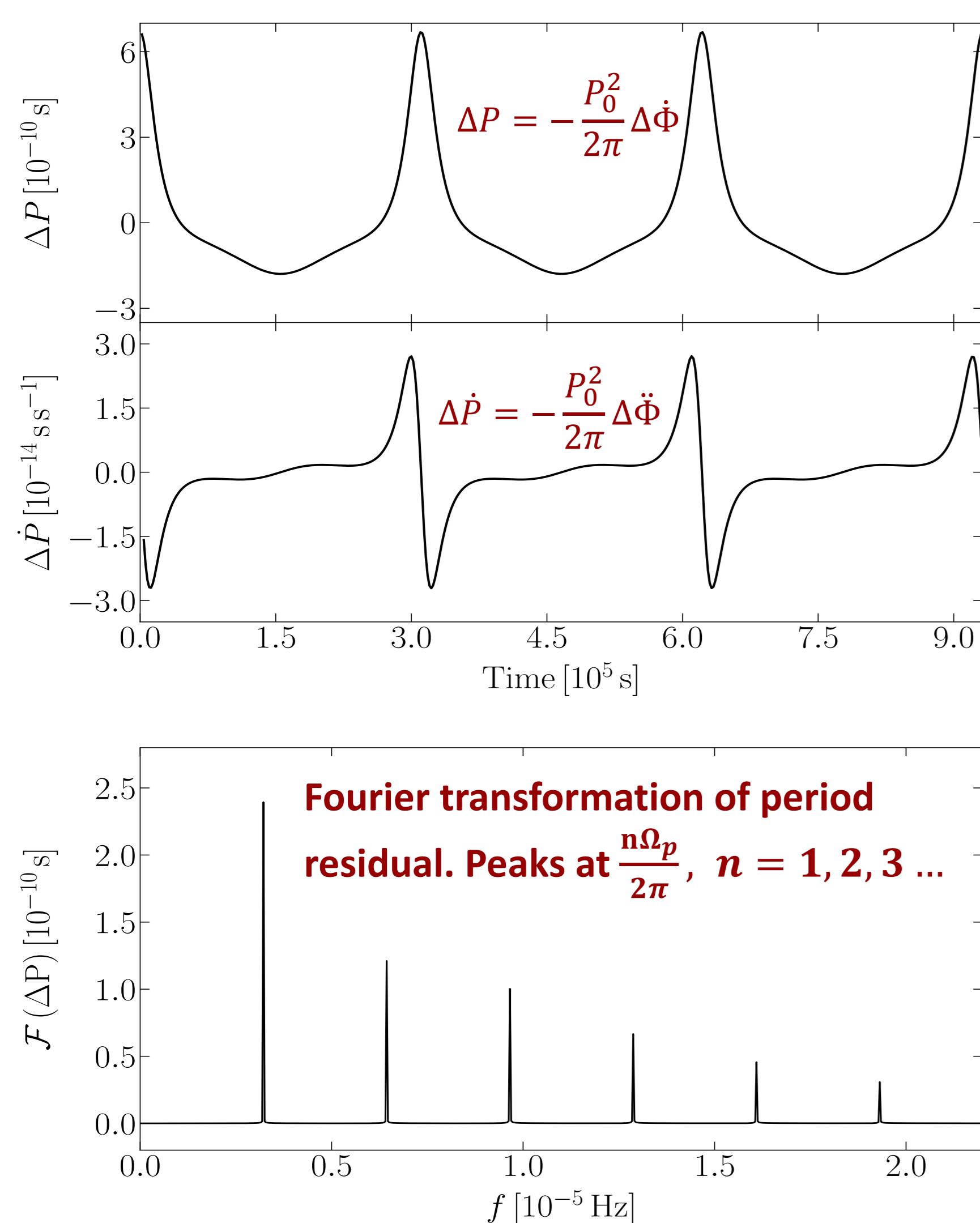
$$W = G(\Theta), \quad \Theta = H(\psi, \theta, \phi, \chi, t)$$

Pulse width = Function of configuration, emission cone, line of sight

### Quadrupole radiation of GW:

$$h_{ij}^{\text{TT}} = \frac{2G}{c^4 r} \frac{d^2 I_{ij}}{dt^2} = -\frac{2G}{rc^4} \mathcal{R}_{ik} \mathcal{R}_{jl} A_{kl}$$

## Result 1: timing residual

Figure 2. Timing residuals for a large wobble angle case.  $\Omega_p$  is the free precession angular frequency.

★ To the second order expansion of  $\epsilon$ ,  $\delta$ , and  $\gamma$  (small wobble case), the period residual is

$$\Delta P \approx \frac{P_0^2}{2\pi} \Omega_p \gamma (\delta + 1) \cot \chi \cos(\Omega_p t) + \frac{P_0^2}{4\pi} \Omega_p \gamma^2 (1 + 2 \cot^2 \chi) \cos(2\Omega_p t)$$

## Result 2: pulse-width modulation

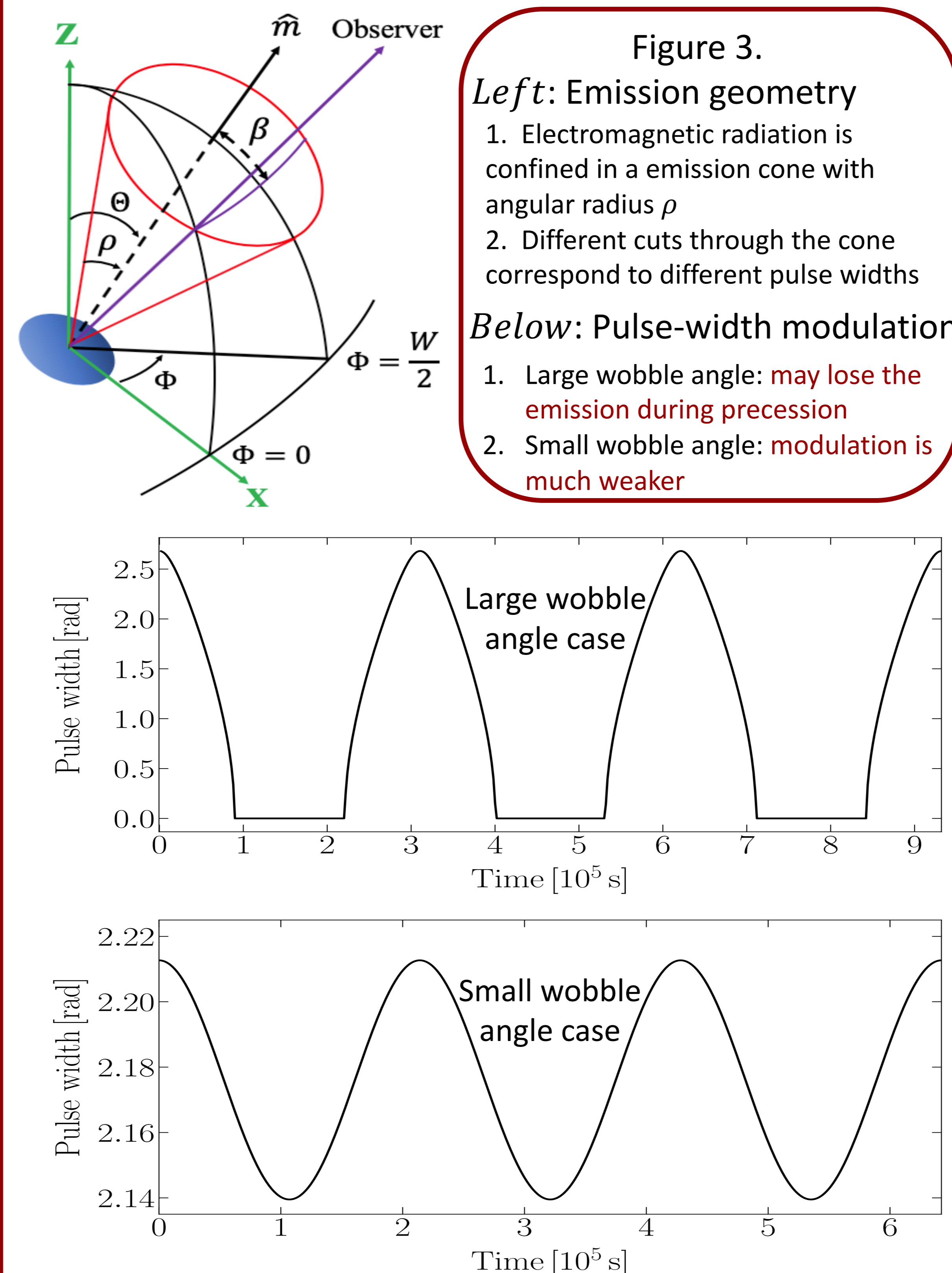


Figure 3.

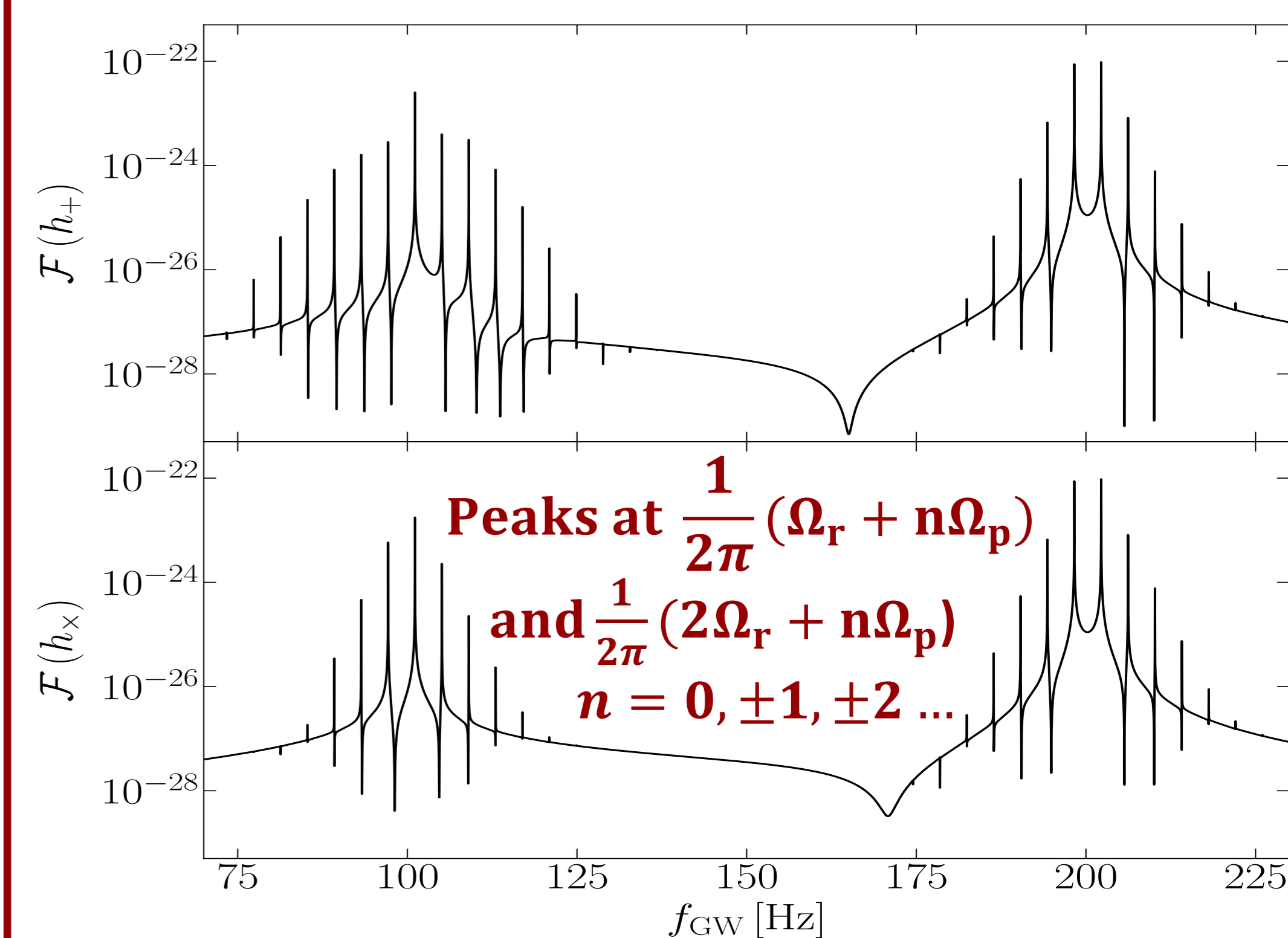
### Left: Emission geometry

- Electromagnetic radiation is confined in a emission cone with angular radius  $\rho$
- Different cuts through the cone correspond to different pulse widths

### Below: Pulse-width modulation

- Large wobble angle: may lose the emission during precession
- Small wobble angle: modulation is much weaker

## Result 3: GW waveform

Figure 4. The Fourier amplitudes of  $h_+$  and  $h_x$ . The parameters  $\epsilon$ ,  $\delta$ , and  $\gamma$  have been exaggerated for illustration. Physical parameter can be easily implemented.  $\Omega_r$  is the rotation angular frequency.

★ To the second order expansion of  $\epsilon$ ,  $\delta$ , and  $\gamma$  (small wobble case), the peaks of GW angular frequencies are at

first order lines

$$\Omega_r + \Omega_p \text{ and } 2\Omega_r$$

second order lines

$$2(\Omega_r + \Omega_p), \Omega_r - \Omega_p, \Omega_r - 3\Omega_p, \text{ and } 2(\Omega_r - \Omega_p)$$

## Conclusion

- Radio/X-ray signals and continuous GWs of freely-precessing NSs will provide valuable information about the wobble angle, the nonaxisymmetry, and the oblateness of the source.
- The next-generation GW detectors may detect the continuous GWs from precessing NSs and constrain the equation of state of NSs.

## References

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