# Study of Ultracompact Stars in General Relativity 

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## Introduction

- It is usually considered $2.16 \mathrm{M}_{\odot}$ as the upper limit mass for a neutron star. Objects with masses between this value and $5 \mathrm{M}_{\odot}$ could be weakly interacting black holes or very massive neutron stars. Motivated by this, the present work's aim is to investigate these possible neutron stars, the so-called ultracompact stars.
- Recently, for example, in a binary system, the inferred mass of the object that accompanied a giant star was $3.3 \mathrm{M}_{\odot}$ [1]. Thus, considering that very intense magnetic fields, rotations, and very high densities can be modeled by anisotropic EoSs or deformed geometries, in this work some modeling possibilities for ultracompact stars are studied.


## Description of the Research

- It is well known that the equations that describe the equilibrium structure in a neutron star are the TOV equations [2]. They describe a static and spherically symmetric star and assume a perfect fluid. Seeking to obtain how pressure $p=p(r)$, mass $m=m(r)$ and density $\varepsilon=\varepsilon(r)$ vary with radius $r$, they are given by (with natural units $c=G=\hbar=1$ ):

$$
\begin{gathered}
\frac{\partial p}{\partial r}=-\frac{(\varepsilon+p)\left(4 \pi r^{3} p+m\right)}{r^{2}\left(1-\frac{2 m}{r}\right)} \\
\frac{\partial m}{\partial r}=4 \pi r^{2} \varepsilon
\end{gathered}
$$

- The first modification to them that we have studied is the one proposed by Zubairi et al. [3]. In this case, we also assume a perfect fluid, but a static and deformed star $(z=\gamma r$, with $z$ representing the vertical axis and $\gamma$ the degree of star deformation). Considering this, we obtain, after using Einstein's equations:

$$
\begin{gathered}
\frac{\partial p}{\partial r}=-\frac{(\varepsilon+p)\left[4 \pi r^{3} p+\frac{1}{2} r\left(1-\left(1-\frac{2 m}{r}\right)^{\gamma}\right)\right]}{r^{2}\left(1-\frac{2 m}{r}\right)^{\gamma}} \\
\frac{\partial m}{\partial r}=4 \pi \gamma r^{2} \varepsilon
\end{gathered}
$$

- Alternatively, in a second modification, which is based in the work of Raposo et al. [4], for the treatment of the so-called "C stars" - again considering a static and spherically symmetric star -, there is the introduction of an anisotropy in the fluid (this is why the pressure is divided into two contributions: radial $P_{r}$ and tangential $P_{t}$ ), and the formula for the perfect fluid is no longer used. Here, $C$ is a parameter that measures the deviation from isotropy, and the function $f(\varepsilon)$ can be chosen as $\varepsilon$. In this case, TOV equations become:

$$
\begin{gathered}
\frac{\partial P_{r}}{\partial r}=-\frac{\left(\varepsilon+P_{r}\right)\left(4 \pi r^{3} P_{r}+m\right)}{r^{2}\left(1-\frac{2 m}{r}\right)\left(\frac{2}{r} C f(\varepsilon) \sqrt{1-\frac{2 m}{r}}+1\right)} \\
\frac{\partial m}{\partial r}=4 \pi r^{2} \varepsilon
\end{gathered}
$$

- Having the TOV equations, what we do now is to present the EoS that we have chosen to use. Motivated by possible evidences from strange stars, and also in theoretical investigation of recent works [5], we have used heretofore only the Bag Model EoS. Eventually, we intend to use other types of EoS as well, not only for strange stars but also for hybrid stars.
- Thus, with $B$ being the bag constant of the model, we have:

$$
p(\varepsilon)=\frac{1}{3}(\varepsilon-4 B)
$$

- In the next section, we shall see how the calculations made using the Zubairi-Weber formalism and the results obtained for the C-stars can be compared numerically. With that in mind, one could also establish connections with other formalisms, such as those that describe stars with significant magnetic fields. It is in this sense that the present work has been developing at the moment.


## Preliminary Results

- In Figure 1 below, we can check, as attested in the previous section, how it is possible to make numerical connections between the two formalisms.


Figure 1:Mass-radius diagram for sequences of strange stars, varying, separately, the values of $\gamma$ and C. "BH" stands for Black Hole.

## Conclusion

- Hitherto it was possible to reproduce both the analytical and the numerical results already known. Moreover it was possible to establish the bases for the beginning of an investigation that uses the contribution of both modified TOV equations and EoSs that make it possible to obtain masses greater than $2.16 \mathrm{M}_{\odot}$ for neutron stars.
- Furthermore, through the use of modified Tolman-Oppenheimer-Volkoff equations and selected EoSs, significant results can be achieved. Interesting purposes - such as comparison with models that have magnetic fields or rotation in their own structure - can begin to be outlined.


## References

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