Beam hosing in plasma wakefield accelerators

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Context

Transverse beam-plasma instability

Competes with the self-modulation instability (SMI): $N_{\mathrm{h}} = 8\, N_{\mathrm{sm}}$

Analogous to hosing of a laser pulse in plasma

This case was studied first and the theory for it is more advanced

We know that laser hosing can grow at wavelengths smaller than λ_p

Theory suggests that the SMI can couple to hosing

This coupling leads to a faster-growing and asymmetric centroid

It seems that we observe this regime in AWAKE

General goals

Understand coupled hosing

Why does the centroid become asymmetric?

Under what conditions does it happen?

Measure/study growth rate distribution

Look into long-wavelength regime

Compare different regimes/assumptions

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Coupled hosing has been discussed before

Equations for the beam centroid and the beam radius

$$\frac{d^2 x_c}{dz^2} = \frac{2\, I_1(r_b)}{r_b} \int_\infty^\xi d\xi' \sin(\xi - \xi') \frac{r_{b0}^2}{r_b(\xi')} K_1(r_b(\xi'))[x_c(\xi') - x_c(\xi)]$$

$$\frac{d^2 r_b}{dz^2} - \frac{\epsilon^2}{r_b^3} = -\frac{8\, I_2(r_b)}{r_b} \int_\infty^\xi d\xi' \sin(\xi - \xi') \frac{r_{b0}^2}{r_b(\xi')} K_1(r_b(\xi'))$$

First try to reproduce results in paper:

solve both coupled equations numerically, with self-consistent evolution of the radius

* C. B. Schroeder et al., Phys. Rev. E **86**, 026402 (2012)

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↑

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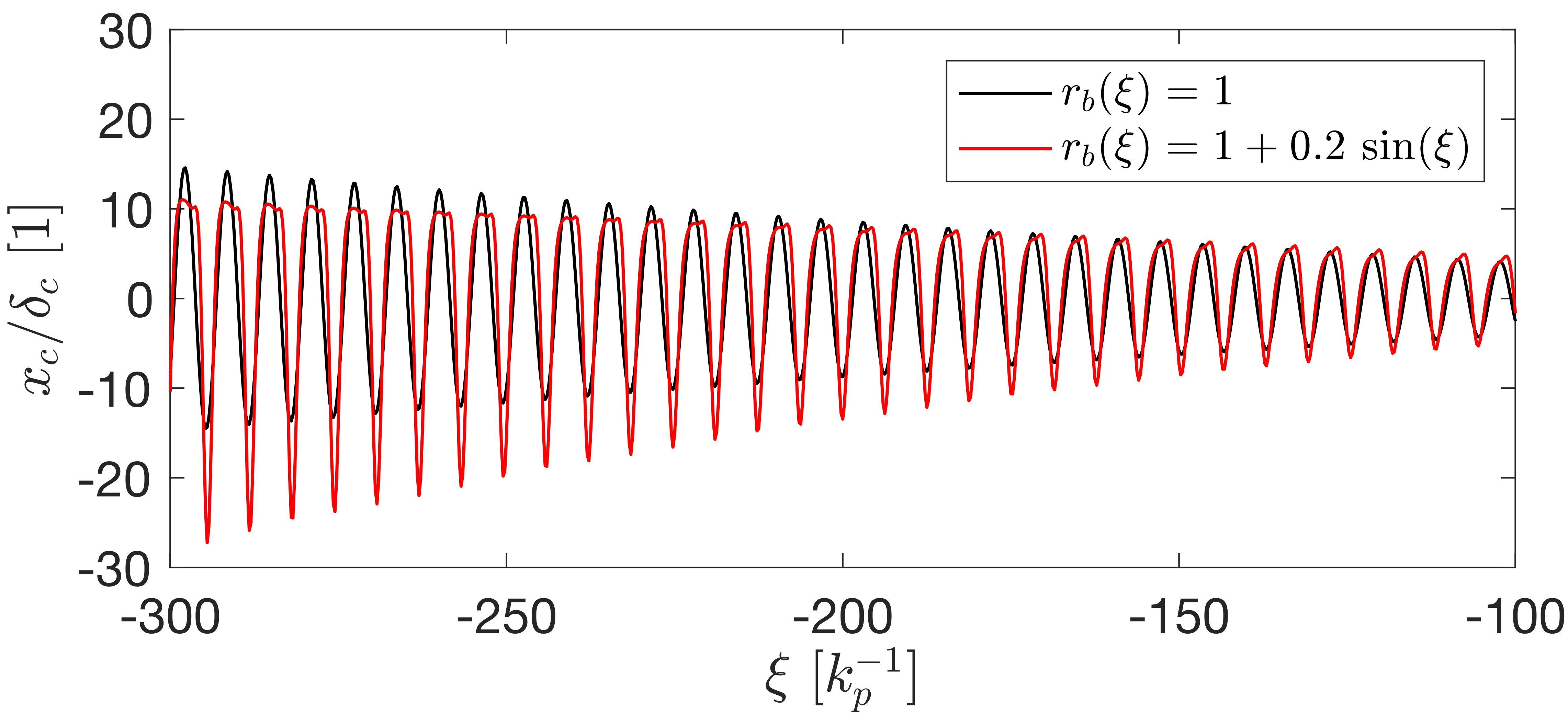
Coupled hosing is asymmetric

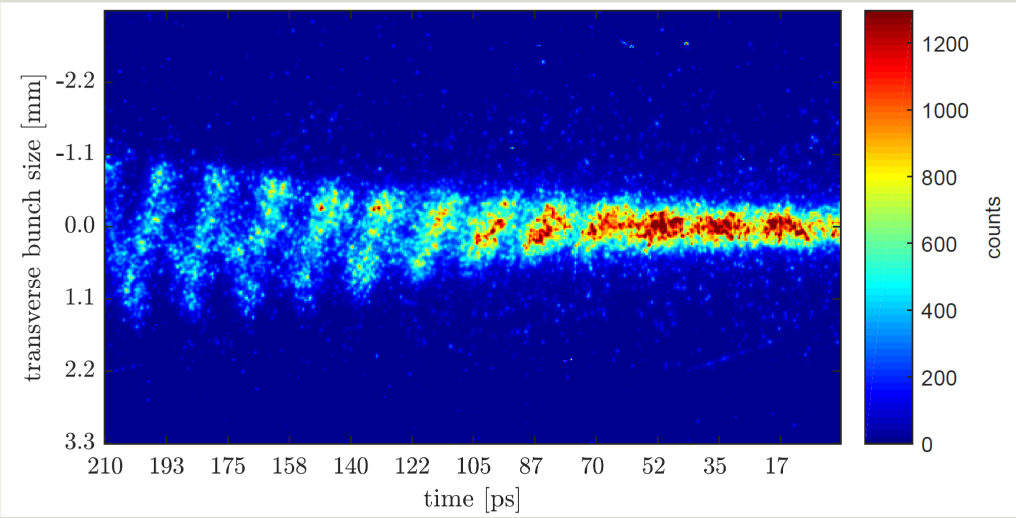
Numerical solution of the centroid equation assuming a non-evolving beam radius (constant and modulated)



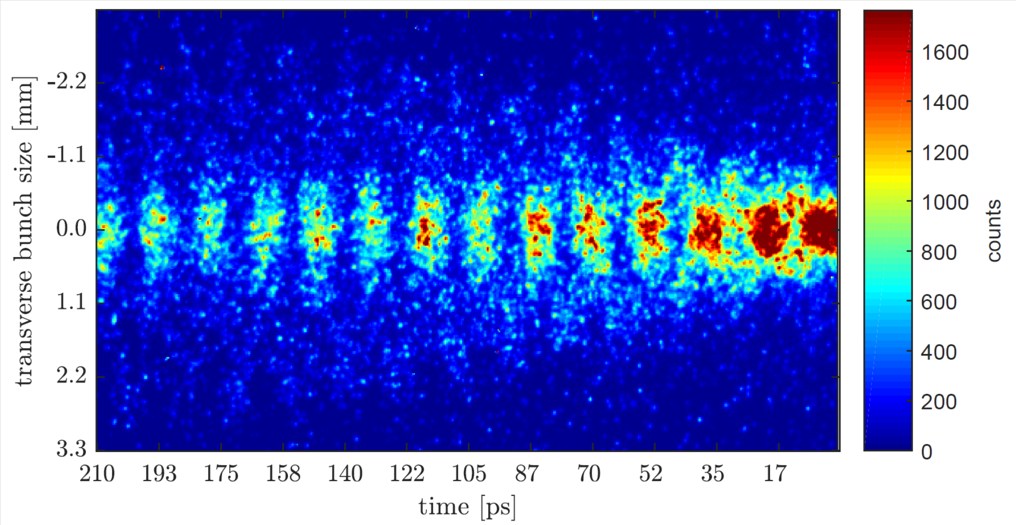
Comparison to Experimental Data

Experimental Data:

- Hosing event:



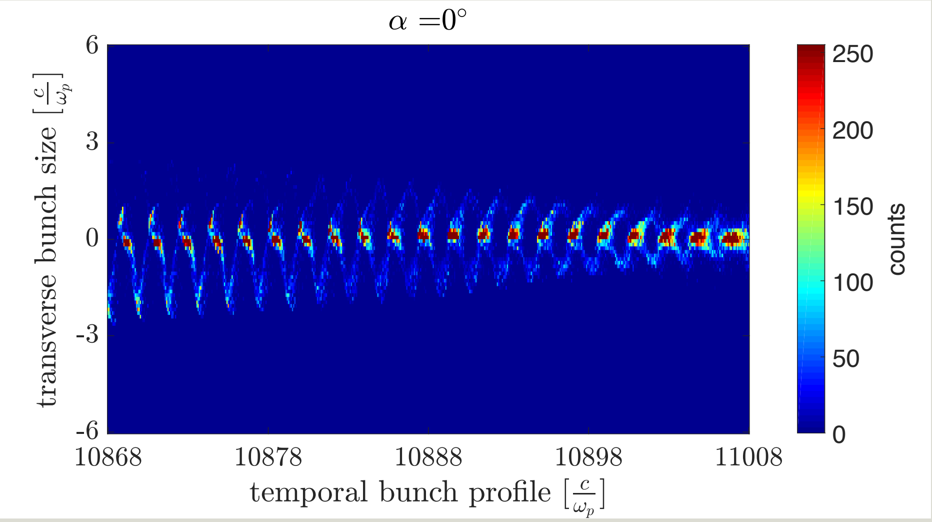
- SSM event:



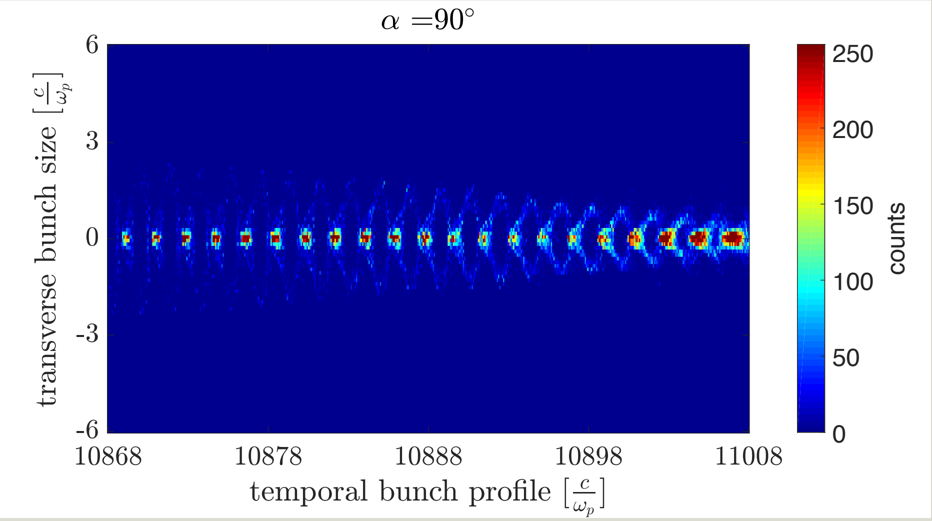
- $n_0 = 0.5 \cdot 10^{14} \text{cm}^{-3}$
- $N \approx 1.0 \cdot 10^{11}$ p^+/bunch
- $\sigma_z = 186$ ps

OSIRIS Simulations:

- Hosing plane:



- SSM plane:

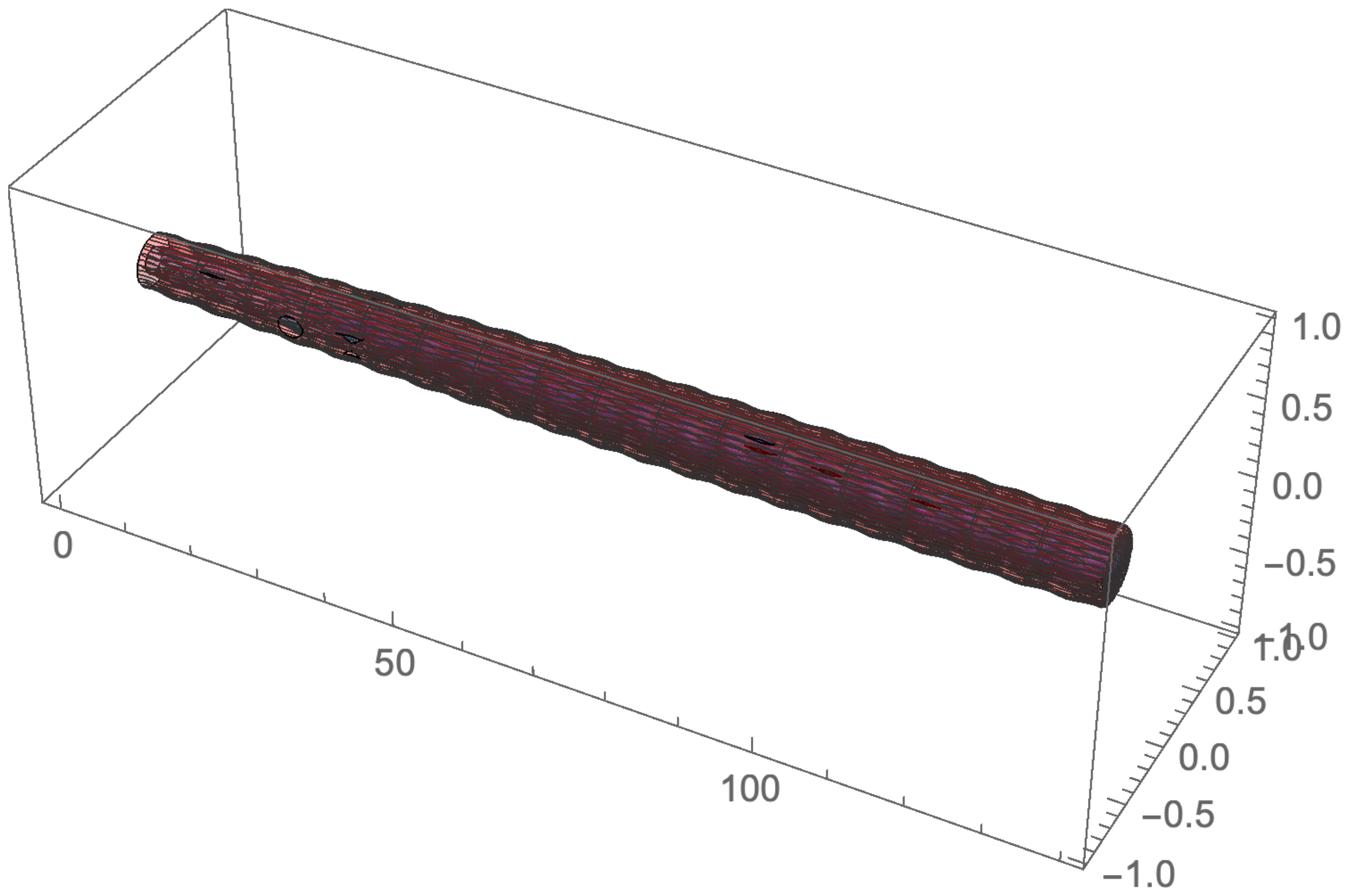


- $n_0 = 0.5 \cdot 10^{14} \text{cm}^{-3}$
- $N = 1.6 \cdot 10^{11}$ p^+/bunch
- $\sigma_z = 200$ ps

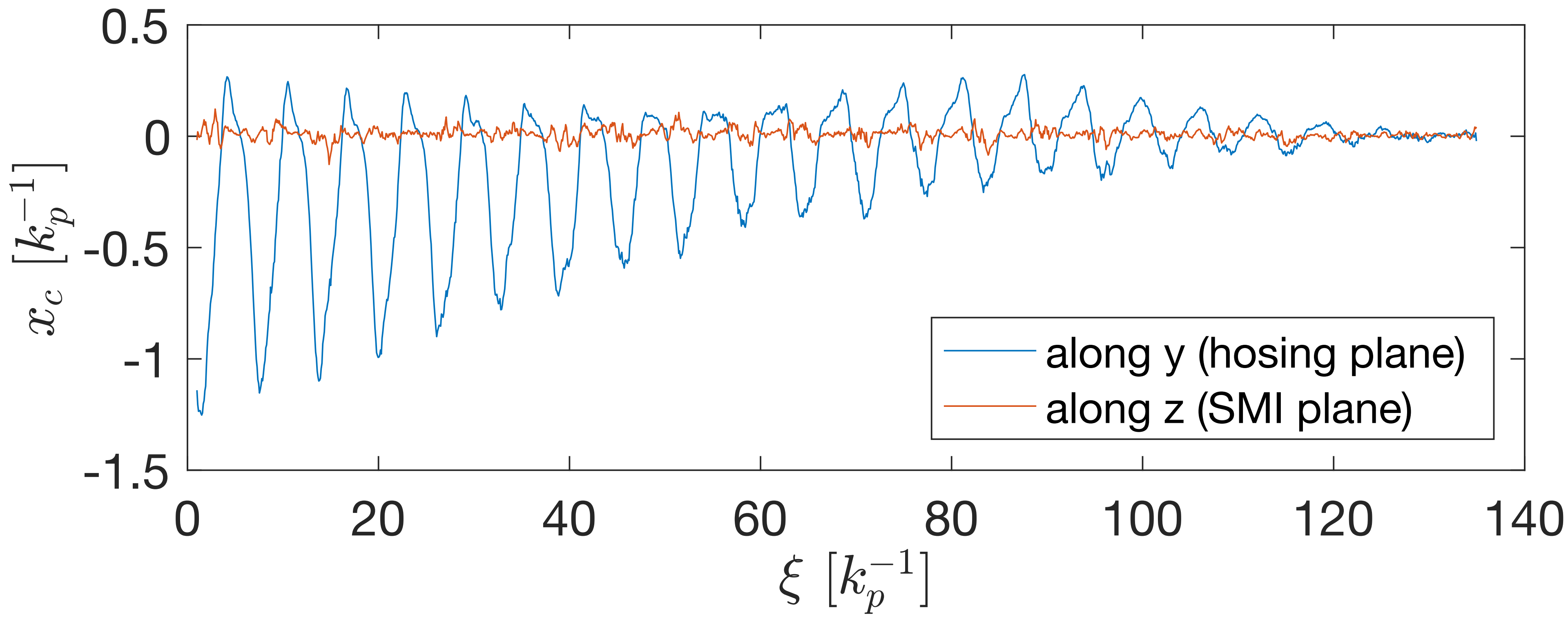
Setting up a PIC simulation for hosing

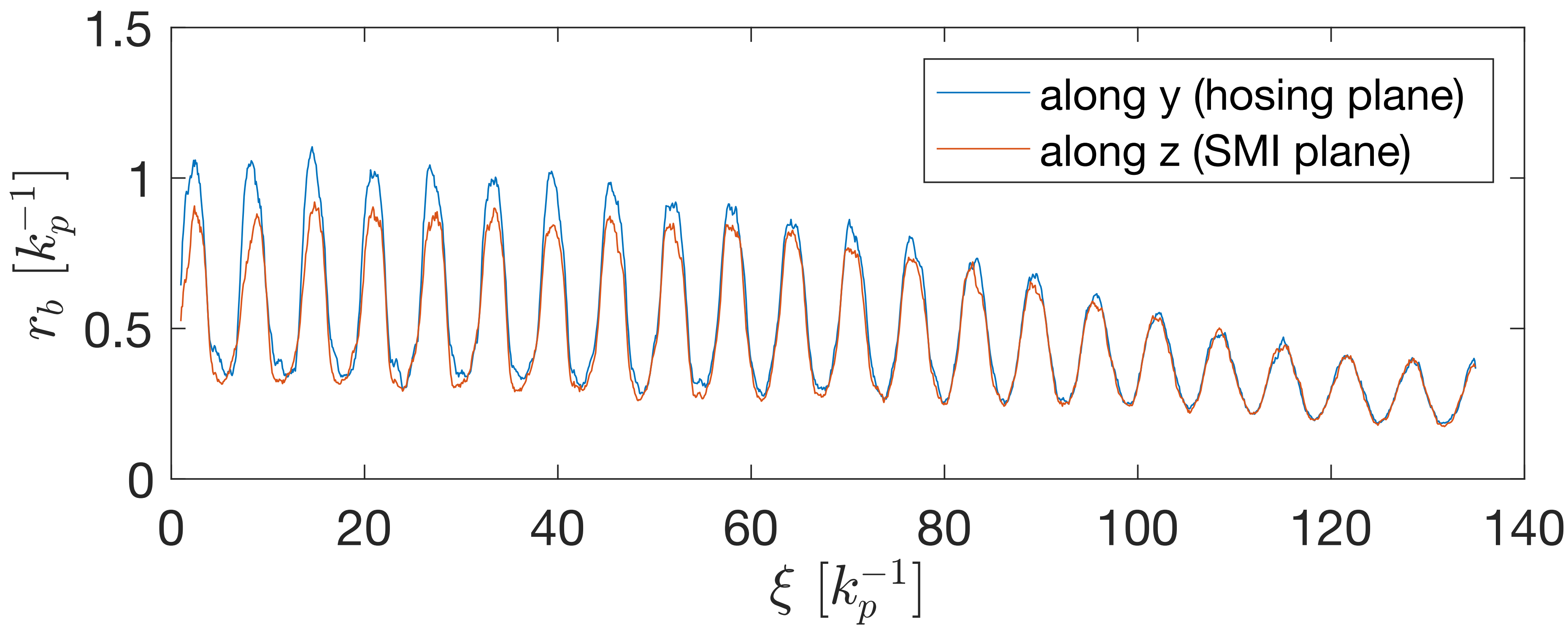
A setup to reproduce the conditions in the experiment

- Seed for the SMI at 100 ps ahead of the middle (cut)
- Wiggle in the centroid of the bunch profile along the y direction, at λ_p



A 3D simulation shows the same behaviour





The asymmetry depends on the seed

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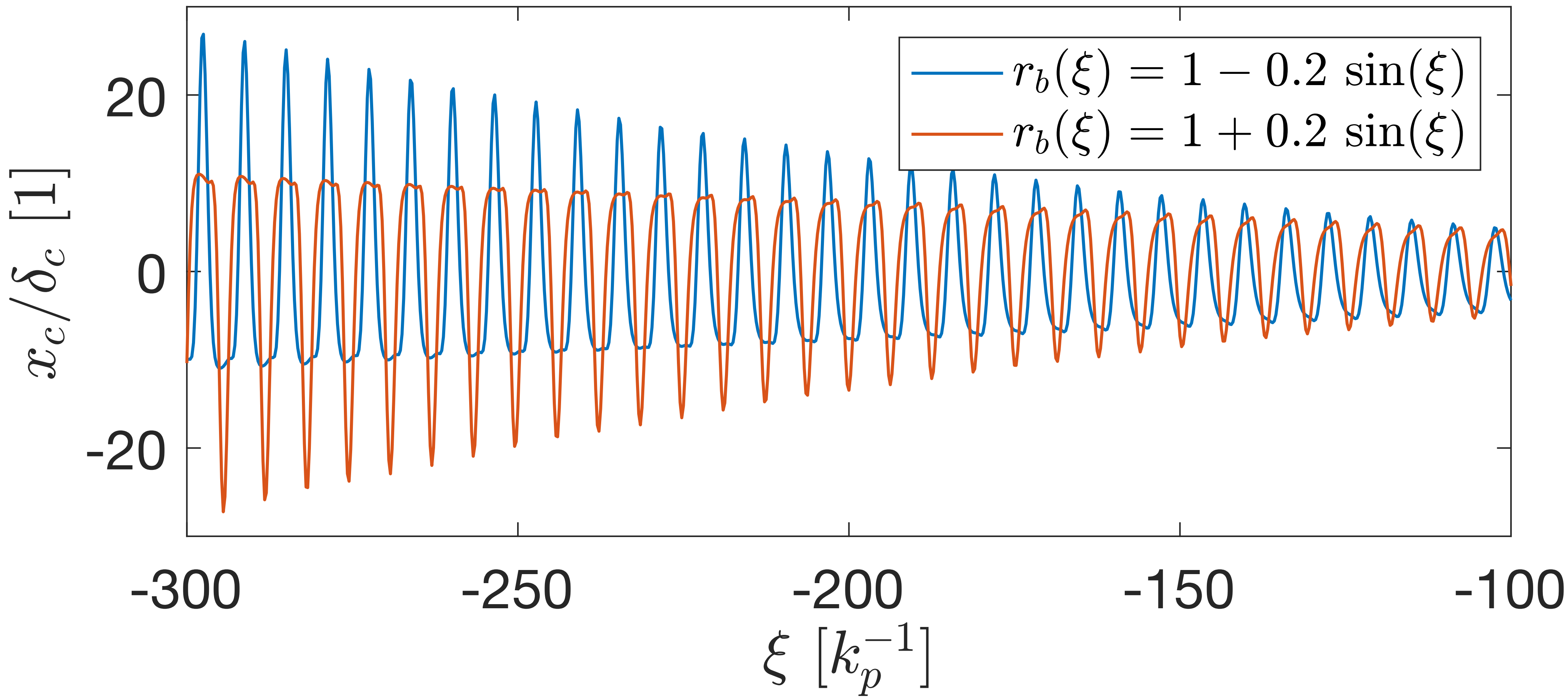
Numerical solution of the centroid equation for opposite signs of the radius modulation



Also demonstrated with the 3D simulation

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Future work:

Use further simulations to study this regime, e.g. track particles and analyse fields

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Long-wavelength hosing in lasers

The hosing growth rate versus wavenumber

- "A long-wavelength hosing instability in laser-plasma interactions" has been studied some time ago
- This regime can be identified by looking at this graph

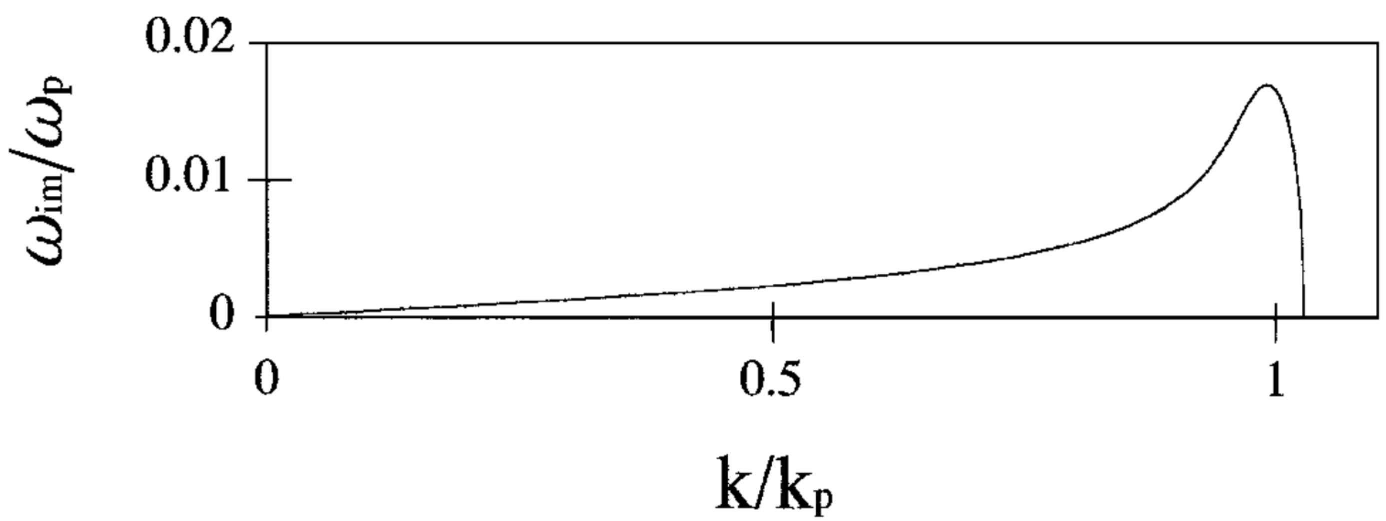

FIG. 2. The growth rate for hosing vs wave number for $\tilde{x}_R = 256$.

* B. J. Duda et al., Phys. Rev. Lett. **83**, 1978 (1999)

Doing the same for beam hosing

How do you obtain the growth rate vs. wavenumber?

- Express original equation as a system of coupled harmonic oscillators
- Convert coordinates to the lab frame

$$\xi = z' - c\,t$$
$$z = z'$$

- Substitute plane wave solutions

$$x_c(x,t) = A\,\exp[i(kx - \omega t)]$$

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  $\xi = z' - ct$
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- Substitute plane wave solutions
  $x_c(x,t) = A\,\exp[i(kx - \omega t)]$
- Obtain a dispersion relation, and solve for Im(**ω**)

System of coupled oscillators

$$\frac{d^2 x_c}{dz^2} = \frac{2\,I_1(r_b)}{r_b}\int_\infty^\xi d\xi' \sin(\xi-\xi')\,\frac{r_{b0}^2}{r_b(\xi')} K_1(r_b(\xi'))[x_c(\xi') - x_c(\xi)]$$

⇕

$$\left(\partial_z^2 + \langle\psi\rangle\right) x_c = \langle w\rangle$$

$$\left(\partial_\xi^2 + 1\right)\left(\nabla_\perp^2 - 1\right) w = \frac{q}{e}\,\frac{n_b}{n_0} x_c$$

$$\left(\partial_\xi^2 + 1\right)\left(\nabla_\perp^2 - 1\right) \psi = \frac{q}{e}\,\frac{n_b}{n_0}$$

Resulting dispersion relation

The dispersion relation depends on the beam parameters and the transverse beam profile

$$\frac{1}{\langle g(r) \rangle} \left(\frac{\hat{\omega}}{c} - \hat{k} \right)^2 \left(\frac{\hat{\omega}^2}{c^2} - 1 \right) + \left(1 - \frac{\hat{\omega}^2}{c^2} \right) Q_b \, \phi(\xi) = Q_b \, S(\xi)$$

$$\left(\nabla_\perp^2 - 1 \right) g(r) = R(r)$$

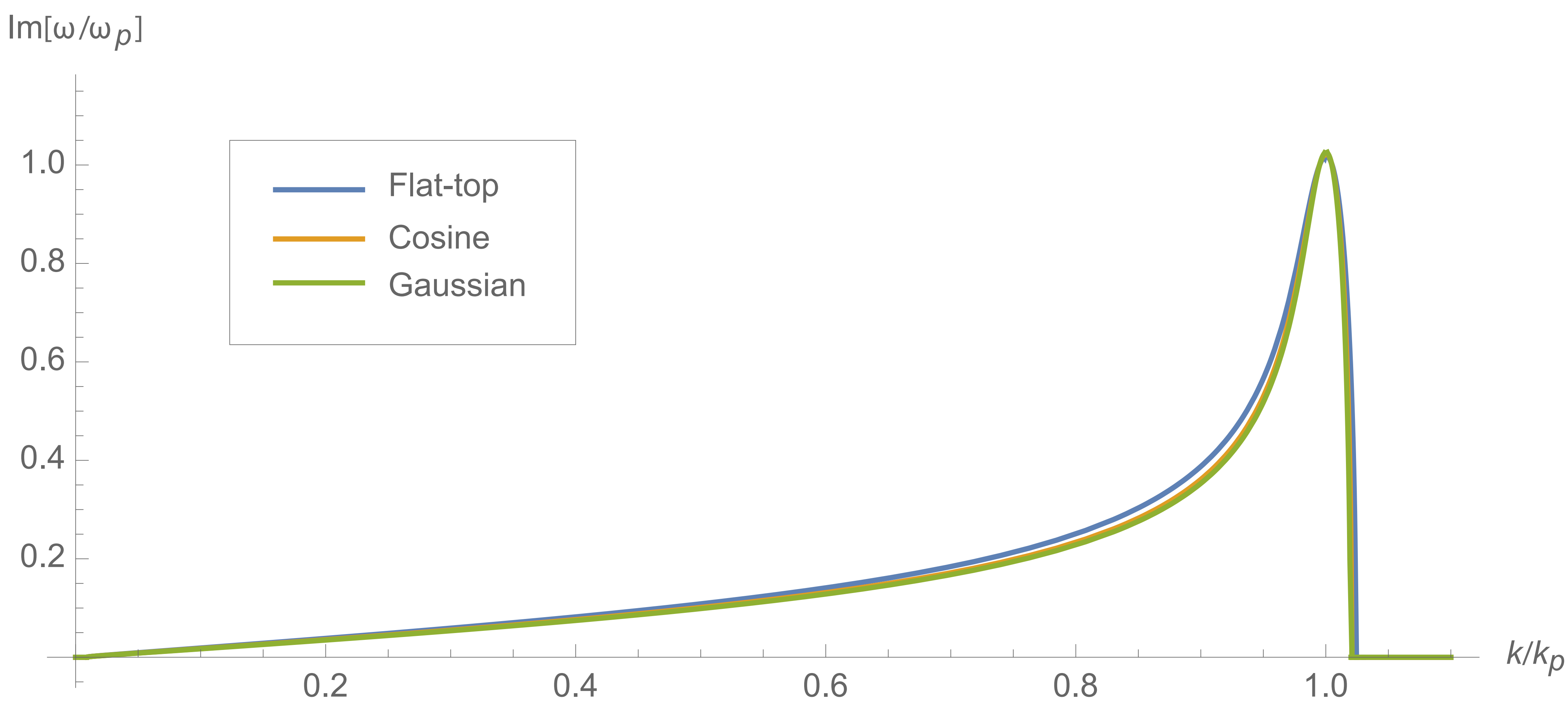
$$S_b(\xi, r) = S(\xi) \cdot R(r)$$

$$\psi(\xi, r) = \phi(\xi) \cdot g(r)$$

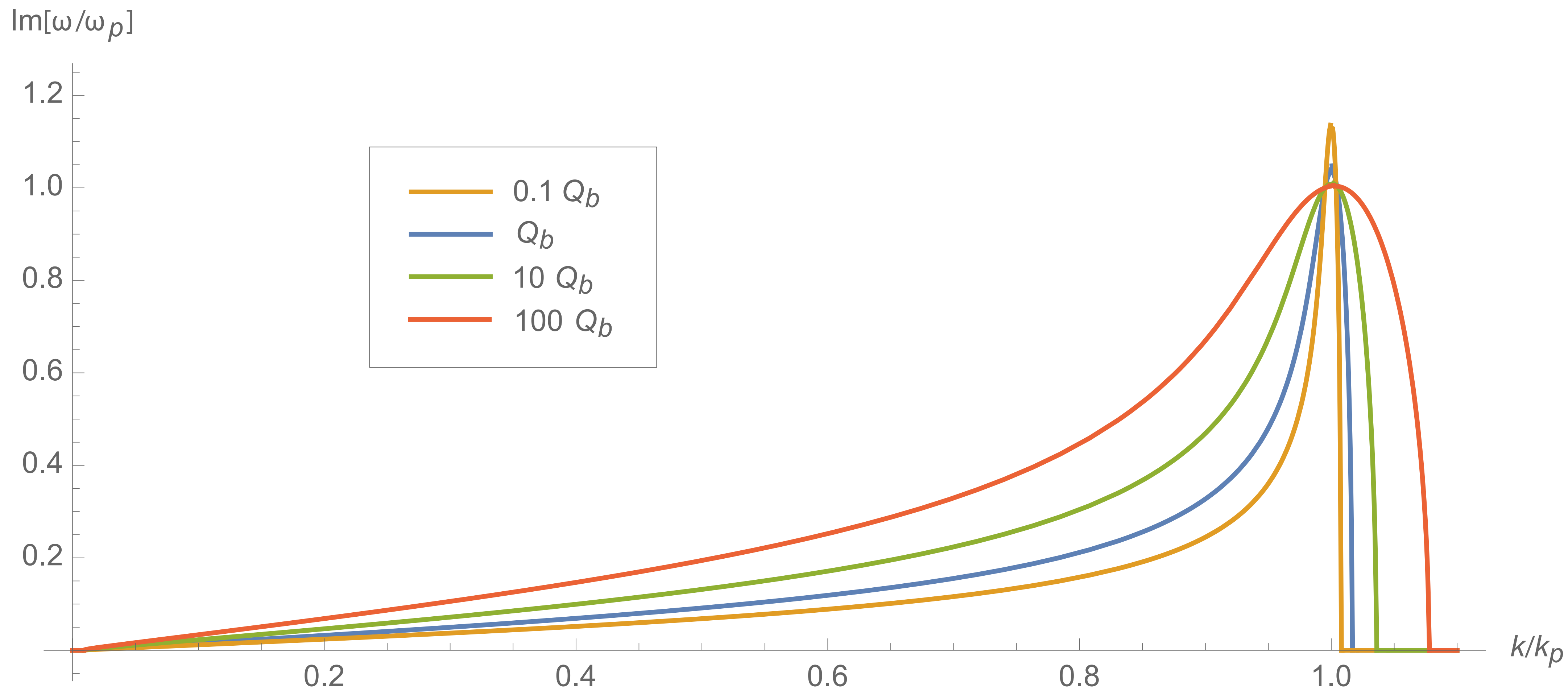
$$Q_b = \left(\frac{q}{e} \right)^2 \frac{m_e}{M_b} \frac{n_b}{n_0} \frac{1}{2\gamma}$$

The transverse profile shape is not too important

Using AWAKE's proton bunch parameters



A different shape would require different parameters by orders of magnitude



$$Q_b = \left(\frac{q}{e}\right)^2 \frac{m_e}{M_b} \frac{n_b}{n_0} \frac{1}{2\gamma}$$

What if we “measured” the growth rate?

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On simulations

- Right now I have only a limited set of data
- **First idea:**
  - FFT of centroid at every time
  - Stack FFTs
  - Fit exponential function to FFT histogram along *z* for every *k*
- Didn’t really work

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Test method on linear theory first

By seeding at different wavenumbers

Solve centroid equation numerically for each sinusoidal seed at a given k

Apply FFT and fit analysis for that k

Slow and unfeasible for simulations

By using a seed with a range of wavenumbers

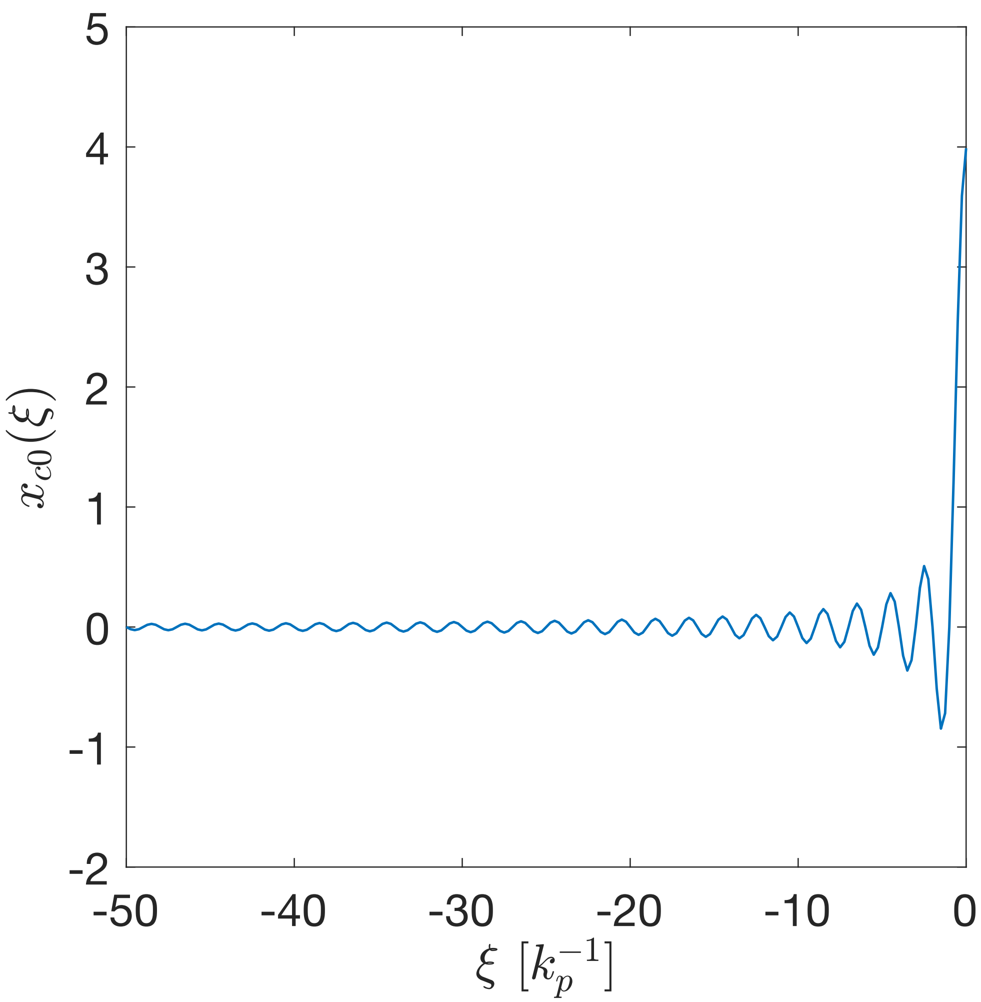
Solve centroid equation once for an initial centroid profile that represents many different wavenumbers

Apply FFT and fit analysis for each k

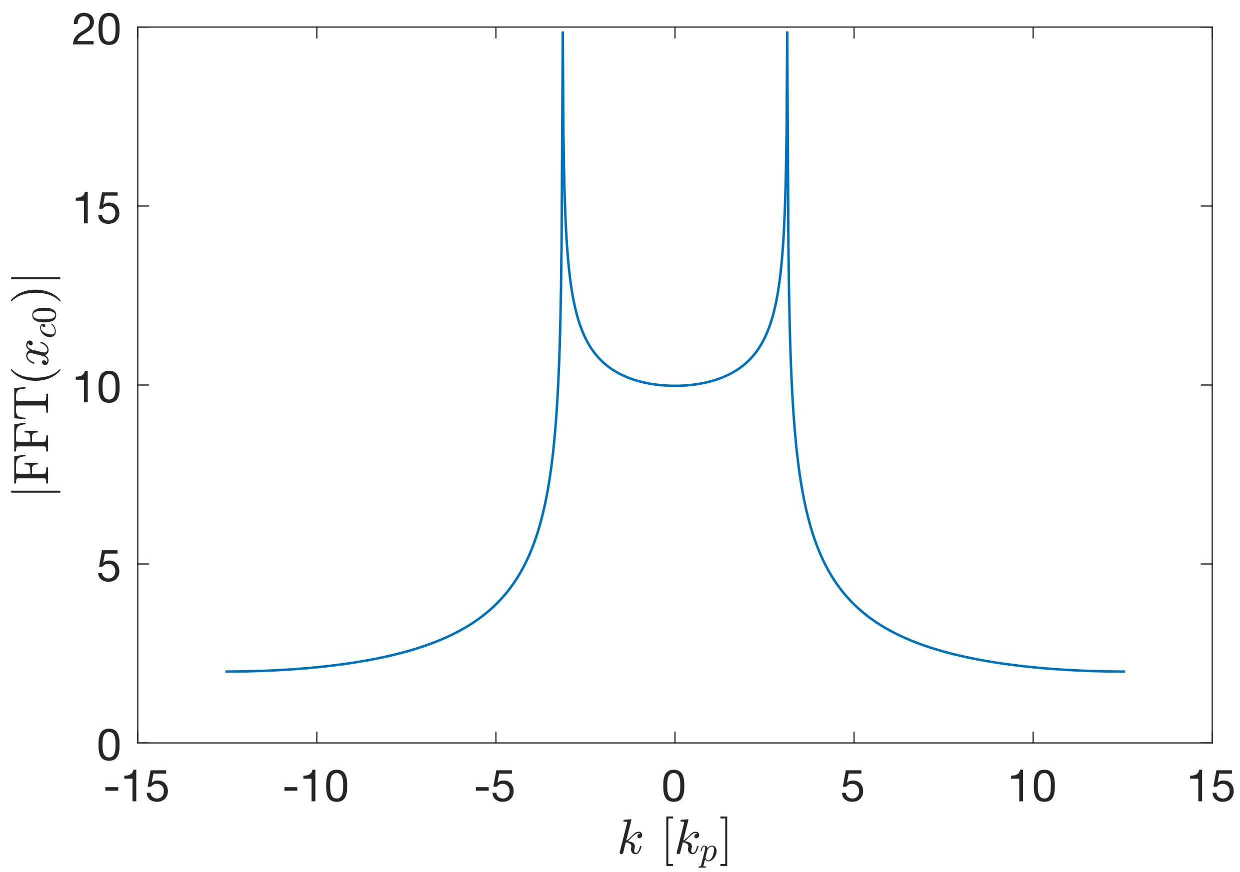
Fast and appropriate for simulations

“Probe” seed (range of wavenumbers)

Initial centroid profile



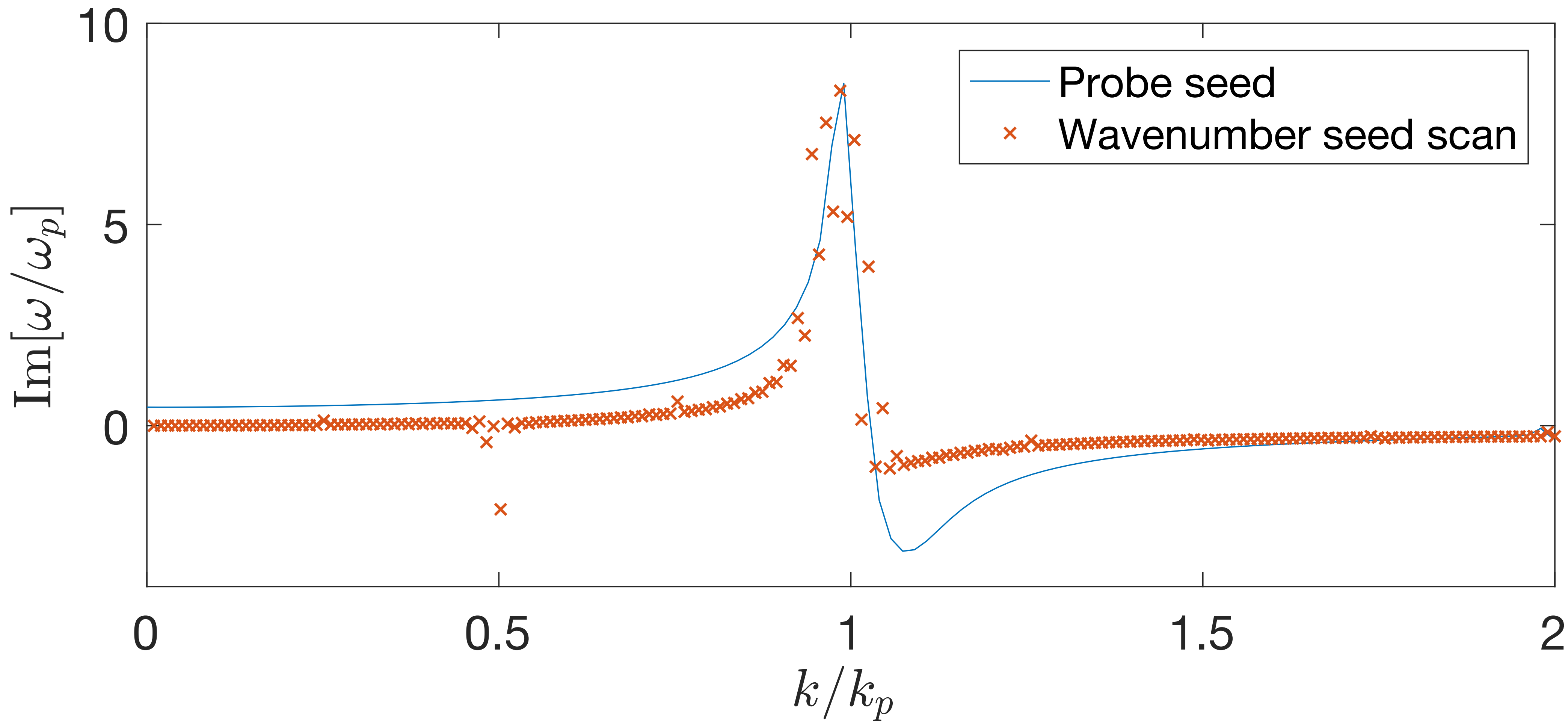
$$\propto \frac{sin(\xi)}{\xi}$$



Ideally a Heaviside function

Result shows interesting features

From an exponential fit to the FFT histogram of x_c along z
From the numerical solution of the centroid equation (pure hosing)



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Future work:

More simulations with different initial conditions

Focus on long-wavelength hosing