Recoil and Kinematics in Parton Showers

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The future of parton showers

Push to higher orders and improved accuracy :

- Beyond $1 \rightarrow 2$ branching in kinematics and splitting kernels
- Global vs. local recoil schemes
- Include more spin and colour interferences
- Use higher-order emission kernels

Will be central to address[2]:

- Singularity structure for NNLO matching, increasingly important
- Lack of systematic expansion of uncertainties at higher orders
- Issues with local recoil schemes

[2] Dasgupta et al. 2018; Bewick et al. 2019.

What is the parton shower made of?

- Key components:
  - Kinematic mapping
  - Emission kernels/splitting functions
  - Choice of evolution variable and recoil scheme

Current implementation in Herwig (dipole shower):

$$\begin{aligned} \textit{Emitter} \rightarrow\ & q_i = zp_i + y(1-z)p_k + kt\ , \\ \textit{Emission} \rightarrow\ & k_1 = (1-z)p_i + zyp_k - kt\ , \\ \textit{Spectator} \rightarrow\ & q_k = (1-y)p_k\ , \end{aligned}$$

- Develop a new mapping in order to:
  - Distribute recoils globally
  - Factorise without explicitly taking soft or collinear limits

Mapping for single emission with global recoil

Mapping with both soft and collinear cases included:

$$q_i^\mu = (1-\alpha_1)\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y(1-\beta_1)n^\mu - \sqrt{y\alpha_1\beta_1}\,n^\mu_{\perp,1}\ ,$$
$$k_1^\mu = \alpha_1\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y\beta_1 n^\mu + \sqrt{y\alpha_1\beta_1}\,n^\mu_{\perp,1}\ ,$$
$$q_k^\mu = \alpha\Lambda^\mu{}_\nu p_k^\nu\ , \qquad (k = 1,..,n \qquad k \neq i)$$

- Above case is single emitter q_i with one emission k_1, in this case $\alpha_1 = 1-\beta_1 \to (1-z)$
- Includes soft limit ($\alpha_1, y \to 0$) and collinear limit ($y \to 0$)
- $n_\perp$ represents the transverse component (kt)
- Includes global treatment of recoil via Lorentz transformation $\Lambda^\mu{}_\nu$

Action of Lorentz transformation

- Momentum conservation requires: $q_i + k_1 + q_k = p_i + p_k$
- Use Lorentz transformation(LT) to distribute recoil
- n vector gives backwards direction $n = Q - \frac{Q^2}{2p_i \cdot Q} p_i$

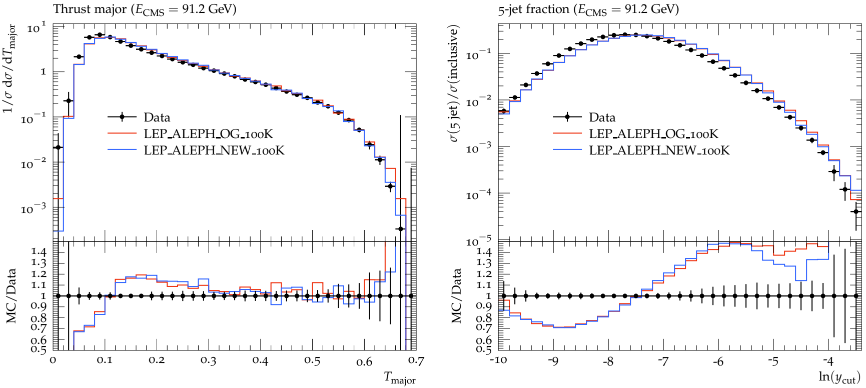
Can be expressed as:

$$\Lambda^{\mu}{}_{\nu} Q^{\nu} = \frac{Q^{\mu} - y n^{\mu}}{\alpha}$$

- $\alpha = \sqrt{1 - y}$ which in collinear limit means that $\alpha \to 1$
- Action on the emitter, p_i, acts as delta function just giving p_i^{μ} (for single emitter case)

Test LEP plots

- Simple test, plugging in new mapping and LT
- ‘ALEPH_2004_S5765862’ Rivet analysis used

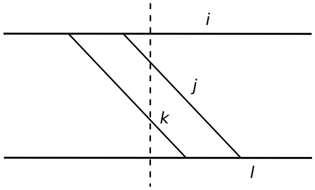


Plot of thrust major and 5-jet fraction for NEvents = 100,000

Mapping for two emissions

Two emissions, one emitter:

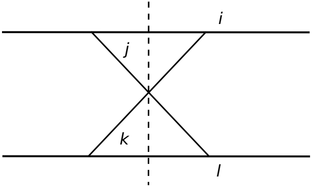
$$
\begin{aligned}
q_i^\mu =& (1-\alpha_{i1}-\alpha_{i2})\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y_i(1-\beta_{i1}-\beta_{i2})n_i^\mu \\
& - \sqrt{y_i\alpha_{i1}\beta_{i1}}\,n^\mu_{\perp,1} - \sqrt{y_i\alpha_{i2}\beta_{i2}}\,n^\mu_{\perp,i2}\,, \\
k_{i1}^\mu =& \alpha_{i1}\;\alpha\Lambda^\mu{}_\nu p_i^\nu + y_i\beta_{i1}n_i^\mu + \sqrt{y_i\alpha_{i1}\beta_{i1}}\,n^\mu_{\perp,i1}\,, \\
k_{i2}^\mu =& \alpha_{i2}\;\alpha\Lambda^\mu{}_\nu p_i^\nu + y_i\beta_{i2}n_i^\mu + \sqrt{y_i\alpha_{i2}\beta_{i2}}\,n^\mu_{\perp,i2}\,, \\
q_k^\mu =& \alpha\Lambda^\mu{}_\nu p_k^\nu\,, \qquad (k=1,..,n \qquad k\neq i)
\end{aligned}
$$



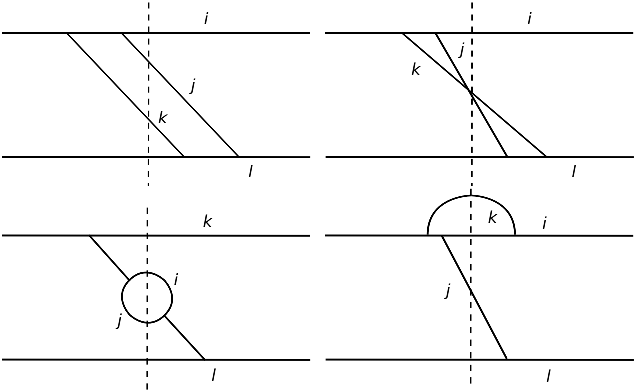
Mapping for two emissions

Two emissions, two emitters:

$$q_i^\mu = (1-\alpha_{i1})\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y_i(1-\beta_{i1})n_i^\mu - \sqrt{y_i\alpha_{i1}\beta_{i1}}\,n^\mu_{\perp,i1}\ ,$$
$$k_{i1}^\mu = \alpha_{i1}\ \alpha\Lambda^\mu{}_\nu p_i^\nu + y_i\beta_{i1}n_i^\mu + \sqrt{y_i\alpha_{i1}\beta_{i1}}\,n^\mu_{\perp,i1}\ ,$$
$$q_j^\mu = (1-\alpha_{j1})\,\alpha\Lambda^\mu{}_\nu p_j^\nu + y_j(1-\beta_{j1})n_j^\mu - \sqrt{y_j\alpha_{j1}\beta_{j1}}\,n^\mu_{\perp,j1}\ ,$$
$$k_{j1}^\mu = \alpha_{j1}\ \alpha\Lambda^\mu{}_\nu p_j^\nu + y_j\beta_j 1 n_j^\mu + \sqrt{y_j\alpha j1 l\beta_{j1}}\,n^\mu_{\perp,j1}\ ,$$
$$q_k^\mu = \alpha\Lambda^\mu{}_\nu p_k^\nu\ , \qquad (k=1,..,n \qquad k\neq i,j)$$



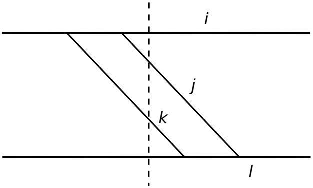
Examples of two emission case



Examples of 2 emission diagrams

- Complicated interaction of soft and collinear singularities for two emissions
- Devise framework for separating singularities

Example: Separating singularities

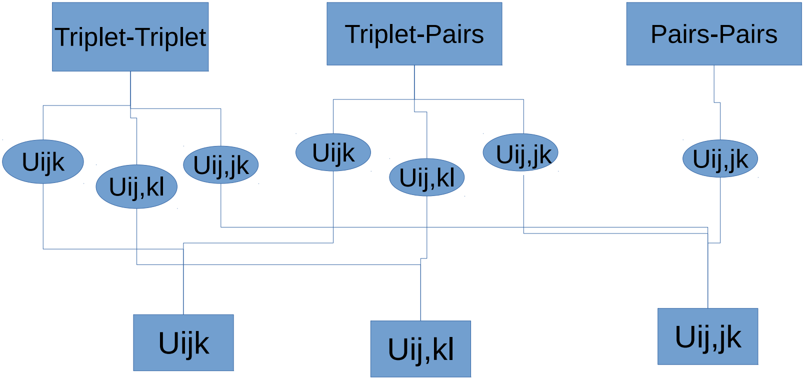


Singularities in the above diagram:

$$\frac{1}{S_{ijk}}\frac{1}{S_{ij}}\frac{1}{S_{kl}}\frac{1}{S_{jkl}}$$

- Where $S_{ijk} = S_{ij} + S_{ik} + S_{jk}$ and $S_{ij} = (q_i + q_j)^2$
- Three different triple collinear cases, (ijk), (jkl) and (ij, kl)
- Aim to partition singularities so can be expressed as a sum

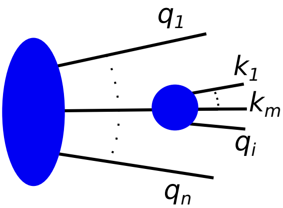
Example: Separating singularities



Overview

Tools:

- New mapping which exposes soft and collinear singularities for up to n-emissions
- Mapping treats recoil globally via Lorentz Transformation
- Comprehensive framework for organising the singularities up to 2-emission case
- Partitioning of singularities identified for diagram topologies



Summary and WIP

Current checks/tests:

- In collinear limit can reproduce CS calculations and splitting functions
- Testing implementation of single emission mapping/recoil in Herwig
- Comparison of event shapes in LEP analysis

Work in progress:

- Inclusion of phase space up to two emissions
- Formal implementation of partitioning
- NNLO splitting functions

Backup slides

Thrust definition

Thrust Major T_{major} : The thrust major vector, n_{Ma} , is defined in the same way as the thrust vector, but with the additional condition that n_{Ma} must lie in the plane perpendicular to n_T ,

$$T_{major} = \max_{\vec{n}_{Ma} \perp \vec{n}_T} \left(\frac{|p_i \cdot n_{Ma}|}{\sum_i |pi|} \right)$$

Determination of emission kernels

- Need to take collinear and soft limits which allow factorisation[3]
- Where q_k is momentum of final state parton k:

  a) Soft limit $q_k = \lambda q$, $\lambda \to 0$, $|\mathcal{M}_{m+1,a...}|^2 \propto 1/\lambda^2$

  b) Collinear limit $q_k \to (1-z)q_i/z$,
  $|\mathcal{M}_{m+1,a...}|^2 \propto 1/q_i.q_k$

Dipole factorisation

- Consider $(m+1)$ partons, factorise out parton k to give $|\mathcal{M}_{m,a...}|^2$

$$|\mathcal{M}_{m+1,a...}|^2 \to |\mathcal{M}_{m,a...}|^2 \otimes V_{ik,j}$$

- $V_{ik,j}$ = singular factor including parton k and it's interaction with partons i and j from the m parton amplitude

[3] Catani and Seymour 1997.

Mapping in two emission case

$$
\begin{aligned}
q_i^\mu =&(1-(\alpha_1+\alpha_2))\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y(1-(\beta_1+\beta_2)n^\mu \\
&-\sqrt{y\alpha_1\beta_1}n^\mu_{\perp,1} - \sqrt{y\alpha_2\beta_2}n^\mu_{\perp,2}\,, \\
k_1^\mu =&\alpha_1\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y\beta_1 n^\mu + \sqrt{y\alpha_1\beta_1}n^\mu_{\perp,1}\,, \\
k_2^\mu =&\alpha_2\,\alpha\Lambda^\mu{}_\nu p_i^\nu + y\beta_2 n^\mu + \sqrt{y\alpha_2\beta_2}n^\mu_{\perp,2}\,, \\
q_k^\mu =&\alpha\Lambda^\mu{}_\nu p_k^\nu\,, \qquad (k=1,..,n \qquad k\neq i)
\end{aligned}
$$

References I

- Gavin Bewick et al. “Logarithmic Accuracy of Angular-Ordered Parton Showers”. In: (2019). arXiv: 1904.11866 [hep-ph].
- S. Catani and M. H. Seymour. “A General algorithm for calculating jet cross-sections in NLO QCD”. In: *Nucl. Phys.* B485 (1997). [Erratum: Nucl. Phys.B510,503(1998)], pp. 291–419. DOI: 10.1016/S0550-3213(96)00589-5,10.1016/S0550-3213(98)81022-5. arXiv: hep-ph/9605323 [hep-ph].
- Mrinal Dasgupta et al. “Logarithmic accuracy of parton showers: a fixed-order study”. In: (2018). arXiv: 1805.09327 [hep-ph].

- QCD and Collider Physics, Ellis, Stirling and Webber. (p.170)