Soft Photon Resummation in SHERPA

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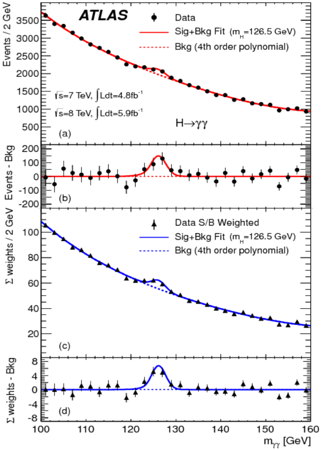
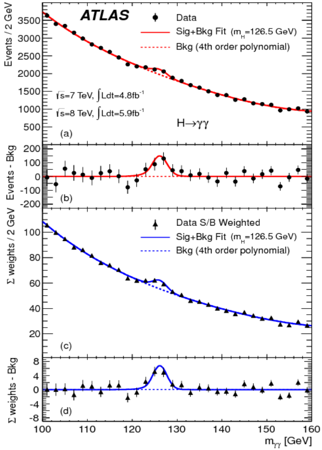
Overview



- For precision physics a lepton-lepton collider is desirable
- The discovery of Higgs Boson tells us at what energy to run

Motivation



- For precision physics a lepton-lepton collider is desirable
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At the FCC-ee there will be,

- 5×10^{12} Z
- 10^8 WW
- 10^6HZ
  $+$ 25k WW $\rightarrow$ H
- Each Higgs will only cost $\approx$ €255

Motivation

We must not forget that an e^+e^- will precisely measure EW observables

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$	Challenge
M_Z [MeV]	Z linesh.	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3	QED Corrections
Γ_Z [MeV]	Z linesh.	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2	QED Corrections
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12	QED Corrections
N_ν	$\sigma(M_Z)$	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6	Bhabha scattering (QED)
M_W [MeV]	ADLO	$80376 \pm 33\{6\}$	0.5	0.3	12	QED Corrections
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100	

Table: Adapted from Arxiv:1903.09895

Precision Machine $\implies$ Precise Calculations

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- Initial state radiation can be calculated la Yennie-Frautschi-Suura
- YFS for FSR already in SHERPA and accurate to NNLO QCD and NLO EW

YFS Exponentiation

- Lets first consider virtual photon corrections in the soft limit

$$
\begin{aligned}
\mathcal{M}_0^0 &= M_0^0 \\
\mathcal{M}_0^1 &= \alpha B M_0^0 + M_0^1 \\
\mathcal{M}_0^1 &= \frac{(\alpha B)^2}{2} M_0^0 + \alpha B M_0^1 + M_0^2 \\
&\cdots \\
\mathcal{M}_0^{n_V} &= \sum_{r=0}^{n_V} M_0^{n_V} \frac{(\alpha B)^r}{r\,!}
\end{aligned}
$$

where B is the virtual infrared factor.

$$
B = 2\alpha \Re \int \frac{d^4 k}{k^2} \frac{i}{(2\pi)^2} \left(\frac{2p_1 - k}{2kp_1 - k^2} - \frac{2p_2 - k}{2kp_2 - k^2} \right)^2
$$

YFS Exponentiation

Summing to infinity yields,

$$\sum_{n_V=0}^{\infty} \mathcal{M}_0^{n_V} = e^{\alpha B} \sum_{n_V=0}^{\infty} M_0^{n_V}$$

This can be generalised to n_R real photons such that,

$$\sum_{n_V=0}^{\infty} \left| \mathcal{M}_{n_R}^{n_V + \frac{1}{2} n_R} \right|^2 = e^{2\alpha B} \sum_{n_V=0}^{\infty} \left| M_{n_R}^{n_V + \frac{1}{2} n_R eal} \right|^2$$

YFS: Real Emissions

For a single photon emission we have,

$$\frac{1}{2(2\pi)^3}\sum_{n_V=0}^{\infty}\left|M_1^{n_V+\frac{1}{2}}\right|^2=\tilde{S}(k)\left|M_0^{n_V+\frac{1}{2}}\right|^2+\sum_{n_V=0}^{\infty}\tilde{\beta}_1^{n_V+1}(k)$$

- Factorisation of real emissions occurs at the amplitude squared level
- Eikonal term $\tilde{S}(k)=-\frac{\alpha}{4\pi^2}\left(\frac{p_1}{p_1k}-\frac{p_2}{p_2k}\right)^2$
- $\tilde{\beta}_{n_R}^{n_V+n_R}$ complete IR finite squared matrix element for born process plus n_V virtual and n_R photons

YFS: Real Emissions

For n_R photons summed over all virtuals,

$$\left(\frac{1}{2(2\pi)^3}\right)^{n_R}\left|\sum_{n_V=0}^{\infty} M_{n_R}^{n_V+\frac{1}{2}n_R}\right|^2$$

$$= \tilde{\beta}_0 \prod_{i=1}^{n_R}\left[\tilde{S}(k_i)\right] + \sum_{i=1}^{n_R}\left[\frac{\tilde{\beta}_1(k_i)}{\tilde{S}(k_i)}\right]\prod_{j=1}^{n_R}\left[\tilde{S}(k_j)\right]$$

$$+ \sum_{\substack{i,j=1\\ i\neq j}}^{n_R}\left[\frac{\tilde{\beta}_2(k_i,k_j)}{\tilde{S}(k_i)\tilde{S}(k_j)}\right]\prod_{l=1}^{n_R}\left[\tilde{S}(k_l)\right] + \dots$$

$$+ \sum_{i=1}^{n_R}\left[\tilde{\beta}_{n_R-1}(k_1,\dots,k_{i-1},k_{i+1},\dots,k_{n_R})\,\tilde{S}(k_i)\right] + \tilde{\beta}_{n_R}(k_1,\dots,k_{n_R})$$

Master Equation

This gives us our cross section

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_q \; e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^{n} \tilde{S}\left(k_j\right) \theta(\Omega; k_j) \left[\tilde{\beta}_0 \left(p_1, p_2; q_1, \cdots, q_{n'}\right) + \sum_{j=1}^{n} \frac{\tilde{\beta}_1 \left(p_1, p_2; q_1, \cdots, q_{n'};, k_j\right)}{S(k_j)} + \sum_{\substack{j,l=1 \\ j \neq l}}^{n} \frac{\tilde{\beta}_2 \left(p_1, p_2; q_1, \cdots, q_{n'}; k_j, k_l\right)}{S(k_j) S(k_l)} + \cdots \right]$$

The exponentiation of real emissions gives a factor

$$\tilde{B} = -\frac{1}{8\pi^2} \int \frac{d^3 k}{k^0} \Theta(\Omega, k) \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2$$

The Algorithm

- The infrared singularities in the Eikonal's $\tilde{S}(k)$ are excluded by a cut-off $k^0 > \epsilon\frac{\sqrt{s}}{2}$
- $\tilde{\beta}$ are infrared finite and are calculated perturbatively *order by order*
  - Non trivial for other methods e.g Structure function
- Photons are explicitly created
  - Allows us to calculate properties of soft photons
  - Algorithm is very MC friendly- computation is quite fast

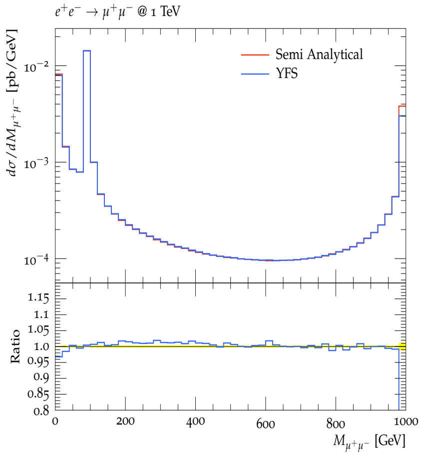
The Algorithm

1. First the phasespace is rearrange by a change of variable $v = 1 - \frac{s'}{s}$
2. v is then generated by standard Monte-Carlo Methods e.g Importance Sampling
3. If $v < \epsilon$ no photons are generate. For $v \geq \epsilon$ the multiplicity is generated according to a Poissonian distribution with average $\mu = \frac{2\alpha}{\pi}\log(\frac{s}{m_e^2})\log(\frac{v}{\epsilon})$
4. The n photon momenta are distributed according to the Eikonal $\tilde{S}$ and then rescaled to ensure 4-momentum conservation
5. The perturbative parts can be calculated separately using ME generator
6. Give everything to the master equation to calculate the cross section

Current Status

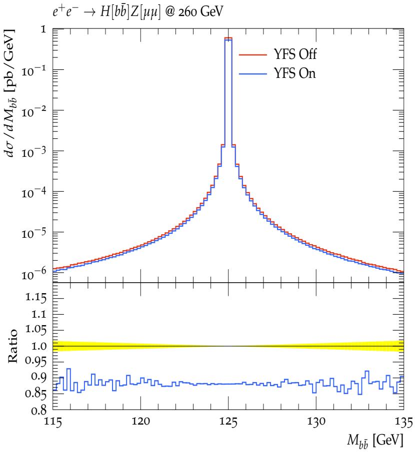
- There is a known issue with the integration limits. Cause a notable deviation. Will be fixed soon
- It can interface with other SHERPA modules e.g ME Generator, PS, Hadronization, UFO...
- On the perturbative side there is a small numerical stability issue from the loop providers (due to small lepton masses)
- However some higher order corrections have been hard coded for $e^+e^- \to f\bar{f}$

Validation



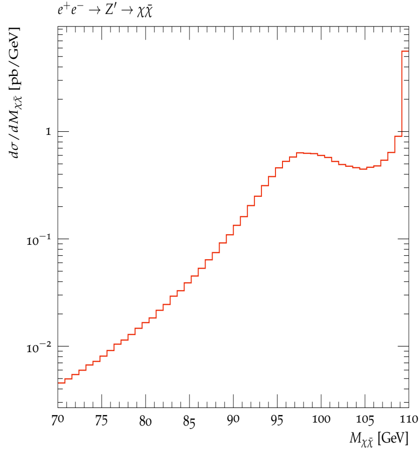
- There is a Analytical formula for $e^+e^- \to f\bar{f}$ Known to $\mathcal{O}(\alpha^2)$
- Comparison is done at $\mathcal{O}(\alpha)$

Higgs-Strahlung



- This is very preliminary
- Fixed order calculation just to test YFS
- Deviation is to be expected, nothing too impressive. The higher order corrections are more important

UFO Interface



- Exaggerated Z' model! Just to test interface
- If you have a realistic UFO for e^+e^- I would to test it

Conclusion and Outlook

1. A very good description of higher order QED effects is needed for precision physics
2. YFS is fully compatible with SHERPA other tools, in particular ME
3. Devation and Loop problem needs to be solved- deviation wont be too tricky
4. For linear collider beam polarisation will be needed. Interface to existing software will not be difficult

Back up slides

Comparison to Data