# On the Mass and Stability of the Sexaquark.

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## Summary

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## Summary

The proposal of a bound state $uuddss$ with a small radius and small mass as a candidate for the dark mass encourages the research on the properties of such a state expected in the framework of QCD . To this extent we determine the chromomagnetic and chromoelectric forces between the constituents, which depend on their distances. To get an qualitative estimate of the distances we propose an analogy with the Bohr atom, where the mass of the ground state and of the distance between the proton and the electron are fixed by the Coulomb interaction.

# HADRON SPECTRUM FROM QCD

After the proposal that QCD *is* the theory of strong interactions De Rujula, Georgi and Glashow realized that the fine structure (the chromomagnetic interaction, CMI) accounts for the mass differences between $\Delta$ and the nucleon and between $\Sigma$ and the $\Lambda$. In this framework, there is the successful prediction :

$$M(\Xi^*) - M(\Xi) = M(Y^*) - M(\Sigma) \qquad (1)$$

which had been previously obtained, by assuming the same coefficients for the mass terms transforming both as an octet for the decuplet and the octet of baryons.

# SPECTRUM OF CHARMED AND BEAUTIFUL HADRONS

Applying the same approach to the charmed baryons $\Sigma_c$ and $\Lambda_c$, they predicted a mass difference high enough to allow the strong decay $\Sigma_c^+ \to \Lambda_c + \pi^+$, in agreement with the discovery of both particles in a neutrino experiment .

The chromomagnetic interaction between the three constituent quarks gives a contribution to the mass of the baryon, they form, which depends on the quadratic Casimir of $SU(6)$ color spin, $SU(6)_{cs}$, $SU(3)$ color, $SU(3)_c$ and $SU(2)$ spin, $SU(2)_s$, and is proportional with a negative factor to (Hogaasen and Sorba) :

$$C_6(qqq) - \frac{C_3(qqq)}{2} - \frac{C_2(qqq)}{3} - 6 \qquad (2)$$

## SPECTRUM OF THE NON STRANGE BARYONS

Since the color singlets built with three quarks with spin $\frac{1}{2}$ and $\frac{3}{2}$ belong to the 70 and 20 with Casimir $\frac{33}{4}$ and $\frac{21}{4}$, respectively, the proportionality factor is :

$$\frac{M_N - M_\Delta}{4} \tag{3}$$

The masses of $N$ and $\Delta$ may be obtained by adding the sum of the effective masses of the constituents, which is given by $\frac{M_N+M_\Delta}{2}$, since the chromagnetic contributions to their masses are opposite.

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# MASSES OF THE STRANGE BARYONS

The mass difference between the $\Sigma$ and $\Lambda$ hyperons is a consequence of the different girochromomagnetic factors of the light (u and d) and strange quark (s). Indeed the total spin of the two light quarks in the $\Lambda$ is 0 and therefore the chromomagnetic contribution to the mass of that particle is equal to the one for the nucleon. Therefore the mass difference between the strange and the light constituents of the baryons is just given by the difference $M_\Lambda - M_N$. Instead for the $\Sigma$ the total spin of the two light quarks is 1, which implies a total chromomagnetic contribution to its mass

$[\frac{1}{6} - \frac{2k_s}{3}](M_\Delta - M_N)$,

where $k_s = 0.6$ is the ratio of the strange and light girochromomagnetic factors .

## MASSES OF THE CHARMED BARYONS

As long as for the charmed baryons the chromomagnetic interaction between a light and a charmed quark is weaker, mainly for the smaller gyro-chromomagnetic factor of the charmed quark inversely proportional to its mass . To reproduce the masses a factor $k_{1c} = 0.24$ is needed for the CMI and an effective mass for the charmed quark of 1715 MeV.
The $\Sigma_b$ and $\Lambda_b$ particles have a mass difference even larger, as expected.

## SPECTRUM OF THE MESONS

For the mesons ($\pi$, $K$ $\rho$, $K^*$) the chromomagnetic contribution is proportional with a positive coefficient to

$$C_6(q\bar{q}) - \frac{C_3(q\bar{q})}{2} - \frac{C_2(q\bar{q})}{3} - 4 \tag{4}$$

The vector and the pseudoscalar mesons belong to the 35 and to the 1 representations of $SU(6)_{cs}$ with quadratic Casimir 6 and 0, respectively, and therefore the proportionality coefficient for the mesons built with light constituents is :

$$\frac{3(M_\rho - M_\pi)}{16} \tag{5}$$

# SPECTRUM OF THE MESONS

Interestingly enough, both the sign and the order of magnitude of the mass differences appearing in Eq. (3) and Eq. (5) have been obtained in as a consequence of the sum rules proposed by Weinberg in the framework of the transformation, which relates constituent and current quarks .
The chromomagnetic contribution for the strange pseudoscalar is proportional to $k_s$ and one expects :

$$\frac{M_{K^*} - M_K}{M_\rho - M_\pi} = k_s \tag{6}$$

The sum of the light constituent masses in the mesons is given by $\frac{3M_\rho + M_\pi}{4}$, while for the strange mesons is given by $\frac{3M_{K^*} + M_K}{4}$ . So one needs a larger chromomagnetic interaction and a smaller effective masses for the light and the strange quarks than in the case of the baryons. Both these properties can be understood by the more intense chromoelectric attraction between a quark and an antiquark, which form a color singlet with respect to two quarks, which combine in a $\bar{3}$ of SU(3) color. Indeed, the stronger attraction implies a smaller constituent mass and a larger contact interaction.

## SPECTRUM OF THE CHARMED PARTICLES

In fact, for the charmed mesons $D$ and $D^*$, a slightly smaller mass, 1615 MeV, and larger $k_{2c} = 0.26$ are needed with respect to the charmed baryons. Also, the values found for the $c\ \bar{c}$ states, 1535 MeV for the mass of the charmed quark and $K_c^2 = 0.186$ for the square of the gyrochromomagnetic factor can be understood as a consequence of the smaller distance between the constituents.
The mass of 3621.40 MeV of the $\Xi_{cc}^{++}$ recently found by $LHCb$ [**?**] implies an effective mass of the constituent charmed quarks of 1665$MeV$, somewhat smaller than the one found for the charmed mesons and $\Lambda_c$.

## SPECTRUM OF THE TETRAQUARKS

For the two nonets of scalar tetraquarks [**?**] [**?**], where the isoscalar states built with the light constituents are the $f^0(600)$ and $f^0(1370)$ resonances, their masses are reproduced with an effective chromomagnetic interaction as for the baryons and with a larger constituent mass. Interestingly enough, this explains why the lowest one, which decays into two pions, has a very large width, while the other one decays mainly into four pions as shown by Gaspero. In fact, the $SU(6)_{cs}$ Casimir, which gives the most important chromomagnetic contribution to the masses, implies that the lighter state is almost a $SU(6)_{cs}$ singlet with an "open channel" (Jaffe) into two pions, which are also $SU(6)_{cs}$ singlets, while the heavier one transforms mainly as a 405 and therefore has an open channel into a pair of $\rho$ mesons, which transform as a 35 of $SU(6)_{cs}$ color spin . A general analysis of the spectrum of negative and positive pentaquarks built with the three lightest quarks and the study of $3q\ 3\bar{q}$ hexaquarks can be found in works in collaboration with Mario Abud, Francesco Tramontano, Domenico Falcone and Giulia Ricciardi . [**?**].

## SPECTRUM OF THE HIDDEN CHARM PENTAQUARKS

Recently the spectrum of the hidden charm $(\frac{3}{2})^{-}$ states with the same $SU(3)_f$ properties of the proton has been studied in this framework with the prediction of two states with a mass around the state found by $LHCb$ with "open channel" into $pJ/\psi$ and $\Lambda_c(\bar{D})^{*0}$ and two higher mass states with "open channel" into $\Sigma_c(\bar{D})^{*}$. Here we want to find the QCD interaction of the constituents of the $SU(3)_c \times SU)2)_s \times SU(3)_f$ singlet proposed by Gladnys Farrar as a candidate for the dark mass to get information both for the mass, the stability and the dimensions of the lowest state with those six constituents.

## Mass of the Sexaquark

Our purpose is to evaluate the mass of the lightest sexaquark starting from Pauli principle, QCD and the undetermination principle. In the following we shall face the problem of determining the chromomagnetic and chromoelectric interaction of the six constituents, which depend on their relative distance and on the way they combine to build a singlet of $SU(3)_c \times SU(2)_s \times SU(3)_f$ . We shall also describe the consequent selection rules for the semileptonic and non-leptonic decays of the sexaquark.

# The QCD chemistry of six quarks

It is istructive to compare the cases, where the six quarks are $uuddss$ combined in a singlet of $SU(3)_c \times SU(2)_s \times SU(3)_f$ and where the six quarks are $uuuddd$ . In this last case they may combine in two color singlets, $uud$, a proton, and $udd$, a neutron combined into a deuteron. One may also consider a totally symmetric spatial wave function : an isospin singlet for the Pauli principle implies that they transform as the ...of $SU(6)_{cs}$ with quadratic Casimir = 12. Instead the $uuddss$ $SU(3)_f$ singlets may combine into a pair ΛΛ, ΣΣ or $N\Xi$, but also into a completely spatial symmetric wave function and in this case Pauli principle implies that a $SU(3)_f$ singlet corresponds to the 490 represntation of $SU(6)_{cs}$ with quadratic Casimir = 18.

## SEXAQUARK BUILT WITH THREE DIQUARKS

There is also the possibility to have three diquarks $ud$, $us$ and $ds$ with color spin $\bar{3}, 0$, advocated by Maiani, Piccinini, Polosa and Riquier to describe the lowest tetraquark nonet, combined into a $SU(3)_f$ singlet. A similar possibility does not exist for three pairs $ud$, since to combine into a color singlet gives rise to a totally antisymmetric wave function, not the right one for three bosons.

## Consequences of the Pauli Principle for the Transformation with Respect to $SU(6)_{cs}$ of the Sexaquark

By assuming a symmetric form for the spatial variables the transformation properties under $SU(3)_{cs}$ of a $uuddss$ singlet under color, spin and flavor is the 490 representation (Young tableaux two rows with three boxes) which combines with the Young tableaux of a $SU(3)_f$ singlet (three rows with two boxes) to give rise to a completely antisymmetric wave function Since the quadratic Casimir of the 490 representation is 18, the contribution of the chromomagnetic contribution would be given by :

- $\frac{(m_{\Delta - m_N)}[6+8k_s+(k_s)^2]}{10}$

## Mass of a Sexaquark with a Symmetric Spatial Function

If we take the same constituent masses and the proportional factor from baryons :

$$m_q = 363 \tag{7}$$

$$m_s = 538 \tag{8}$$

and $k_s = 0.6$, we get for the mass of the sexaquark :

$4m_q + 2m_s - \frac{(m_{\Delta - m_N})[6+8k_s+(k_s)^2]}{10} =$

about 2 GeV .

## Mass of a Sexaquark Built with three Diquarks

Let us compute the mass of a sexaquark built with three $\bar{3}, 0$ of $SU(3_)c \times SU(2)_s$ . First we should evaluate the mass of the three diquarks. The mass of the non strange diquark $ud$ will be deduced from the mass of the Λ particle, which is the combination of the diquark and of the strange quark in a color singlet . Therefore the mass of the Λ is given by the sum of the masses of the strange quark and of the diquark diminuished of the chromoelectric binding energy, $-71MeV$. This implies for the diquark a mass of $(1116 + 71 - 609)MeV = 577MeV$ . The masses of the strange diquarks is given by : $(609 + 434 - 148 \times 0.6 - 142\frac{1218}{1043} = 780Mev$ .

## Mass of a Sexaquark Built with Three Diquarks

The mass of the sexaquark obtained by the combination into a singlet of $SU(3)_c \times SU(2)_s \times SU(3)_f$ is given by the sum of the masses of the three diquarks diminuished by the chromoelectric binding energy. We fix the parameters, $m_u$, $m_s$ , $k_s$ and the binding energy of the light quarks in the baryons to reproduce the masses of the states of the 56 of $SU(6)_{fs}$, namely the octet with spin $frac12$ and the decuplet with spin $frac32$ . We keep into account that for the $(\Omega)^-$ the chromoelectric contribution is enhanced by the factor $\frac{m_s}{m_u}$ with respect to the nucleon and the $\Delta$ since the higher mass of the strange quark implies a smaller distance by the constituents . For the other strange particles we assume the factor $\frac{2m_u+m_s}{3m_u}$ for the baryons with strangeness $-1$ and the factor $\frac{m_u+2m_s}{3m_u}$ for the baryons with strangeness $-2$ .

## QCD Evaluation of the Masses of Ordinary Hadrons

The different values of the same constituent masses and the proportionality factors for the baryons and for the mesons depend on the relative distance of the constituents, which is smaller for the mesons as a consequence of the stronger chromoelectric coupling of a $q$ and a $\bar{q}$, which form a $SU(3)_c$ singlet with respect to two quarks transforming as a $\bar{3}$. While a $q\bar{q}$ pair lies on a segment, three $q$'s on a plane, the geometry of six constituents is more complicate .

The sum of the kinetic energy and of the potential energy is :

$$-e_s^2/(2r) = -(me_s^4)/2[h/(2\pi)]^2 \tag{9}$$

## QCD and the Bohr Model

Let us first consider the mesons, a pair $q\bar{q}$ interacting according to quantum chromodynamics . We make the analogy with the Bohr model, which is able to predict the spectrum of the hydrogen atom as in quantum mechanics. The QCD interaction has a Coulomb term, for which we may apply the Bohr rule for the lowest state :

$$rp = h/(2\pi) \tag{10}$$

By the dynamic rule

$$P^2/(mr) = e_s^2(r^2) \tag{11}$$

one gets :

$$r = [h/(2\pi)]^2/(me_s^2) \tag{12}$$

# Bohr Approach to the Masses of the Mesons

Let us first consider the mesons, a pair $q\bar{q}$ interacting according to quantum chromodynamics . We make the analogy with the Bohr model, which is able to predict the spectrum of the hydrogen atom as in quantum mechanics. The QCD interaction has a Coulomb term, for which we may apply the Bohr rule for the lowest state :

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By the dynamic rule

$$P^2/(mr) = e_s^2(r^2) \tag{14}$$

one gets :

$$r = [h/(2\pi)]^2/(me_s^2) \tag{15}$$

## Pion, $\rho$, N and $\Delta$

For the pion and the $\rho$ built with a pair $q\bar{q}$ of light quark of mass $m_q$ , since $e_s^2 = \frac{2\alpha_s}{3}$, the effective mass of the two light constituents is:

$$(m_\pi + 3m_\rho)/4 = 2m_q(1 - (\alpha_s)^2/18) \quad (16)$$

For the nucleon and the $\Delta$, with the three constituents on the vertices of an equilater triangle let us first consider the mass of the diquark :

$$m_{qq} = 2m_q(1 - \frac{(\alpha_s)^2}{72} \quad (17)$$

The strength acting on the third quark is $\sqrt{3} frac\alpha_s/(4h^2)$, where h, the height of the triangle, is the distance of the third quark from the center of mass of the diquark.

## N and $\Delta$

For the masses of the nucleon and of the $\Delta$ one has :

$$(m_N + m_\Delta)/2 = 3m_q[1 - \frac{2m_q(1 - \frac{(\alpha_s)^2}{72})}{32(\alpha_s)^2(3 - \frac{\alpha_s}{36})}] \qquad (18)$$

So one gets :

$$m_q = 434MeV \qquad (19)$$

and

$$\alpha_s = 5.4 \qquad (20)$$

## Radius of the Pion and of the Nucleon

It is important to find the radius of the pion and of the nucleon from the Bohr relationship :

$$e^2 rm = (\frac{h}{2\pi})^2 \qquad (21)$$

which implies :

$$r_\pi = \frac{1}{2} r_{q\bar{q}} \frac{hc}{3\pi e_s^2 217 MeV} = 0.45 fermi \qquad (22)$$

$$r_p = \frac{1.38 r_{q\bar{q}}}{\sqrt{3}} = 0.717 fermi \qquad (23)$$

## Masses of the Isospin Multiplets of the Octet

We have the following expression for the masses of the eight isospin multiplets with $m_u = 434 MeV$, $m_s = 638 MeV$ and $k_s = 0.6$ :

$$m_N = 3(m_u - 71 MeV) - 148 MeV = 941 MeV \tag{24}$$

$$m_\Lambda = 2(m_u - 71 MeV) + m_s(1 - \frac{71 MeV}{m_u} - 148 MeV = 1115 MeV \tag{25}$$

$$m_\Sigma = 2(m_u - 71 MeV) + m_s(1 - \frac{71 MeV}{m_u} - 148(\frac{4}{3}k_s - \frac{1}{3}) = 1794 \tag{26}$$

$$m_\Xi = (m_u - 71 MeV) + 2m_s(1 - \frac{71 MeV}{m_u}) - 148(\frac{4}{3}k_s - \frac{1}{3}(k_s)^2 = 1337 MeV \tag{27}$$

## Masses of the Isospin Multiplets of the Decuplet

$$m_{\triangle} = 3(m_u - 71MeV) + 148MeV = 1237MeV \tag{28}$$

$$m_{Y^*} = 2(m_u - 71MeV) + m_s(1 - \frac{71MeV}{m_u} + 148(\frac{2}{3}k_s + \frac{1}{3}) = 1371MeV \tag{29}$$

$$m_{(\Xi)^*} = (m_u - 71MeV) + 2m_s(1 - \frac{71MeV}{m_u}) + 148(\frac{2}{3}k_s + \frac{1}{3}(k_s)^2) = 1514MeV \tag{30}$$

$$m_{(\Omega)^+} = 3m_s(1 - \frac{71MeV}{m_u}) + 148(k_s)^2) = 1864MeV \tag{31}$$

## Mass of the non Strange Diquark

The mass of the diquark with color spin $\bar{3}, 0$ may be obtained from the one for the $\Lambda$, which consists of that diquark and of a strange quark from :

$$m_\Lambda = m_s(1 - \frac{71MeV}{m_u} + m_{ud} = 1115MeV \qquad (32)$$

which implies $m_{ud} = 578MeV$

and the same result follows from :

$$m_{ud} = 2(m_u - 71MeV) - 148MeV = 578MeV \qquad (33)$$

## Mass of the Strange Diquarks

For the strange diquarks we may write :

$$m_{us} = (m_u - 71MeV) + m_s(1 - \frac{71MeV}{m_u}) - k_s 148MeV = 845MeV \tag{34}$$

The mass of the sexaquark built with the three diquarks is given by the sum of the masses of the three diquarks and of the chromo-electric binding energy . To evaluate the binding energy one may relate to the binding energy of the $\Xi$ by multiplying by the ratio of the masses of the light (strange) diquark and $m_u$ $(m_s) = 1.32$ . The mass of the sexaquark is given by :

$$m_{sexa} = (m_u - 71MeV) + m_s(1 - \frac{71MeV}{m_u}) - k_s 148MeV = (578 + 1690 - 371)MeV \tag{35}$$

## Selections rules for the weak decays of the sexaquark

The strangness changing weak current :

$$\bar{u}_L(x)\gamma_\mu s_L(x) \tag{36}$$

changes the $s$ into a $u$ quark, while the non-leptonic strangeness changing hamiltonian changes the $s$ into a $d$ quark : in both case the sexaquark is turned into a state forbidden by the spin statistic relationship. This property implies selection rules for the processes, which turn the sexaquark into a state with different strangness and increase the stability of this state .

## Selections rules for the weak decays of the sexaquark

The Gamow-Teller contribution may transform the $ds$ diquark into a $du$ diquark transforming as a $\bar{3}, S = 1$ of $SU(3)_c \times SU(2)_s$ with a change of its chromomagnetic energy from negative to positive, which almost compensates the decreasing of its mass, which is the consequence of the difference between the strange and the light quarks . Exchanges of gluons transforming the transformation properties of the diquarks with respect to $SU(3)_c \times SU(2)_s$ increase their mass.

## Stability of a Sexaquark Built with a Symmetric wave function

The Gamow-Teller weak interaction transforms a strange quark $s$ into a $u$ quark and consequently for the Pauli principle changes the transformation property with respect to $SU(3)_c \times SU(2)_s \times SU(3)_f$ of the resulting state $uuudds$ with the consequence of a smaller chromomagnetic contribution to the binding energh, which almost compensates the mass difference between $s$ and $u$ quarks.

## Conclusions

The chromoelectric and chromomagnetic interactions between the six constituents, $uuddss$ of the sexaquark proposed by [**?**] depend on their relative distances . A qualitative idea on them may be achieved by the analogy with the Bohr atom, for which these distances depend on the strenght of the chromoelectric force between the constituents. If we assume that the sexaquark is built with three diquarks, $ud$, $us$ and $ds$, trasforming as a $\bar{3}, 0$ of $SU(3)_{cs}$ in a symmetric S-wave combined in a singlet, we get a qualitative evaluation of their mass around 1900$MeV$ and selections rules for the Fermi transitions, but not for the Gamow-Teller . So the construction of a sexaquark with a sufficiently small mass and the stability necessary to be a candidate for the dark matter is still open .

## Conclusions

Since the mass depends strongly on the distances we agree with the statement by Farrar that a smallness of the dimension of the sexaquark would imply a smaller value for its mass and therefore we think that a more precise evaluation of the relative distances is needed to get a reliable information on the very important issue of the mass of the sexaquark. As long as for the stability one has to worry about the Gamow-Teller transitions.