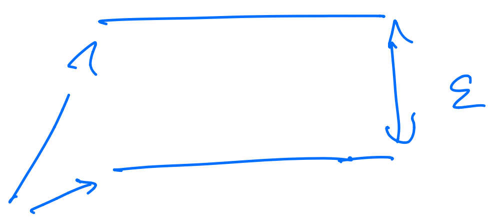
Problem

Evaluate the energy to break a Cooper pair!

Cooper pair

$$\text{const}\ \hat{S}_+|0\rangle = \text{const} \sum_{m>0} a_m^\dagger a_{-m}^\dagger |0\rangle$$

Problem



N-particles

both levels N-fold degenerate

Study this case as a function of $\frac{\varepsilon}{V_0}$!

$\hat{V}$ – pairing interaction the same between pairs in either levels or between them.

Quasispin operators

$$\hat{S}_+ = \sum_{m>0} a_m^+ a_{-m}^+ = \hat{S}_x + i\hat{S}_y$$

$$\hat{S}_- = \sum_{m>0} a_{-m} a_m = \hat{S}_x - i\hat{S}_y$$

$$\hat{S}_0 = \frac{1}{2} \sum_{m>0} \left[a_m^+ a_m + a_{-m}^+ a_{-m} - 1 \right] = \hat{S}_z$$

$$[\hat{S}_+, \hat{S}_-] = 2\hat{S}_0$$

$$[\hat{S}_0, \hat{S}_\pm] = \pm \hat{S}_\pm$$

$$\hat{V} = -V_0 \hat{S}_+ \hat{S}_- = -V_0 \left[\hat{\vec{S}}^2 - \hat{S}_0^2 + \hat{S}_0 \right]$$

$$\begin{cases} \hat{S}_0 |0\rangle = -\frac{\Omega}{2} |0\rangle \\ \vec{S}^2 |0\rangle = \frac{\Omega}{2}\left(\frac{\Omega}{2}+1\right)|0\rangle \end{cases}$$

$$E_{S,S_0} = \langle S, S_0 | \hat{V} | S, S_0 \rangle$$

$$= -V_0 \left[S(S+1) - S_0^2 + S_0 \right]$$

$$\left| S = \frac{\Omega}{2}, S_0 = \frac{N-\Omega}{2} \right\rangle = \text{const} \left(\hat{S}_+ \right)^{\frac{N}{2}} \left| S = \frac{\Omega}{2}, S_0 = -\frac{\Omega}{2} \right\rangle$$

$$= \text{const} \left(\hat{S}_+ \right)^{\frac{N}{2}} | 0 \rangle$$

Here N - even

vacuum

Problem

Consider a degenerate 2Ω-level and N (even) $< 2\Omega$ fermions.

The Hamiltonian is

$$\hat{H} = \sum_{k,\ell} \langle k,-k|V|\ell,-\ell\rangle$$

with
$$\begin{cases} \langle k,-k|V|\ell,-\ell\rangle = -V_0 \\ \text{all other matrix elements vanishing} \end{cases}$$

Total number of states $= \binom{2\Omega}{N} = \dfrac{(2\Omega)!}{N!\,(2\Omega-N)!}$

List all possible eigenenergies of this system!

Other suggested problems

In file all_problems.pdf consider only these problems depending on the section:

Set 2: Prob 5 , Prob 2

Set 3: Prob.6 , Prob. 14

Set 4: Prob. 2 ,

Set F1: Prob. 3, Prob. 4 , Prob. 5
Prob. 6, Prob.7, Prob. 9

$$|2\rangle = \Big(\prod_{m>0} u_m\Big)^2 \underbrace{\Big(\sum_m \frac{v_m}{u_m} a^+_m a^+_{-m}\Big)}_{\text{Cooper pair (unnormalized)}} |0\rangle$$

Problem Calculate the normalization!

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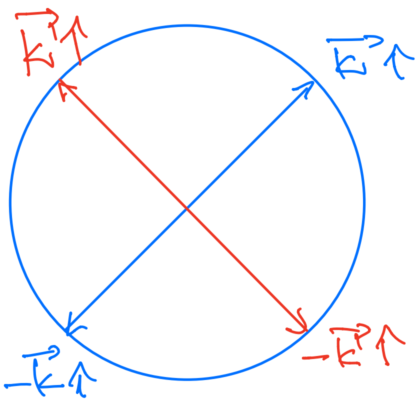
Problem



Now $(\vec{k}\uparrow, -\vec{k}\uparrow)$ are not anymore time-reversed pairs.

- Can one make such Cooper pairs?

If YES, how? Under what conditions?

If NOT, why not?

Describe their properties if such Cooper pairs exist!