Investigating resonant shattering flares by modelling the vibrational modes within neutron stars

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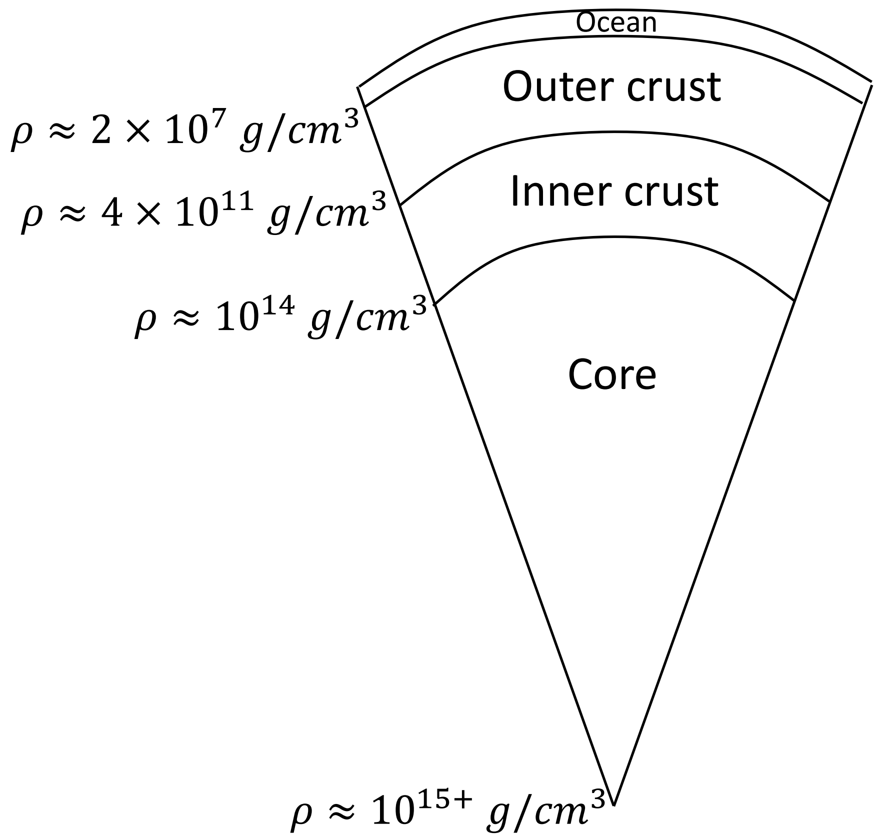
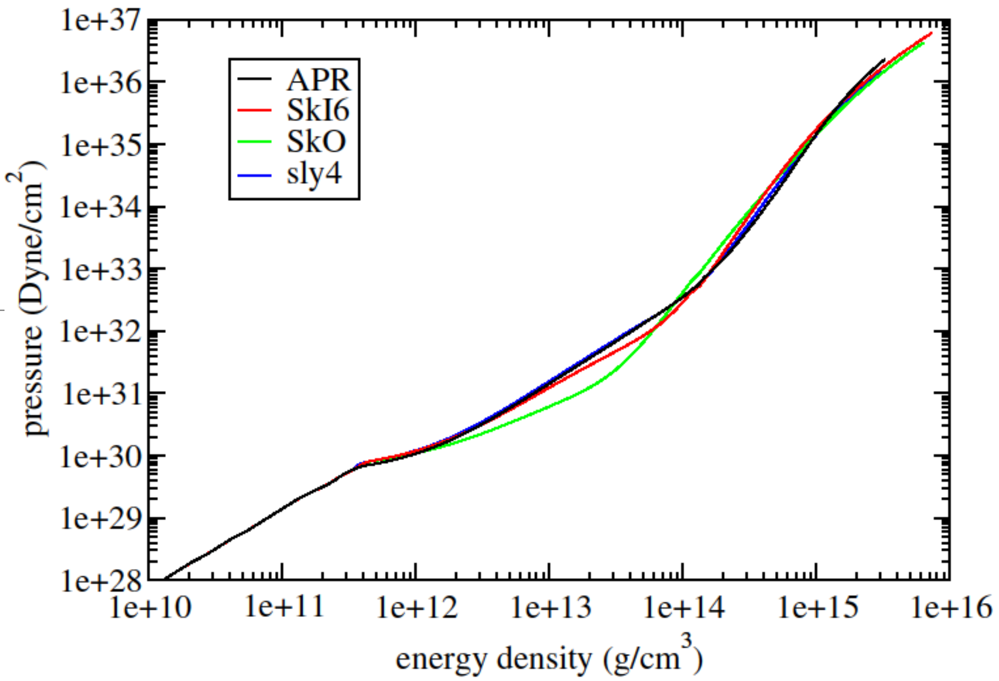
Neutron star structure

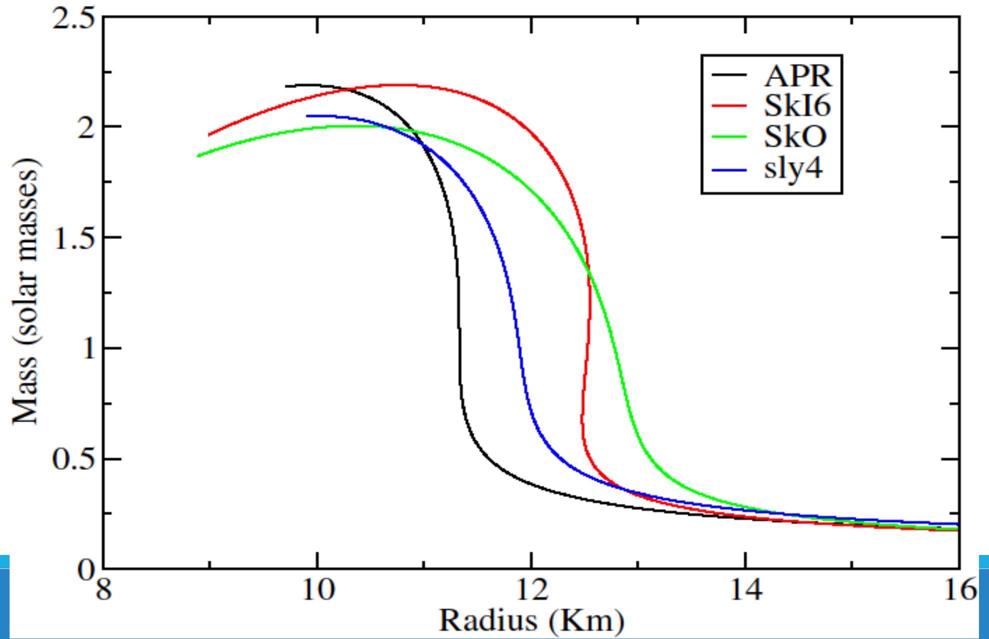
- The crust is solid and has composition discontinuities (different nuclei)
- The core has a smooth change in composition
- A star with higher central density has a smaller crust
- Deeper in the star, less is known about its structure



Neutron star structure

- At the densities in neutron stars (10^{15} g/cm^3) the equation of state is purely theoretical
- There are many different equations of state that can be used to describe matter at high densities
- The equation of state is constrained by experimental data
- Is the core a fluid? Superfluid? Baryonic matter? Hyperonic matter? Quark-gluon plasma?
- We need to use several equations of state and compare the results



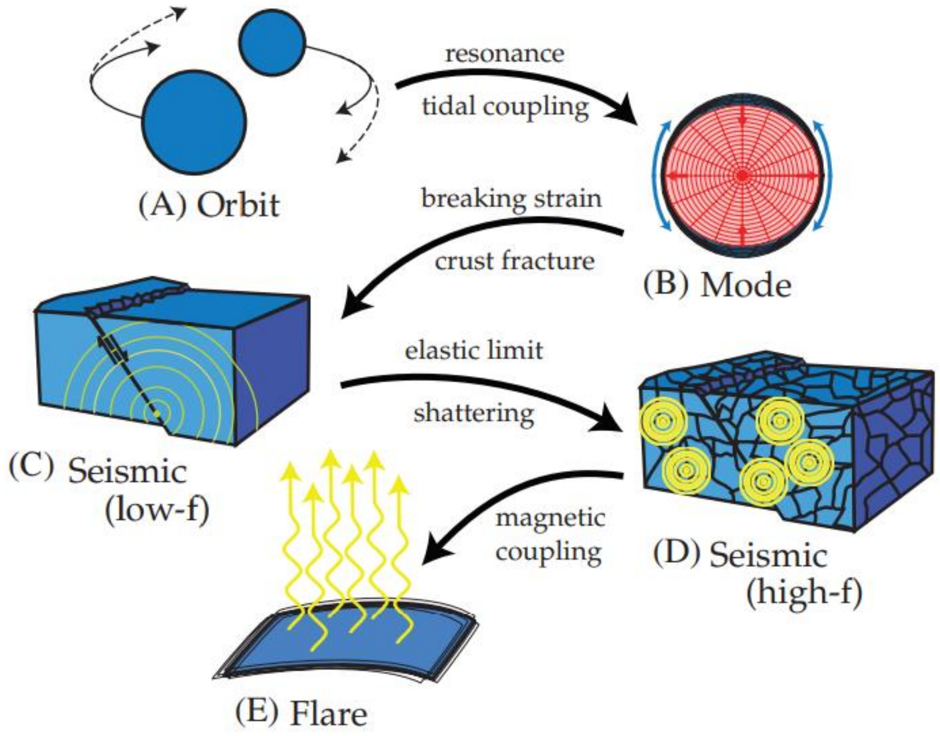


Vibrational modes

- Tidal forces cause the matter in stars to oscillate
- Boundary conditions in the core and at the surface constrain the oscillations of the star to specific vibrational modes
- Each vibrational mode displaces the matter in the star in different ways
- The modes are grouped based on the restoring force
- We are mainly interested in the interface modes (i-modes)

Resonant shattering flares

- Resonance can occur between the orbit of binary neutron stars and their vibrational modes
- Energy is deposited at the base of the crust
- Eventually the crust shatters, releasing a flare
- How long does this occur before the neutron stars merge?



Resonant shattering flares

The overlap integral between the tidal field and the i-mode is given by:

$$Q = \frac{1}{MR^2} \int d^3x \, \rho \xi^* \cdot \nabla [r^2 Y_{2\pm 2}(\theta, \varphi)]$$

We estimate the energy deposited in the crust using:

$$E_{max} \propto f^{\frac{1}{3}} Q^2 M^{-\frac{2}{3}} R^2 q \left(\frac{2}{1+q} \right)^{\frac{5}{3}}$$

The energy required to shatter the crust is:

$$E_b \approx \left(\frac{0.1}{\epsilon_b} \right)^2 (2\pi f)^2 \int d^3x \, \rho \xi_b^* \cdot \xi_b$$

Only i-modes give $E_b \ll E_{max}$ at low frequency. t-modes have too low Q values, g-modes do not penetrate the crust, and the f-mode has too high a frequency for the flare to be distinguished from the merger.

Fluid stars

- In a purely fluid star the shear modulus is zero
- This removes transverse traction and makes the radial traction dependant on the radial and transverse displacement
- This simplifies the equations, so there is only one input parameter

$$(1+\tilde{V})\frac{dy_1}{dx} = \left(\frac{\tilde{V}}{\Gamma_1} - 3\right)y_1 + \left[\frac{l(l+1)}{c_1\Omega^2} - \frac{\tilde{V}}{\Gamma_1}\right]y_2$$

$$(1+\tilde{V})\frac{dy_2}{dx} = (c_1\Omega^2 + Ar)y_1 + (1 - \tilde{U} - Ar)y_2$$

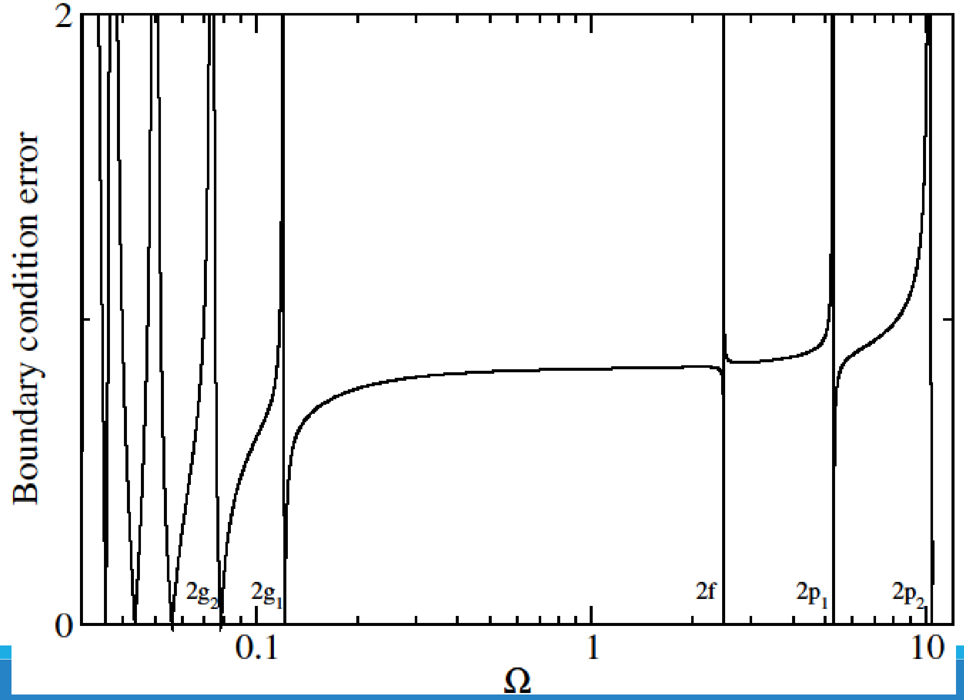
(McDermott, 1988)

- These equations must be solved for the entire radius of the star
- The aim is to find an Ω value for which y_1 and y_2 are continuous and satisfy the boundary equations

At r=0: $\frac{c_1\Omega^2}{l}y_1 - y_2 = 0$, at r=R_*: $(\tilde{V} - c_1\Omega^2 - 4 + \tilde{U})y_1 + \left[\frac{l(l+1)}{c_1\Omega^2} - \tilde{V}\right]y_2 = 0$

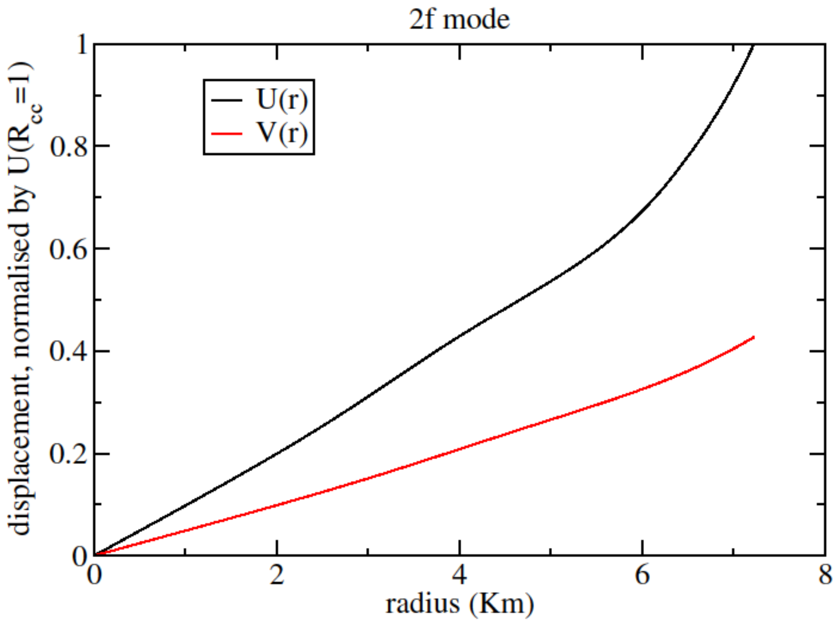
Fluid stars

- The input parameter Ω is the dimensionless angular frequency
- Scale is arbitrary
- Modes are found when the error is zero

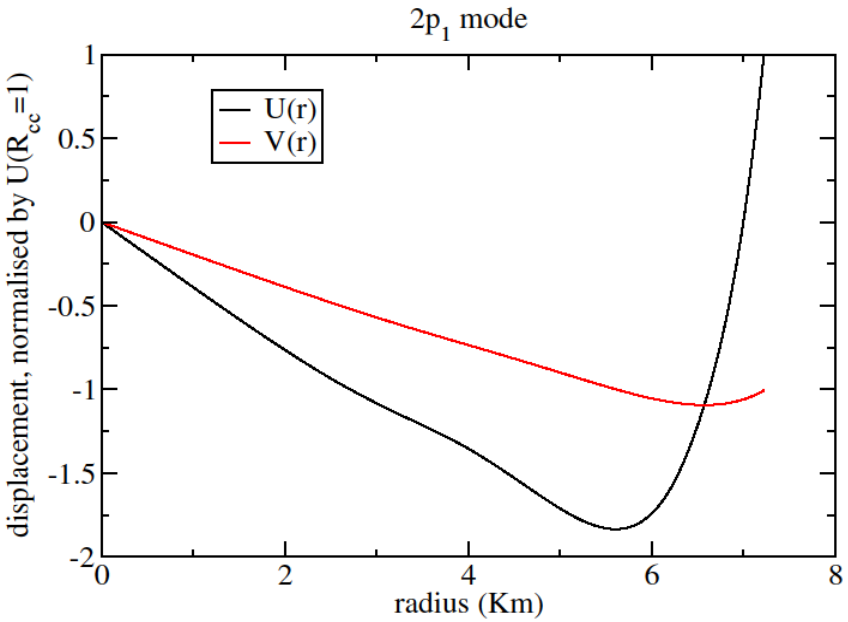


Fluid stars

For a star with 0.503 solar masses, using the BPS equation of state:



Frequency = 2539 Hz



Frequency = 4730 Hz

$$(U(r) = y_1 r)$$

$$(V(r) = \frac{r}{c_1 \Omega^2} y_2)$$

- There is no i-mode for a fluid star

Adding the crust

- An i-mode appears at the crust core boundary
- Need to calculate the shear modulus of the crust
- The crust is solid, so traction must now be considered
- There are now 2 input parameters, frequency and the transverse displacement at the surface of the star
- In the crust we use:

At R_{cc}: $z_1 = y_1$

$$z_2 = \tilde{V}(y_1 - y_2)$$

$$z_4 = 0$$

$$(1+\tilde{V})\frac{dz_1}{dx} = -\left(1 + 2\frac{\alpha_2}{\alpha_3}\right)z_1 + \frac{1}{\alpha_3}z_2 + l(l+1)\frac{\alpha_2}{\alpha_3}z_3$$

$$(1+\tilde{V})\frac{dz_2}{dx} = \left(-c_1\tilde{V}\Omega^2 - 4\tilde{V} + \tilde{U}\tilde{V} + 12\Gamma_1\frac{\alpha_1}{\alpha_3}\right)z_1 + \left(\tilde{V} - 4\frac{\alpha_1}{\alpha_3}\right)z_2 + l(l+1)\left(\tilde{V} - 6\Gamma_1\frac{\alpha_1}{\alpha_3}\right)z_3 + l(l+1)z_4$$

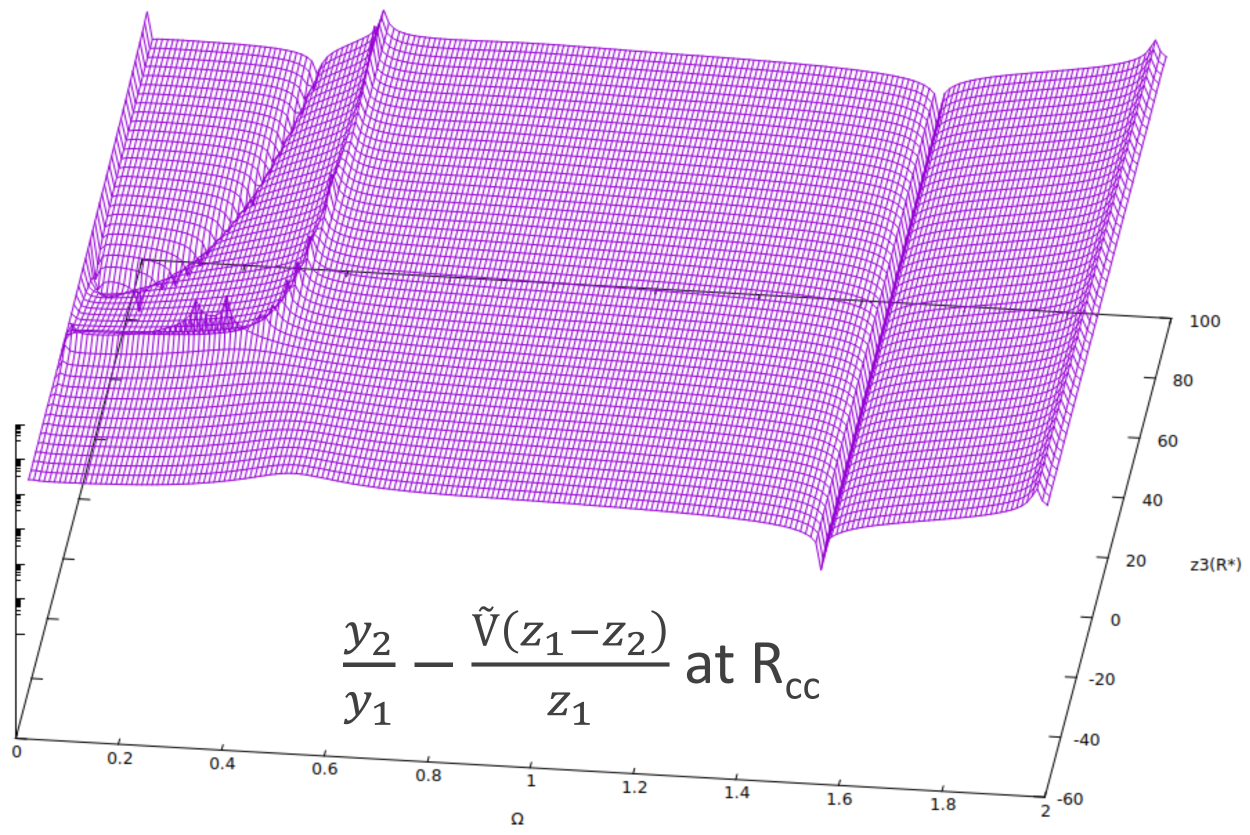
$$(1+\tilde{V})\frac{dz_3}{dx} = -z_1 + \frac{1}{\alpha_1}z_4$$

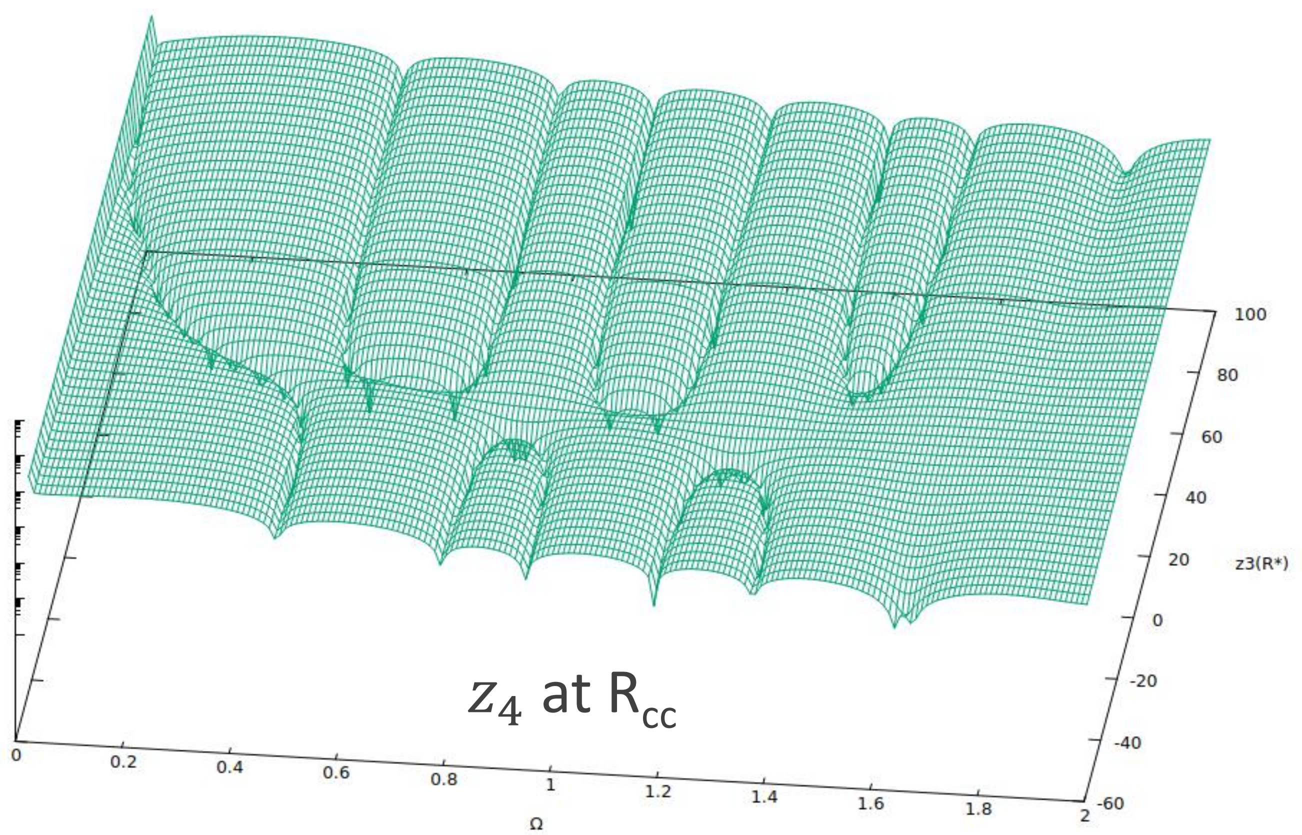
$$(1+\tilde{V})\frac{dz_4}{dx} = \left(\tilde{V} - 6\Gamma_1\frac{\alpha_1}{\alpha_3}\right)z_1 - \frac{\alpha_2}{\alpha_3}z_2 + \left[-c_1\tilde{V}\Omega^2 + \frac{2}{\alpha_3}\left([2l(l+1)-1]\alpha_1\alpha_2 + 2[l(l+1)-1]\alpha_1^2\right)\right]z_3$$

(McDermott, 1988)

Adding the crust

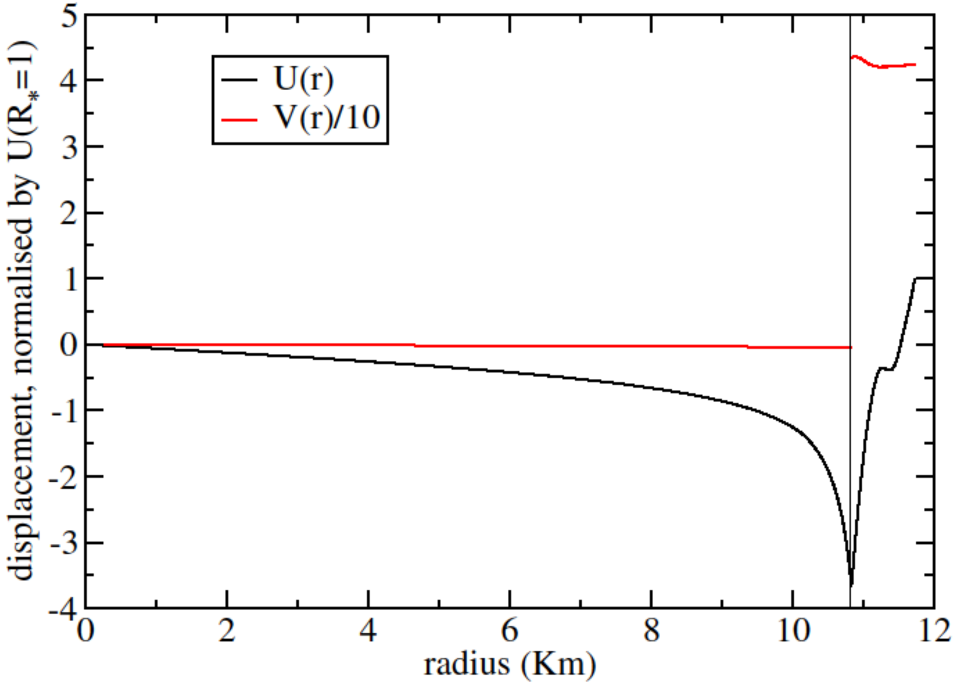
- There are 2 values being minimised
- A mode is found when the minima coincide





Adding the crust

For a neutron star with 1.4 solar masses, using the sly4 equation of state the 2i mode is:



Frequency = 188.5 Hz

$$(U(r) = y_1 r = z_1 r)$$

$$(V(r) = \frac{r}{c_1 \Omega^2} y_2 = z_3 r)$$

- The displacement is largest at the crust-core boundary

Further work

- Modify to core equations to use superfluidity
- Find the i-mode for different equations of state
- Use the range of frequencies to calculate the energy transferred into the mode by resonance
- Investigate the flare emitted by the shattering of the crust
- What is the composition of the inner crust (nuclear pastas)
- How often do resonant shattering flares occur?