Structure Properties of Strange Quark Star in the presence of Strong Magnetic Fields

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Why the strange quark stars?

Contents

A brief story about strange quark stars

- Hybrid Stars: Compact stars with a quark core
- Pure strange quark stars (SQS):
  - They are created after the second explosion in the end of the life of super massive stars (The super luminous explosion is called Quark-Nova)
  - They are completely made up of quarks with a small fraction of electrons.

http://www.quarknova.ca/

Why strong magnetic fields?

- Compact Objects like neutron stars and strange quark stars have strong magnetic fields.
- Neutron stars with very strong magnetic fields are called magnetars.
- Strong magnetic fields may originate from magnetic flux.
- Using observational data and applying virial theorem, the maximum magnetic field for this objects is $B_{max} \sim 2 \times 10^{18} G$ (Observational data *RX J1856.5−3754*: $M = 1.7 M_{sun}$ and $R = 8$ km)

Prakash, et al., 2003

$$B_{max} = B_{sun}\left(\frac{M}{M_{sun}}\right)\left(\frac{R_{sun}}{R}\right)^2$$

- Surface magnetic field is in O($\sim 10^{15}$). It is predicted the interior magnetic field is in O($\sim 10^{18}$).

Lai, Shapiro, 1991

Our system: Magnetized SQM

- **SQM with spin-up (+) and spin-down (−) quarks (u, d, and s) in a uniform magnetic field**

- **MIT bag model (with two cases of the bag constant)**

- **The spin polarization parameter:** $\xi = \frac{\rho^{(+)} - \rho^{(-)}}{\rho^{(+)} + \rho^{(-)}}$

- **The density of spin-up quarks:** $\rho^{(+)}$ **and the density of spin-down quarks:** $\rho^{(-)}$

F. Kayanikhoo, K. Naficy, G. H. Bordbar, Eur. Phys. J. A , 56:2 (2020)

Formalism of calculations

- *Energy density of single particle:*

$$E^i = [{p_i}^2 c^2 + {m_i}^2 c^4 (1 + 2\,J B_D)]^{1/2}$$

$$B_D = \mathrm{B}/B_c \qquad B_c = \frac{m_i^2 c^3}{q_i \hbar}$$

- *Number density of particles:*

$$\rho = \sum_{\mathrm{J}=0}^{J_{max}} \frac{2qB}{h^2 c} g_J p_F\,(\mathrm{J})$$

- *Upper limit of Landau level*

$$J_{max} = \frac{\varepsilon_F^2 - 1}{2B_D} \qquad \varepsilon_\mathrm{F} = \frac{\mathrm{E_F}}{\mathrm{mc}^2}$$

Formalism of calculations

- *Total energy density of System:* $\varepsilon_{tot} = \varepsilon_i^+ + \varepsilon_i^- + B_{bag}$

- *Kinetic energy density of each particle (i):*

$$\varepsilon_i{}^{\pm} = \frac{2B_D}{(2\,\pi)^2\lambda^3} m_i c^2 \sum_J g_\vartheta (1 + 2\,JB_D)\eta\left(\frac{x_F{}^{\pm}}{(1 + 2\,JB_D)^{1/2}}\right)$$

$$\eta(x) = \frac{1}{2}[x\sqrt{1 + x^2} + \ln\,(x\sqrt{1 + x^2})] \qquad x_F = \frac{p_F}{m_q c} = (\varepsilon_F^2 - 1 - 2\,JB_D)^{1/2}$$

- *The equation of state of SQM (EOS):* $P(\rho) = \rho\left(\frac{\partial\varepsilon_{tot}}{\partial\rho}\right) - \varepsilon_{tot}$

Formalism of calculations

Magnetic field energy density and pressure

- The contribution of the electromagnetic field is directly added to the EoS and energy density to give the total energy density and pressure:

$$\varepsilon_B = P_B = \frac{B^2}{8\pi}$$

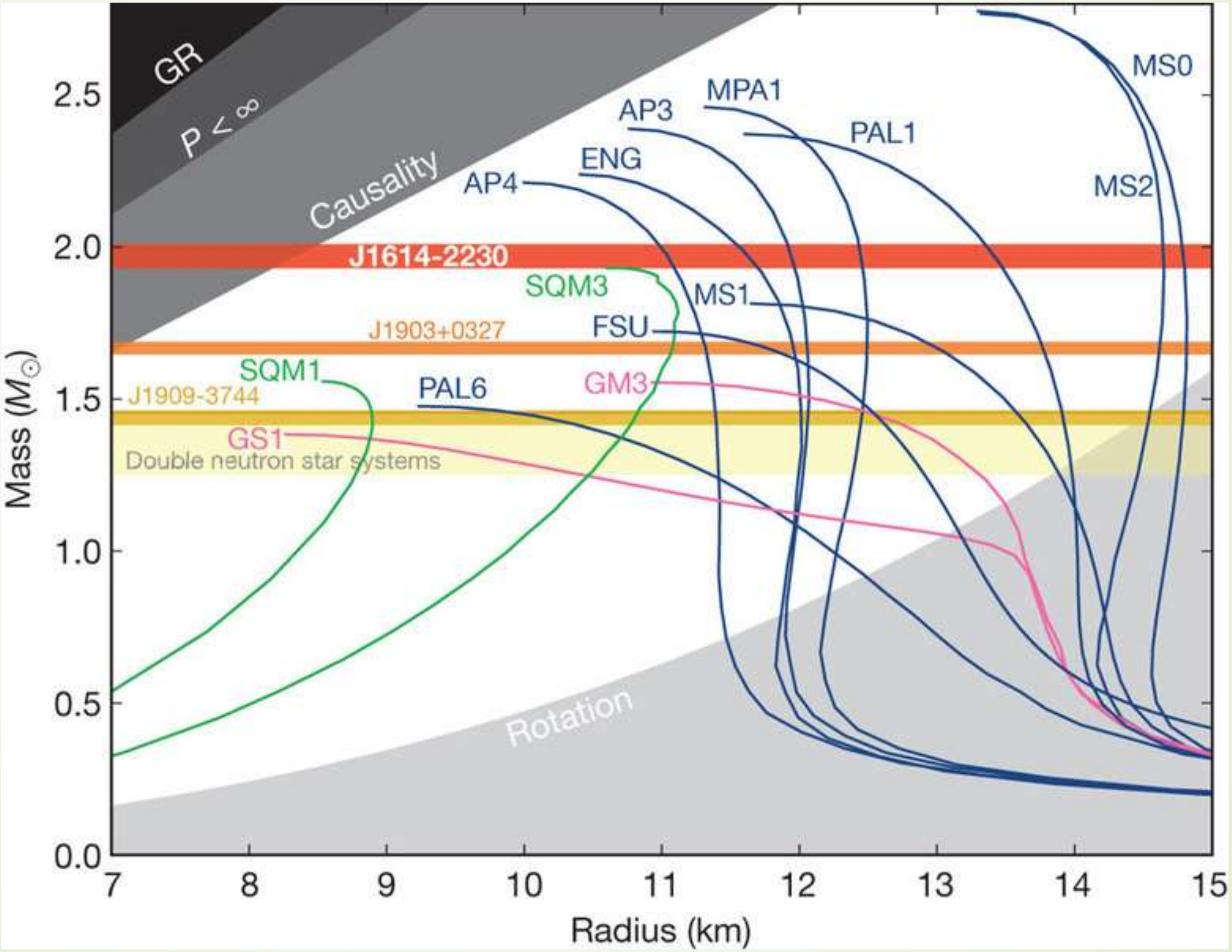
Structure of SQS

- **To investigate the structure of compact objects we use hydrostatic equilibrium equations (HEE).**
- **The first attempt for obtaining Einsteinian HEE for stars was done by Tolmann-Oppenhimer-Volkoff.**
- **TOV equations are used for the case of spherical symmetric compact objects.**

$$\frac{dp}{dr} = -\frac{G[\varepsilon(r) + \frac{P(r)}{c^2}][m(r) + \frac{4\pi r^3 P(r)}{c^2}]}{r^2[1 - \frac{2Gm(r)}{rc^2}]}$$

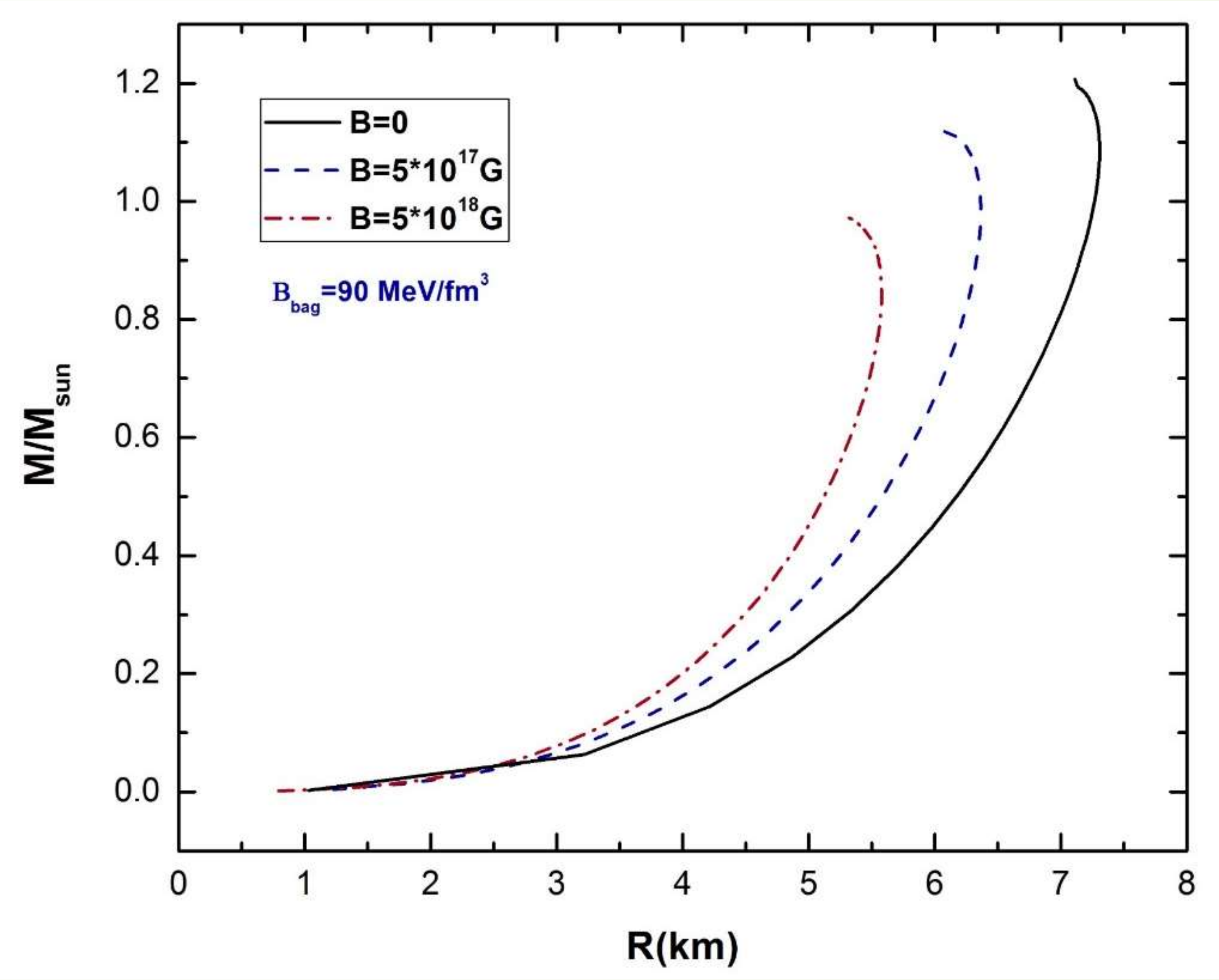
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r)$$

Mass radius relation



Results

- The mass radius relation follows $M \propto R^{\alpha}$, $\alpha \sim 3.5$
- Increasing the magnetic field leads to the smaller mass and radius for SQS

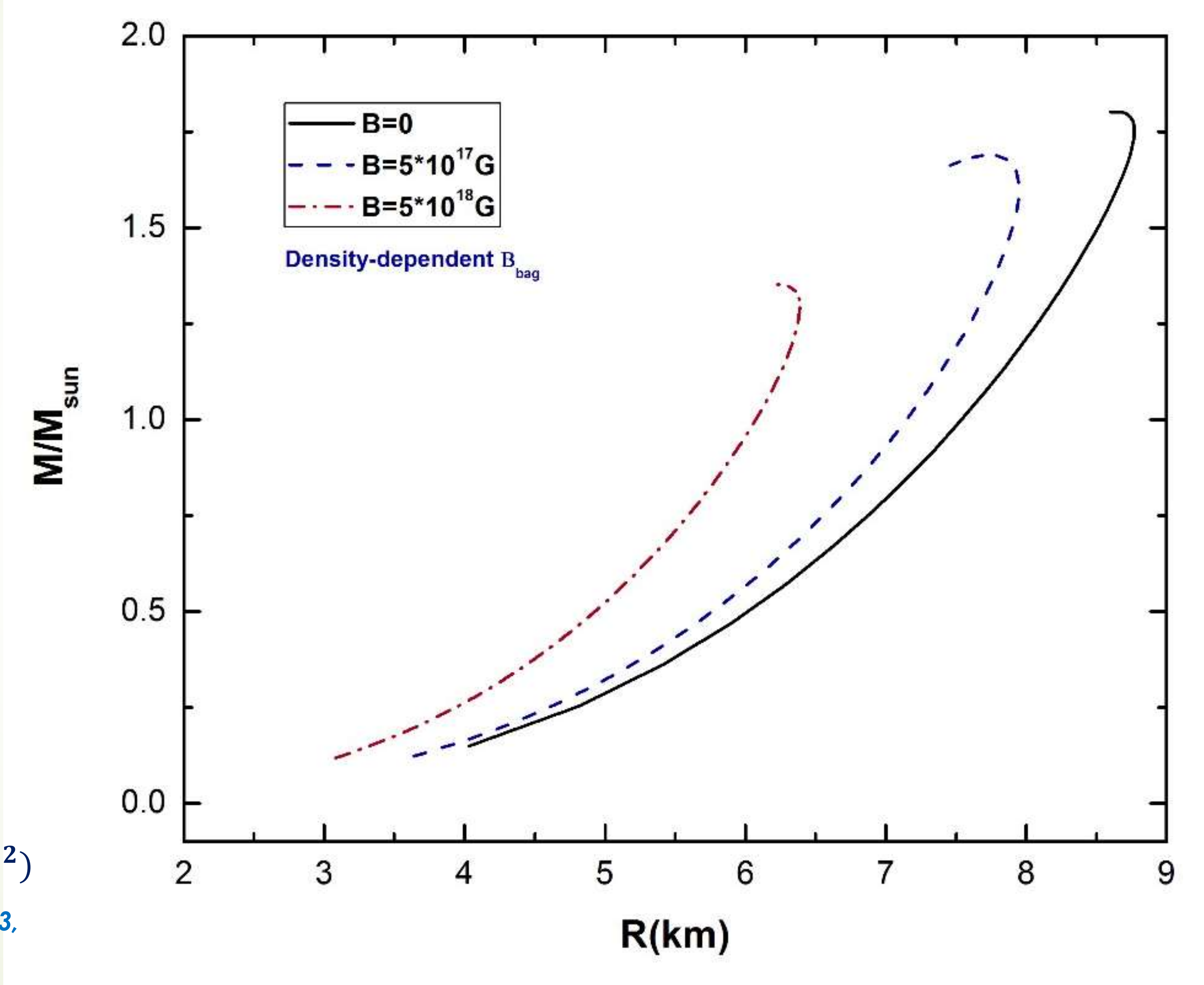


Results

- The mass radius relation follows $M \propto R^{\alpha}, \alpha \sim 3.2$
- In comparison with the case of $fixed\ B_{bag}$ the radius increases with the less steep slope.
- The maximum gravitational mass increases in comparison with the case of $fixed\ B_{bag}$.

$$B_{bag} = B_{\infty} + (B_0 - B_{\infty})\exp(-\beta(\frac{\rho}{\rho_0})^2)$$

G. H. Bordbar, and M. Modarres, J. Phys. G23, 1631 (1997).



Results

$B_{bag} = 90\ MeV/fm^3$

Magnetic field	$\frac{M}{M_{sun}}$	R (km)	$\frac{2M}{R}$	Z_s
0	1.22	7.10	0.68	0.42
$5 \times 10^{17}\ G$	1.11	6.06	0.72	0.47
$5 \times 10^{18}\ G$	0.97	5.32	0.73	0.47

- **The Buchdahl condition ($2M/R \leq 8/9$) is established for all magnetic fields.**

  H.A. Buchdahl, Phys. Rev. 116, 1027 (1959)

- **The gravitational redshift value ($Z_s = [1 - 2\frac{GM}{c^2R}]^{\frac{1}{2}} - 1$) is larger than that value for neutron stars.**

Results

Density – dependent B_{bag}

Magnetic field	$\frac{M}{M_{sun}}$	R (km)	$\frac{2M}{R}$	Z_s
0	1.80	8.65	0.82	0.61
5×10^{17} G	1.65	7.43	0.88	0.70
5×10^{18} G	1.42	6.37	0.88	0.71

- **The Buchdahl condition ($2M/R \leq 8/9$) is established for all magnetic fields.**
- **The gravitational redshift value is larger than that for neutron stars.**
- **The compactification factors and corresponding surface redshifts are larger than those in the case of fixed bag constant model.**

Results

Strange quark star candidates

Observed stars	$\frac{M}{M_{sun}}$	R (km)	$\frac{2M}{R}$	Z_s
$RX\,J\,185635-3754$ [1]	1.20	8.00	0.60	0.34
$Her\,X-1$ [65]	1.10	7.70	0.57	0.31
$4U\,1608-52$ [66]	1.74	9.3	0.74	0.49
$4U\,1820-30$ [67]	1.58	9.1	0.69	0.43
$SAX\,J\,1808.4-3659\,(SS1)$ [68]	1.44	7.07	0.80	0.57
$SAX\,J\,1808.4-3659\,(SS2)$ [68]	1.32	6.35	0.83	0.62

1. M. Prakash, J.M. Lattimer, A.W. Steiner, D. Page, Nucl. Phys. A, 715, 835 (2003).
65. Bombaci, (2002). arXiv:astro-ph/0201369.
66. T. Guver et al., Astrophys. J. 712, 964 (2010)
67. T. Guver et al., Astrophys. J. 719, 1807 (2010)
68. X.D. Li et al., Phys. Rev. Lett. 83, 3776 (1999)

Conclusion

- We have considered the pure SQM (u, d, s quarks), Isotropic EOS and spherical symmetry HEE.
- The maximum gravitational masses and radii decrease by increasing the magnetic field.
- The SQS becomes more compact by increasing the magnetic field.
- The density-dependent bag constant model leads to more compact SQS in comparison with the fixed bag constant model.

A brief introduction of current work

Einstein-Maxwell field equations

$$R_{\mu\vartheta} - \frac{1}{2} R g_{\mu\vartheta} = \frac{8\pi G}{c^4} T_{\mu\vartheta}$$

$$T_{\mu\vartheta} = (\rho c^2 + p) u_\mu u_\vartheta + p g_{\mu\vartheta} + \frac{1}{4\pi} \left(F_{\mu\alpha} F_\vartheta^\alpha - \frac{g_{\mu\vartheta}}{4} F_{\alpha\beta} F^{\alpha\beta}\right)$$

- $T_{\mu\vartheta}$ is the energy-momentum tensor,
- The two first terms are the perfect fluid contribution (ρ: The matter energy density, p: Isotropic fluid pressure, u_μ: 4-vector fluid velocity)
- The third term is the Maxwell energy momentum tensor.
- $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ (A_α: The magnetic vector potential)
- Lorene Library: http ://www.lorene.obspm.fr

D. Chaterjee, et al. MNRAS 447, 3785–3796 (2015)

Future ideas…

- Anisotropic EOS of magnetized SQM
- Rotating Magnetized SQS
- …

Thank you for your attention

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