

Quark-flavor dependence of the transport parameters of the quark-gluon plasma

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Motivation

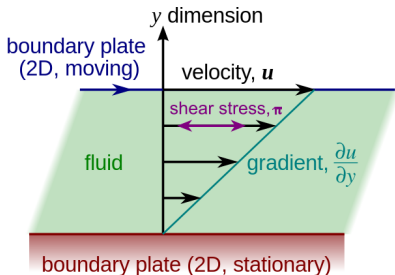
QGP is a nearly perfect fluid → transport properties?

- Hydrodynamical simulations
- Perturbative QCD
- Green-Kubo formulas (integrals of correlation functions)
- Lattice QCD
- Holographic QCD
- Kinetic theory (Boltzmann equation)
- Effective models (quasiparticle model)

Transport parameters

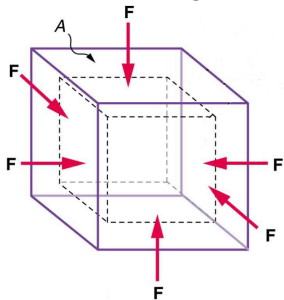
Shear viscosity η :

reaction to a change of shape



Bulk viscosity ζ :

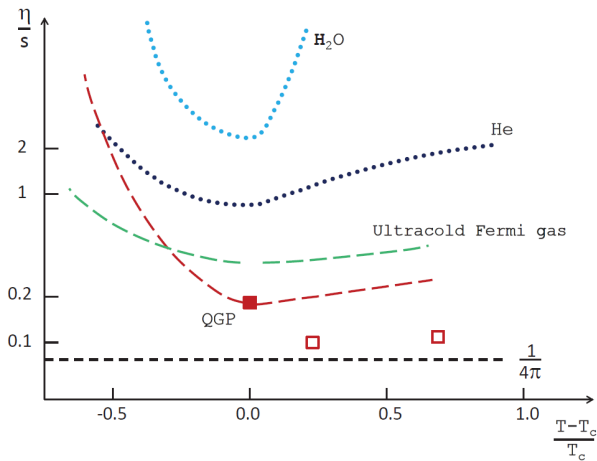
reaction to a change of volume



+ electrical conductivity σ

[Wikimedia commons; archive.cnx.org]

Shear viscosity: QGP as „the most ideal fluid”



[Cremonini, Guersoy, Szepietowski, JHEP08 '12]

Kinetic theory

Fluid in equilibrium is described by stress-energy tensor $T^{\mu}_{\nu}(f_0)$.

Out of equilibrium: $f_0 \rightarrow f$;

$$f - f_0 = \delta f \Rightarrow \delta T^{\mu}_{\nu};$$

$$\delta f = ?$$

→ Boltzmann equation!

Boltzmann equation: relaxation time approximation

The evolution of one-particle distribution function

$$p^\mu \partial_\mu f = -C[f] \sim -\frac{\overbrace{f - f_0}^{\delta f}}{\tau}$$

Relaxation time approximation: after eliminating all driving forces, the distribution function f comes back to its equilibrium form f_0 in time τ :

$$\tau^{-1} = n_0 \bar{\sigma}$$

$$n_0 = d \int \frac{d^3 p}{(2\pi)^3} f_0$$

Relaxation time: $N_f = 0$ vs $N_f = 2 + 1$

$$\tau^{-1} = n_0 \bar{\sigma}$$

Gluons in gluonic plasma (pure SU(3) Yang-Mills theory):

$$\tau_g^{-1} = n_g \bar{\sigma}_{gg \rightarrow gg}$$

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Gluons in quark-gluon plasma with 2+1 quark flavors:

$$\begin{aligned} \tau_g^{-1} = & n_g [\bar{\sigma}_{gg \rightarrow gg} + \bar{\sigma}_{gg \rightarrow u\bar{u}} + \bar{\sigma}_{gg \rightarrow d\bar{d}} + \bar{\sigma}_{gg \rightarrow s\bar{s}}] + \\ & + n_l \bar{\sigma}_{gl \rightarrow gl} + n_{\bar{l}} \bar{\sigma}_{g\bar{l} \rightarrow g\bar{l}} + n_s \bar{\sigma}_{gs \rightarrow gs} + n_{\bar{s}} \bar{\sigma}_{g\bar{s} \rightarrow g\bar{s}} \end{aligned}$$

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Feynman diagram for $qq' \rightarrow qq'$ scattering

[Hosoya, Kajantie, NPB250 '85; V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

Transport coefficients in kinetic theory

Shear viscosity

$$\eta = \frac{d}{15T} \int \frac{d^3 p}{(2\pi)^3} \frac{\bar{p}^4}{E^2} \tau f_0 (1 \pm f_0)$$

[Hosoya, Kajantie, NPB250 '85; Danielewicz, Gyulassy, PRD31 '85; Sasaki, Redlich, PRC 79 '09]

Bulk viscosity

$$\zeta = \frac{d}{T} \int \frac{d^3 p}{(2\pi)^3} \frac{\tau}{E^2} f_0 (1 \pm f_0) \left\{ \left(E^2 - T^2 \frac{\partial m^2}{\partial T^2} \right) \frac{\partial P}{\partial \epsilon} - \frac{1}{3} \bar{p}^2 \right\}^2$$

[Bluhm, Kämpfer, Redlich, PRC 84 '11]

Electrical conductivity

$$\sigma = \frac{d q^2}{T} \int \frac{d^3 p}{(2\pi)^3} \frac{\bar{p}^2}{E^2} \tau f_0 (1 - f_0)$$

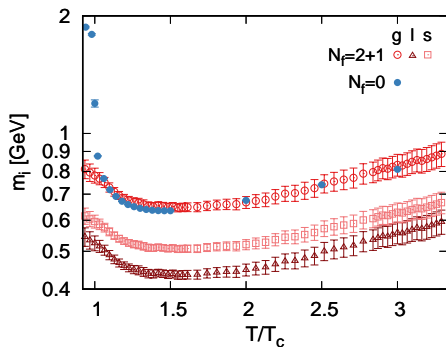
[L. Thakur et al., PRD 95 '17]

Quasiparticle Model

Weakly interacting particles with effective masses, $m[g(T), T]$:

$$m_q^2 = m_0^2 + 2m_0 \sqrt{\frac{g^2 T^2}{6}} + \frac{g^2 T^2}{3}$$

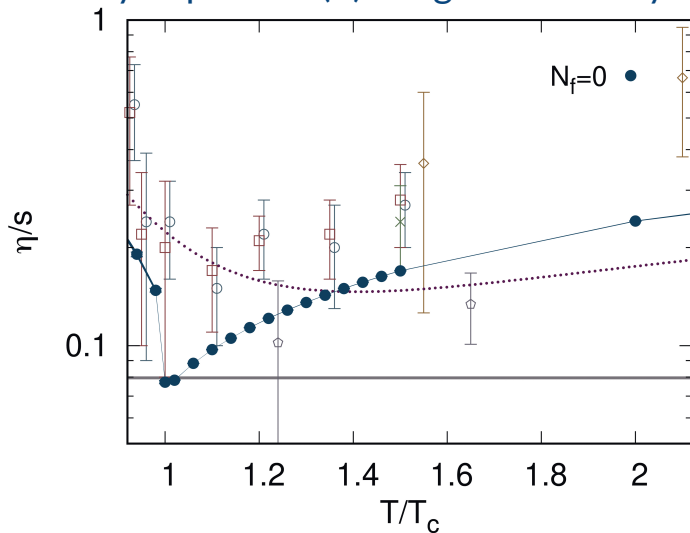
$$m_g^2 = \frac{1}{6} \left(3 + \frac{1}{2} N_f \right) g^2 T^2$$



→ temperature-dependent dispersion relations, $E^2(T) = p^2 + m^2(T)$

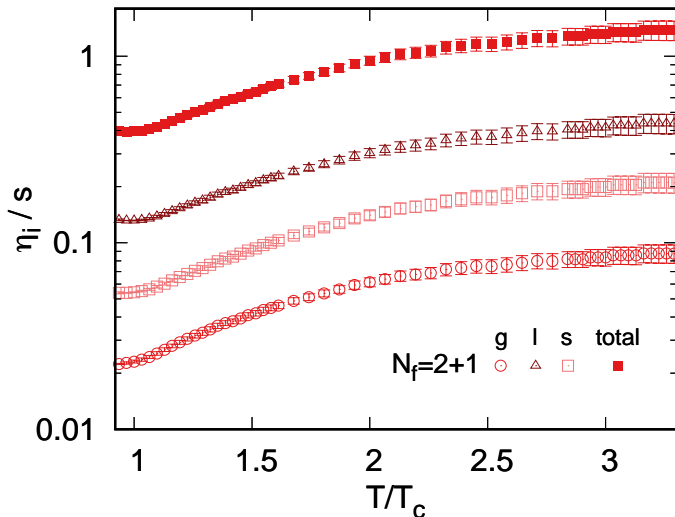
[Pisarski, Nucl. Phys. A498, 423C '89; Bluhm, Kämpfer, Redlich, PRC 84 '11]

Shear viscosity in pure SU(3) Yang-Mills theory



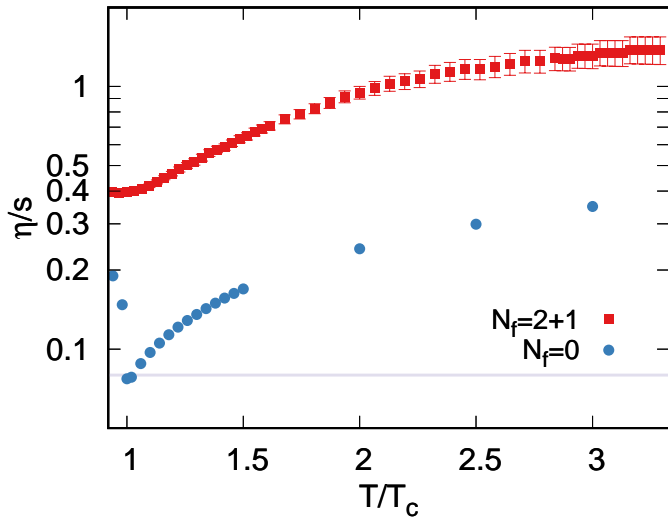
[dashed: Christiansen et al., PRL 115 '15 + various LQCD data cited in V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

Shear viscosity in QCD for $N_f = 2 + 1$

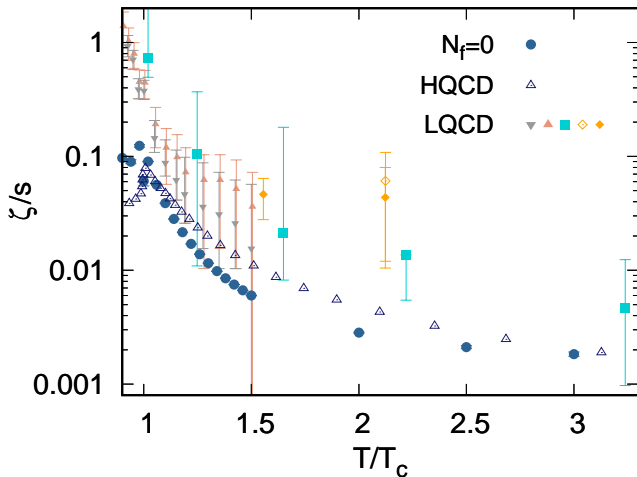


[V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

Shear viscosity: $N_f = 0$ vs $N_f = 2 + 1$

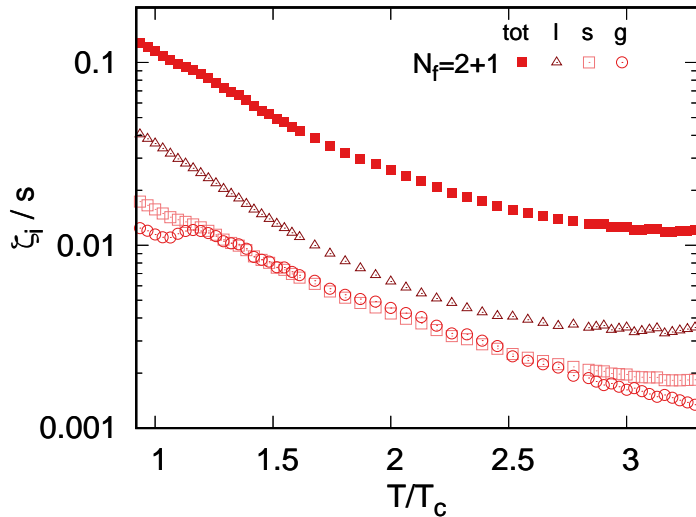


Bulk viscosity in pure SU(3) Yang-Mills theory

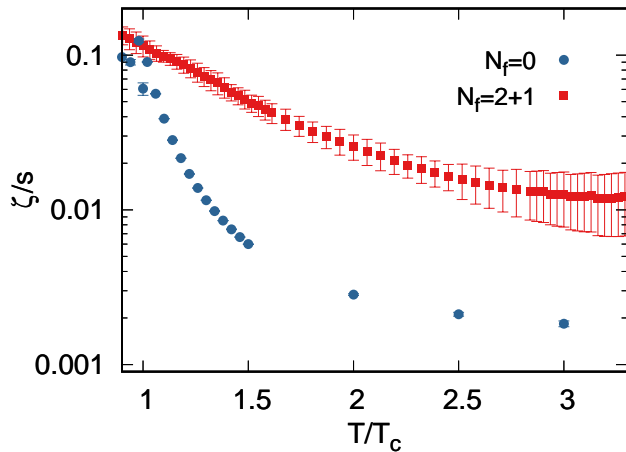


[HQCD: Li, He, Huang, JHEP 06 '15; LQCD: Astrakhantsev et al., JHEP 101 '17, Meyer, PRL 100 '08; Sakai, Nakamura, PoS LAT2007 '07]

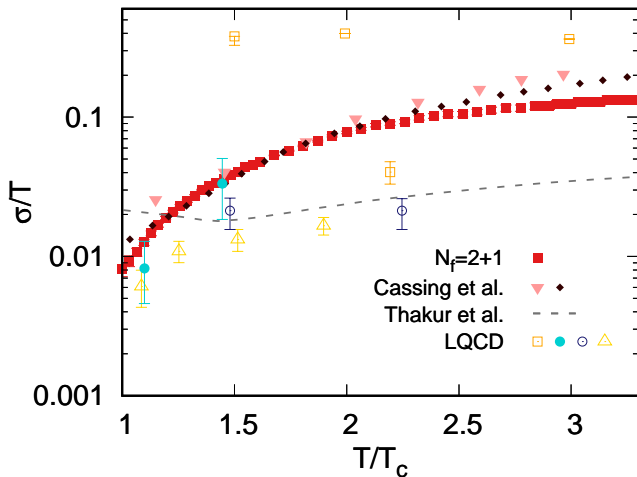
Bulk viscosity in QCD for $N_f = 2 + 1$



Bulk viscosity: $N_f = 0$ vs $N_f = 2 + 1$

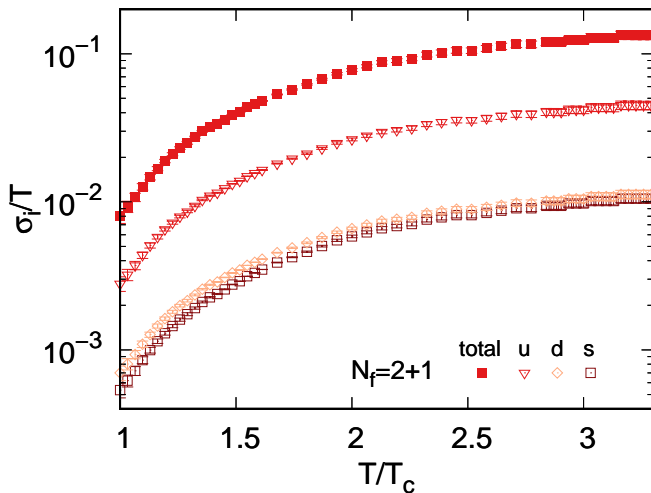


Electrical conductivity: $N_f = 2 + 1$



[Cassing et al., PRL 110 '13; L. Thakur et al., PRD 95 '17 Amato et al., PRL 111 '13; Aarts et al., PRL 99 '07; Ding et al., PoS 185 '11; Gupta et al., PLB 97' 04]

Electrical conductivity: $N_f = 2 + 1$



Summary

- Additional d.o.f. increase viscosity and conductivity of the plasma
- $\eta/s, \zeta/s$ comparable to other results for $N_f = 0$
- σ/T lays in between available LQCD data and agrees with different effective models

★ More flavors, non-zero chemical potential, direct solution of the Boltzmann equation...

Quasiparticle Model

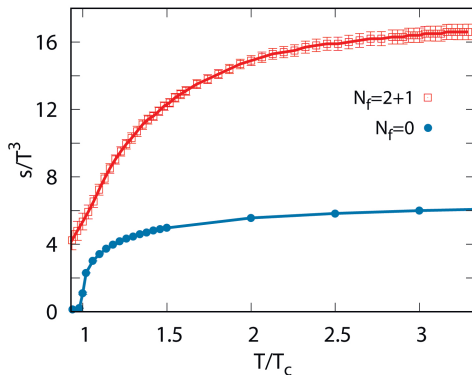
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$$M_g^2 = \frac{1}{6} \left(3 + \frac{1}{2} N_f \right) G^2 T^2$$

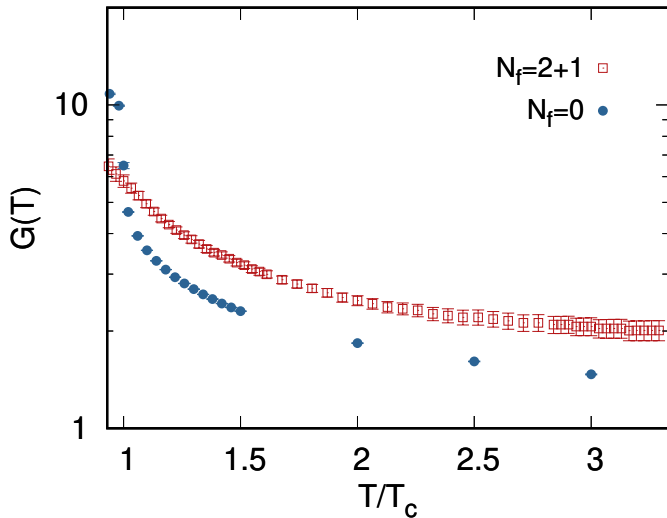
[Pisarski, Nucl. Phys. A498, 423C '89;

Bluhm, Kämpfer, Redlich, PRC 84 '11]



[IQCD: $N_f=2+1$: Borsanyi et al., Phys. Lett. B730 '14; $N_f=0$: Borsanyi et al., JHEP1207, 056 '12]

Effective running coupling



[V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

Relaxation time:

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Light quarks in full QCD:

$$\begin{aligned} \tau_l^{-1}(g(T), T, m_l, m_s, m_g) = & 12 n_l (\bar{\sigma}_{ud \rightarrow ud} + \bar{\sigma}_{uu \rightarrow uu}) + \\ & 12 n_{\bar{l}} (\bar{\sigma}_{u\bar{u} \rightarrow d\bar{d}} + \bar{\sigma}_{u\bar{d} \rightarrow u\bar{d}} + \bar{\sigma}_{u\bar{u} \rightarrow u\bar{u}} + \bar{\sigma}_{u\bar{u} \rightarrow g\bar{g}}) + \\ & 16 n_g \bar{\sigma}_{ug \rightarrow ug} + 6 n_s \bar{\sigma}_{us \rightarrow us} + 6 n_{\bar{s}} \bar{\sigma}_{u\bar{s} \rightarrow u\bar{s}}. \end{aligned}$$



Feynman diagram for $qq' \rightarrow qq'$ scattering

Perturbative QCD expectations for $N_f = 3$

Perturbative parametrization:

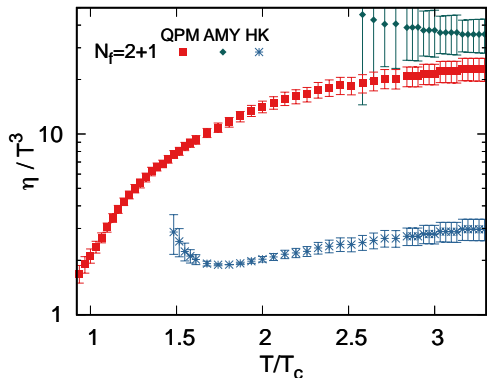
$$\eta_{\text{HK}} = \frac{64\pi^4}{675} \frac{T^3}{g^4 \ln(4\pi/g^2)} \left[\frac{21N_f}{6.8 [1 + 0.12(2N_f + 1)]} + \frac{16}{15 [1 + 0.06N_f]} \right]$$

NLL expansion in g for $N_f = 3$:

$$\eta_{\text{AMY}} = \frac{T^3}{g^4} \left[\frac{\eta_1}{\ln(\mu_*/m_D)} \right],$$

$$\eta_1 = 106.66, \mu_*/T = 2.957$$

$$m_D^2 = \frac{1}{2} g^2 T^2$$



[Hosoya, Kajantie, Nucl.Phys. B 250 '85; Arnold, Moore, Yaffe, JHEP 030 '03]