Inverse-Chirp Imprint of GW in Scalar Tensor Theory

Hao-Jui Kuan

Collaborators: Chiao-Qiang Geng, Ling-Wei Luo

56 Karpacz Winter School in Theoretical Physics
Superfluidity and Transport for Multimessenger Physics
of Compact Stars

TSING HUA UNIVERSITY

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Outline

- Brief Intro. on Scalar Tensor Theory (ST)
- Equivalence of Two Frames in ST
- Simulation of Supernova Core Collapse in ST
- Results
- Conclusion

Scalar Tensor Theory

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Mach's Principle and a Relativistic Theory of Gravitation*

C. BRANS† AND R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received June 23, 1961)

The role of Mach's principle in physics is discussed in relation to the equivalence principle. The difficulties encountered in attempting to incorporate Mach's principle into general relativity are discussed. A modified relativistic theory of gravitation, apparently compatible with Mach's principle, is developed.

... the geometrical and inertial properties of space are meaningless for an empty space, the physical properties of space have their origin in the matter contained therein ...

Scalar Tensor Theory

They consider a kind of action with a scalar field non-minimal coupling to the Ricci Curvature:

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left[\phi R - \frac{\omega}{\phi} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - U(\phi)\right] + S_m[\psi_m, g_{\mu\nu}]$$

which retains the weak equivalent principle with an effective gravitational coupling strength

$$G_{eff} := G/\phi$$

The action in the Jordan frame:

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left[F(\phi)R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - U(\phi)\right] + S_m[\psi_m, g_{\mu\nu}]$$

1. Jordan-Brans-Dicke theory: $F(\phi) = \phi$, $\omega(\phi) \equiv \omega_{BD}$ and $U(\phi) \equiv 0$
2. Brans-Dicke-Bergmann-Wagoner(BDBW): $F(\phi) = \phi$
3. In Fujii's book (2007) : $F(\phi) = \xi\phi^2$, $\omega(\phi) = \frac{1}{2}\phi$ and $U(\phi) \equiv 0$

Scalar Tensor Theory

The action in the Jordan frame can be cast into the Einstein frame, where the scalar field minimally couples to $\boldsymbol{R}$

$$S = \frac{1}{16\pi G}\int dx^4\sqrt{-g_\star}\,[R_\star - 2g_\star^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)] + S_m[\psi_m, A(\varphi)^2 g_{\star\mu\nu}]$$

with $F = A^{-2}$ and the redefinitions of

$$V := A^4 U$$

$$\partial_\mu\varphi = \frac{d\varphi}{d\phi}\partial_\mu\phi := \left(\pm\sqrt{\frac{3(F_{,\phi})^2}{2F^2} + \frac{\omega}{\phi F}}\right)\partial_\mu\phi \qquad (\bigstar)$$

One can describe GW in the Jordan frame in terms of the scalar field in the Einstein frame in 1PN ! T. Damour and G. Esposito-Farese, Class. Quant. Grav. 9, 2093 (1992)

(i.e. assume the coupling function to be $\alpha := \dfrac{d\ln A}{d\varphi} = \alpha_0 + \beta_0(\varphi - \varphi_0)$)

Equivalence of Two Frames

To have the equation of motions of the scalar fields in the Jordan and the Einstein frame be equivalent

$$\frac{\delta}{\delta\varphi} = \frac{\delta}{\delta\phi}\frac{d\phi}{d\varphi}$$

implies that one should prevent the Jacobian of scalar fields from vanishing.

What it means by saying a φ is physical or not?

We define a solution of EoM of φ is physical if there exists a scalar field ϕ in the Jordan frame such that ($\bigstar$) holds and $\dfrac{d\varphi}{d\phi}$ is regular (i.e. nonzero and finite).

$$\partial_\mu\varphi = \tfrac{d\varphi}{d\phi}\partial_\mu\phi := \left(\pm\sqrt{\frac{3(F_{,\phi})^2}{2F^2} + \frac{\omega}{\phi F}}\right)\partial_\mu\phi \qquad (\bigstar)$$

Equivalence of Two Frames

From the definition of coupling function, one have the relation

$$\alpha(\varphi) := \frac{d\ln A}{d\varphi} = -\frac{1}{2F}\frac{d\phi}{d\varphi}\frac{dF}{d\phi}$$

The condition $dF/d\phi \neq 0$ makes the action possible to be reduced to BDBW case. Under this assumption and W.L.O.G. we set $\varphi_0 = 0$, the Jacobian of two scalar fields is non-vanishing iff

$$\alpha = \alpha_0 + \beta_0\varphi \neq 0 \Rightarrow \varphi \neq \varphi_c := -\frac{\alpha_0}{\beta_0}$$

Note that this critical value of the scalar field in the Einstein frame is independent of the potential !

Simulation Setup

Long-Lived Inverse Chirp Signals from Core-Collapse in Massive Scalar-Tensor Gravity

Ulrich Sperhake,[1,2,*] Christopher J. Moore,[1,3] Roxana Rosca,[1] Michalis Agathos,[1] Davide Gerosa,[2] and Christian D. Ott[2]
[1]*DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*
[2]*TAPIR 350-17, Caltech, 1200 East California Boulevard, Pasadena, California 91125, USA*
[3]*IST-CENTRA, Departamento de Física, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*
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This Letter considers stellar core collapse in massive scalar-tensor theories of gravity. The presence of a mass term for the scalar field allows for dramatic increases in the radiated gravitational wave signal. There are several potential *smoking gun* signatures of a departure from general relativity associated with this process. These signatures could show up within existing LIGO-Virgo searches.

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1. Spherically symmetric situation: $ds^2 = -F\alpha^2 dt^2 + FX^2 dr^2 + r d\Omega^2$
2. Hybrid EoS which is of the form $p = p_c + p_{th}$ and $\epsilon = \epsilon_c + \epsilon_{th}$, where

$$p_c = K_1 \rho^{\Gamma_1}, \quad \epsilon_c = \frac{K_1}{\Gamma_1 - 1}\rho^{\Gamma_1 - 1}, \quad \text{as} \quad \rho \le \rho_{nuc} \quad K_1 = 4.9345 \times 10^{14}\ [\text{cgs}]$$

$$p_c = K_2 \rho^{\Gamma_2}, \quad \epsilon_c = \frac{K_1}{\Gamma_2 - 1}\rho^{\Gamma_2 - 1} + E_3, \quad \text{as} \quad \rho > \rho_{\text{nuc}}$$

$$p_{th} = (\Gamma_{th} - 1)\rho\epsilon_{th}$$

Simulation Setup

3. Perfect fluid: $T_{\alpha\beta} = \rho H u_\alpha u_\beta + P g_{\alpha\beta}$

4. Consider the action

$$S = \frac{1}{16\pi G}\int dx^4 \sqrt{-g_\star}\,[R_\star - 2g_\star^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)] + S_m[\psi_m, A(\varphi)^2 g_{\star\mu\nu}]$$

where $V = 2m^2\varphi^2$

5. Simulation parameters are $\{\alpha_0, \beta_0, m, \Gamma_1, \Gamma_2, \Gamma_{th}\}$

The EoM of φ with $\alpha = \alpha_0 + \beta_0\varphi$ is

$$\boxed{\Box^\star\varphi = -4\pi G(\alpha_0 + \beta_0\varphi)T^\star + m^2\varphi.}$$

- GW detectable inside the LIGO sensitivity window: m < 10^{-13} eV
- Strong scalarization and satisfy binary pulsar constraints: m > 10^{-15} eV

We fix $\{m, \Gamma_1, \Gamma_2, \Gamma_{th}\} = \{10^{-14}, 1.3, 2.5, 1.35\}$

and require α_0 (β_0) is positive (negative)

Results

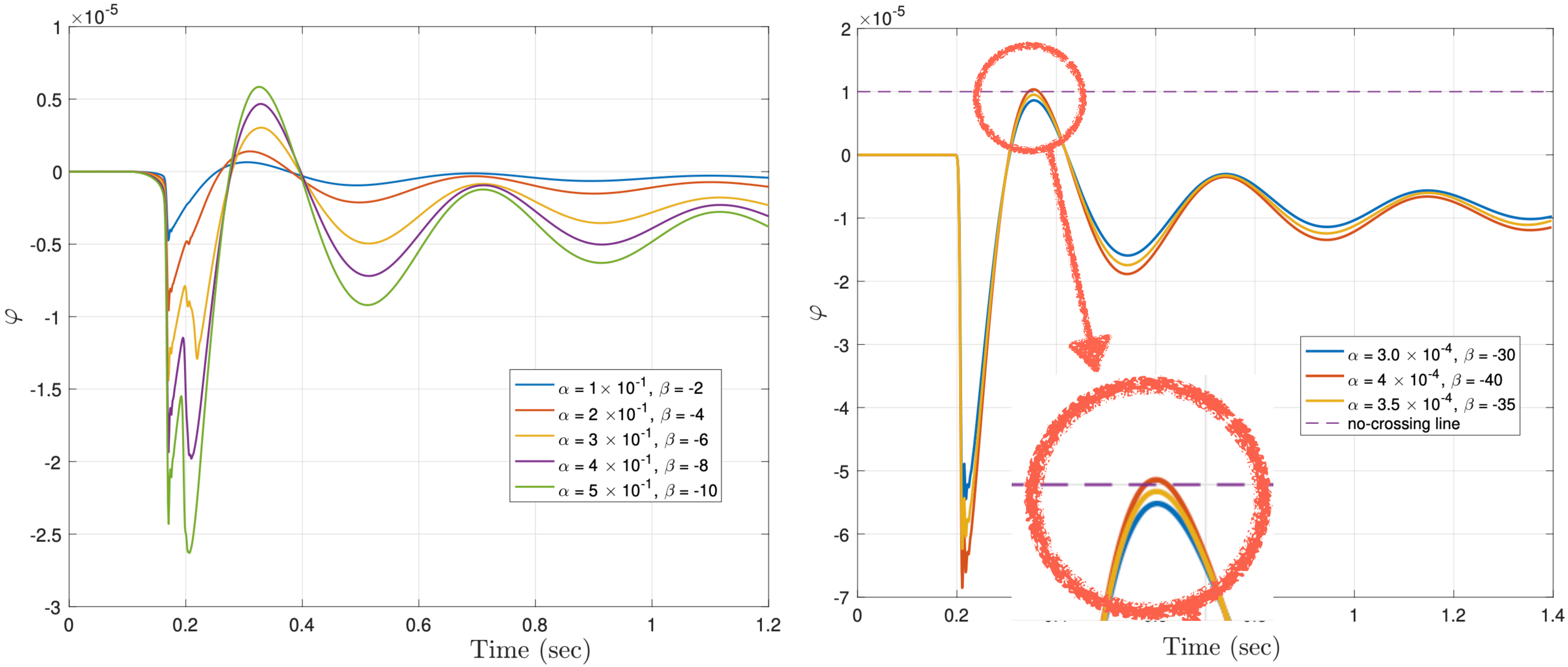


FIG. 1. Waveforms of the scalar field, extracted at $r_{ex} = 5 \times 10^9$ cm away from the supernovae core. In the above figure, the critical value is $\varphi_c = k = 0.05$, which is so high to have a constraint on the scalar field while in the below figure, $\varphi_c = k = 1 \times 10^{-5}$ rules out the scalar field labeled by $\beta_0 = -40$.

Results

Recall the EoM of the scalar field: $\Box^\star\varphi = -4\pi G(\alpha_0 + \beta_0\varphi)T^\star + m^2\varphi.$

From which one can define an effective mass of the scalar field

$$m_{eff}^2 := -4\pi G\beta_0 T^\star + m^2 \approx 4\pi G\beta_0\rho + m^2$$

FAR AWAY

$$\left(\partial_t^2 - \nabla^2 + m_{eff}^2\right)\varphi = 0$$

It defines a cutoff, i.e. the modes whose angular frequencies above it will propagate outward, but for the others they will exponentially damped out.

Results

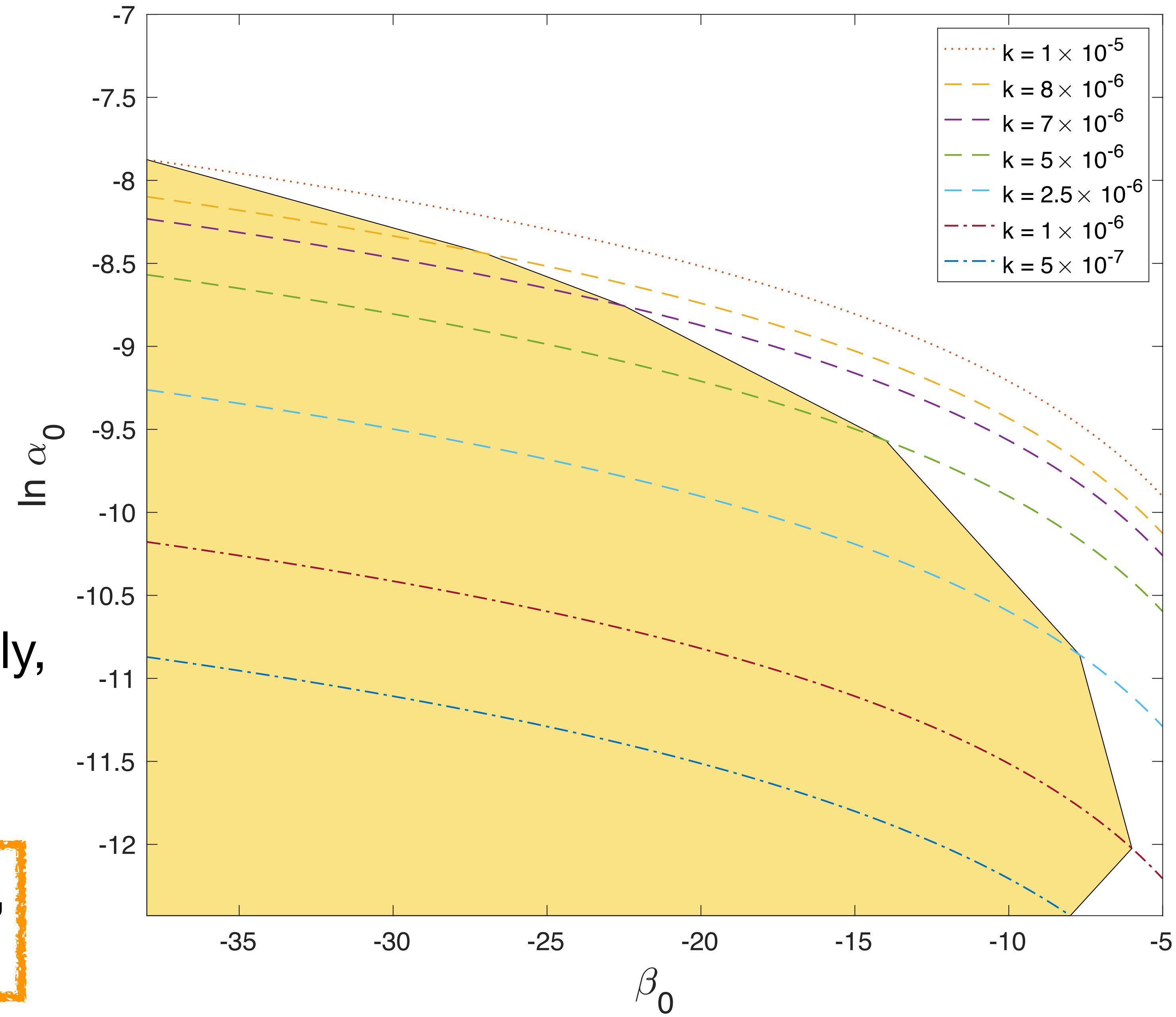
- Consider the sets defined by

$$S_k = \left\{ (\alpha_0, \beta_0) \middle| -\alpha_0/\beta_0 = k \right\}$$

- For each there exists a unique pair of (α_0, β_0) labeling the maximum relevant value of β_0

- Trace out the limit of β_0 continually, one can draw a boundary.

Remark: For different initial data, the boundary would be different



Conclusion

- In the first order post newtonian approximation of coupling function, the criterion to ensure the scalar field in the Einstein frame has a reflection in the Jordan frame manifest itself as a critical value of the scalar field.

- For a specific initial data, the criterion induce a inviable region on the parameter space of ST theory in 1PN.

- Using the parameters in the relevant area of the parameter space, the results of the simulation show that the generic inverse-chirp signal of GWs.

Thank you for your attention!