Karpacz, 28 February 2020

Universality of the Relativistic correction to glitch rise-times

Lorenzo Gavassino

Nicolaus Copernicus Astronomical Center of the Polish Academy of Sciences

Based on

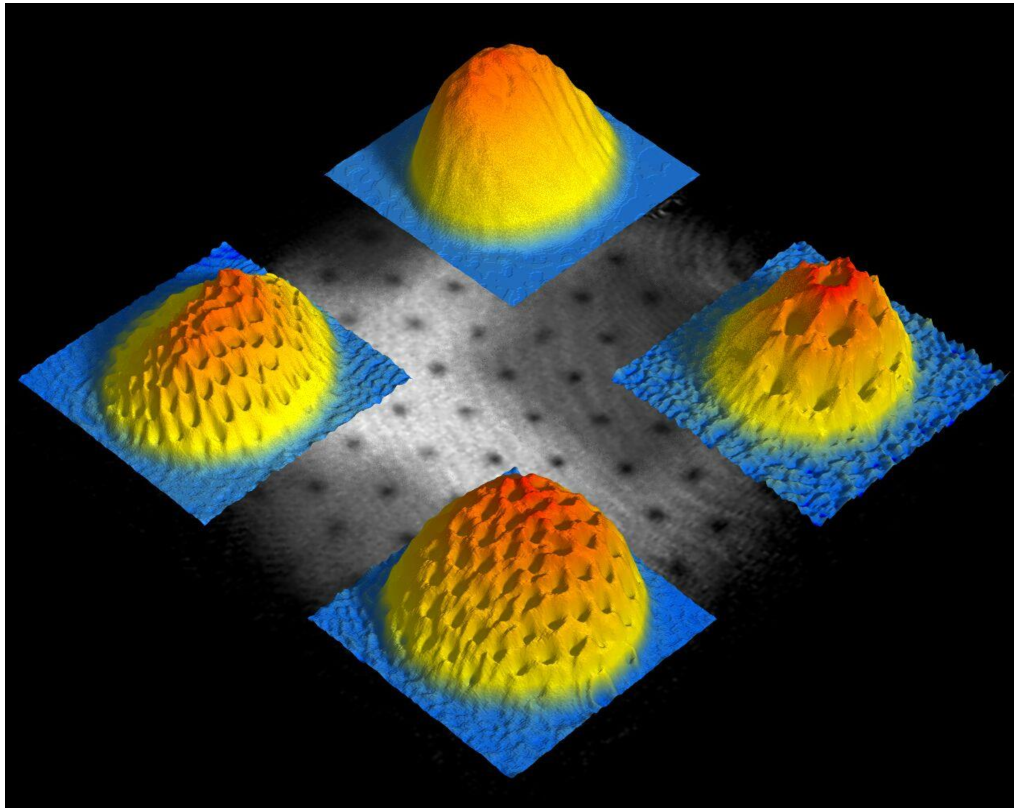
arXiv:2001.08951

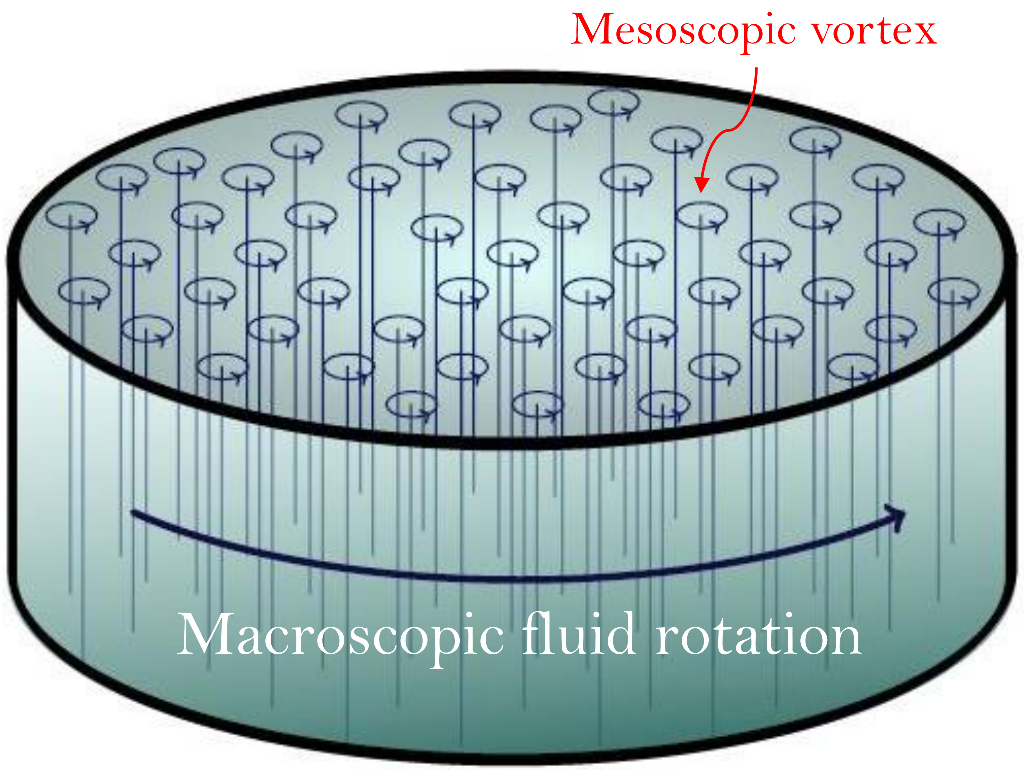
In collaboration with
Dr. Marco Antonelli
Prof. Pierre Pizzochero
Prof. Brynmor Haskell

CAMK PAN

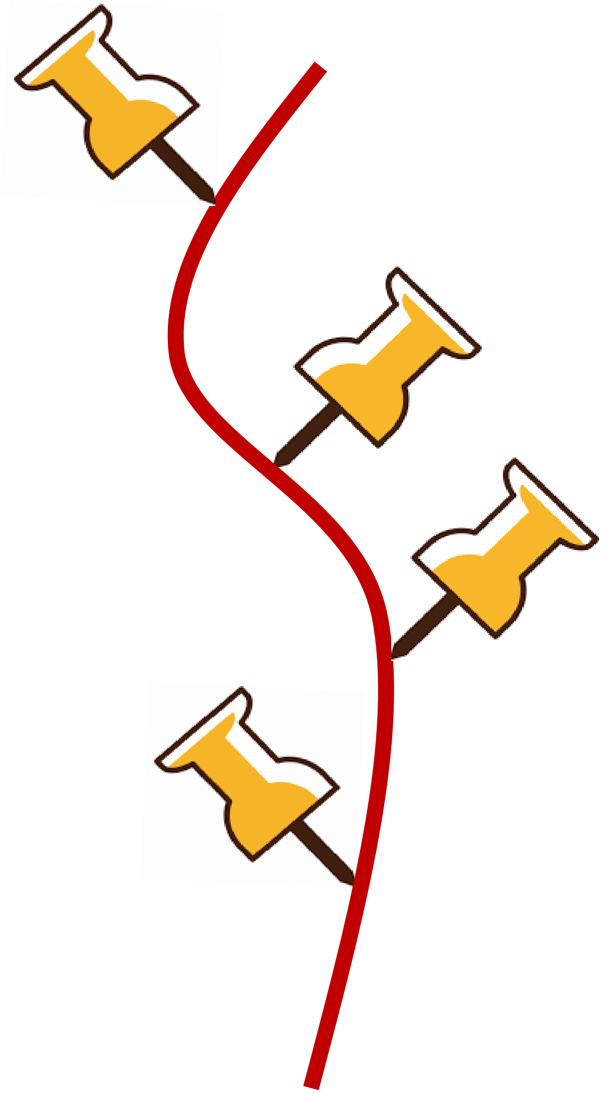
Quantized vortices

To mimic rigid-body rotation a superfluid must be crossed by an array of vortices of quantized circulation





Angular momentum reservoir



In some regions of the star vortices have the tendency to "pin" to other components of the system (see e.g. Alpar 1977, Link 2012, Seveso et al. 2016, Wlazłowski et al. 2016)

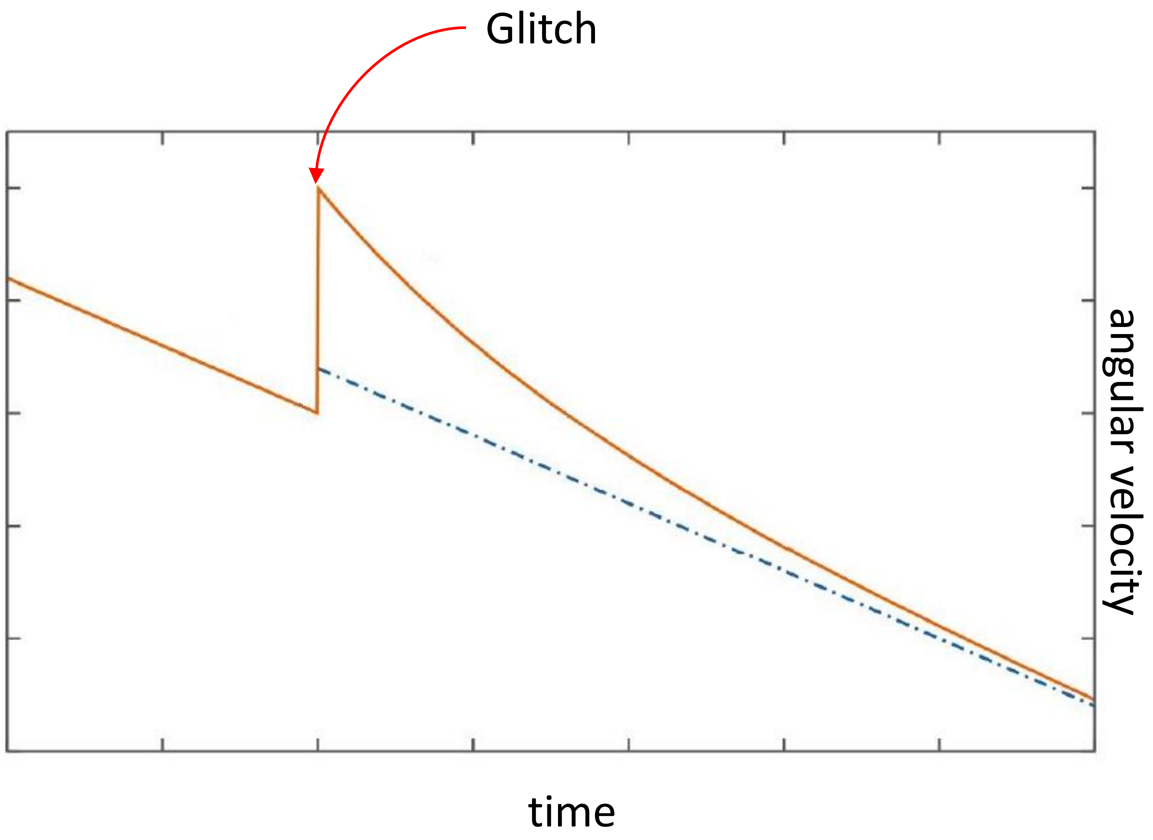
- Vortex pinning implies frozen vortices
- Frozen vortices imply frozen momentum circulation
- Frozen momentum circulation implies that the superfluid cannot slow down!

The glitch theory in few words

- The neutron star loses angular momentum: the p (visible) component slows down, the n (superfluid) cannot, because the vortices are pinned.

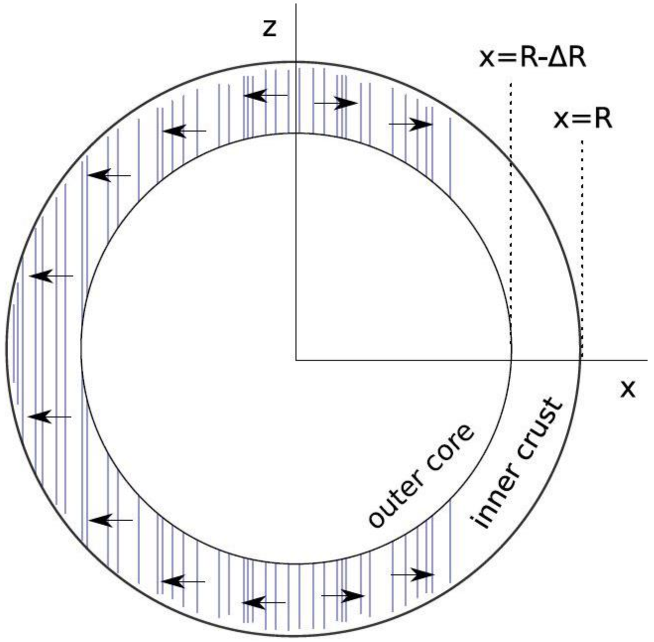
- At a certain point the velocity lag is too large, the vortices are eradicated and the reservoir is released.

- Sudden spin-up of the rotational frequency… the glitch!



This explanation has first been proposed in Anderson & Itoh (1975)

Recent Newtonian models



Mutual friction coupling

$$\partial_t \Omega_{vp} = -\dot{\Omega}_p - \mathcal{B}(2\Omega_p + 2\Omega_{vp} + x\partial_x \Omega_{vp})\Omega_{vp}$$

$$I\,\dot{\Omega}_p + \int dI_v\, \partial_t \Omega_{vp} = 0$$

Angular momentum conservation

From Antonelli & Pizzochero (2017).
See also e.g. Andersson et al. (2012), Chamel (2013), Haskell & Melatos (2015)

But Newtonian models are not enough anymore. Sourie et al. (2017) have shown that the relativistic corrections to the rise time can be of the order of the 50%.

What does General Relativity add?

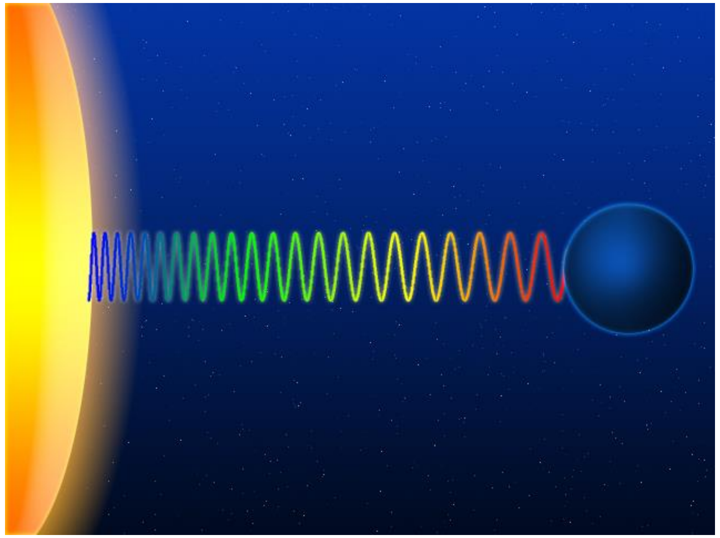
Kerr-like metric in quasi-Schwarzschild coordinates in Hartle's approximation:

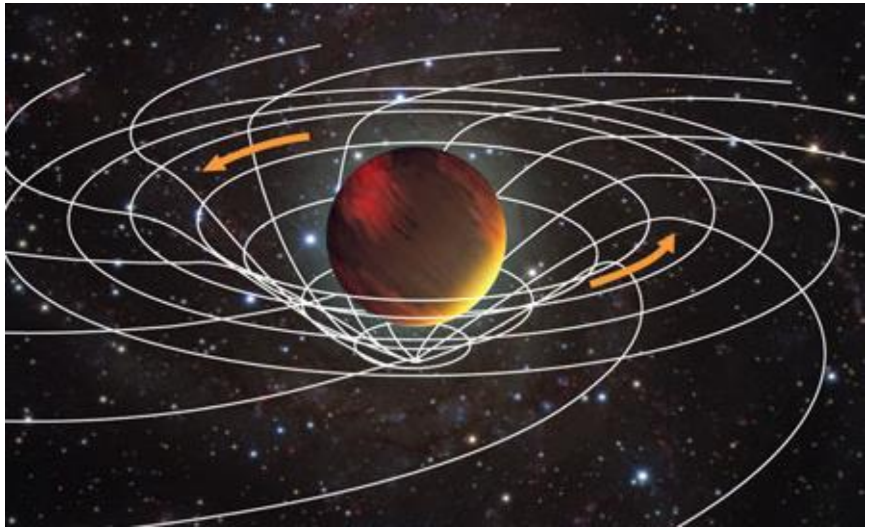
$$ds^2 = -e^{2\Phi}dt^2 + r^2 \sin^2\theta\,(d\varphi - \omega dt)^2 + e^{2\Lambda}dr^2 + r^2 d\theta^2$$

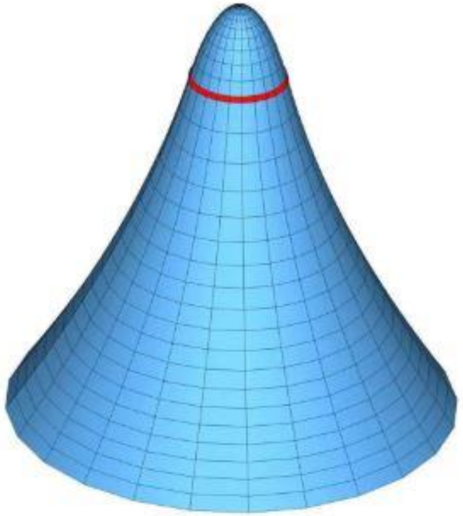
Gravitational time dilation (redshift effect)

Frame dragging (Lense-Thirring effect)

Space curvature ($C \neq 2\pi R$)



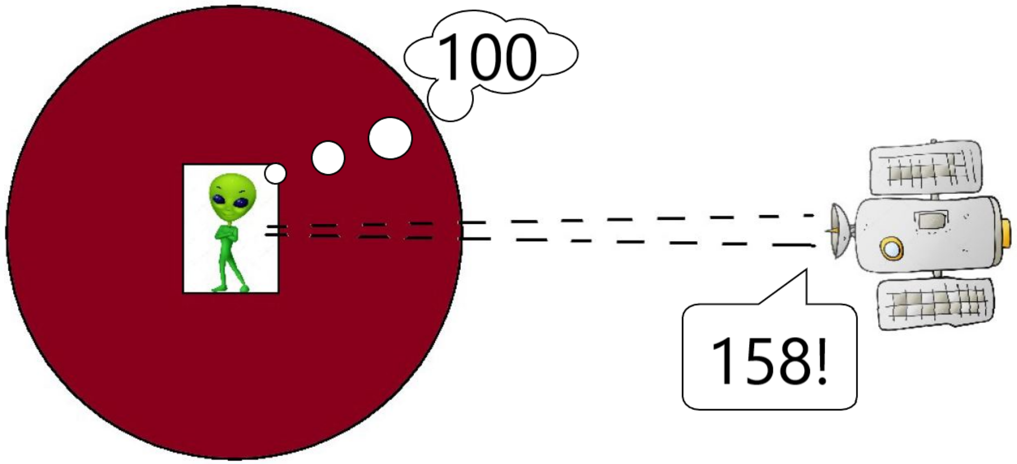




Time dilation and frame dragging

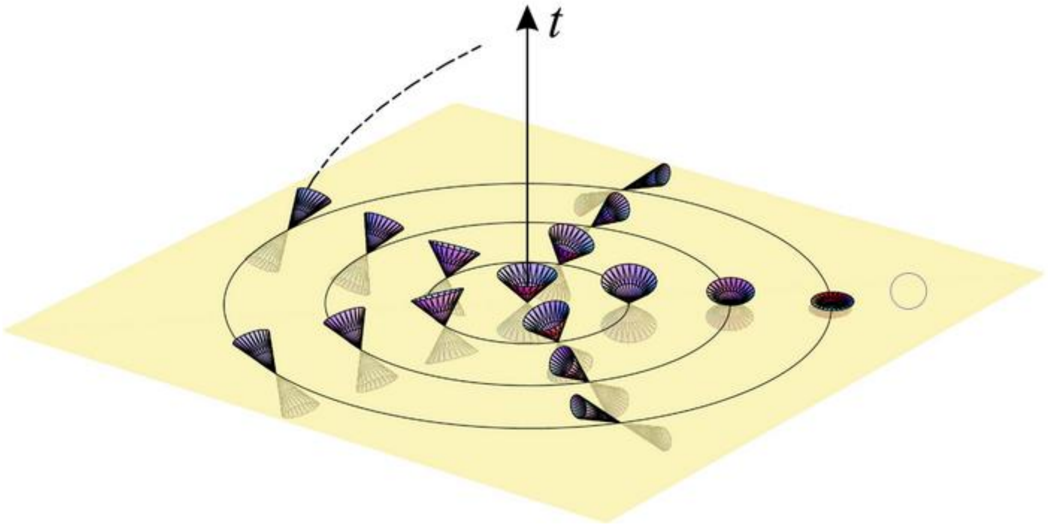
An observer on the Earth sees the internal events in slow motion:

$$d\tau = e^{\Phi} dt$$



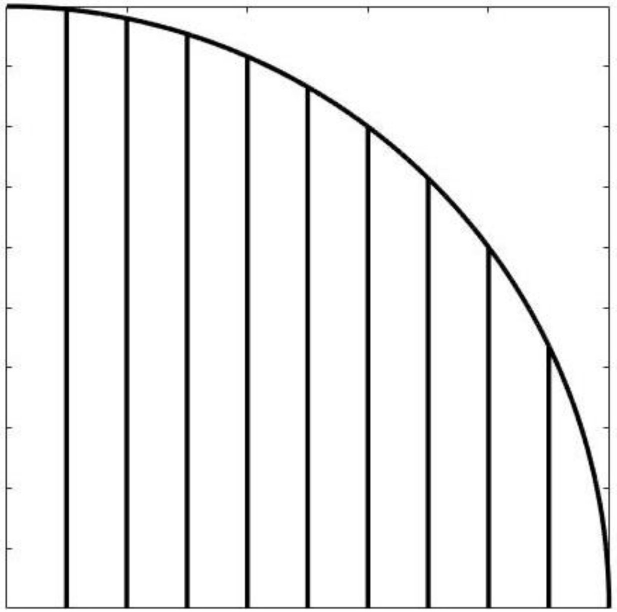
The Eulerian observer (ZAMO) rotates from the point of view of the observer on the Earth:

$$u_Z = e^{-\Phi}\left(\partial_t + \omega \partial_\varphi\right)$$



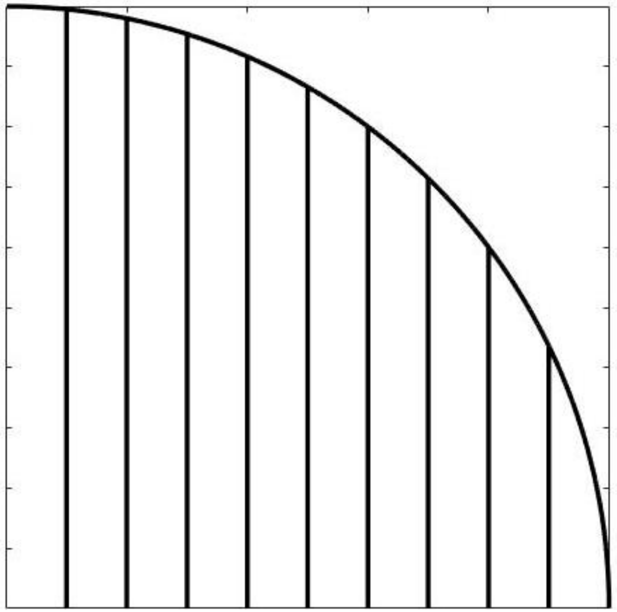
Geometry of vortices

Newtonian prediction with
any equation of state

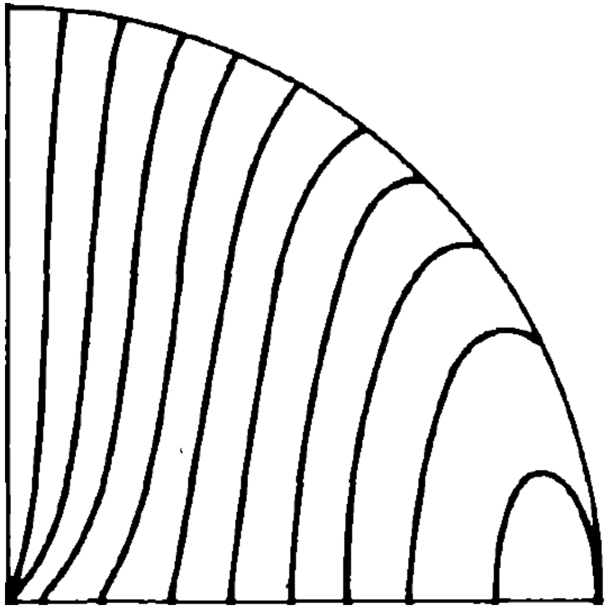


Geometry of vortices

Newtonian prediction with any equation of state

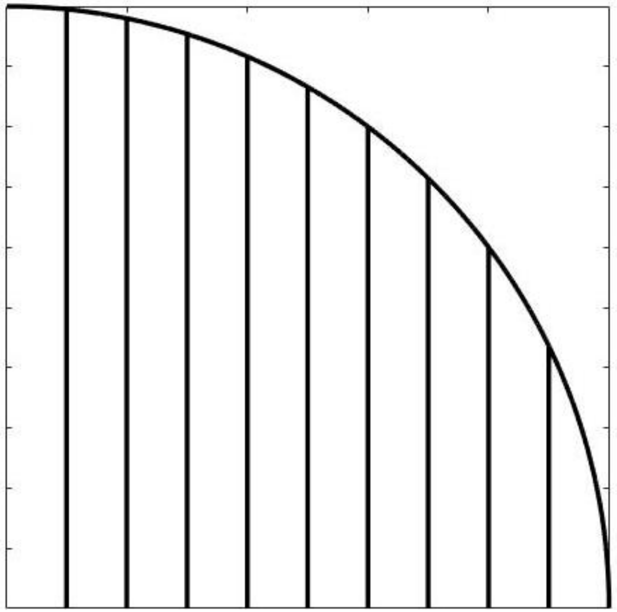


Full GR with non-realistic equation of state (Rothen 1980)

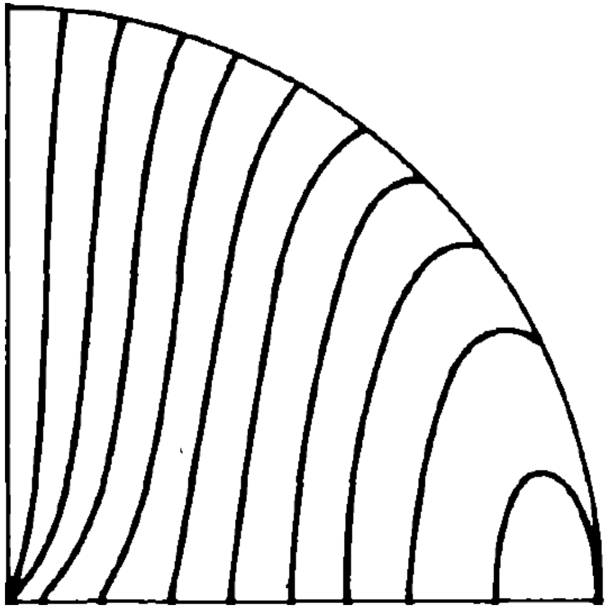


Geometry of vortices

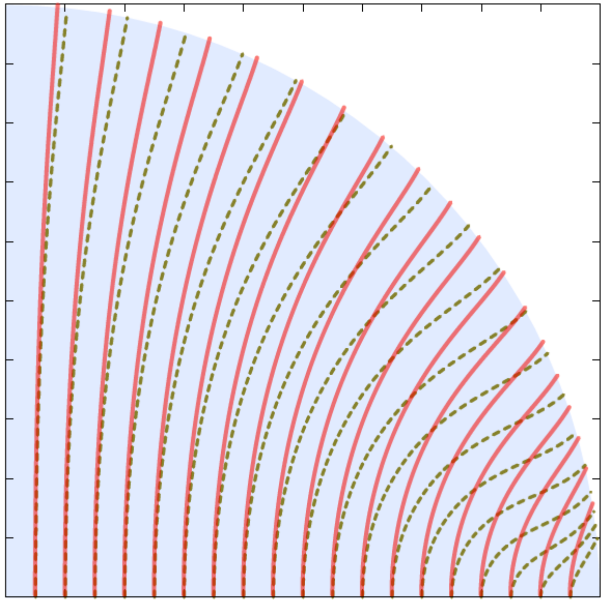
Newtonian prediction with any equation of state



Full GR with non-realistic equation of state (Rothen 1980)

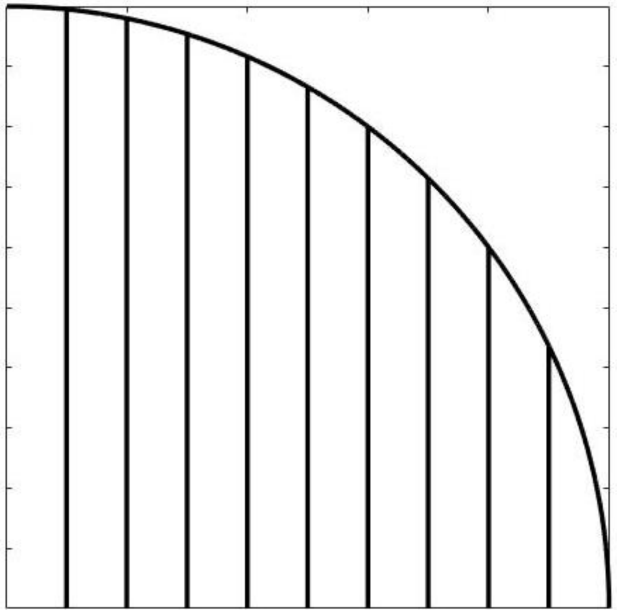


Schwarzschild-like metric with realistic equation of state (Antonelli 2017)

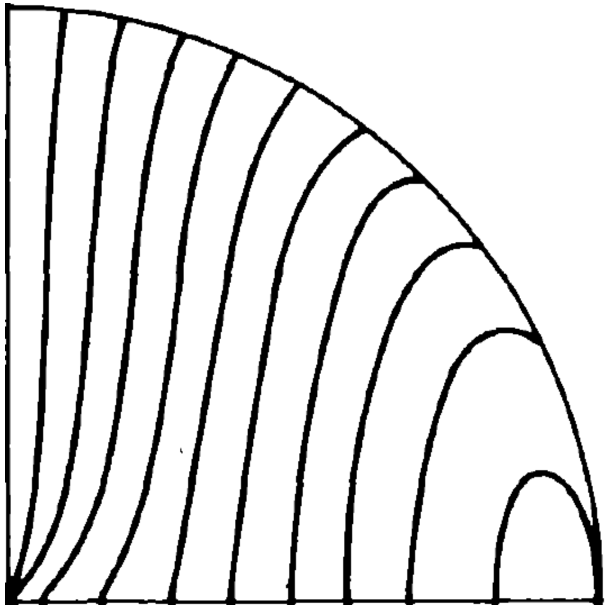


Geometry of vortices

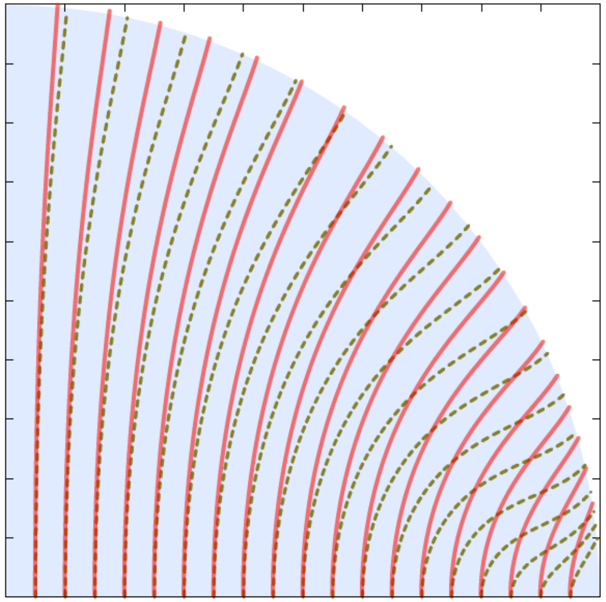
Newtonian prediction with any equation of state



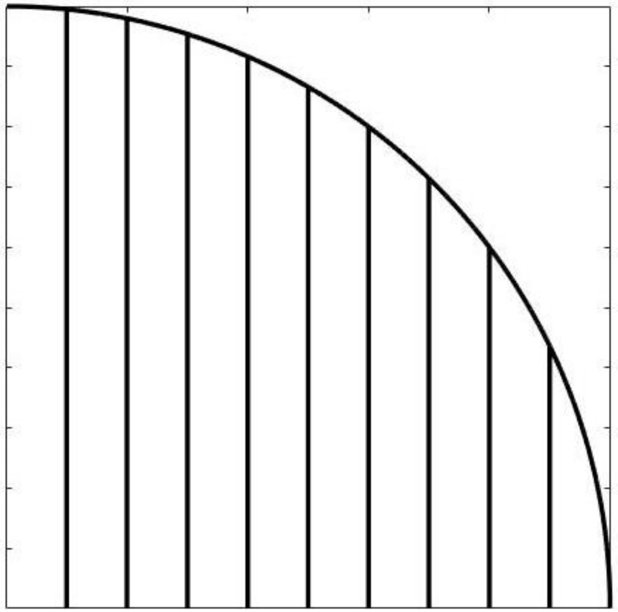
Full GR with non-realistic equation of state (Rothen 1980)



Schwarzschild-like metric with realistic equation of state (Antonelli 2017)



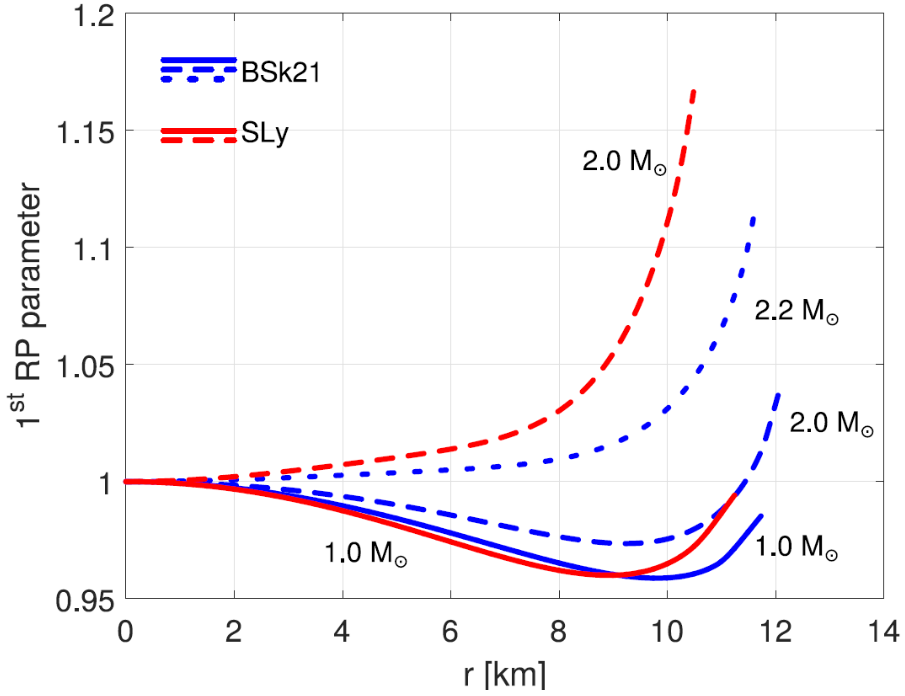
Full GR with realistic equation of state (Gavassino et al. 2020)



Time dilation and frame dragging correction cancel out, leaving the vortex profile almost identical to Newtonian predictions!

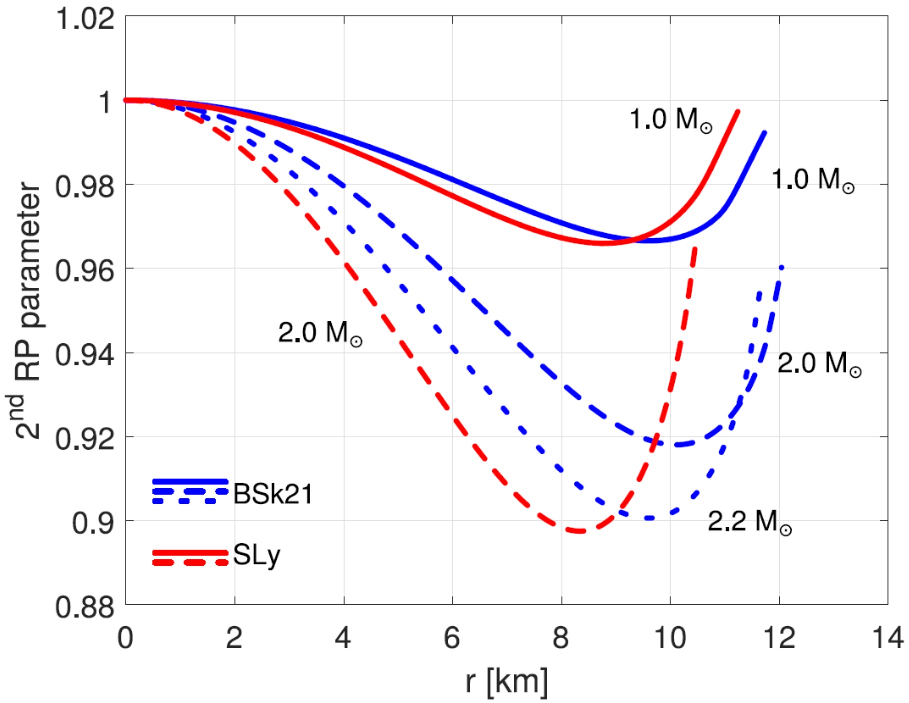
The Ravenhall and Pethick approximation

$e^{\Phi-\Lambda} \approx const$



It happens because mass-energy density and pressure have the same order of magnitude ($P_c \sim \rho_c/3$)

$e^{-\Phi-\Lambda}(1-\tilde{\omega}) \approx const$

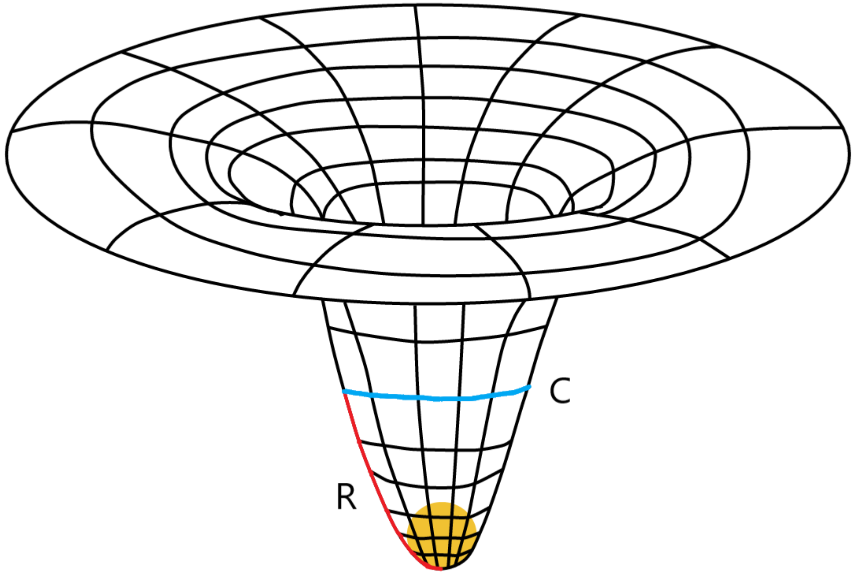


It is a general property of Hartle's slow-rotation approximation

First noted in Ravenhall & Pethick (1994)

Curvature of space and vortex density

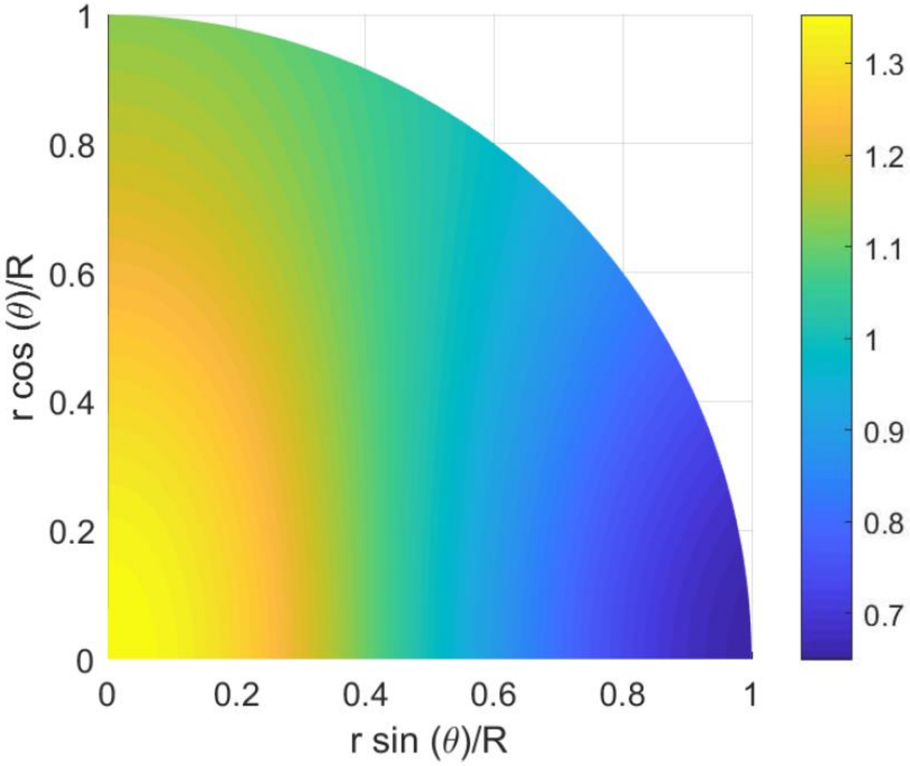
In a curved space $C \neq 2\pi R$



It affects the vortex density in a latitude-dependent way

$$n_v^{GR} \approx n_v^{Nw} e^{\Phi_D - 2\Phi}(1-\tilde{\omega})\sqrt{1+\sin^2\theta\ \left(-1+e^{-2\Lambda}\right)}$$

Colormap of n_v^{GR}/n_v^{Nw} (n_v^{Nw} is uniform)



Gavassino et al. (in preparation)

Local correction to the coupling time-scale

In Newtonian models one finds that the mutual friction coupling time-scale is

$$t \sim \frac{m_n(1-\epsilon_n)}{Bk n_v}$$

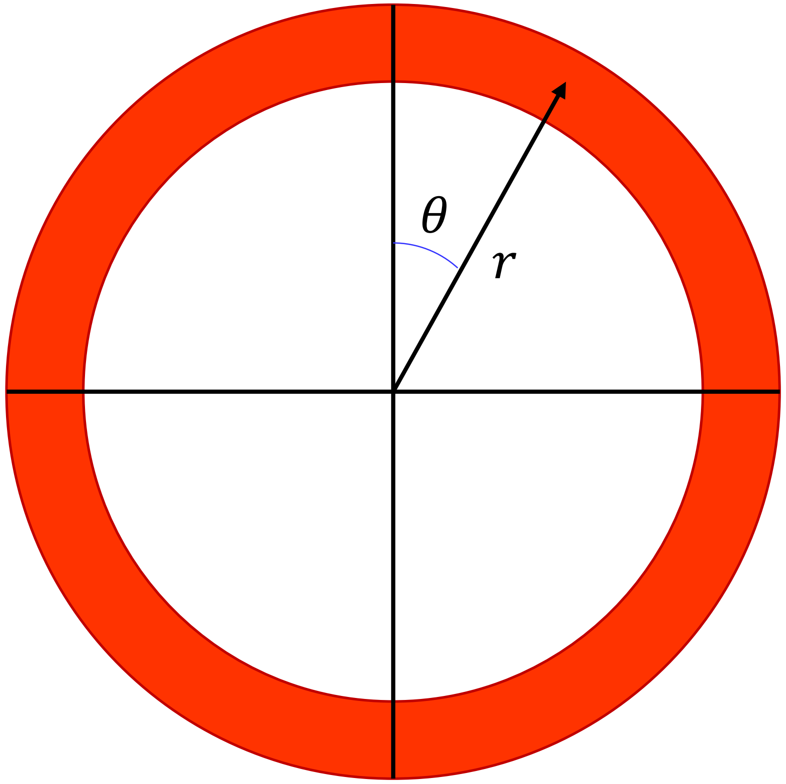
It is inversely proportional to the vortex density

The formula for the relativistic correction at a given radius and latitude assuming the superfluid to be located in a thin shell near the surface

$$t_{GR} = \frac{t_{Nw}}{(1-\tilde{\omega})\sqrt{1+\sin^2\theta\ \left(-1+e^{-2\Lambda}\right)}}$$

Frame dragging: part of the angular velocity is only apparent

Curvature of space: larger distances imply smaller vortex density



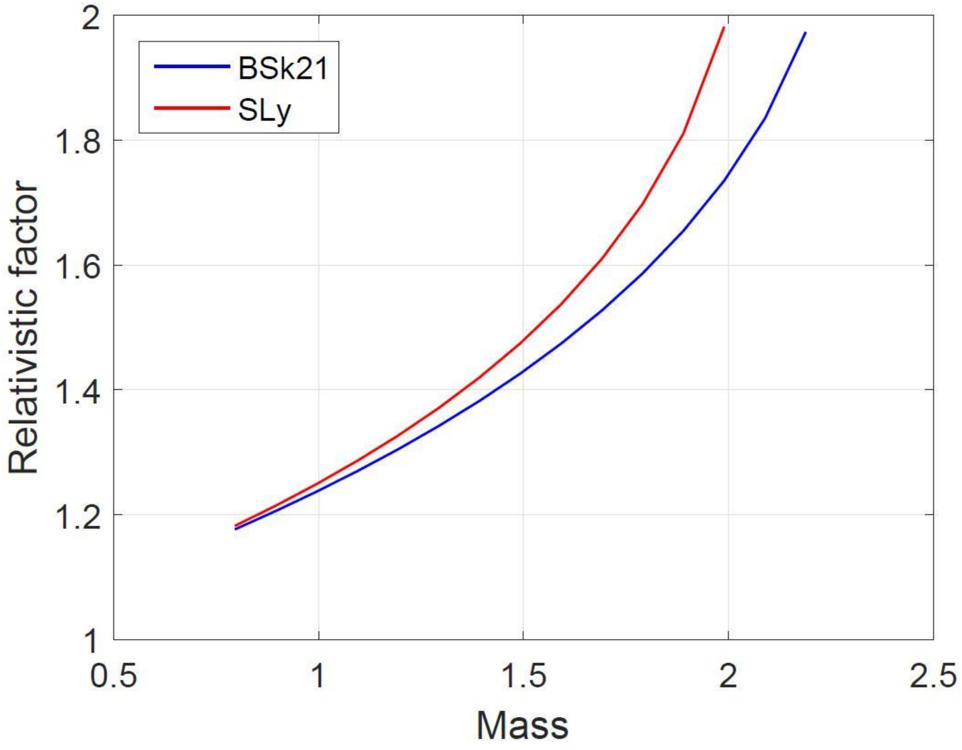
Thin shell assumption

Global correction to the rise time

We can, finally, calculate the global correction to glitch rise-times in the thin shell assumption

M: Mass R: Radius I: Moment of Inertia

$$\frac{t_{GR}}{t_{Nw}} = \left(1 - \frac{2M}{R}\right)^{-1/2} \left(1 - \frac{2I}{R^3}\right)^{-1}$$



Global correction to the rise time

We can, finally, calculate the global correction to glitch rise-times in the thin shell assumption

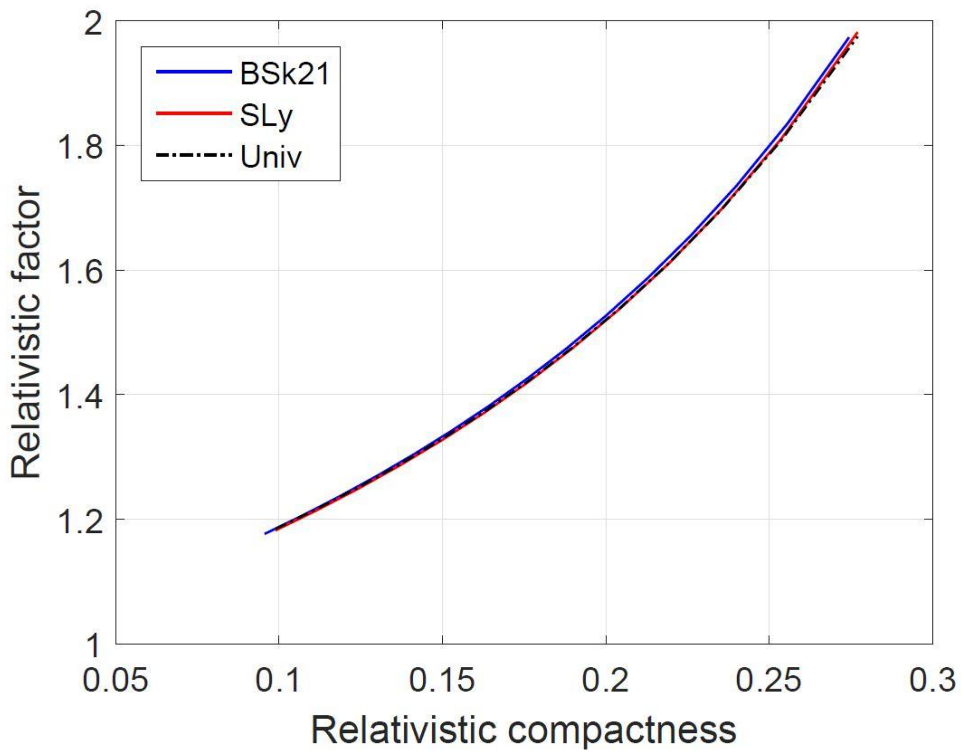
M: Mass R: Radius I: Moment of Inertia

$$\frac{t_{GR}}{t_{Nw}} = \left(1 - \frac{2M}{R}\right)^{-1/2} \left(1 - \frac{2I}{R^3}\right)^{-1}$$

But there is the universal relation (Breu & Rezzolla 2016)

$$\frac{I}{R^3} = \bar{a}_1 \left(\frac{M}{R}\right)^2 + \bar{a}_2 \left(\frac{M}{R}\right)^1 + \bar{a}_3 \left(\frac{M}{R}\right)^0 + \bar{a}_4 \left(\frac{M}{R}\right)^{-1}$$

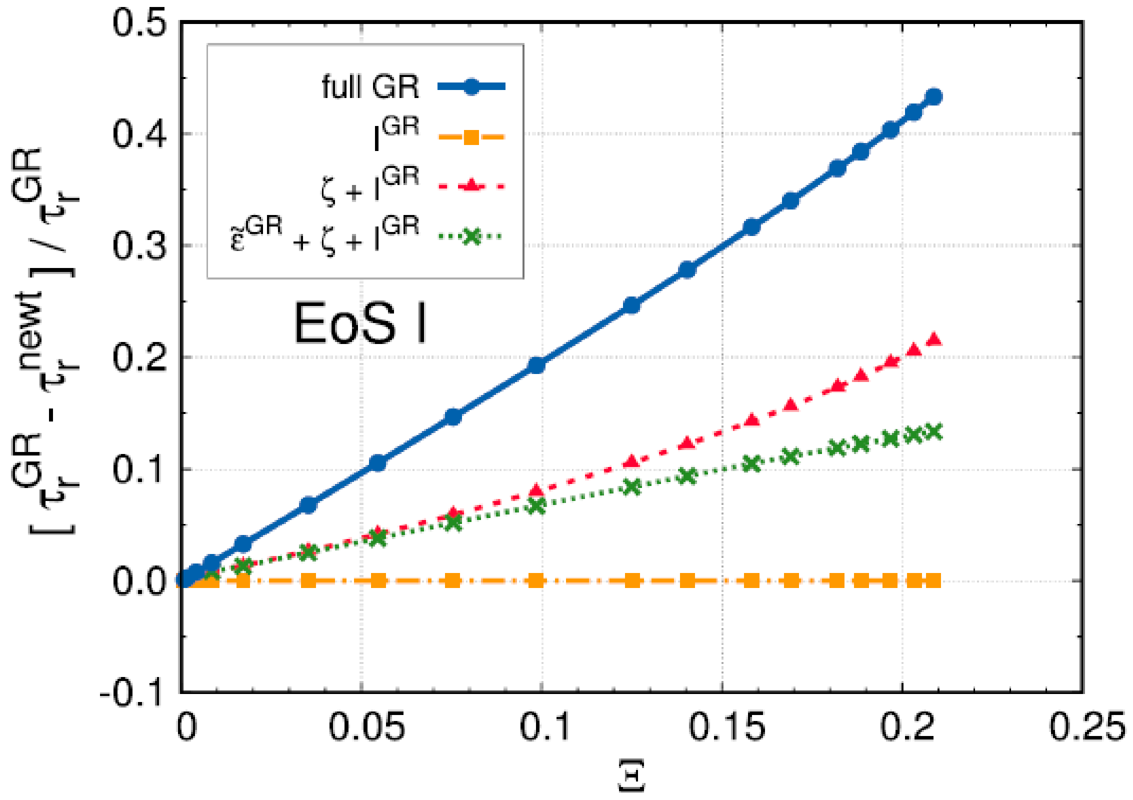
$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$\bar{a}_4$
8.13×10^{-1}	2.10×10^{-1}	3.18×10^{-3}	-2.72×10^{-4}

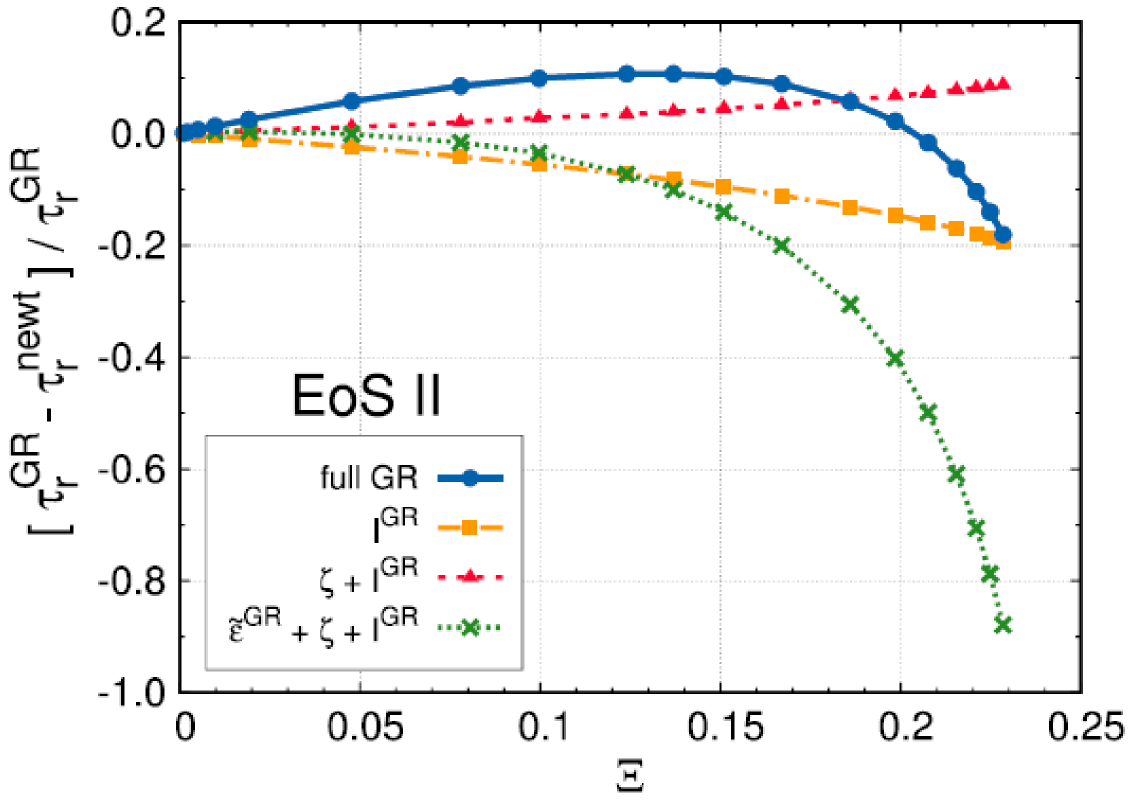


Written as a function of the compactness it is EOS independent!

Outside the thin shell limit there is no hope to have a universal formula (Sourie et al. 2017)

But if the reservoir is in the whole core…





Plots from Sourie at al. 2017

In conclusion

- We have made an analytical study of the relativistic effects on the geometry and density of vortices.
- Newtonian predictions for the rise-time can be corrected using a universal formula.
- The analysis we made constitutes the basis for the construction of relativistic glitch models.

In conclusion

- We have made an analytical study of the relativistic effects on the geometry and density of vortices.
- Newtonian predictions for the rise-time can be corrected using a universal formula.
- The analysis we made constitutes the basis for the construction of relativistic glitch models.

Thank you for your attention!