Core and crust contributions in overshooting glitches: the Vela pulsar 2016 glitch

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Pulsar glitches

Four glitches of the Crab pulsar, Espinoza et al. (2011)

- Sudden acceleration in the rotation of a pulsar
- Two components, one spins down (normal), and one does not (superfluid)
- For some reason, the superfluid component occasionally releases angular momentum to the normal component
- The glitch trigger is still unknown
How does angular velocity evolve on short timescales?

Figure taken from:
- Haskell et al. (2012)

See also:
- Antonelli & Pizzochero (2017)
- Graber et al. (2018)
- Ashton et al. (2019)

This talk is based on:

Rigidly-rotating components

\[ \dot{\Omega}_p = -\frac{1}{x_p} \left( x_1 \dot{\Omega}_1 + |\dot{\Omega}_\infty| \right) \]

\[ \dot{\Omega}_1 = -b_1 (\Omega_1 - \Omega_p) \]
Rigidly-rotating components

\[
\dot{\Omega}_p = -\frac{1}{x_p} \left( x_1 \dot{\Omega}_1 + x_2 \dot{\Omega}_2 + |\dot{\Omega}_\infty| \right)
\]

\[
\dot{\Omega}_1 = -b_1 (\Omega_1 - \Omega_p)
\]

\[
\dot{\Omega}_2 = -b_2 (\Omega_2 - \Omega_p)
\]
Working assumptions and some considerations

- The components are rigidly rotating
- We assume that \( x_1 + x_2 + x_p = 1 \)
- We are using a Newtonian model (the rise time should be corrected by a factor of \( \lesssim 2 \), see Sourie et al. 2017 and Gavassino et al. 2020)
- There are two different initial lags \( \Omega_{1p}^0, \Omega_{2p}^0 \) between the two superfluid components the normal component

\[
\dot{\Omega}_p = -\frac{1}{x_p} \left( x_1 \dot{\Omega}_1 + x_2 \dot{\Omega}_2 + |\dot{\Omega}_\infty| \right)
\]
\[
\dot{\Omega}_1 = -b_1 (\Omega_1 - \Omega_p)
\]
\[
\dot{\Omega}_2 = -b_2 (\Omega_2 - \Omega_p)
\]
It is possible to solve analytically the system, in order to obtain the angular velocity of the normal component with respect to the spin down of the star.

\[ \Delta \dot{\Omega}_p(t) = \Delta \dot{\Omega}_p^\infty \left[ 1 - \omega e^{-t\lambda_+} - (1 - \omega) e^{-t\lambda_-} \right] \]

where \( \lambda_\pm, \Delta \dot{\Omega}_p^\infty, \omega \) are analytical functions of \( x_{1,2}, \tau_{1,2}, \Omega_{1,2p}^0 \).
An overshoot can occur when there is a maximum in $\Delta \Omega_p(t)$.

$\Rightarrow \omega > 1$

It can be shown that, with some simplifying assumptions, the overshoot condition corresponds to:

$b_1/b_2 < 1$
Palfreyman et al. (2018) measured a glitch on the Vela pulsar with the best precision ever achieved. Our aim is that of fitting the data with the three-component model in order to obtain the value of the parameters $x_{1,2}$, $b_{1,2}$, $\Omega_{1,2}^0$. 

Palfreyman et al. 2018
Bayesian analysis

\[ P(\mathcal{P} | \mathcal{D}) = \frac{P(\mathcal{D} | \mathcal{P}) P(\mathcal{P})}{P(\mathcal{D})} \]

- \( \mathcal{P} \): parameters of the model \((x_{1,2}, b_{1,2}, \Omega_{1,2p}, t_g, t_M, \Delta r_0)\).
- \( \mathcal{D} \): the data in Palfreyman et al. (2018)
- \( P(\mathcal{D} | \mathcal{P}) \): likelihood
- \( P(\mathcal{P}) \): prior
- \( P(\mathcal{D}) \): marginalised likelihood, a.k.a. evidence
- \( P(\mathcal{P} | \mathcal{D}) \): posterior

We keep the prior as uninformative as possible.
Marginalised posterior distribution (for some of the variables)

- $b_1$ (s$^{-1}$)
- $x_1$
- $\Omega_{1p}^0$ ($10^{-5}$ rad/s)
- $b_2$ (s$^{-1}$)
- $x_2$
- $\Omega_{2p}^0$ ($10^{-3}$ rad/s)
Are we completely sure about it?

The fit has been performed with a prior on the moments of inertia fraction given by:

\[
x_1, x_2 \sim \begin{cases} 
\text{Unif}(0, 1)\text{Unif}(0, 1) & \text{if } x_1 + x_2 < 1 \\
0 & \text{elsewhere}
\end{cases}
\]

Tightening the prior to 0.05 would yield a much lower evidence for this model, and a natural logarithm of the Bayes factor of 5.6 proves it to be very unlikely.
Normal component

Charged moment of inertia fraction

Mass (M_{\odot})

SLy4
BSk20
BSk21
DDME2

\( P(x_p) \)
Other variables

1/$\lambda_+$ [s]

1/$\lambda_-$ [s]

$\omega$

$\Delta\Omega_p^{\infty}[10^{-4} \text{ rad/s}]$
Conclusions

- The results obtained in this work, although with a simple toy model, is compatible with much of the literature.
- The pinned superfluid cannot be contained just in the crust of the star (see also works on the activity parameter).
- It is also possible to obtain an estimate of the coupling parameters between the superfluid components and the normal component.
- Importance of a Bayesian approach: the output of this inference can be reused for another Vela glitch.
Thank you for your attention!
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<tr>
<th>Variable</th>
<th>16th percentile</th>
<th>Median</th>
<th>84th percentile</th>
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<td>$\Omega^0_{1p}$</td>
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<td>$\Delta r_0$</td>
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<td>$t_M$</td>
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Fit of the residuals